Consolidation

Based on part of the GeotechniCAL reference package
by Prof. John Atkinson, City University, London

When soil is loaded undrained, the pore pressures increase. Then, under site conditions, the excess pore pressures dissipate and water leaves the soil, resulting in **consolidation** settlement. This process takes time, and the rate of settlement decreases over time.

The amount of settlement which occurs in a given time depends on the

1. permeability of the soil
2. length of the drainage path
3. compressibility of the soil

If soil is unloaded (e.g. by excavation) the excess pore pressures may be negative.

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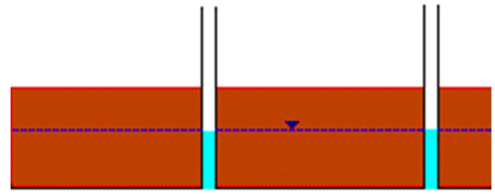
The process of consolidation and settlement

In coarse soils (sands and gravels) any volume change resulting from a change in loading occurs immediately; increases in pore pressures are dissipated rapidly due to high permeability. This is called **drained** loading.
In fine soils (silts and clays) - with low permeabilities - the soil is **undrained** as the load is applied. Slow seepage occurs and the excess pore pressures dissipate slowly, consolidation settlement occurs.
The rate of volume change diminishes with time; about one-half of the total consolidation settlement occurs in one-tenth of the total time.

Back to The process of consolidation and settlement

The basic consolidation process and terminology

Consider a site on clay soil with initial steady-state groundwater conditions. An embankment is built, the loading is undrained: the pore pressure in the soil increases, seepage flow and therefore volume changes commences. As consolidation takes place, settlement occurs, and continues at a decreasing rate until steady-state conditions are regained.



Click on the buttons to see the sequence of loading and pore pressure changes.

Terms and symbols

Seepage refers to the flow of groundwater in a saturated soil.

q = rate of seepage flow

Excess pore pressure ($\bar{u}$)

is the difference between the current pore pressure (u) and the steady state pore pressure (u_o).

$\bar{u} = u - u_o$

Hydraulic gradient (i)

is the difference in total head between two points in the soil.

Permeability or the coefficient of permeability (k)

relates to flow in a given direction, i.e. along a given drainage path.

Back to The process of consolidation and settlement

One-dimensional consolidation

A general theory for consolidation, incorporating three-dimensional flow vectors is complicated and only applicable to a very limited range of problems in geotechnical engineering. For the vast majority of practical settlement problems, it is sufficient to consider that both seepage and strains take place in one direction only; this usually being vertical.

One-dimensional consolidation specifically occurs when there is no lateral strain, e.g. in theoedometer test

One-dimensional consolidation can be assumed to be occurring under wide foundations.

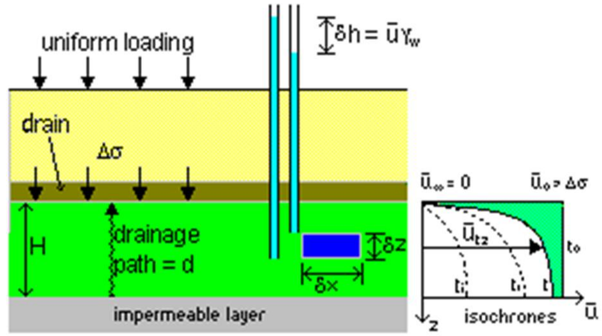
Back to Consolidation

One-dimensional consolidation theory

- Mathematical model and equation
- Isochrones

- Terzaghi's solution
- Solution using parabolic isochrones

A simple one-dimensional consolidation model consists of rectilinear element of soil subject to vertical changes in loading and through which vertical (only) seepage flow is taking place.



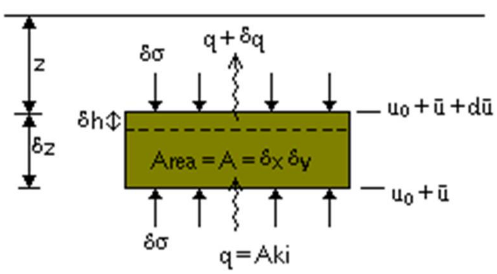
There are three variables:

1. the excess pore pressure ($\bar{u}$)
2. the depth of the element in the layer (z)
3. the time elapsed since application of the loading (t)

- The total stress on the element is assumed to remain constant.
- The coefficient of volume compressibility (m_v) is assumed to be constant.
- The coefficient of permeability (k) for vertical flow is assumed to be constant.

Back to One-dimensional consolidation theory

Mathematical model and equation



Consider the element of consolidating soil. In time δτ:
· the seepage flow is δθ
(q = A k i = A k δη/δζ)
· the change in excess pressure is
$\delta\bar{u} = \delta h\, \gamma_w$
· the thickness changes by
$\delta H = -m_v\, \delta\zeta\delta\sigma'$
It can be shown that the basic equation for one-dimensional consolidation is:

$$\frac{k}{m_v \gamma_w} \frac{\partial^2 \bar{u}}{\partial z^2} = \frac{\partial \bar{u}}{\partial t}$$

By defining the **coefficient of consolidation** as

$$c_v = \frac{k}{m_v \gamma_w}$$

this can be written:

$$c_v \frac{\partial^2 \bar{u}}{\partial z^2} = \frac{\partial \bar{u}}{\partial t}$$

Back to One-dimensional consolidation theory

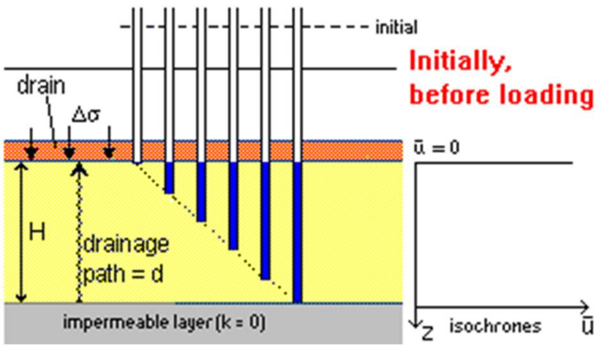
Isochrones

- Properties of isochrones

Solutions to the one-dimensional consolidation equation can be obtained by plotting the variation of $\bar{u}$with the depth in the layer at given elapsed times. The resulting curves are called **isochrones.** (Gk. *iso* = equal; *kronos* = time)

The figure shows a set of supposed standpipes inserted into a consolidating layer.
Before loading, the pore pressure in the drain is zero. At the base of each standpipe there is some initial pore pressure u= u_o, the excess pore pressure $\bar{u}$= 0.

Immediately after the loading is applied the standpipes will each show an initial excess pore pressure of $\bar{u}_i$, thereafter the excess pore pressure will dissipate.



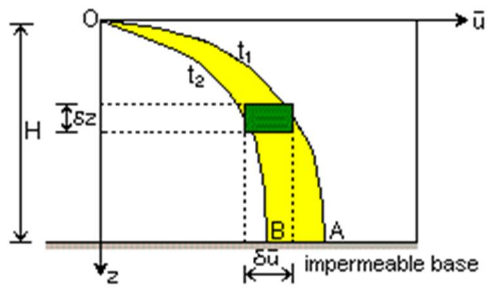
Click on the following time intervals to observe the changes in $\bar{u}$across the thickness of the layer with time.

1. Before loading $\bar{u}$= 0
2. Initial (after loading) when time = 0 $\bar{u}$= Δσ
3. 0 < time < t_c
4. time = t_c *(still no change at the bottom)*
5. t_c < time < t_∞
6. Finally at time = ∞

Adjacent to the drain (at the top) the excess pore pressure drops to zero almost immediately
At the bottom of the layer the dissipation is quite slow.

Back to Isochrones

Some properties of isochrones



The **gradient** of an isochrone is related to the hydraulic gradient (i):

$$\frac{\partial \bar{u}}{\partial z} = -\gamma_w i$$

At the drainage surface, isochrones are steepest and $\bar{u} = 0$.
At the impermeable (k = 0) base the seepage velocity is zero since V = ki; the isochrones will therefore be at 90° to the impermeable boundary.
Between two isochrones the change in thickness in time $\delta\tau$, i.e. $(t_2 - t_1)$, is $\delta H = -m_v\, \delta\zeta\, \delta\bar{u}$,
where $\delta z.\delta\bar{u}$ is the shaded area.
Thus, the **settlement** at the surface of the layer is given by:
$\rho = \Delta H = m'_v$ area OAB

Back to One-dimensional consolidation theory

Terzaghi's solution

- General solution
- Drainage path length

The basic equation is

$$c_v \frac{\partial^2 \bar{u}}{\partial z^2} = \frac{\partial \bar{u}}{\partial t}$$

$\bar{u}(z,t)$ is excess pore pressure at depth z after time t.
The solution depends on the boundary conditions:
The general solution is obtained for an overall (average) degree of consolidation using non-dimensional factors.

Back to Terzaghi's solution

General solution

The following non-dimensional factors are used in order to obtain a solution:

· Degree of consolidation at depth z

$$U_z = \frac{\bar{u}_0 - \bar{u}}{\bar{u}_0}$$

· Time factor

$$T_v = \frac{c_v t}{d^2}$$

· Drainage path ratio

$$Z = \frac{z}{d}$$

The differential equation can now be written as:

$$\frac{\partial^2 U_z}{\partial Z^2} = \frac{\partial U_z}{\partial T_v}$$

If the excess pore pressure is uniform with depth, the solution is:

$$U_z = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M} \sin(MZ) \exp(-2M^2 T_v)$$

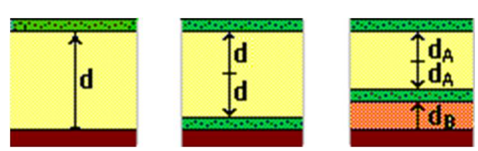
$$\text{where } M = \frac{\pi}{2}(2m+1)$$

Putting $U_t = \rho_t/\rho$¥ = average degree of consolidation in the layer at time t:

$$U_t = 1 - \sum_{m=0}^{m=\infty} \frac{2}{M^2} \exp(M^2 T_v)$$

Back to Terzaghi's solution

Drainage path length



During consolidation water escapes from the soil to the surface or to a permeable sub-surface layer above or below (where $\bar{u} = 0$). The rate of consolidation depends on the longest path taken by a drop of water. The length of this

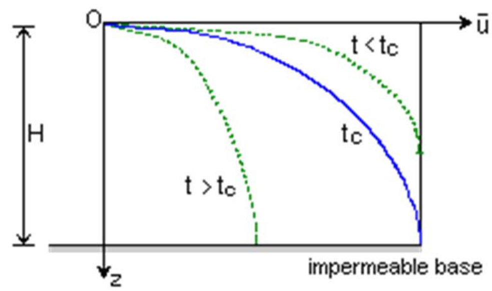
longest path is the **drainage path length, d**. Typical cases are:
An open layer, a permeable layer **both** above and below (d = H/2)
A half-closed layer, a permeable layer **either** above or below (d = H)
Vertical sand drains, horizontal drainage (d = L/2)

Back to One-dimensional consolidation theory

Solution using parabolic isochrones

- Solution for $t < t_c$ case
- Solution for $t > t_c$ case

Isochrones can be approximated to parabolas, affording reasonably accurate solutions to the differential equation for one-dimensional consolidation. Solutions must be obtained for two separate, but adjoining, cases:
· When the elapsed time (t) is less than the **critical time (t_c)**
· When the elapsed time (t) is greater than the **critical time (t_c)**



The **critical time** is the time that must elapse before the excess pore pressures at the impermeable base first begin to dissipate.

Back to Solution using parabolic isochrones

Solution for t < tc case

Putting time factor

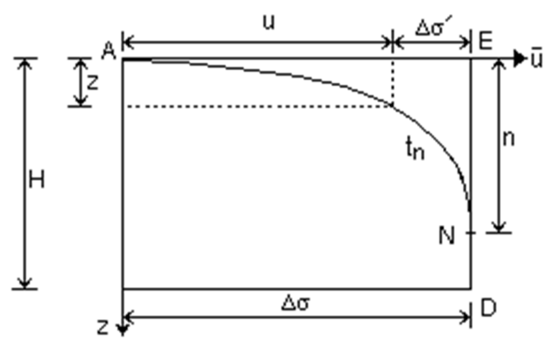
$$T_v = \frac{c_v t}{d^2}$$

and average degree of consolidation,

$$U_t = \frac{\Delta\rho_t}{\Delta\rho_\infty}$$

the general solution is

$$U_t = \frac{2}{\sqrt{3}}\sqrt{T_v}$$



This is valid for 0 < t < tc
At t = t_c, n = H = $\sqrt{12c_v t}$
Giving

$$T_v = \frac{1}{12}$$

and

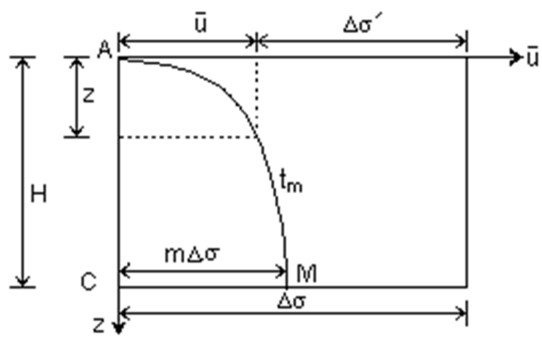
U_t = 0.3333

Back to Solution using parabolic isochrones

Solution for t > tc case

Putting time factor

$$T_v = \frac{c_v t}{d^2}$$



and average degree of consolidation,

$$U_t = \frac{\Delta\rho_t}{\Delta\rho_\infty}$$

the general solution is

$$U_t = 1 - \frac{2}{3}\exp\left(\frac{1}{4} - 3T_v\right)$$

This is valid for t_c < t < t¥
At t = t_c, n = H = $\sqrt{12c_v t}$
Giving

$$T_v = \frac{1}{12}$$

and

U_t = 0.3333

Back to Consolidation

The oedometer test

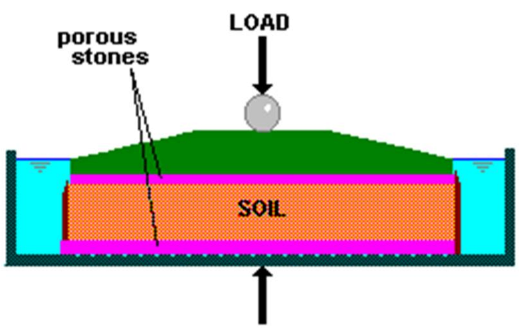
- Apparatus and procedure

The one-dimensional compression and swelling characteristics of a soil may be measured in the laboratory using the **oedometer test** (from the Greek: *oidema* = a swelling).
A cylindrical specimen of soil enclosed in a metal ring is subjected to a series of increasing static loads, while changes in thickness are recorded against time.
From the changes in thickness at the end of each load stage the **compressibility** of the soil may be observed, and parameters measured such as Compression Index (C_c) and Coefficient of Volume Compressibility (m_v).
From the changes in thickness recorded against time during a load stage the **rate** of consolidation may be observed and the **coefficient of consolidation (c_v)** measured.
The test is fully detailed in BS 1377.

Back to The oedometer test

Apparatus and procedure

The saturated specimen is usually 75 mm diameter and 15-20 mm thick, enclosed in a circular metal ring and sandwiched between porous stones.



Vertical static load increments are applied at regular time intervals (e.g. 12, 24, 48 hr.). The load is doubled with each increment up to the required maximum (e.g. 25, 50, 100, 200, 400, 800 kPa). During each load stage thickness changes are recorded against time.

After full consolidation is reached under the final load, the loads are removed (in one or several stages - to a low nominal value close to zero) and the specimen allowed to swell, after which the specimen is removed and its thickness and water content determined. With a porous stone both above and below the soil specimen the drainage will be *two-way* (i.e. an open layer in which the drainage path length, d = H/2)

Back to Consolidation

Determination of c_v from test results

- The Root-Time method
- The Log-Time method

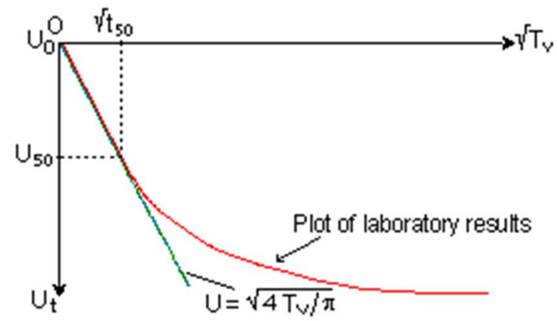
The recorded thickness changes during one of the load stages in an oedometer test are used to evaluate the **coefficient of consolidation (c_v)**.
The procedure involves plotting thickness changes (i.e. settlement) against a suitable function of time [either Ötime or log(time)] and then fitting to this the theoretical T_v:U_t curve.
In this way known intercepts of T_v:U_t are located from which c_v may be calculated.

Back to Determination of c_v from test results

The Root-Time method

- Curve fitting based on Terzaghi's equation
- Curve fitting based on parabolic isochrones

The first portion of the curve of settlement against Ötime is approximately a straight line. The U_0 (U_t = 0) point is located at the intercept with the U_t axis. A second point is required: suppose this is U_{90}/Öt_{90} (point C). The location of this point depends on the equation for the curved portion [See curve fitting methods: Terzaghi or parabolic isochrones]. Once U_{90} has been located other values follow since the U_t axis scale is linear. The coefficient of consolidation is therefore:



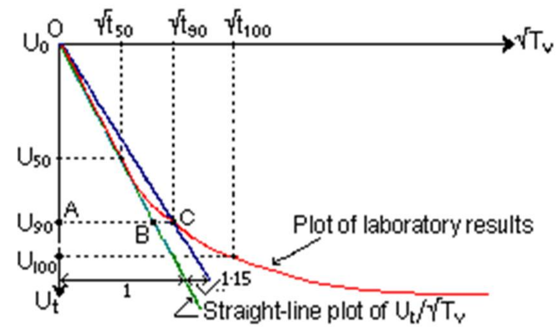
$$c_v = \frac{T_{90}\, d^2}{t_{90}}$$

where d = drainage path length
[d = H for one-way drainage, d = H/2 for two-way drainage]
Other appropriate time-interval values could be used:
e.g. U_{50}, ÖT_{50}, Öt_{50} , etc.

Back to The Root-Time method

Curve fitting based on Terzaghi's equation

From Terzaghi's analysis, the straight-line portion is:
For 0 < Ut < 0.6,



$$U_t = \sqrt{\frac{4T_v}{\pi}}$$

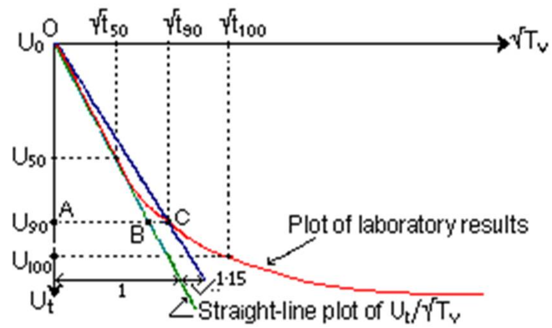
On the straight line:
ÖT_{90} = AB = 0.9 x Ö(π/4) = 0.7976
On the curved portion:
ÖT_{90} = AC = Ö0.848 = 0.9209
Thus, a line drawn through points O and C has abscissae 1.15 times greater than those of the straight line (OB). [0.9209/0.7976 = 1.15]
After the laboratory results curve has been plotted, line OB is drawn, followed by line OC: this crosses the laboratory curve at point (ÖT_{90},U_{90}) and locates Öt_{90}

The coefficient of consolidation is therefore:

$$c_v = \frac{T_{90}\, d^2}{t_{90}} = \frac{0.848\, d^2}{t_{90}}$$

Back to The Root-Time method

Curve fitting based on parabolic isochrones



From the parabola equation the straight-line portion is:
For 0 < Ut < 0.333,

$$U_t = \sqrt{\frac{4T_v}{3}}$$

On the straight line:
ÖT_{90} = AB = 0.9 x Ö(3/4) = 0.7794
On the curved portion:
ÖT_{90} = AC = Ö0.716 = 0.8462
Thus, a line drawn through points O and C has abscissae 1.086 times greater than those of the straight line (OB). [0.8462/0.7794 = 1.086]
After the laboratory results curve has been plotted, line OB is drawn, followed by line OC: this crosses the laboratory curve at point (ÖT_{90},U_{90}) and locates Öt_{90}

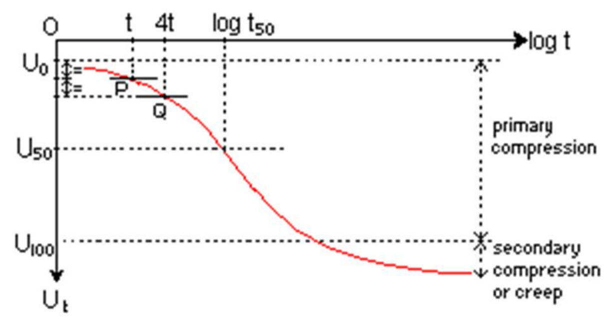
The coefficient of consolidation is therefore:

$$c_v = \frac{T_{90}\, d^2}{t_{90}} = \frac{0.716\, d^2}{t_{90}}$$

Back to Determination of c_v from test results

The Log-Time method

An alternative to the Root-Time method, that is particularly useful when there is significant secondary compression (creep). The U_o point is located by selected two points on the curve for which the times (t) are in the ratio 1:4, e.g. 1 min and 4 min; or 2 min and 8 min.; the vertical intervals AP and PQ will be equal.



The U_{100} point can be located in the final part of the curve flattens sufficiently (i.e. no secondary compression). When there is significant secondary compression, U_{100} may be located at the intercept of straight line drawn through the middle and final portions of the curve.
Now U_{50} and log t_{50} can be located.
The coefficient of consolidation is therefore:

$$c_v = \frac{T_{50}\, d^2}{t_{50}} = \frac{0.196\, d^2}{t_{50}}$$

Back to Consolidation

Calculation of settlement times

- Prediction of time for given settlement
- Prediction of settlement amount at given time

After the coefficient of consolidation (c_v) has been determined from laboratory data calculations are possible for site settlements. It is important to note that c_v is *not a constant*, but varies with both the level of stress and degree of consolidation. For practical site settlement calculations, however, it is sufficiently accurate to measure c_v relative to the loading range applicable on site and then assume this value to be approximately constant for all degrees of consolidation (except for very low values).
The basic equation used is:

$$c_v = \frac{T_v\, d^2}{t}$$

where d = drainage path length
[d = H for one-way drainage, d = H/2 for two-way drainage]
T_v and t are coupled to a given degree of consolidation

Back to Calculation of settlement times

Prediction of time for given settlement

Example

The final consolidation settlement of a layer of clay 5.0 m thick is calculated to be 280mm. The coefficient of consolidation for the loading range is 0.955 mm^2/min. There is two-way drainage, upward and downward. Calculate the time required for (a) 90% consolidation settlement, (b) a settlement of 100 mm.

(a) Drainage path length, d = 5.0/2 = 2.50 m = 2500 mm
For U_{90}, $T_{90} = 0.848$. Then

$$t_{90} = \frac{0.848 \times 2500^2}{0.955} = 5.55 \times 10^6 \text{ min} = 10.55 \text{ yr}$$

(b) For 100 mm settlement, $U_t = 100/280 = 0.357$
and since $U_t < 0.6$, $T_v = 0.357^2 \times \pi/4 = 0.100$
Then time for 100mm settlement

$$= \frac{0.100 \times 2500^2}{0.955} = 0.654 \times 10^6 \text{ min} = 1.24 \text{ yr}$$

Back to Calculation of settlement times

Prediction of settlement amount at given time

Example

A layer of clay has a thickness of 4.0 m and drains both upward and downward. A laboratory test has yielded a coefficient of consolidation for the appropriate loading range of 0.675 mm^2/min. The final consolidation settlement has been calculated to be 120mm. Provide estimates of the consolidation settlement that may be expected 1yr, 2yr, 5yr and 10yr after construction.

Drainage path length, d = 2000 mm
When $U_t < 0.6$, use $U_t = Ö(4T_v/\pi)$
When $U_t > 0.6$,

$$\text{use } = U_t = 1 - \frac{8}{\pi^2}\left\{\exp\left(-\frac{\pi^2}{4T_v}\right) + \frac{1}{9}\exp\left(-\frac{9\pi^2}{4T_v}\right) + \frac{1}{25}\exp\left(-\frac{25\pi^2}{4T_v}\right) + \ldots\right\}$$

c_v = 0.645 mm²/min = 928.8 mm²/day

time t (years)	time t (days)	$T_v = c_v t/d^2$	Ut (<0.6)	Ut (>0.6)	r_c (mm) at time t
1	365	0.0848	0.328		39
2	730	0.1965	0.465		56
5	1825	0.4238	0.735	0.715	86
10	3650	0.8475		0.900	108
23.6	8613	2.0		0.994	119

Back to Consolidation

Reliability for design purposes

Laboratory measurements of stress-strain parameters (C_c, Cs, m_v) are generally acceptable, provided sampling quality is good, e.g. minimal disturbance, valid representation of strata, maintenance of structure and water content, careful preparation, etc.
Measurements of strain/time relationships (c_v) and permeability (k) are not so reliable.
Observed rates of settlement are generally greater than values based on oedometer test results.
Reliability is compromised by factors such anisotropy (e.g. silt/sand layers, varves, fissures, etc), presence of roots, organic matter and voids, and also the effects of secondary compression.
Loads are not often applied instantaneously, and so due allowance should be for the gradual application of loading.

Back to Consolidation

Secondary compression or creep

- Coefficient of secondary compression
- Overconsolidation due to creep

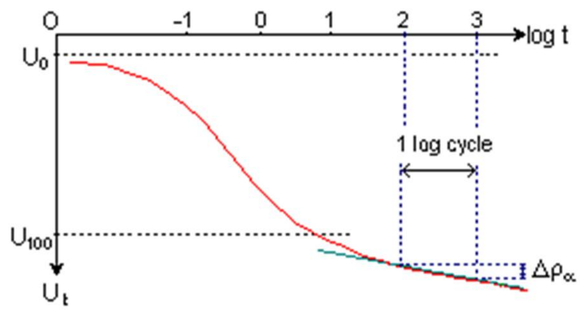
In some soils (especially recent organic soils) one-dimensional compression continues under constant loading after all of the excess pore pressure has dissipated, i.e. after primary consolidation has ceased - this is called **secondary compression** or **creep**.
It is generally thought that creep is due to changes in soil structure, although no reliable theory has been proposed as yet.
It is likely that some creep is occurring due primary consolidation, affecting the linearity of the r/Ötime curve and thus making the accurate prediction of settlement difficult and possibly

unreliable.
For practical purposes, the Log-Time plot (described elsewhere) can be used to estimate a **coefficient of secondary compression ($C\alpha$)**.

Back to [Secondary compression or creep]()

Coefficient of secondary compression

The **amount** of secondary compression is the settlement occurring *after* t_{100}, i.e. after full dissipation of excess pressures
= ρ_a (or s_a).
The ρ/log t curve after t_{100} can be approximated to a straight line, the slope of which gives the **coefficient of secondary compression (C_a)**.
The slope of the laboratory curve is measured over one log-time cycle, e.g.1000 to 10000 mins.



$$C_\alpha = \frac{\Delta\rho}{\Delta \log t}\frac{1}{H}$$

[i.e. change in unit thickness per log cycle]

$$\text{or } C_\alpha = \frac{\Delta e}{\Delta \log t}$$

[i.e. change in void ratio per log cycle]

Back to [Secondary compression or creep]()

Over consolidation due to creep

Creep (secondary compression) is basically similar to compaction, except it takes place slowly.
The result of creep is a change in volume (also water content and void ratio).
The soil is in effect further consolidated, and therefore if unloaded is left *overconsolidated.*
The phenomenon of overconsolidation due to creep is noticeable in soft clays.