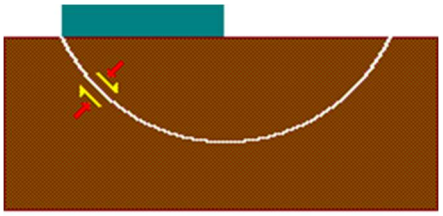
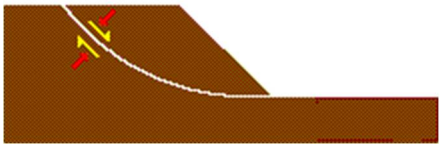
Shear strength

- Common cases of shearing
- Strength

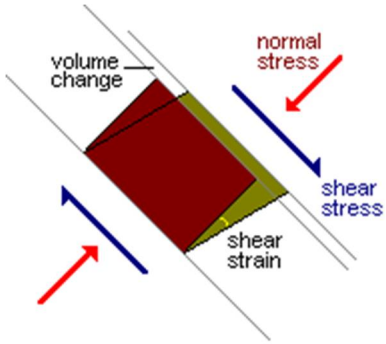




Near any geotechnical construction (e.g. slopes, excavations, tunnels and foundations) there will be both mean and normal stresses and shear stresses. The mean or normal stresses cause volume change due to compression or consolidation.

The shear stresses prevent collapse and help to support the geotechnical structure. Shear stress may cause volume change.

Failure will occur when the shear stress exceeds the limiting shear stress (strength).



Common cases of shearing

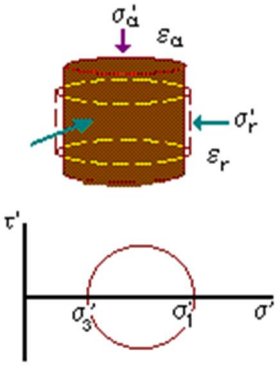
Back to Shear strength

In practice, the state of stress in the ground will be complex.

There are simple theories for two special cases.

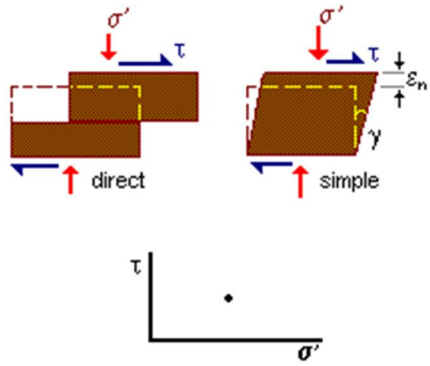
Triaxial (axial symmetry)
Parameters used for analysis:

- deviator stress
- shear strain
- normal stress
- volumetric strain
- specific volume



Direct or simple shear
Parameters used for analysis:

- · shear stress
- · shear strain
- · normal stress
- · volumetric (normal) strain
- · void ratio



It is not possible to draw a Mohr circle for a shear test unless stresses on vertical planes are measured.

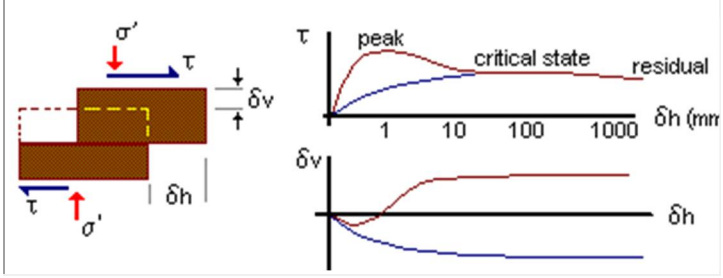
Strength

Back to Shear strength

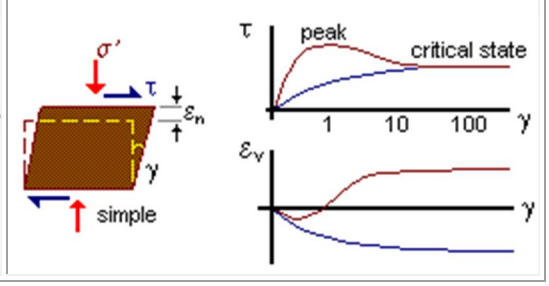
- Peak strength
- Critical state strength
- Residual strength

In very simple terms, the strength of soil is the **maximum shear stress (τ_f)** it can sustain, or the shear stress acting on a shear slip surface along which it is failing. There are three distinct strengths: peak, critical (or ultimate) and residual. Shearing may be simple or direct.

Drained direct (ring) shear



Drained simple shear



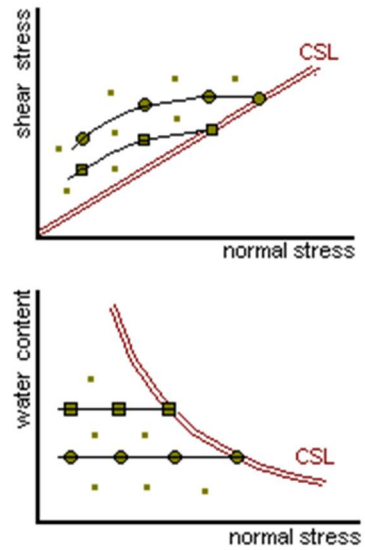
We explore the relationship between the maximum shear stress and the effective normal stress (σ') by ploting a graph of τ_f against σ'.

Some aspects of the behaviour show up more clearly if we normalise the data by plotting τ_f / σ' against σ' / σ'

Peak strength

Back to Strength

- Peak strength in shear tests
- Peak strength in triaxial tests
- Peak strength and dilatancy

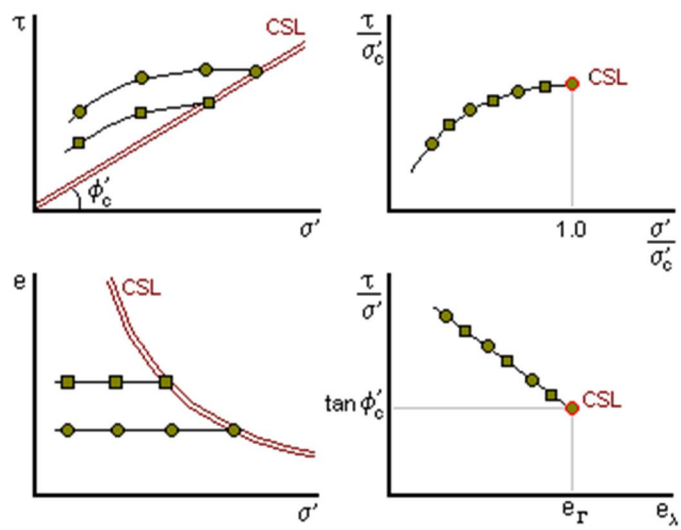


The peak strength is the maximum value of the shear stress or the maximum value of the ratio of shear stress to effective mean or normal stress. For drained tests these will occur simultaneously, for undrained tests they may occur at different points and the definition used here is the maximum stress ratio.

- Peak strengths can only occur at shear stresses above the critical state line and at water contents below the CSL.
- Peak states can occur anywhere in the regions above and below the CSL.
- Peak states at the same water content fall on unique smoth envelopes.
- The peak states can be represented on a graph in 3 dimensions.
- All peak states fall on a surface in this graph.

Peak strength in shear tests

Back to Peak strength Forward to Equations



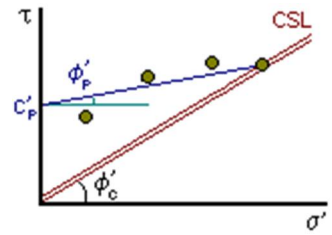
The circle represent the results of a set of shear tests on samples at the same moisture content but different normal stresses. The squares represent the results of a second set of tests at a different moisture content.

We normalise the data by plotting τ_f / σ' against σ' / σ'

The basic peak states, before normalisation, fall on different curves each for a particular water content or void ratio.

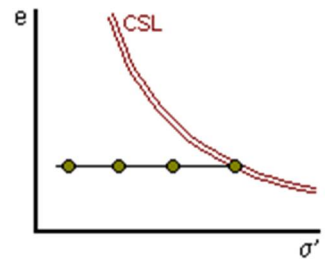
After normalisation all the peak states fall on a single unique envelope.

Equations



At a given water content or void ratio, all the peak states fall on a single smooth envelope.
This may be represented in one of two ways:

As a power law
$\tau'_p = B\sigma'^{\,a}$



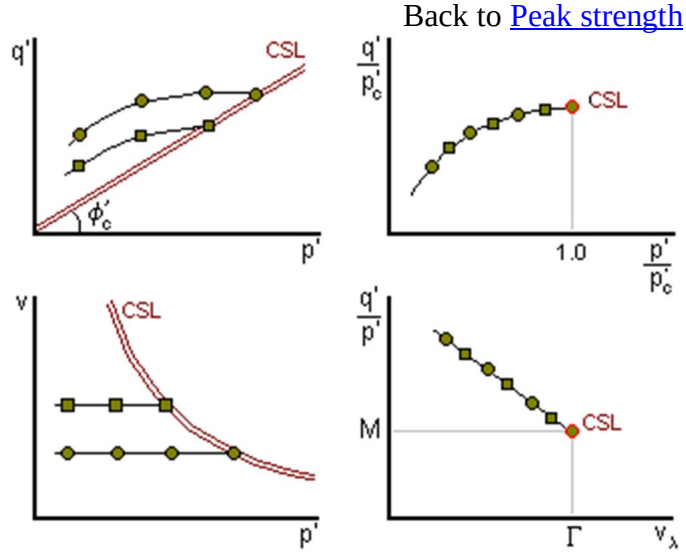
As a linear (Mohr-Coulomb) envelope
if the curvature is relatively small over a given range.
$\tau'_p = c'_p + \sigma' \tan \phi'_p$

The parameters a, B and c'_p, ϕ'_p depend on the water content or void ratio.

Even at a given water content or voids ratio, the parameters c'_p and ϕ'_p depend on the range of stress for linear approximation.

Back to Peak strength

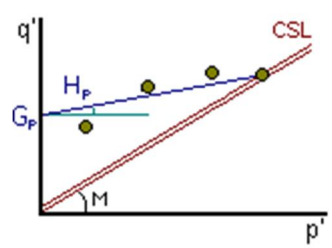
Peak strength in triaxial tests



The basic peak states, before normalisation, fall on different curves each for a particular water content or specific volume.

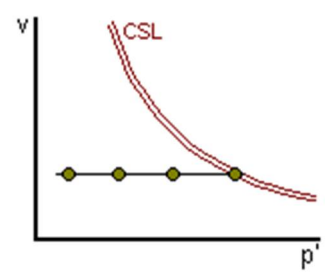
After normalisation all the peak states fall on a single unique envelope.

equations



At a given water content or specific volume all the peak states fall on a single smooth envelope. This may be represented in one of two ways:

As a power law
$q'_p = \beta\, p'^{\,\alpha}$



As a linear envelope
if the curvature is relatively small over a given range.
$q'_p = G_p + H_p\, p'$

The parameters α, β and G_p, H_p depend on the water content or voids ratio.

Even at a given water content or voids ratio, the parameters G_p and H_p depend on the range of stress for linear approximation.

Back to Peak strength

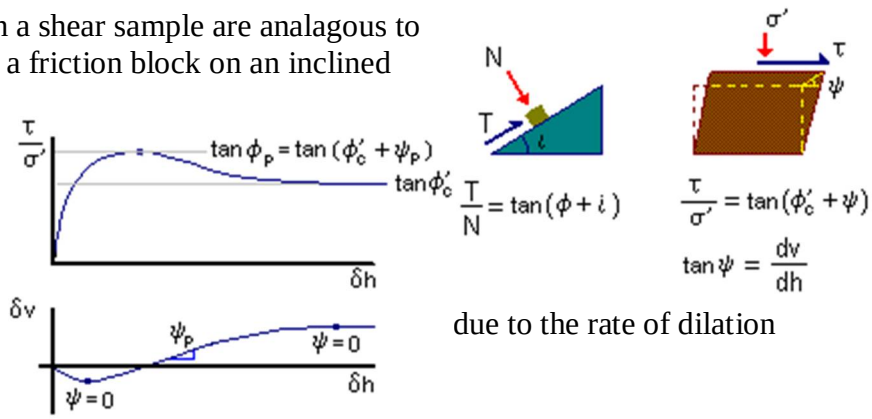
Peak strength and dilatancy

- Peak state and initial state

Stresses and displacements in a shear sample are analagous to the forces and movements of a friction block on an inclined plane.

At critical state
$\psi = 0$ and $\tau' = \sigma' \tan\phi'_c$

The additional stress ratio (above the critical state) is $\tan \psi = dv/dh$

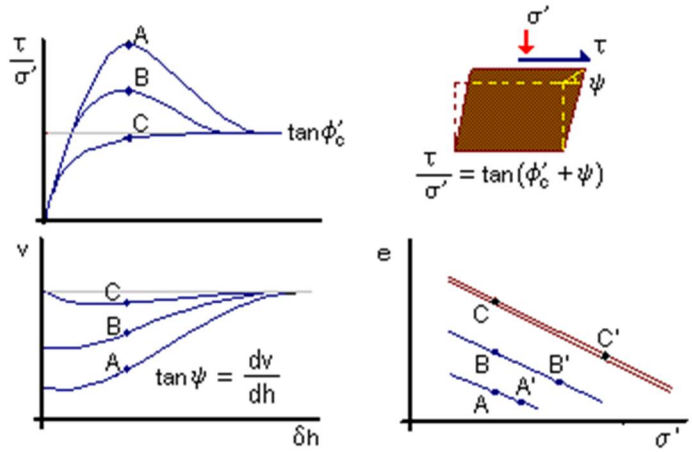


due to the rate of dilation

Back to Peak strength and dilatancy

Peak state and initial state

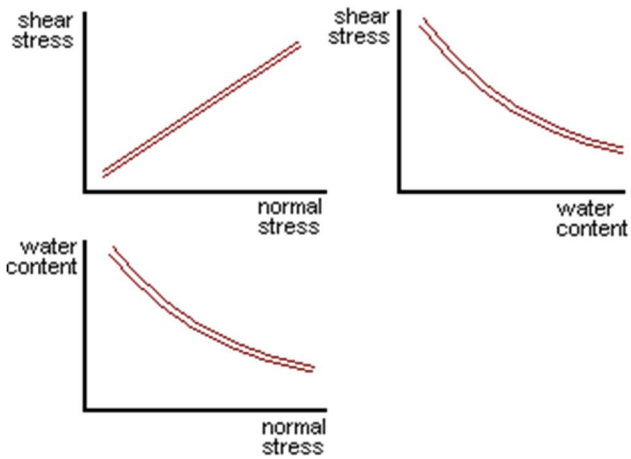
The peak stress ratio depends on the initial state given by the initial overconsolidation ratio. The maximum rate of dilation increases with overconsolidation ratio. For the same initial overconsolidation ratio (i.e. A and A') the peak stress ratio is the same.



Back to Strength

Critical state strength

At its critical state soil continues to distort at constant effective stress and at constant volume. This applies for turbulent flow of the particles: if the flow becomes laminar, as in clays at large strain, the strength falls to the residual.

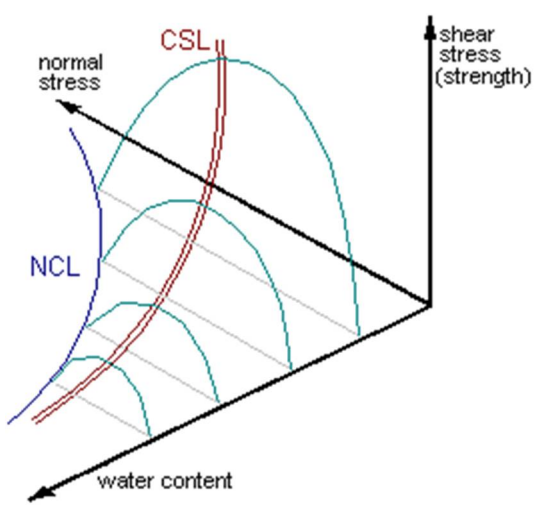


When soil is at its critical state there is a unique relationship between shear stress, effective normal stress and water content (or specific volume or void ratio). Critical states are unique and do not depend on initial state or stress path.

Critical states correspond to shear strains typically 10% to 40%.

Critical shear stress (critical state strength) increases with increasing effective normal stress and with decreasing water content.

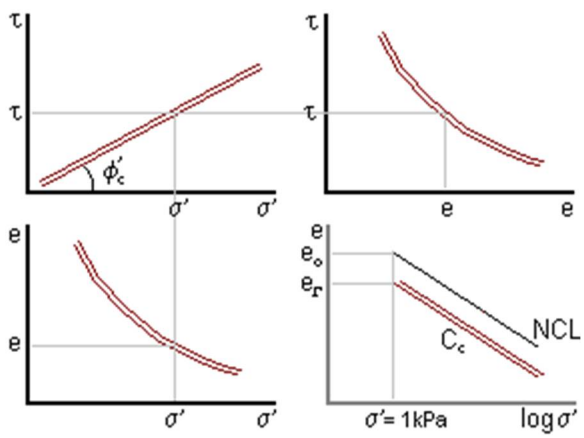
The critical state line can be represented as a graph in 3 dimensions. For isotropic compression, shear stresses are zero and the isotropic normal compression line can also be represented.



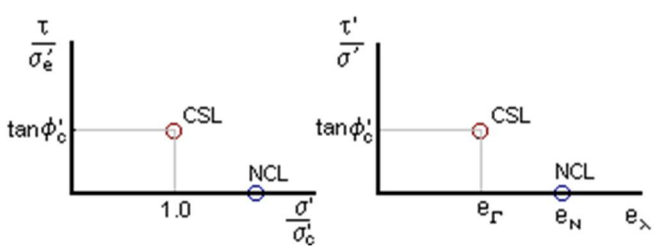
Back to Critical state strength

Critical state strength in shear tests

The graphs show the critical state line. If you know either σ' or e at the critical state you can calculate the critical state strength τ'.
ϕ'_c c_c and e_G are soil parameters.
The one-dimensional normal compression line (NCL) for zero shear stress is also shown.



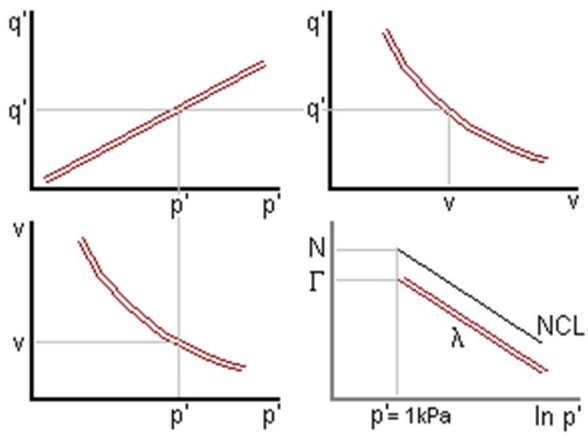
The critical state line can be normalised with respect to the critical pressure σ'_c or the equivalent void ratio e_l. The critical state line and the isotropic normal compression line both reduce to single points.



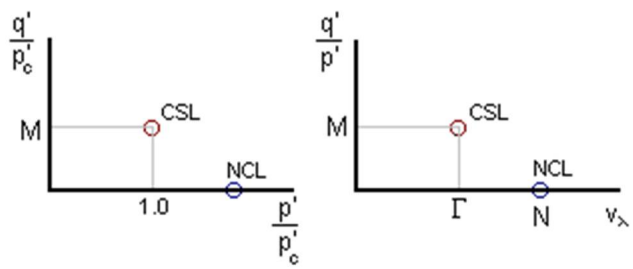
Back to Critical state strength

Critical state strength in triaxial tests

The graphs show the critical state line. If you know either p' or v at the critical state you can calculate the critical state deviation stress q'.
M λ and Γ are soil parameters.
We should really use subscripts c and e for compression and extension as the values are slightly different.
The isotropic normal compression line corresponds to zero deviator stress



The critical state line can be normalised with respect to the critical pressure ϕ'_c or the equivalent specific
volume v_l. The critical state line and the isotropic normal compression line both reduce to single points.



Back to Critical state strength

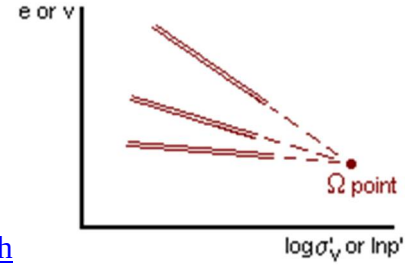
Typical values of critical state strength parameters

The critical state parameters are basic soil parameters and they depend principally on the nature of the soil grains. For fine grained soils the CS parameters are related to the Atterberg limits; for coarse-grained soils they are related to the mineralogy and shape of the grains.

Typical values	λ	G	M	ϕ'
high plasticity clay	0.16	2.45	0.89	23º
low plasticity clay	0.10	1.80	1.18	29º
quartz sand	0.16	3.00	1.28	32º
carbonate sand	0.34	4.35	1.65	40º

For fine-grained soils the gradient λ or C_c of the critical state line is related to the Atterberg limits by
$C_c = (I_p \times G_s) / 200$
$\lambda = (I_p \times G_s) / 460$

For many soils the critical state lines all pass through a single point called the Ω (omega) point.
$v(_W) = 1.25\ p'(_W) = 10$MPa
$e(_W) = 0.25\ \sigma'(_W) = 15$MPa.



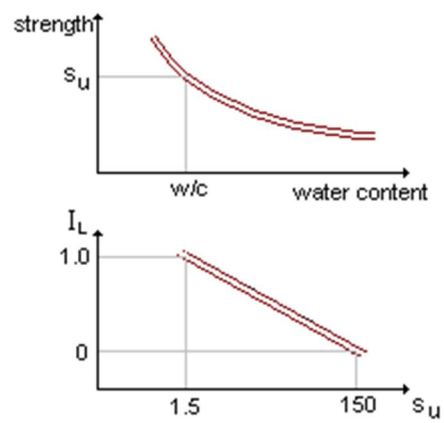
Back to Critical state strength

Undrained strength

The critical state strength is uniquely related to the water content.

If the soil is sheared without change of water content (i.e. undrained) its strength remains the same. This is called the undrained strength s_u. But if the soil is not undrained and the water content changes the strength will also change.

The undrained strength is directly related to the liquidity index I_L. Some authors give slightly different values for s_u but s_u at PL (i.e. I_L=0) is always 100 times s_u at LL (i.e. I_L=1)

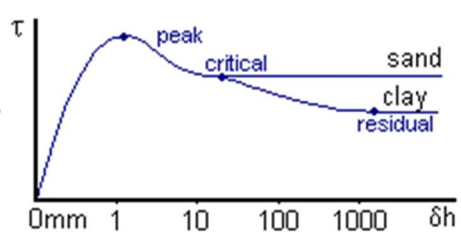


Back to Strength

Residual strength

- Residual strength: equations

This is the very lowest strength which occurs after very large displacements. For sands the residual strength is the same as the critical state strength. For clays the residual is about ½ the critical state strength. For clays the flat clay particles become aligned parallel to the direction of shear.

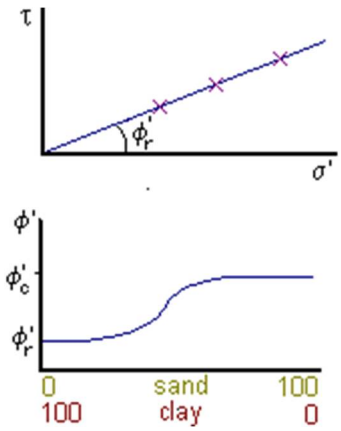


The residual strength occurs after very large (>1m) movements and is not usually relevant for geotechnical engineering where generally ground movements must be small. However, on old landslides there may have already been very large

movements and in such cases the strength may already be at the residual before construction starts.

Back to Residual strength

Residual strength: equations



Residual strength applies to clays after very large shear displacements when clay particles have become aligned in well-defined shear zones or slip planes.

Drained case

$\tau = \sigma' \tan \phi'_r$

ϕ'_r = residual friction angle

In clays ϕ'_r can be less than ½ϕ'_c.

For London Clay,

ϕ'_c »22º and ϕ'_r»10º.

In mixed soils ϕ'_r depends on the quantity of clay present.

Undrained case

$\tau = s_{ur}$

s_{ur} = undrained residual strength

(depends on water content)