Shear strength

- Common cases of shearing
- Strength

Near any geotechnical construction (e.g. slopes, excavations, tunnels and foundations) there will be both mean and normal stresses and shear stresses. The mean or normal stresses cause volume change due to compression or consolidation.

The shear stresses prevent collapse and help to support the geotechnical structure. Shear stress may cause volume change.

Failure will occur when the shear stress exceeds the limiting shear stress (strength).

Common cases of shearing

In practice, the state of stress in the ground will be complex.

There are simple theories for two special cases.

Triaxial (axial symmetry)

Parameters used for analysis:

- deviator stress
- shear strain
- normal stress
- volumetric strain
- specific volume
**Direct or simple shear**
Parameters used for analysis:
- shear stress
- shear strain
- normal stress
- volumetric (normal) strain
- void ratio

It is not possible to draw a Mohr circle for a shear test unless stresses on vertical planes are measured.

**Strength**

- Peak strength
- Critical state strength
- Residual strength

In very simple terms, the strength of soil is the **maximum shear stress** ($\tau_f$) it can sustain, or the shear stress acting on a shear slip surface along which it is failing. There are three distinct strengths: peak, critical (or ultimate) and residual. Shearing may be simple or direct.

We explore the relationship between the maximum shear stress and the effective normal stress ($\sigma'$) by plotting a graph of $\tau_f$ against $\sigma'$.

Some aspects of the behaviour show up more clearly if we normalise the data by plotting $\tau_f / \sigma'$ against $\sigma' / \sigma'$

**Peak strength**
- **Peak strength in shear tests**
- **Peak strength in triaxial tests**
- **Peak strength and dilatancy**

The peak strength is the maximum value of the shear stress or the maximum value of the ratio of shear stress to effective mean or normal stress. For drained tests these will occur simultaneously, for undrained tests they may occur at different points and the definition used here is the maximum stress ratio.

- Peak strengths can only occur at shear stresses above the critical state line and at water contents below the CSL.
- Peak states can occur anywhere in the regions above and below the CSL.
- Peak states at the same water content fall on unique smooth envelopes.
- The peak states can be represented on a graph in 3 dimensions.
- All peak states fall on a surface in this graph.

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**Peak strength in shear tests**

The circle represent the results of a set of shear tests on samples at the same moisture content but different normal stresses. The squares represent the results of a second set of tests at a different moisture content.

We normalise the data by plotting $\tau / \sigma$ against $\sigma / \sigma'$

The basic peak states, before normalisation, fall on different curves each for a particular water content or void ratio.

After normalisation all the peak states fall on a single unique envelope.
**Equations**

At a given water content or void ratio, all the peak states fall on a single smooth envelope. This may be represented in one of two ways:

**As a power law**

\[ \tau_p' = B \sigma_p'^a \]

**As a linear (Mohr-Coulomb) envelope**

if the curvature is relatively small over a given range.

\[ \tau_p' = c_p' + \sigma_p' \tan \phi_p' \]

The parameters \( a, B, c_p', \phi_p' \) depend on the water content or void ratio.

Even at a given water content or voids ratio, the parameters \( c_p', \phi_p' \) depend on the range of stress for linear approximation.

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**Peak strength in triaxial tests**

The basic peak states, before normalisation, fall on different curves each for a particular water content or specific volume.

After normalisation all the peak states fall on a single unique envelope.

**equations**

At a given water content or specific volume, all the peak states fall on a single smooth envelope. This may be represented in one of two ways:

**As a power law**

\[ q_p' = \beta p'^\alpha \]

**As a linear envelope**

if the curvature is relatively small over a given range.

\[ q_p' = G_p + H_p p' \]
The parameters $\alpha$, $\beta$ and $G_p$, $H_p$ depend on the water content or voids ratio.

Even at a given water content or voids ratio, the parameters $G_p$ and $H_p$ depend on the range of stress for linear approximation.

Peak strength and dilatancy

- **Peak state and initial state**

Stresses and displacements in a shear sample are analogous to the forces and movements of a friction block on an inclined plane.

At critical state

$\psi = 0$ and $\tau' = \sigma' \tan \phi'_c$

The additional stress ratio (above the critical state) is

$tan \psi = \frac{dv}{dh}$

due to the rate of dilation

Peak state and initial state

The peak stress ratio depends on the initial state given by the initial overconsolidation ratio. The maximum rate of dilation increases with overconsolidation ratio. For the same initial overconsolidation ratio (i.e. A and A') the peak stress ratio is the same.
Critical state strength

- Critical state strength in shear tests
- Critical state strength in triaxial tests
- Undrained strength
- Typical values of critical state strength parameters

At its critical state soil continues to distort at constant effective stress and at constant volume. This applies for turbulent flow of the particles: if the flow becomes laminar, as in clays at large strain, the strength falls to the residual.

When soil is at its critical state there is a unique relationship between shear stress, effective normal stress and water content (or specific volume or void ratio). Critical states are unique and do not depend on initial state or stress path.

Critical states correspond to shear strains typically 10% to 40%.

Critical shear stress (critical state strength) increases with increasing effective normal stress and with decreasing water content.

The critical state line can be represented as a graph in 3 dimensions. For isotropic compression, shear stresses are zero and the isotropic normal compression line can also be represented.
Critical state strength in shear tests

The graphs show the critical state line. If you know either $\sigma'$ or $e$ at the critical state you can calculate the critical state strength $\tau'$. $\phi'_c$, $c_c$ and $e_G$ are soil parameters. The one-dimensional normal compression line (NCL) for zero shear stress is also shown.

The critical state line can be normalised with respect to the critical pressure $\sigma'_c$ or the equivalent void ratio $e_i$. The critical state line and the isotropic normal compression line both reduce to single points.

Critical state strength in triaxial tests

The graphs show the critical state line. If you know either $p'$ or $v$ at the critical state you can calculate the critical state deviation stress $q'$. $M \lambda$ and $\Gamma$ are soil parameters. We should really use subscripts $c$ and $e$ for compression and extension as the values are slightly different. The isotropic normal compression line corresponds to zero deviator stress.
The critical state line can be normalised with respect to the critical pressure $\phi'_c$ or the equivalent specific volume $v_i$. The critical state line and the isotropic normal compression line both reduce to single points.

### Typical values of critical state strength parameters

The critical state parameters are basic soil parameters and they depend principally on the nature of the soil grains. For fine grained soils the CS parameters are related to the Atterberg limits; for coarse-grained soils they are related to the mineralogy and shape of the grains.

<table>
<thead>
<tr>
<th>Typical values</th>
<th>$\lambda$</th>
<th>G</th>
<th>M</th>
<th>$\phi'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>high plasticity clay</td>
<td>0.16</td>
<td>2.45</td>
<td>0.89</td>
<td>23º</td>
</tr>
<tr>
<td>low plasticity clay</td>
<td>0.10</td>
<td>1.80</td>
<td>1.18</td>
<td>29º</td>
</tr>
<tr>
<td>quartz sand</td>
<td>0.16</td>
<td>3.00</td>
<td>1.28</td>
<td>32º</td>
</tr>
<tr>
<td>carbonate sand</td>
<td>0.34</td>
<td>4.35</td>
<td>1.65</td>
<td>40º</td>
</tr>
</tbody>
</table>
For fine-grained soils the gradient $\lambda$ or $C_c$ of the critical state line is related to the Atterberg limits by

\[ C_c = \frac{I_p \times G_s}{200} \]

\[ \lambda = \frac{I_p \times G_s}{460} \]

For many soils the critical state lines all pass through a single point called the $\Omega$ (omega) point.

\[ v(w) = 1.25 \times p'(w) = 10\text{MPa} \]

\[ e(w) = 0.25 \times \sigma'(w) = 15\text{MPa} \]

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**Undrained strength**

The critical state strength is uniquely related to the water content.

If the soil is sheared without change of water content (i.e. undrained) its strength remains the same. This is called the undrained strength $s_u$. But if the soil is not undrained and the water content changes the strength will also change.

The undrained strength is directly related to the liquidity index $I_L$. Some authors give slightly different values for $s_u$ but $s_u$ at PL (i.e. $I_L=0$) is always 100 times $s_u$ at LL (i.e. $I_L=1$)

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**Residual strength**

- **Residual strength: equations**

This is the very lowest strength which occurs after very large displacements. For sands the residual strength is the same as the critical state strength. For clays the residual is about $\frac{1}{2}$ the critical state strength. For clays the flat clay particles become aligned parallel to the direction of shear.

The residual strength occurs after very large (>1m) movements and is not usually relevant for geotechnical engineering where generally ground movements must be small. However, on old landslides there may have already been very large
movements and in such cases the strength may already be at the residual before construction starts.

Residual strength: equations

Residual strength applies to clays after very large shear displacements when clay particles have become aligned in well-defined shear zones or slip planes.

**Drained case**
\[ \tau = \sigma' \tan \phi'_r \]
\( \phi'_r \) = residual friction angle
In clays \( \phi'_r \) can be less than \( \frac{1}{2} \phi'_c \).
For London Clay,
\( \phi'_c \geq 22^\circ \) and \( \phi'_r \geq 10^\circ \).
In mixed soils \( \phi'_r \) depends on the quantity of clay present.

**Undrained case**
\[ \tau = s_{ur} \]
\( s_{ur} \) = undrained residual strength
(depend on water content)