ANALYSIS OF PROJECT PLANNING

USING CPM AND PERT

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Spring 2010

In Partial Fulfillment of Math 4395–Senior Project

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ABSTRACT

This project presents an analysis of the Critical Path Method (CPM) and Project Evaluation Review Technique (PERT) in Project Planning. It highlights the means by which a network diagram is constructed from a list of project activities and the computation involved for each method. The project will consist of examples problems, along with a complex problem constructed from scratch, using both CPM and PERT, and compare results highlighting advantages and disadvantages of each methods.

ACKNOWLEDGEMENTS

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1.1 INTRO

Working Americans typically follow a morning routine that begins with waking up, and ends with arriving at work. While everyone's routine will vary in number of tasks, and tasks themselves, the majority of routine include two things: a clean shower and getting dressed. Its obvious getting dressed for work before showering is not efficient and can cause you to be late for work, but what is not so apparent is our routine is actually a project. The complete project begins with waking up and is completed when we arrive at work. Tasks like a morning jog, eating breakfast, taking a shower, and getting dressed are activities which must be completed in order to complete the project. This is project planning in its simplest form. Whether we do it mentally or use pen and paper, we plan for every project in an effort to save time, money, and avoid headaches. On the corporate level giving incorrect estimates, failing to meet deadlines, and oversight can lead to loss of revenue and marginal profit. Project planning is a vital tool, without it constructing a 20-story skyscraper would be nearly impossible.

In the 1950's both government agencies and the private sector became more interested in efficiency pertaining to large projects. One example was a study conducted in Great Britain by

the Operational Research Section of the Central Electricity

Generating Board, which examined problems with refurbishing a

generating plant. By 1957 they devised a technique which reduced

the overhaul time to 42% of the average time required for the

project. There are two main techniques pertaining to project

planning in use today: the Critical Path Method (CPM) and the

Project Evaluation and Review Technique (PERT).

1.2 HISTORY

Critical Path Method (CPM)

The Critical Path Method, sometimes referred to as Critical Path

Analysis (CPA) was developed in the 1950's by DuPont Corporation

and Remington Rand Corporation. It was specifically developed to

manage power plant maintenance projects. They wanted to develop

a management tool that would help in the scheduling of chemical

plant shut downs for maintenance and then restarting them once

maintenance was complete. The CPM methods saved the company one

million dollars in the first year of use.

Project Evaluation and Review Technique (PERT)

Separate but similar work was also being conducted in the mid

1950's by the United States Navy. The US government discovered

the Russians were developing their own missile technology, and

because national security was at stake the Navy immediately

launched their own program to close the missile gap.

The project was enormous, and so it was important for the Navy to conduct research on planning and controlling complicated projects. The research was referred to as the Program Evaluation Research Task (code-name PERT). In February of 1958, Dr. C.E. Clark, from the PERT team, introduced the first arrow diagram. PERT, later referred to as the Program Evaluation and Review Technique, was applied to the Fleet Ballistic Missile Program later that year. With over 3,000 contractors, vendors, and other teams involved, it was of strategic importance to complete the project quickly and efficiently. PERT proved its worth, and was given credit for taking two years off the estimated time needed to develop the Polaris missile, and is still the standard for all Navy projects today.

1.3 THIS PROJECT

CPM and PERT were developed independently, but their main difference is that CPM uses deterministic (known) activity durations and PERT consists of probabilistic activity durations. In this project I will be analyzing both methods and demonstrating their application through example. I will also create an example from scratch, solve using both methods, and analyze the results, highlighting advantages and disadvantages of both methods.

SECTION 2
DESCRIPTION OF METHODS

2.1 NETWORK BASICS

Both CPM and PERT are network based techniques. They are vital tools in the planning, scheduling, and control of projects. A project is a collection of interrelated activities with each activity consuming time and resources. In order to use either method you must:

(1) Define the activities involved for each project.

(2) Define the precedence relationship of activities:

- Can the activity be completed independently of other activities?

- Must the activity be preceded by another activity?

Once you have this information a network can be created showing the precedence relationship. The next step involves specific computations to develop a time schedule for the project.

CONSTRUCTING THE NETWORK

Each activity is represented by an arc pointing in the direction of progress. The nodes establish precedence relationships among different activities. There are three rules for constructing the network:

RULE 1: Each activity is represented by one, and only one, arc.

RULE 2: Each activity must be identified by two distinct

end nodes.

RULE 3: Maintain correct precedence relationship as you add

activities to network. The following must be

considered:

1) What activities must immediately precede the

current activity?

2) What activities must follow the current

activity?

What activities must occur concurrently with

current activity?
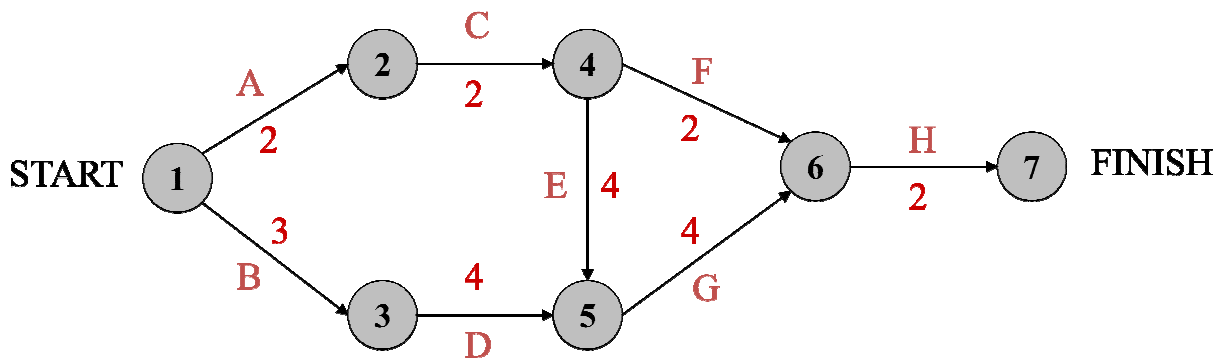
FIGURE 2.1.1 NETWORK

DUMMY ACTIVITIES

Dummy activities are used to show correct precedence. They

consume no time or resources and are usually represented with a

dashed arc and labeled with D_i, where (i=1,2,3,…,n). There is no

set number of how many dummy activities you can use.

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Suppose you have the following network to construct:

Activity	Predecessors	Duration
A	–	2 weeks
B	–	1 week
C	A,B	3 week
E	B	1 week

FIGURE 2.1.2 DUMMY ACTIVITY TABLE

Without using a dummy arc your network can look like this:
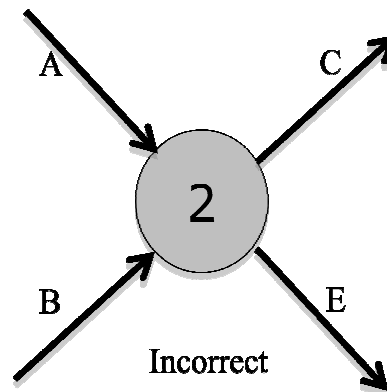
FIGURE 2.1.3 INCORRECT DUMMY

This diagram shows that activity E cannot begin until both activity A and B are completed, which is incorrect. According to our activity table, for activity E to begin it only needs activity B (not A) to be completed. The use of a dummy arc can correct as demonstrated in FIGURE 2.1.4.

A

C

D_1

B

E
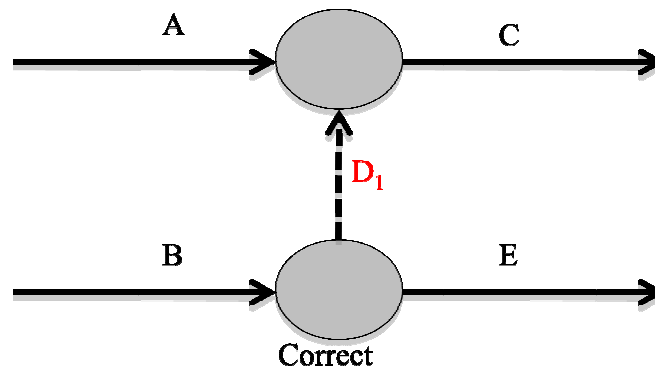
Correct

FIGURE 2.1.4 CORRECT DUMMY

2.2 CPM COMPUTATIONS

A critical activity is an activity that has no leeway in determining its start and finish times. If a critical activity runs late, then the entire project will run late.
A noncritical activity is an activity that allows some scheduling slack, meaning it can be advanced or delayed (within limits) without affecting the completion time of the project.
An event is defined as a point in time when activities are completed and another activity is started. In terms of a network, an event corresponds to a node.

E_j = Earliest occurrence time of event j.

L_j = latest occurrence time of event j.

D_{ij} = Duration of activity.

The critical path calculations involve two passes: The *forward pass* determines the earliest occurrence times of the

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events, and the **backward pass** calculates their latest occurrence times.

Forward Pass (Earliest Occurrence Times).

The computation starts at node 1 and advance recursively to end node n.

Step 1: Set $E_1=0$ (indicates the project starts at node 1 and time 0). The first node of the network will never have any nodes going into it, so that is why it will always be zero.

General step j. Given that nodes **p, q, ...**, and **v** are linked directly to node j by incoming activities **(p, j), (q, j), ...**, and **(v, j)** and that the earliest occurrence times of events (nodes) **p, q, ...**, and **v** have already been computed, then the earliest occurrence time of event **j** is computed as

$$E_j = max\{E_p + D_{pj}, E_q + D_{qj}, ..., E_v + D_{vj}\}$$

Step 3: Calculate E_j for every node until you reach the final node, **n**.

Backward Pass (Latest Occurrence Times L_j).

Once you have computed the forward pass , the backward pass computations start at the final node n and end at node 1.

Step 1: Set $L_n = E_n$ (indicates that the earliest and latest occurrence of the last node of the project are the same.

General step j. Given that nodes p, q,..., and v are

        linked directly to node j by outgoing activities

        (j,p) , (j,q) ,..., (j,v) and that the latest

        occurrence times of events (nodes) p, q,..., and v

        have already been computed, then the latest

        occurrence time of event j is computed as

$$L_j = min\{L_p + D_{jp},\ L_q + D_{jq},...,L_v + D_{jv}\}$$

Step 3: Calculate E_j for every node until you reach the

        final node, n.

Based on the preceding calculations, an activity (i,j) will be

critical if it satisfies three conditions:

1) $L_i = E_i$

2) $L_j = E_j$

3) $L_j - L_i = E_j - E_i = D_{ij}$

The three conditions state that the earliest and latest

occurrence times of end nodes i and j are equal and the duration

D_{ij} fits "tightly" in the specified time span. An Activity that

does not satisfy all three conditions is thus noncritical. By

definition, the critical activities of a network must constitute

an uninterrupted path that spans the entire network from start

to finish.

Determination of the Slacks (S_{ij}):

Slack times give you the amount of time an activity can run late without delaying your project. Consequently a slack time of zero identifies a critical activity.

$$\underline{S_{ij}} = L_j - E_i - D_{ij}$$

2.3 PERT COMPUTATIONS

Pert differs from CPM in that it bases the duration of an activity on three estimates:

1. **Optimistic time, a,** which occurs when execution goes extremely well.

2. **Most likely time, m,** which occurs when execution is done under normal conditions.

3. **Pessimistic time, b,** which occurs when execution goes extremely poorly.

The PERT method was motivated by the assumption that the activity time was a random variable with a beta distribution. Unlike the normal distribution, which has an infinite range and is symmetrical, the **beta distribution** has a minimal and maximum value, and is capable of assuming a wide variety of shapes. A typical beta distribution can be seen in FIGURE 2.3.1.
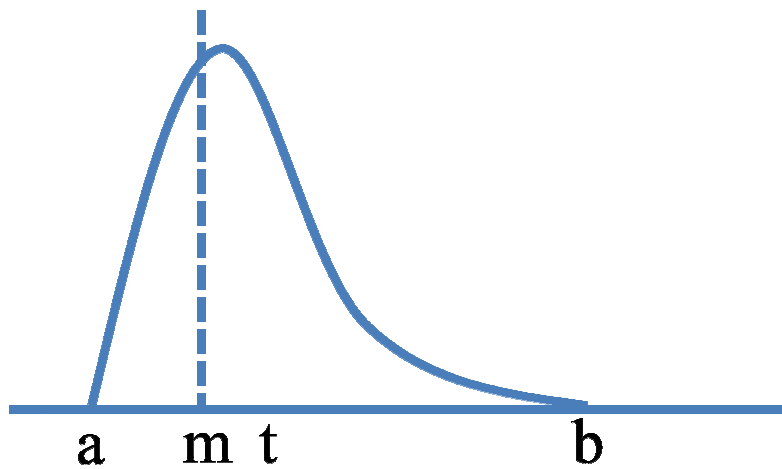
FIGURE 2.3.1 BETA DISTRIBUTION

The **expected value (the average duration time, mean,)** of beta

distribution is approximately:

$$\bar{E} = \underline{\hspace{3em}}$$

Note: the estimate is a weighted average of the values of a, m,
and b, where the weights (---) sum to 1.   This means that the
estimate will always lie between a and b.

The *standard deviation* of an activity time is estimated by

assuming that there are six standard deviations between the

optimistic and pessimistic times:

$$ij = \underline{\hspace{2em}}$$

Once we compute the standard deviation we can compute the

variance of the activity by $Var_{ij} = (\quad_{ij})^2 = \underline{\hspace{2em}}$

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The purpose of PERT is to analyze the probability that a critical path will be finished by *any given time.* The analysis proceeds as followed:

1) Let T equal the total time that will be taken by the activities on a critical path.

2) Find the probability that the value of T will turn out to be less than or equal to any specified value of interest.

    a. The activity times are independent random variables. This is a valid assumption for most PERT networks.

    b. The random variable T has an approximately normal distribution. This assumption relies on the central limit theorem, which in broad terms states that the sum of independent random variables is approximately normally distributed.

Then $T \leq 22$

We will want to convert T to a standard normal random variable and use a Z table. The first step is to find the standard deviation of T (standard deviation of the path). To do this we need the variance of T (variance of the path).

To find the **_variance of a path_**, we add all the variance for a path we wish to take. For example if we have a path consisting of activities starting at activity A and finishing at activity D, such that, A→B→C→D, and we wish to find the probability that we can complete activity D within 5 days, we

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will need to know the standard deviation of the path, but to do that we must know the variance of the path. The variance of the path is just calculated by adding the variance of each activity.

Var T=(var for activity A)+(var for activity B)+(var for activity C)+(var for activity D)

We can now calculate the **standard deviation of the path**, which is simply: $\sqrt{Var\,T}$

Finally, we now need to convert T to a standard normal random variable, Z, in the usually way: $Z=\dfrac{T-\mu}{\sigma}$

Recall that **μ** is mean (the expected completion time). We know use the Z score to calculate the probability.

2.4 EXAMPLE OF METHODS

We will now apply both CPM and Pert to the following example:

A publisher has a contract with an author to publish a textbook. The (simplified) activities associated with the production of the textbook are given. The author is required to submit to the publisher a hard copy and a computer file of the manuscript.

CPM Method

A list of activities is provided. Using the CPM method we can find out the completion time of the project. We will also determine which activities are critical activities–activities

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that if delayed would delay the completion of the project, and
slack times for non-critical activities–the amount of time a
non-critical activity can be delayed (run late) without
effecting the completion time of the project.

ACTIVITY	DESCRIPTION	PREDECESSOR(S)	DURATION (days)
A	Manuscript proofreading by editor	-	3
B	Sample pages preparation	-	2
C	Book cover design	-	4
D	Artwork preparation	-	3
E	Author's approval of A, B	A, B	2
F	Book formatting	E	4
G	Author's review of formatted pages	F	2
H	Author's review of artwork	D	1
I	Production of printing plates	G, H	2
J	Book production and binding	C, I	4

FIGURE 2.4.1 EXAMPLE CPM ACTIVITY TABLE

Based on the table we can now construct a network, shown in
FIGURE 2.4.2.
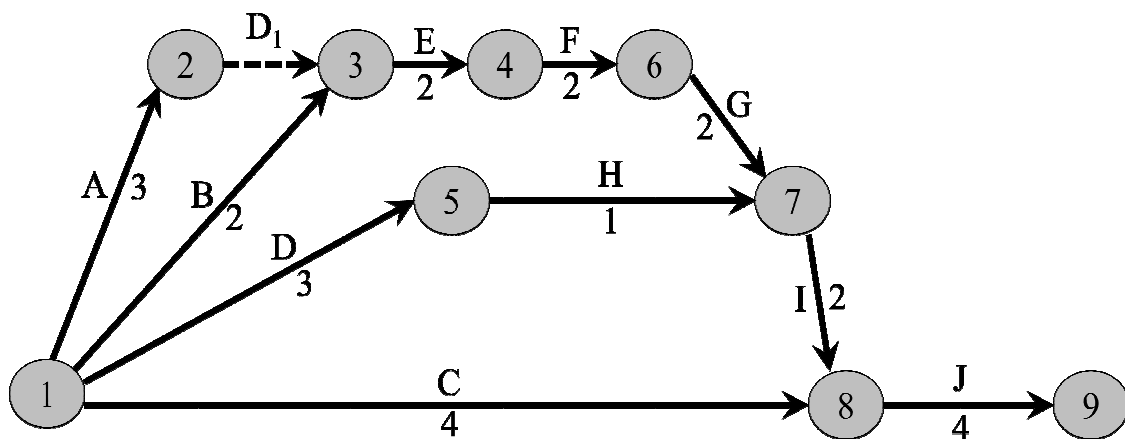
FIGURE 2.4.2 EXAMPLE NETWORK

In our example one dummy arc was needed to show the correct
precedence for activity E to begin. The next step is to

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calculate the forward pass to determine the latest times an

event can begin.

We will begin by calculating the Earliest Occurrence Time, E_j, by doing a forward pass. So, we will first set $E_1=0$. Since there is only one arc coming into Node 2 (From Node A) the earliest occurrence time to $E_2=3$. If E_j has more than one are coming in to it, E_j = maximum of the sums (E_i + T_{ij}) coming into E_j. FIGURE 2.4.3 shows the complete Earliest Occurrence Time computations.
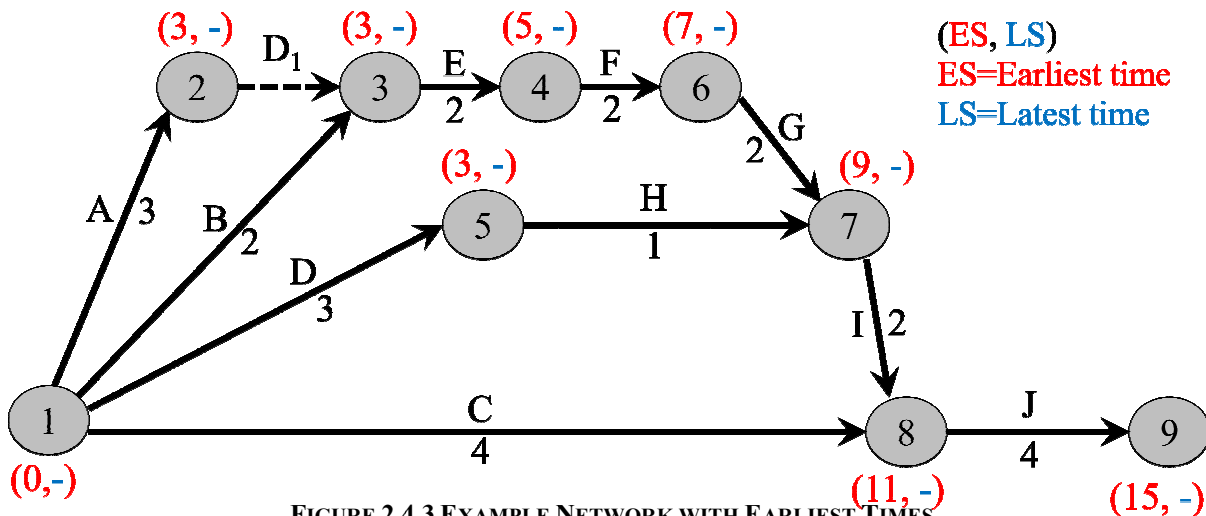
FIGURE 2.4.3 EXAMPLE NETWORK WITH EARLIEST TIMES

The results have given us $E_9=15$

Now we can compute the Latest Occurrence Times, L_j, using a Backwards Pass. We will set $L_n= E_n$ (with n being the completion node, the last node). So $L_9= E_9=15$.

FIGURE 2.4.4 has the complete Backwards pass of our example.

FIGURE 2.4.4 EXAMPLE NETWORK WITH LATEST TIMES

Now we can compute the slack times, S_{ij}, for each activity. To

do this we will use the formula $S_{ij}=L_j-E_i-D_{ij}$. So for example we

will calculate

    S_{18} (Slack time for activity C)= $L_8-E_1-D_{18}=11-0-4=7$

So $S_{18}=7$, which means that activity C can be delayed 7 days and

it will not make the activity late shown in FIGURE 2.4.5.

FIGURE 2.4.5 EXAMPLE NETWORK WITH SLACK TIMES

Once we compute the slack times for all activities we can now

identify the critical activities, which are all activities with

a slack time=0. The critical path is identified in red in

FIGURE 2.4.6.
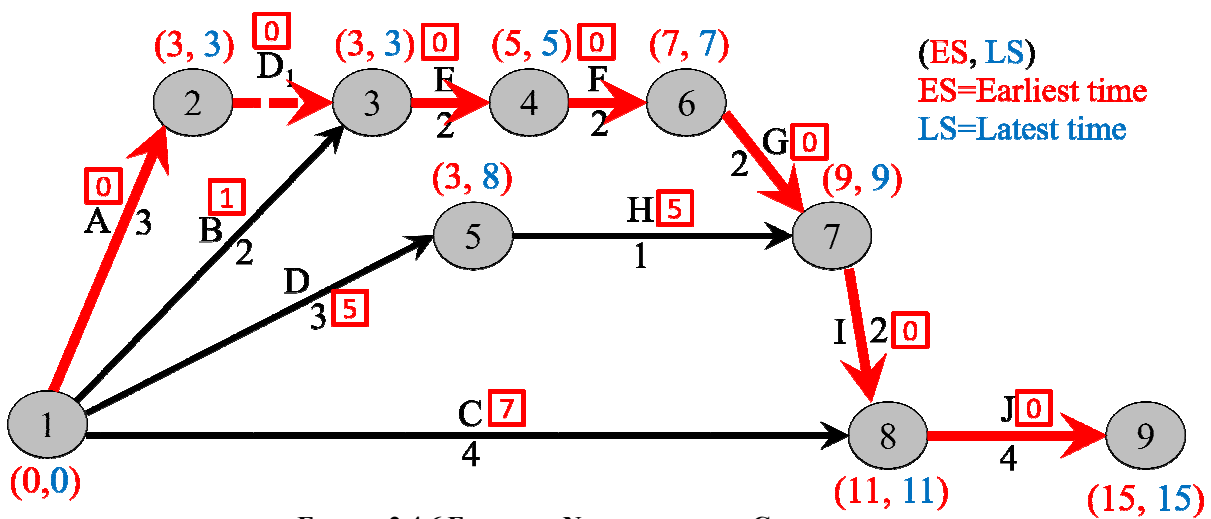
FIGURE 2.4.6 EXAMPLE NETWORK WITH CRITICAL PATH

We know have some very valuable information.

The **critical path** is (1,2)→(2,3)→(3,4)→(4,6)→(6,7)→(7,8)→(8,9).

Activities that can run late (activities with a slack time   0)

are activities B, D, H, C

The completion time of our project=15 days.

PERT METHOD

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Using the PERT method our activity chart would need three deterministic time for duration of the project shown in FIGURE 2.4.7.

ACTIVITY	DESCRIPTION	PREDECESSOR(S)	DURATION (a, m, b) in days
A	Manuscript proofreading by editor	-	2, 3, 4
B	Sample pages preparation	-	1, 2, 3
C	Book cover design	-	3, 4, 5
D	Artwork preparation	-	2, 3, 4
E	Author's approval of A, B	A, B	1, 2, 3
F	Book formatting	E	3, 4, 5
G	Author's review of formatted pages	F	1, 2. 3
H	Author's review of artwork	D	1, 1, 1
I	Production of printing plates	G, H	1, 2, 3
J	Book production and binding	C, I	3, 4, 5

FIGURE 2.4.7 EXAMPLE PERT ACTIVITY TABLE

Notice that for activity H, Author's review of artwork, the duration is 1 for optimistic, most likely, and pessimistic times. This implies an activity that will always take 1 day regardless of the three estimates. It should also be obvious that the μ, the mean, will equal 1, and σ^2, the variance, will equal 0.

For simplicity we used the duration time from the CPM activity chart as the most likely time, and, with the exception of H, we added one day for the pessimistic time, and likewise, subtracted one day for the optimistic time. So when we

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calculate the mean using our formula $\mu = (\frac{a+4m+b}{6})$ for each activity

we will get the most likely time for that activity as the

result.

   We can now calculate the standard deviation and variance

using the formula for standard deviation $\sigma = (\frac{b-a}{6})$ and the variance

being σ^2. We will calculate the σ_{13} and Var_{13} for activity B as

followed: $\sigma_{13} = \frac{3-1}{6} = \frac{1}{3} = .333$ and $Var_{13} = (\sigma_{13})^2 \approx .11$

The following table has all the computations:

ACTIVITY	i-j	MEAN	VARIANCE
A	1-2	3.00	0.11
B	1-3	2.00	0.11
DUMMY1	2-3	0.00	0.00
C	1-8	4.00	0.11
D	1-5	3.00	0.11
E	3-4	2.00	0.11
F	4-6	4.00	0.11
G	6-7	2.00	0.11
H	5-7	1.00	0.11
I	7-8	2.00	0.11
J	8-9	4.00	0.11

FIGURE 2.4.8 PERT EXAMPLE MEAN AND VARIANCE

The next step is to calculate the Mean of the path and the

standard deviation of the path.

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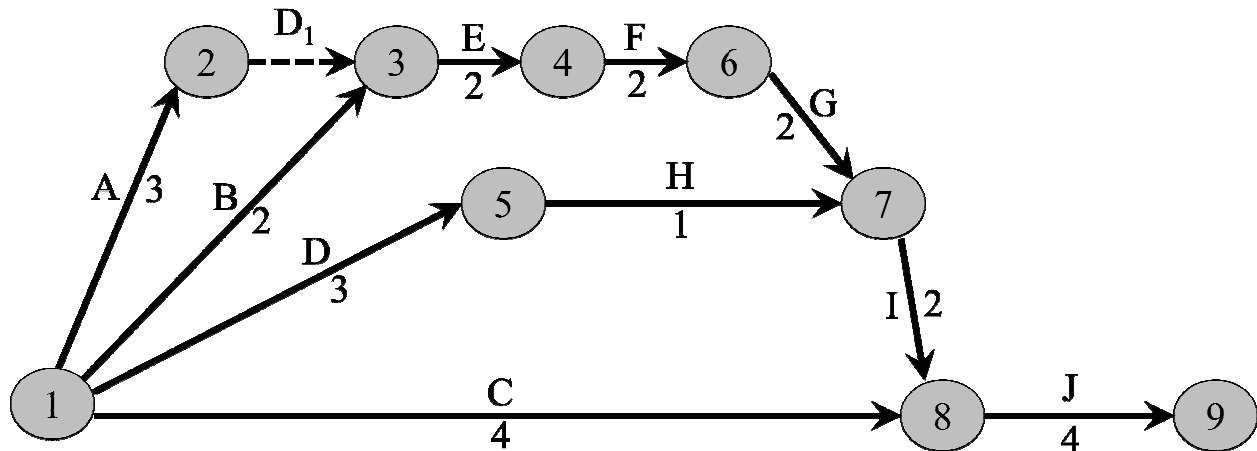
FIGURE 2.4.9 PERT EXAMPLE NETWORK

Mean of the path is simply just adding the the mean of each activity to get to a certain node. For example is we wish to calculate the Mean of the Path to node 4, we would simply add 3+0(Dummy mean)+2=5. Likewise the standard deviation of the path is just a sum of the standard deviation of the path. Figure 2.4.10 should better demonstrate this.

The table is also giving T_{ij} which is the Time in which we would like to know the probability of completing the project in. As noted in the final row, the probability of completing the project in 18 days is 88%. In order to arrive at this, we use the Path Mean and Path Standard Deviation to calculate a Z-score (Z_{ij}) and use the Normal Distribution Table to find our probability.

NODE	LONGEST PATH	PATH MEAN	PATH STANDARD DEVIATION	T_{ij}	z_{ij}	$P\{Z \le Z_{ij}\}$
2		**3.00**	**0.33**	4.00	3.03	99%
3	$1 \to 2 \to 3$	**3.00**	**0.33**	3.00	0.00	50%
4	$1 \to 2 \to 3 \to 4$	**5.00**	**0.47**	7.00	4.26	99%
8	$1 \to 2 \to 3 \to 4 \to 6 \to 7 \to 8$	**13.00**	**0.75**	12.00	-1.33	10%
9	$1 \to 2 \to 3 \to 4 \to 6 \to 7 \to 8 \to 9$	**17.00**	**0.82**	15.00	-2.44	1%
9	$1 \to 2 \to 3 \to 4 \to 6 \to 7 \to 8 \to 9$	**17.00**	**0.82**	16.00	-1.22	12%
9	$1 \to 2 \to 3 \to 4 \to 6 \to 7 \to 8 \to 9$	**17.00**	**0.82**	17.50	0.61	72%
9	$1 \to 2 \to 3 \to 4 \to 6 \to 7 \to 8 \to 9$	**17.00**	**0.82**	19.00	2.43	99%
9	$1 \to 2 \to 3 \to 4 \to 6 \to 7 \to 8 \to 9$	**17.00**	**0.82**	18.00	1.21	88%

FIGURE 2.4.10 PERT EXAMPLE RESULTS

SECTION 3
MY PROJECT

3.1 CREATING MY PROJECT

When deciding to do this project I wanted to be able to apply a mathematical concept to something I am interested in. I have a desire to one day build my home, and immediately saw how using CPM and PERT could help me in managing such a project.

The core of my project was to create a network from scratch, create the network, use both methods, and analyze my results.

3.2 TASK

CPM
I will first begin using the CPM to do my project. So I had to create fixed durations for my activities which starts at Activity A: Site Work, and finishes upon completion of Activity Y: Move in **FIGURE 3.2.1.** Using the rules described in previous sections I constructed a network shown in **Figure 3.2.1.**

Now that we have a network we will begin by computation the Earliest Occurrence Times (Forward Pass), these calculations can be seen in **FIGURE 3.2.3.** Notice that we get a Project Completion Time of 128 working days.

Once all Earliest Occurrence Times are calculated we can start computing our Latest Occurrence Time, recall that we begin by setting E_{23}=L_{23}=128, then using a backwards pass to calculate all Latest Occurrence Times (FIGURE).

Slack times are calculated using both the Earliest Occurrence Time and Latest Occurrence Times. The slack time computations tell us which activities are critical—have a slack time of zero, and which activities are non-critical—have a slack time greater than zero. The critical path is a path on the network from the start node to finish node, which contain slack times of zero. Note that there can be more than one pass, and a critical path can contain a dummy activity.

ACTIVITY	DESCRIPTION	DURATION (WEEKS)	PREDECESSORS
A	Site Work	7	–
B	Foundation	30	A
C	Framing	22	B
E	Plumbing Rough-in	10	C
F	Roofing	4	C
G	Install Exterior Windows and Doors	2	F
H	Electric Rough-in	8	G
I	H.V.A.C.	8	G
J	Siding	5	G
K	Insulation	2	I, E, H
L	Drywall	10	K
M	Paint Exterior (Caulk/Prime/Paint)	3	J
N	Paint Interior Drywall/Stain Trim	5	L
O	Install Interior Trim	7	N
P	Install Tile, Carpet, Hardwoods	10	O
Q	Install Fixtures / Connect Appliances	6	P
R	Install Kitchen Cabinets/countertops	4	P
S	Exterior Landscaping	7	M
T	Electrical Final Trim	4	R
U	Hardware (cabinets, doors)	6	R
V	Walk through/final touch ups	4	S, Q, U, T
W	Cleaning	2	V
X	Final Walk-through	1	W
Y	Move-in	4	X

FIGURE 3.2.1 PROJECT CPM NETWORK

FIGURE 3.2.1 PROJECT CPM NETWORK

FIGURE 3.2.3 PROJECT CPM COMPLETE COMPUTATIONS

ACTIVITY	*i-j*	DESCRIPTION	(a, m, b) IN DAYS	PREDECESSORS
A	1-2	Site Work	4, 7, 10	-
B	2-3	Foundation	28, 30, 35	A
C	3-4	Framing	15, 22, 30	B
E	4-5	Plumbing Rough-in	7, 10, 12	C
F	4-6	Roofing	2, 4, 7	C
G	6-7	Install Exterior Windows and Doors	1, 2, 3	F
H	7-5	Electric Rough-in	5, 8, 10	G
I	7-8	H.V.A.C.	5, 8, 9	G
J	7-9	Siding	2, 5, 7	G
K	5-10	Insulation	1, 2, 3	I, E, H
L	10-11	Drywall	7, 10, 14	K
M	9-12	Paint Exterior (Caulk/Prime/Paint)	2, 3, 5	J
N	11-13	Paint Interior Drywall/Stain Trim	3, 5, 7	L
O	13-14	Install Interior Trim	4, 7, 9	N
P	14-15	Install Tile, Carpet, Hardwoods	8, 10, 12	O
Q	15-16	Install Fixtures / Connect Appliances	3, 6, 8	P
R	15-17	Install Kitchen Cabinets/countertops	3, 4, 5	P
S	12-19	Exterior Landscaping	4, 7, 10	M
T	17-18	Electrical Final Trim	2, 4, 6	R
U	17-19	Hardware (cabinets, doors)	3, 6, 8	R
V	19-20	Walk through/final touch ups	2, 4, 6	S, Q, U, T
W	20-21	Cleaning	1, 2, 3	V
X	21-22	Final Walk-through	.5, 1, 2	W
Y	22-23	Move-in	3, 4, 5	X

FIGURE 3.2.4 PROJECT PERT ACTIVITY TABLE

ACTIVITY	i-j	MEAN	VARIANCE
A	1-2	7	1
B	2-3	30.5	1.36
C	3-4	22.17	6.25
E	4-5	9.83	.69
F	4-6	4.17	.69
G	6-7	2.00	.11
H	7-5	7..83	.69
I	7-8	7.67	.44
J	7-9	4.83	.69
K	5-10	2.00	.11
L	10-11	10.17	1.36
M	9-12	3.17	.25
N	11-13	5.00	.44
O	13-14	6.83	.69
P	14-15	10.00	.44
Q	15-16	5.83	.69
R	15-17	4.00	.11
S	12-19	7.00	1
T	17-18	4.00	.44
U	17-19	5.83	.69
V	19-20	4.00	.44
W	20-21	2.00	.11
X	21-22	1.00	0
Y	22-23	4.00	.11

FIGURE 3.2.5 PROJECT PERT MEAN AND VARIANCE TABLE

NODE	LONGEST PATH	‾PATH MEAN	PATH STANDARD DEVIATION	T_{ij}	Z_{ij}	$P\{z \le z_{ij}\}$	%
1		0.00	0.00				
2	1→2	7.00	1.00	7.00	0.00	0.5000	50%
3	1→2→3	37.50	1.54	40.00	1.62	0.9474	95%
4	1→2→3→4	59.67	2.93	50.00	-3.30	0.0005	0%
5	1→2→3→4→6→7→5	73.67	3.18	85.00	3.56	0.9998	100%
16	1→2→3→4→6→7→5→10→11→13→14→15→16	113.50	3.72	115.00	0.40	0.6664	66%
17	1→2→3→4→6→7→5→10→11→13→14→15→17	111.67	3.64	108.00	-1.00	0.1587	15%
18	1→2→3→4→6→7→5→10→11→13→14→15→17→18	115.67	3.70	113.00	-2.67	0.0038	0%
19	1→2→3→4→6→7→5→10→11→13→14→15→17→19	117.50	3.74	116.00	-1.50	0.0668	6%
20	1→2→3→4→6→7→5→10→11→13→14→15→17→19→20	121.50	3.80	122.00	0.50	0.6915	69%
21	1→2→3→4→6→7→5→10→11→13→14→15→17→19→20→21	123.50	3.81	125.00	0.39	0.6517	65%
22	1→2→3→4→6→7→5→10→11→13→14→15→17→19→20→21→22	124.50	3.81	120.00	-1.18	0.1190	12%
23	1→2→3→4→6→7→5→10→11→13→14→15→17→19→20→21→22→23	128.50	3.83	125.00	-0.91	0.1841	18%
23	1→2→3→4→6→7→5→10→11→13→14→15→17→19→20→21→22→23	128.50	3.83	127.00	-0.39	0.3483	34%
23	1→2→3→4→6→7→5→10→11→13→14→15→17→19→20→21→22→23	128.50	3.83	129.00	0.13	0.5517	55%
23	1→2→3→4→6→7→5→10→11→13→14→15→17→19→20→21→22→23	128.50	3.83	131.00	0.65	0.7422	74%
23	1→2→3→4→6→7→5→10→11→13→14→15→17→19→20→21→22→23	128.50	3.83	132.00	0.91	0.8186	82%
23	1→2→3→4→6→7→5→10→11→13→14→15→17→19→20→21→22→23	128.50	3.83	133.00	4.31	0.9997	100%

FIGURE 3.2.6 PROJECT PERT COMPUTATIONS

PERT

As shown in FIGURE 2.3.4 we first started with the Activity Table showing a, m, and b. In the following table FIGURE 3.2.5 we began calculated the mean and variance and creating a table to better organize our computations.

Once the mean and variance table was constructed (FIGURE 3.2.5) we could then calculate the Path Mean and Path Standard Deviation.

Looking at FIGURE 3.2.6, the most important computation is the Path Mean to reach our final node, completing the entire project (Notice that our CPM calculation gave us a completion time of 128 days, and our PERT calculation gives us a completion time of 128.50 days).

The table also tells us the probability that we complete the project by a specific time frame T_{ij}. This is one of the advantages to using the PERT Method.

4.1 Compare and Contrast

The PERT Method seems to work better for projects that the project manager is not familiar with. Using my example, say I decide to want to add built-in customized solid wood cabinets. I might know how to make them, but have only done it on a small scale, so PERT would compute better results. While doing this project I also realized some other advantages of the PERT method not visible in my report. If a general contractor needed to build a home for a customer, assuming he has built a home numerous times, he can use previous project times to compute a completion time for the customer using CPM, but what if the customer threw in a stipulation? Let's say the customer wanted the home built by a specific date? The contractor can use the PERT method and determine optimistic and pessimistic times to take into consideration weather conditions for the time frame of project. If the contractor usually uses a 6 man crew in constructing the home, PERT would calculate the probability of finishing the home in the requested time, and help determine how big a crew he needs, and a very important key to the success of his project, how much he will have to charge for the additional crew.

One advantage to CPM is the slack time calculations. For activity M (Painting exterior of home), let's say it rains for a week straight, or it is too cold to paint (because paint will not dry) then we know that activity M can take an extra 37 days to be completed, and we will still be on target to complete project as planned. CPM will help project managers identify activities that will delay a project from being completed on time. In building a home, the contractor can pay extra attention to critical activities and if they seem to be running behind, he can act quickly by adding workers, to minimize losses. Instead of causing a critical activity to be late (possibly increasing cost of other critical activities) he can minimize it by only having to increase the cost of that activity.

CONCLUSIONS

In conclusion if you as a project manager are working with a project for the first time it would probably be best to use PERT. After using the method and getting more consistent completion times for activities, they at that point can become the duration of those activities using CPM. While both methods were invented separate from each other I observed a definite link between the two.

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