The most common type of structure an engineer will analyze lies in a plane subject to a force system in the same plane.

In general, it is not possible to perform an exact analyze of a structure. Approximations for structure geometry, material parameters, and loading type and magnitude must be made.

Support connections - Structural members may be joined in a variety of methods, the most common are pin and fixed joints.

A pin connection confines deflection; allows rotation.

A fixed connection confines deflection and rotation.

However, in reality, a pin connection has some resistance against rotation due to friction, therefore, a torsional spring connection may be more appropriate. If the stiffness $k = 0$ the joint is a pin, if $k = \infty$, the joint is fixed.

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Support Idealizations

- Weightless link or light cable
- Rollers and rockers

Support Idealizations

- Smooth contacting surface
- Smooth pin or hinge

Support Idealizations

- Sliders and collar
- Fixed support

Support Idealizations

- Smooth pin

Support Idealizations

- New friction pendulum bearings on the I-40 bridge

Support Idealizations

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Support Idealizations

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Support Idealizations

- Smooth hinge
Support Idealizations

- Roller support

A complex structure may be idealized as a line drawing where orientation of members and type of connections are assumed.

- Fixed support

In many cases, loadings are transmitted to a structure under analysis by a secondary structure. In a line drawing, a pin support is represented by lines that do not touch and a fixed support by connecting lines.
The dead load on the roof is 72 lb/ft².

**Tributary Loadings** - When frames or other structural members are analyzes, it is necessary to determine how walls, floors, or roofs transmit load to the element under consideration.

- **A one-way system** is typically a slab or plate structure supported along two opposite edges.
- **Examples**, a slab of reinforced concrete with steel in one direction or a with steel in both directions with a span ratio $L_2/L_1 > 2$.
- **A two-way system** is typically defined by a span ratio $L_2/L_1 < 2$ or if all edges are supported.
**Loading Idealizations**

- Basis for the theory of linear elastic structural analysis:
  - The total displacement or stress at a point in a structure subjected to several loadings can be determined by adding together the displacements or stresses caused by each load acting separately.
  - There are two exceptions to these rules:
    - If the material does not behave in a linear–elastic manner
    - If the geometry of the structure changes significantly under loading (example, a column subjected to a buckling load)

**Equations of Equilibrium**

- From statics the equations of equilibrium are:
  \[
  \sum F_x = 0 \quad \sum F_y = 0 \quad \sum F_z = 0 \\
  \sum M_x = 0 \quad \sum M_y = 0 \quad \sum M_z = 0
  \]
- However, since we are dealing with co-planar structures, the equations reduce to:
  \[
  \sum F_x = 0 \quad \sum F_y = 0 \quad \sum M_z = 0
  \]
Determinacy and Stability

- **Determinacy** - provide both necessary and sufficient conditions for equilibrium.

- When all the forces in a structure can be determined from the equations of equilibrium, then the structure is considered **statically determinate**.

- If there are more unknowns than equations, the structure is **statically indeterminate**.

For co-planar structures, there are three equations of equilibrium for each FBD, so that for \( n \) bodies and \( r \) reactions:

- \( r = 3n \)  **statically determinate**
- \( r > 3n \)  **statically indeterminate**
Determinacy and Stability

- **Stability** - Structures must be properly held or constrained by their supports
  - **Partial Constraints** - a structure or one of its members with fewer reactive forces than equations of equilibrium
  - **Improper Constraints** - the number of reactions equals the number of equations of equilibrium, however, all the reactions are concurrent. In this case, the moment equations is satisfied and only two valid equations of equilibrium remain.

- Another case is when all the reactions are parallel
  - In general, a structure is *geometrically unstable* if there are fewer reactive forces than equations of equilibrium.
  - An unstable structure must be avoided in practice regardless of determinacy.

\[
\begin{align*}
r < 3n & \quad \text{unstable} \\
r \geq 3n & \quad \text{unstable if members reactions are concurrent or parallel or contains a collapsible mechanism}
\end{align*}
\]
Determinacy and Stability

- **Unstable** The three reactions are concurrent

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Application of the Equations of Equilibrium

- **Free–Body Diagram** - disassemble the structure and draw a free–body diagram of each member.

- **Equations of Equilibrium** - The total number of unknowns should be equal to the number of equilibrium equations

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Determinacy and Stability

- **Unstable** The three reactions are parallel

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Application of the Equations of Equilibrium

- **Free–Body Diagram**
  - Disassemble the structure and draw a free–body diagram of each member.
  - It may be necessary to supplement a member free-body diagram with a free-body diagram of the entire structure.
  - Remember that reactive forces common on two members act with equal magnitudes but opposite direction on their respective free bodies.
  - Identify any two-force members

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Determinacy and Stability

- **Unstable** \( r < 3n \) and member CD is free to move horizontally

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Application of the Equations of Equilibrium

- **Equations of Equilibrium**
  - Check if the structure is determinate and stable
  - Attempt to apply the moment equation \( \Sigma M = 0 \) at a point that lies at the intersection of the lines of action of as many forces as possible
  - When applying \( \Sigma F_x = 0 \) and \( \Sigma F_y = 0 \), orient the \( x \) and \( y \) axes along lines that will provide the simplest reduction of forces into their \( x \) and \( y \) components
  - If the solution of the equilibrium equations yields a *negative* value for an unknown, it indicates that the direction is *opposite* of that assumed
Application of the Equations of Equilibrium

- Draw the free-body diagram and determine the reactions for the following structures

![Free-body diagram](image)

Analysis of Statically Determinate Structures

Any Questions?