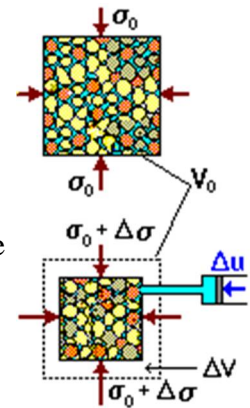


Compression and swelling

- [Mechanisms](#)
- [Common cases](#)
- [Isotropic](#)
- [One-dimensional](#)
- [Wet and dry states](#)

The relationship between volume change and effective stress is called compression and swelling. (Consolidation and compaction are different.) The volume of soil grains remains constant, so change in volume is due to change in volume of water.

Compression and swelling results from [drained](#) loading and the pore pressure remains constant. If saturated soil is loaded [undrained](#) there will be no volume change.



Back to [Compression and swelling](#)

Mechanisms of compression

Compression of soil is due to a number of mechanisms:

rearrangement of grains



fracture and rearrangement of grains



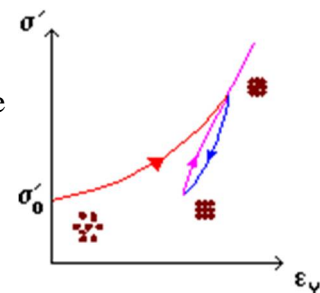
distortion or bending of grains



On unloading, grains will not unfracture or un-rearrange, so volume change on unloading and reloading (swelling and recompression) will be much less than volume change on first loading (compression).

In compression, soil behavior is:

- non-linear
- mostly irrecoverable



Back to [Compression and swelling](#)

Common cases of compression and swelling

In practice, the state of stress in the ground will be complex. These are simple theories for two special cases.

Isotropic:

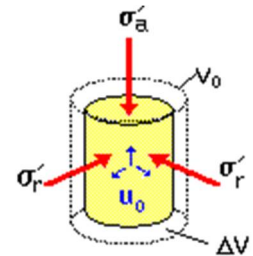
Equal stress in all directions. Applicable to triaxial test before shearing.

$$p' = (\sigma'_a + 2\sigma'_r) / 3$$

= mean stress

$$\varepsilon_v = \Delta V / V_0$$

= volumetric strain



One-dimensional:

Horizontal strains are zero. Applicable to oedometer test and in the ground below wide foundations, embankments and excavations.

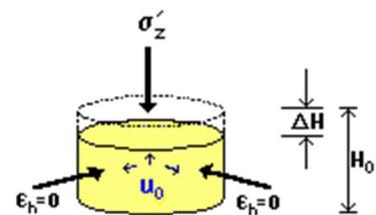
σ'_z = vertical stress

$$\varepsilon_v = \Delta V / V_0$$

$$= \Delta H / H_0$$

$$= \Delta \varepsilon / (1 + e_0)$$

= volumetric strain



Back to [Compression and swelling](#)

Isotropic compression and swelling

- [Equations](#)
- [Over consolidation](#)
- [State](#)

[Isotropic](#) compression and swelling is applied at the start of a triaxial test.

$$p' = (\sigma'_a + 2\sigma'_r) / 3$$

= mean stress

$$V = V_0 - \Delta V_w$$

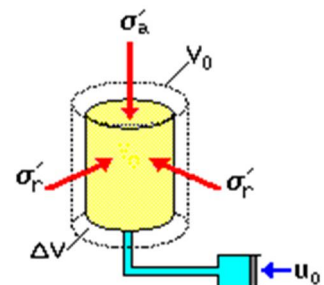
= volume

$$\varepsilon_v = \Delta V / V_0 = \Delta v / v_0$$

= volumetric strain

$$v = V / V_s$$

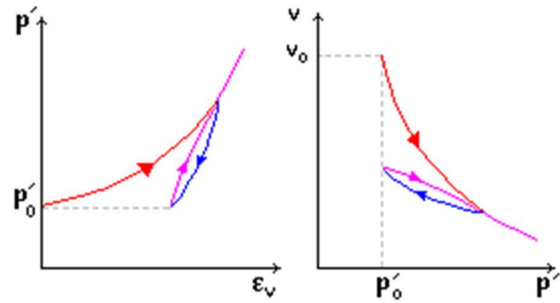
= specific volume



As the mean stress p' is raised and lowered there are volumetric strains and the specific volume changes.

p'_0 = initial mean stress
 v_0 = initial specific volume

Note the paths of compression, swelling and re-loading.



Back to [Isotropic compression and swelling](#)

Equations

- [Bulk modulus](#)
- [Typical values for compression parameters](#)

For isotropic compression and swelling there are simple relationships between specific volume v and (the natural logarithm of) the mean stress p' .

First loading

normal compression line

OAD on the graph

$$v = N - \lambda \ln p'$$

Unloading and reloading

swelling line

BC on the graph

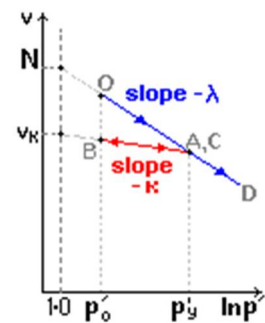
$$v = v_k - \kappa \ln p'$$

N , λ and κ are soil parameters.

v_k and p'_y locate the particular swelling line.

p'_y is referred to as the **yield stress**.

If the current stress and the history of loading/unloading are known, the current specific volume can be calculated.



Back to [Equations for isotropic compression](#)

Bulk modulus

Typical values for isotropic compression parameters

The compression and swelling parameters λ and κ are soil properties and the values depend on the nature of the soil.

Typical values	w_L	I_p	λ
very high plasticity clay	80	50	0.29
high plasticity clay	60	34	0.20
intermediate plasticity clay	42	23	0.14
low plasticity clay	30	12	0.07
quartz sand			0.15
carbonate sand			0.34

For clays $\lambda \approx I_p / 170$.

κ / λ is relatively large (e.g. 0.25 - 0.35) because clay particles can bend and distort.

For sands λ is relatively large due to particles crushing (but states only reach NCL at high pressure).

κ / λ is relatively small (e.g. 0.1) because sand particles crush and rearrange during first compression.

Back to [Isotropic compression and swelling](#)

Over consolidation

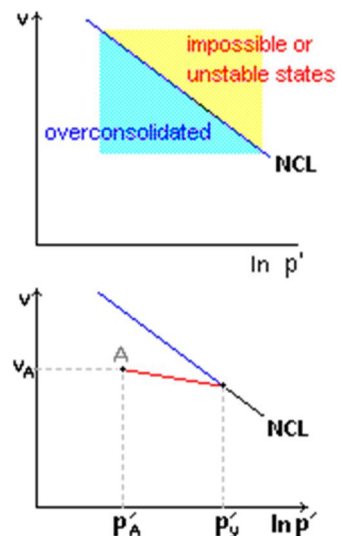
If the current state of soil is on the normal compression line it is said to be **normally consolidated**. If the soil is unloaded it becomes **over consolidated**.

(Soil cannot usually be at a state outside the normal compression line unless it is bonded or structured).

At a state A the over consolidation ratio is

$$R_p = p'_y / p'_a$$

(on NCL $R_p = 1.0$ and soil is normally consolidated).



Note: p'_y is the point of intersection of the swelling line through A and the NCL. This is usually close to the maximum past stress.

Back to [Isotropic compression and swelling](#)

State

- [Change of state](#)
- [Critical state](#)
- [Normalising parameters](#)

The current state of a soil is described by the stress p' , the specific volume v and the overconsolidation ratio R_p (for a complete description the shear stress q' is required).

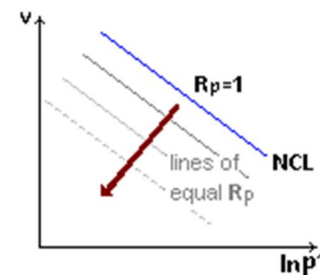
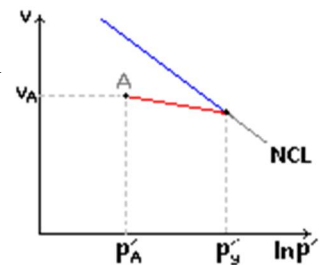
The state at A is given by any two of v_a , p'_a , $R_p = p'_y / p'_a$

All states with the same R_p fall on the lines parallel with the NCL.

$$\ln R_p = \ln (p'_y / p'_a)$$

$$= \ln p'_y - \ln p'_a$$

Many features of soil behaviour, especially shear modulus and peak strength, increase with increasing overconsolidation.



Back to [Isotropic compression: state](#)

Change of state

Loading and unloading

(relevant to all soils)

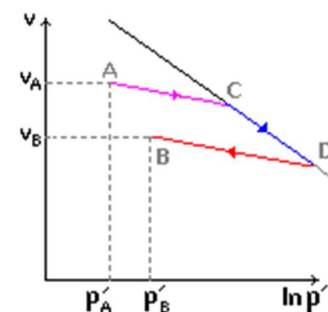
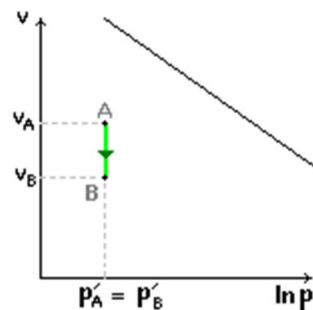
Change of state A to B can only be achieved by normal compression along CD followed by swelling along DB. Note that the yield stress corresponding to B is larger than the yield stress corresponding to A.

Vibration or compaction

(relevant to sands)

or creep

Change of state can occur directly from A to



(relevant to clays) B. Note that the yield

stress corresponding to B is larger than the yield stress corresponding to A.

Back to [Isotropic compression: state](#)

Critical state

There is a critical over consolidation ratio which separates states in which the soil will either compress or dilate during shear. This corresponds to the critical state line CSL. Look at the possible specific volumes (v) that can occur at a mean effective stress p' .

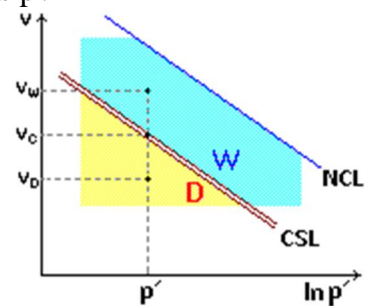
wet side of critical

(W on the graph)

$v_w > v_c$ at stress p'

water content w_w is larger than critical w_c

- loose
- normally consolidated
- or lightly over consolidated
- compress during drained shear



dry side of critical

(D on the graph)

$v_d < v_c$ at stress p'

water content w_d is smaller than critical w_c

- dense
- heavily over consolidated
- dilate during drained shear

Back to [Isotropic compression: state](#)

Normalizing parameters

Normalizing parameters change the current state to a normalized state so that all states with the same over consolidation ratio have the same value.

Equivalent specific volume

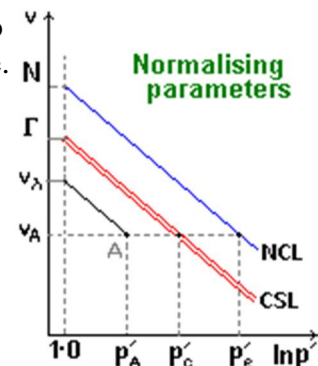
$$v_l = v_a + \lambda \ln p'_a$$

Equivalent pressure

$$\ln p'_e = (N - v_a) / \lambda$$

Critical pressure

$$\ln p'_c = (\Gamma - v_a) / \lambda$$



If A is on the wet side of critical

$$v_e > \Gamma$$

$$p'_a / p'_c > 1$$

If A is on the dry side of critical

$$v_e < \Gamma$$

$$p'_a / p'_c < 1$$

Back to [Compression and swelling](#)

One-dimensional compression and swelling

- [Equations](#)
- [Overconsolidation](#)
- [Horizontal stress](#)
- [State](#)

One-dimensional loading is applied in an oedometer and occurs in the ground beneath wide foundations, embankments or excavations.

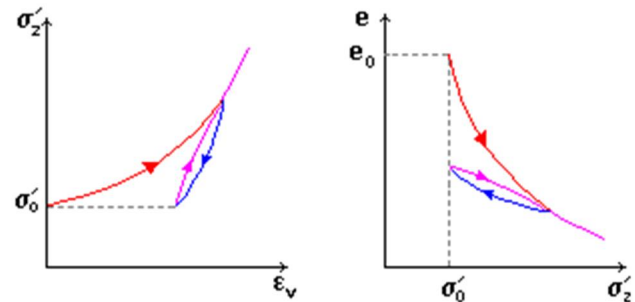
σ'_z = vertical effective stress

H = height or thickness

vertical strain = volumetric strain

$$\varepsilon_v = \Delta H / H_0 = \Delta \varepsilon / (1 + e_0)$$

where H_0 , e_0 and σ'_0 are initial values.



As the vertical stress σ'_z is raised and lowered the top of the sample settles or heaves, or the layer contracts or expands.

Note that the compression-swelling-recompression curve is similar to that for isotropic compression, but the axes used are (σ'_z, e) rather than (p', v) .

Back to [One-dimensional compression and swelling](#)

Equations

- [One-dimensional modulus and compressibility](#)

For one-dimensional compression and swelling there are simple relationships between the void ratio and the logarithm of the vertical effective stress σ'_z .

First loading:

normal compression line (NCL)

OAD on the graph

$$e = e_N - C_c \log \sigma'_z$$

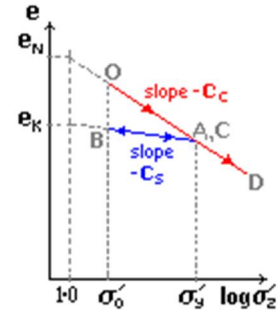
Unloading and reloading:

swelling-recompression line (SRL)

BC on the graph

$$e = e_k - C_s \log \sigma'_z$$

- e_N , C_c and C_s are soil parameters
- e_k and σ'_y locate a particular swelling line



If the current stress σ'_o and the history of loading and unloading are known, the current void ratio can be calculated. e.g.

$$e_o = e_N - C_c \log \sigma'_y + C_s (\sigma'_y - \sigma'_o)$$

Back to [One-dimensional compression: equations](#)

One-dimensional modulus and compressibility

The one-dimensional stiffness modulus is the slope of the stress/strain curve:

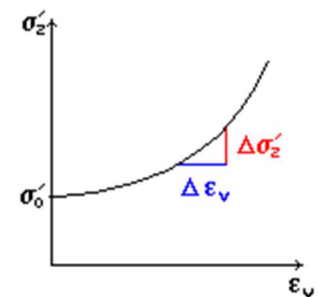
$$M' = \Delta \sigma'_z / \Delta \epsilon_v \text{ or}$$

$$E'_o = \Delta \sigma'_z / \Delta \epsilon_z \text{ (since } \epsilon_h = 0)$$

The reciprocal of stiffness is compressibility. The one-dimensional coefficient of compressibility is the slope of the strain/stress curve:

$$m_v = \Delta \epsilon / (\Delta \sigma'_z (1+e))$$

$$= 1 / E'_o$$



E'_o and m_v apply for the normal compression line and for swelling and recompression lines, and depend on the current state, on the history and on the increment of loading, so they are not soil constants.

Since m_v varies with σ'_z , its value is often quoted for $\sigma'_z = 100\text{kPa}$.

Back to [One-dimensional compression and swelling](#)

Overconsolidation

If the current state of soil is on the normal compression line it is said to be **normally consolidated**. If the soil is unloaded it becomes **overconsolidated**.

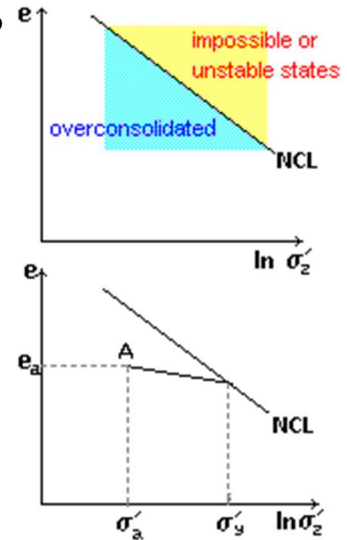
Soil cannot usually be at a state outside the normal compression line unless it is bonded or structured.

At a state A the overconsolidation ratio is

$$R_o = \sigma'_y / \sigma'_a$$

(on NCL $R_o = 1.0$ and soil is normally consolidated).

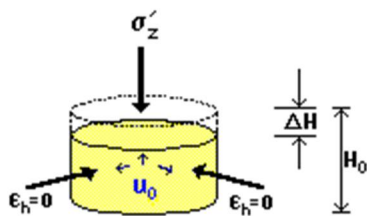
Note: σ'_y is the point of intersection of the swelling line through A and the NCL. This is usually, but not always, close to the maximum past stress (see change of state).



Back to [One-dimensional compression and swelling](#)

Horizontal stress in one-dimensional loading

During one-dimensional loading and unloading the horizontal effective stress σ'_h will change since the condition of zero horizontal strain ($\epsilon_h = 0$) is imposed.



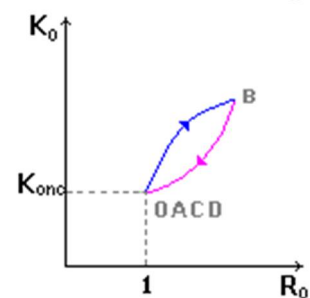
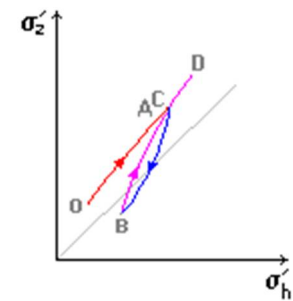
The ratio $K_o = \sigma'_h / \sigma'_z$ is known as the coefficient of earth pressure at rest.

K_o depends on

- the type of soil
- the overconsolidation ratio (R_o)
- the loading or unloading cycle

Approximations

normally consolidated soils:



$K_{onc} \gg 1 - \sin \phi'_c$
 overconsolidated soils:
 $K_o \gg K_{onc} \ddot{O}R_o$

Back to [One-dimensional compression and swelling](#)

State

- [Change of state](#)
- [Critical state](#)
- [Normalising parameters](#)

The current state of a soil is described by the stress σ' , the void ratio e and the overconsolidation ratio R_o (for a complete description the shear stress τ' is required).

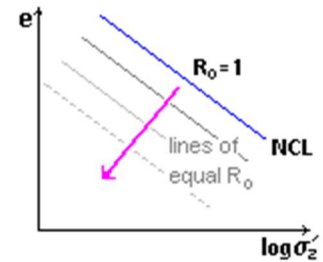
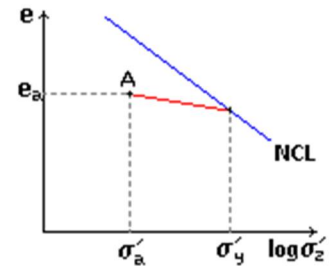
The state at A is given by any two of e_a , σ'_a , $R_o = \sigma'_y / \sigma'_a$

All states with the same R_o fall on the lines parallel with the NCL.

$$\log R_o = \log (\sigma'_y / \sigma'_a)$$

$$= \log \sigma'_y - \log \sigma'_a$$

Many features of soil behaviour, especially shear modulus and peak strength, increase with increasing overconsolidation.



Back to [One-dimensional compression: state](#)

Change of state

Loading and unloading

(relevant to all soils)

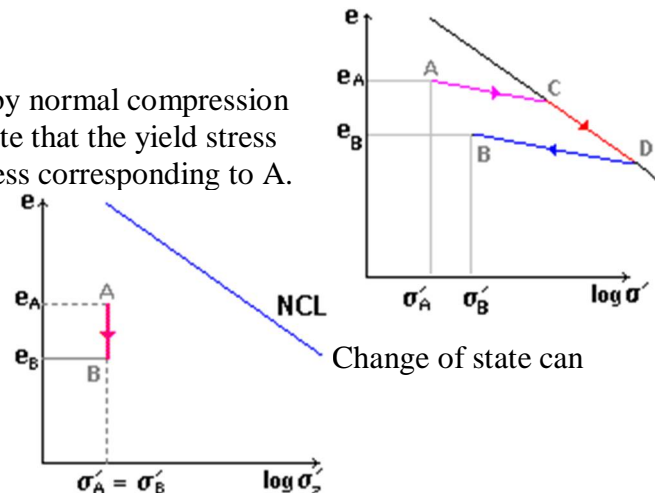
Change of state A to B can only be achieved by normal compression along CD followed by swelling along DB. Note that the yield stress corresponding to B is larger than the yield stress corresponding to A.

Vibration or compaction

(relevant to sands)

or creep:

(relevant to clays)



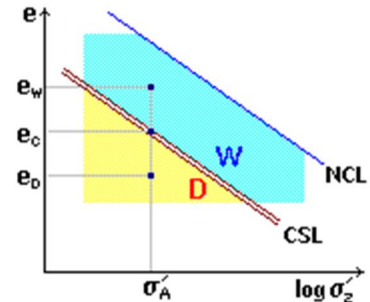
Change of state can

occur directly from A to B. Note that the yield stress corresponding to B is larger than the yield stress corresponding to A.

Back to [One-dimensional compression: state](#)

Critical state

There is a critical overconsolidation ratio which separates states in which the soil will either compress or dilate during shear. This corresponds to the critical state line CSL. Look at the possible void ratios (e) that can occur at an effective stress σ'_a .



wet side of critical

(W on the graph)

$e_w > e_c$ at stress σ'_a

water content w_w is larger than critical w_c

- loose
- normally consolidated or lightly overconsolidated
- compress during drained shear

dry side of critical

(D on the graph)

$e_d < e_c$ at stress σ'_a

water content w_d is smaller than critical w_c

- dense
- heavily overconsolidated
- dilate during drained shear

Back to [One-dimensional compression: state](#)

Normalizing parameters

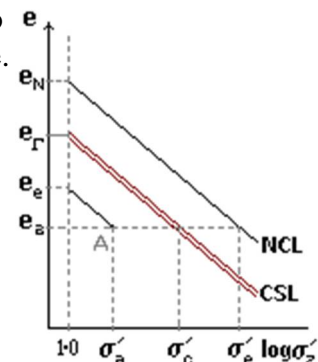
Normalizing parameters change the current state to a normalized state so that all states with the same over consolidation ratio have the same value.

Equivalent void ratio

$$e_l = e_a + C_c \log \sigma'_a$$

Equivalent stress

$$\log \sigma'_e = (e_N - e_a) / C_c$$



Critical stress

$$\log \sigma'_c = (e_G - e_a) / C_c$$

If A is on the wet side of critical

$$e_1 > e_G$$

$$\sigma'_a / \sigma'_c > 1$$

If A is on the dry side of critical

$$e_1 < e_G$$

$$\sigma'_a / \sigma'_c < 1$$

Back to [Compression and swelling](#)

Wet and dry states

- [State parameters](#)

Soils whose states lie on the normal compression line (NCL) are **normally consolidated**. There is a critical over consolidation ratio that corresponds with the critical state line (CSL).

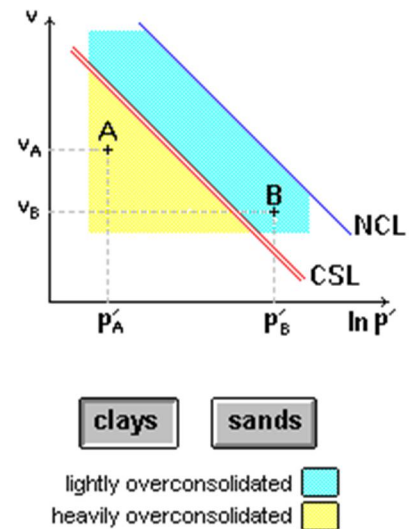
A **lightly** over consolidated soil has a state which lies **above** the CSL.

A **heavily** over consolidated soil has a state which lies **below** the CSL.

States lying **above** the CSL are said to be on the **wet side** of critical.

States lying **below** the CSL are said to be on the **dry side** of critical.

In the diagrams: $v_a > v_b$, and yet since the stress at B is greater, state B is on the wet side of critical, while state A is on the dry side of critical.



Back to [Compression and swelling](#)

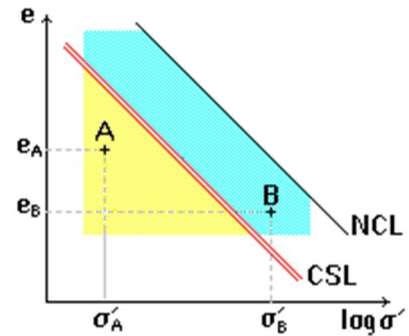
Wet and dry states

- [State parameters](#)

Soils whose states lie on the normal compression line (NCL) are **normally consolidated**. There is a critical over consolidation ratio that corresponds with the critical state line (CSL).

A **lightly** over consolidated soil has a state which lies **above** the CSL.

A **heavily** over consolidated soil has a state which lies **below** the CSL.



States lying **above** the CSL are said to be on the **wet side** of critical.

States lying **below** the CSL are said to be on the **dry side** of critical.

In the diagrams: $v_a > v_b$, and yet since the stress at B is greater, state B is on the wet side of critical, while state A is on the dry side of critical.

Back to [Wet and dry states](#)

State parameters

A measure of the initial state of a soil are the distances it lies at from the CSL, in terms of either volume or stress. These distances are expressed as **state parameters**:

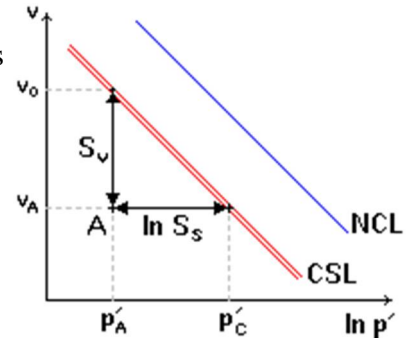
Stress state parameter

$$S_s = p'_a / p'_c$$

$$\ln S_s = \ln p'_a - \ln p'_c$$

Volume state parameter

$$S_v = v_a - v_c$$



The state parameters are related:

$$S_v = \ln S_s$$

Normally consolidated state:

$$S_v = \lambda \ln S_s = 0$$

States on the **wet** side of critical:

S_v and $\ln S_s$ are positive

States on the **dry** side of critical:

S_v and $\ln S_s$ are negative