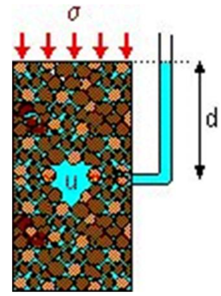
Pore water pressure

- Water table
- Elevation, pressure and total head
- Hydraulic gradient
- Effective stress



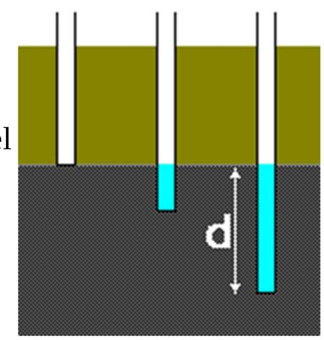
In general, the water in the voids of an element of saturated soil will be under pressure, either due to the physical location of the soil or as a result of external forces. This pressure is the pore water pressure or **pore pressure** u. It is measured relative to atmospheric pressure.

When there is no flow, the pore pressure at depth **d** below the water surface is:
$\mathbf{u} = \gamma_w \mathbf{d}$

Pore water pressure

Water table

- Fine-grained soils
- Coarse-grained soils
- Perched water table



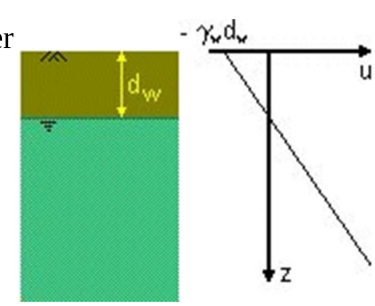
The level in the ground at which the pore pressure is zero (equal to atmospheric) is defined as the **water table** or **phreatic surface**.

When there is no flow, the water surface will be at exactly the same level in any stand pipe placed in the ground below the water table. This is called a **hydrostatic** pressure condition.

The pore pressure at depth **d** below the water table is: : $\mathbf{u} = \gamma_w \mathbf{d}$

Water table

Fine-grained soils



In fine grained soils, surface tension effects can cause capillary water to rise above the water table. It is reasonable to assume that the pore pressure varies linearly with depth, so the pore pressure above the water table will be negative.

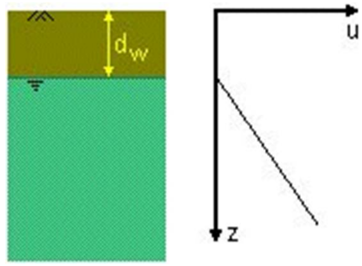
If the water table is at depth d_w then the pore pressure at the ground

surface is $\mathbf{u_o = - \gamma_w . d_w}$
and the pore pressure at depth z is $\mathbf{u = \gamma_w (z - d_w)}$

Where the water table is deeper, or where evaporation is taking place from the surface, saturation with capillary water may not occur. The height to which the soil remains saturated with negative pore pressures above the water table is called the capillary rise.

Water table

Coarse-grained soils



Below the water table the soil can be considered to be saturated. In coarse-grained soils, water will drain from the pores and air will therefore be present in the soil between the ground surface and the water table.

Consequently, pore pressures above the water table can usually be ignored. Below the water table, hydrostatic water pressure increases linearly with depth.

With the water table at depth d_w

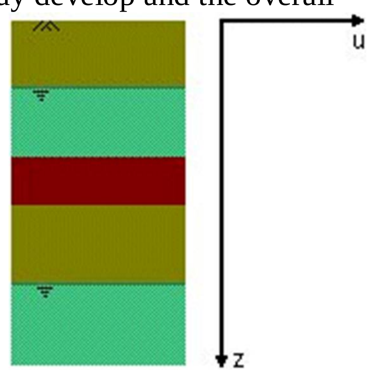
$u = 0$ for $z < d_w$

$u = \gamma_w(z - d_w)$ for $z > d_w$

Water table

Perched water table

Where the ground contains layers of permeable soil (e.g. sands) interspersed with layers of much lower permeability (e.g. clays) one or more **perched water tables** may develop and the overall distribution of pore pressure with depth may not be exclusivelyly linear.



Detection of perched water tables during site investigation is important, otherwise erroneous estimates of *in-situ* pore pressure distributions can arise.

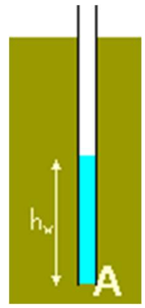
Pore pressure conditions below perched water tables may be affected by local infiltration of rainwater or localised seepage and therefore may not be in hydrostatic equilibrium.

Show the graph of water pressure
Click on a soil layer to display a description

The description appears in the status bar when the mouse is held over a layer.
In Internet Explorer is also appears as a popup label

Pore water pressure

Elevation, pressure and total head

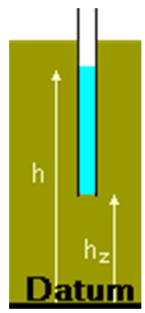


Pore pressure at a given point (e.g. point A in the diagram) can be measured by the height of water in a standpipe located at that point.

Pore pressures are often indicated in this way on diagrams.

The height of the water column is the **pressure head** (h_w)

$$\mathbf{h_w = \frac{u}{\gamma_w}}$$

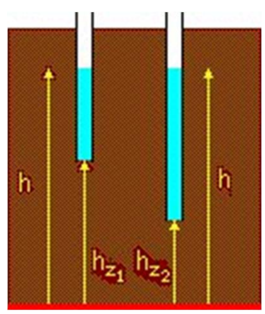


To identify significant differences in pore pressure at different points, we need to eliminate the effect of the points' position. A height datum is required from which locations are measured. The **elevation head** (h_z) of a point is its height above the datum line. The height above the datum of the water level in the standpipe is the **total head** (h).

$$\mathbf{h = h_z + h_w}$$

Pore water pressure

Hydraulic gradient

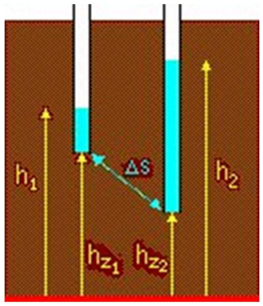


Flow of pore water in soils is driven from positions of higher total head towards positions of lower total head. The level of the datum is arbitrary. It is **differences** in total head that are important. The hydraulic gradient is the rate of change of total head along the direction of flow.

$$\mathbf{i = \Delta h / \Delta s}$$

In each diagram there are two points, a small distance Δs apart, h_{z1} and h_{z2} above datum.
In the first diagram, the total heads are **equal**. The difference in pore pressure is entirely due to the difference in altitude of the two points and the pore water has no tendency to flow.

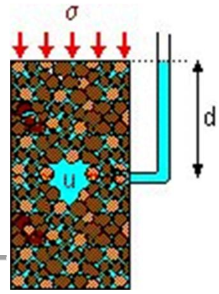
In the second diagram, the total heads are **different**. The hydraulic gradient is
$i = (h_2 - h_1) / \Delta s$
and the pore water tends to flow.



[Pore water pressure](#)

Effective stress

All strength and stress:strain characteristics of soils can be linked to changes in [effective stress](#)



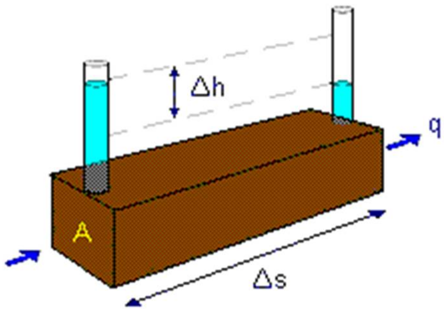
Effective stress (σ') = total stress (σ) - pore water pressure (u)
$\sigma' = \sigma - u$

[Groundwater](#)

Permeability

- [Void ratio](#)
- [Stratified soil](#)
- [Seepage velocity](#)
- [Temperature](#)

Darcy's law
The rate of flow of water q (volume/time) through cross-sectional area A is found to be proportional to hydraulic gradient i according to [Darcy's](#) law:



$$v = \frac{q}{A} = k.i \quad i = \frac{\Delta h}{\Delta s}$$

*where **v** is flow velocity and **k** is coefficient of permeability with dimensions of velocity (length/time).*

The coefficient of permeability of a soil is a measure of the conductance (i.e. the reciprocal of the resistance) that it provides to the flow of water through its pores.

The value of the coefficient of permeability k depends on the average size of the pores and is [related to the distribution of particle sizes](#), particle shape and soil structure. The ratio of permeabilities of typical sands/gravels to those of typical clays is of the order of 10^6. A small proportion of fine material in a coarse-grained soil can lead to a significant reduction in permeability.

Permeability

Void ratio and permeability

Permeability of all soils is strongly influenced by the density of packing of the soil particles which can be simply desrcibed through void ratio e or porosity n.

Sands

For filter sands it is found that $k \approx 0.01 (d_{10})^2$ m/s where d_{10} is the effective particle size in mm. This relationship was proposed by Hazen.

The Kozeny-Carman equation suggests that, for laminar flow in saturated soils:

$$k = \frac{1}{k_0 k_T S_S{}^2} \cdot \frac{e^3}{1+e} \cdot \frac{\gamma_w}{\eta}$$

where k_o and k_T are factors depending on the shape and tortuosity of the pores respectively, S_s is the surface area of the solid particles per unit volume of solid material, and γ_w and η are unit weight and viscosity of the pore water. The equation can be written simply as

$$k = C.\frac{e^3}{1+e} \approx C.e^2$$

Clays

The Kozeny-Carman equation does not work well for silts and clays. For clays it is typically found that

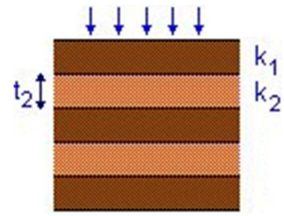
$$\log_{10} k = \frac{e - e_k}{C_k}$$

where C_k is the permeability change index and ek is a reference void ratio.
For many natural clays C_k is approximately equal to half the natural void ratio.

Permeability

Stratified soil and permeability

Consider a stratified soil having horizontal layers of thickness t_1, t_2, t_3, etc. with coefficients of permeability k_1, k_2 k_3, etc.



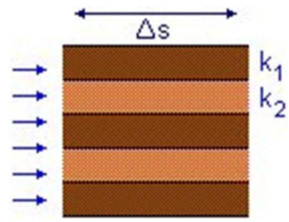
For **vertical flow**, the flow rate q through area A of each layer is the same. Hence the head drop across a series of layers is

$$\Delta h = \frac{qt_1}{Ak_1} + \frac{qt_2}{Ak_2} + \frac{qt_3}{Ak_3}$$

The average coefficient of permeability is

$$k_v = \frac{t_1 + t_2 + t_3}{\frac{t_1}{k_1} + \frac{t_2}{k_2} + \frac{t_3}{k_3}}$$

For **horizontal flow**, the head drop Δh over the same flow path length Δs will be the same for each layer. So $i_1 = i_2 = i_3$ etc. The flow rate through a layered block of soil of breadth B is therefore



$$q = Bt_1k_1i_1 + Bt_2k_2i_2 + Bt_3k_3i_3$$

The average coefficient of permeability is

$$k_h = \frac{t_1k_1 + t_2k_2 + t_3k_3}{t_1 + t_2 + t_3}$$

Permeability

Seepage velocity

Darcy's Law relates flow velocity (v) to hydraulic gradient (i). The volume flow rate q is calculated as the product of flow velocity v and total cross sectional area:
q = v.A

At the particulate level the water follows a tortuous path through the pores. The average velocity at which the water flows through the pores is the ratio of volume flow rate to the average area of voids A_v on a cross section normal to the macroscopic direction of flow. This is the **seepage velocity** v_s

$$v_s = \frac{q}{A_v}$$

Porosity of soil is related to the volume fraction of voids

$$n = \frac{V_v}{V} \approx \frac{A_v}{A} v_s \approx \frac{v}{n}$$

Seepage velocity can be measured in laboratory models by injecting dye into the seeping pore water and timing its progress through the soil.

Permeability

Temperature and permeability

The flow of water through confined spaces is controlled by its viscosity η and the viscosity is controlled by temperature.

An alternative permeability K (dimensions: length^2) is sometimes used as a more absolute coefficient depending only on the characteristics of the soil skeleton.

$$K = \frac{\eta k}{\gamma_w}$$

The values of k at 0°C and 10°C are 56% and 77% respectively of the value measured at 20°C.

Groundwater

Analytical solutions

- Steady one-dimensional flow
- Quasi-one-dimensional and radial flow
- Two-dimensional flow, Laplace
- Transient flow, consolidation

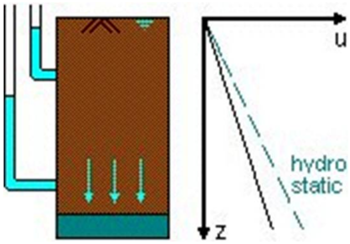
In **steady-state** flow, the pressures and flow rates remain constant over time. In **transient** flow, the pressures and flow rates are time-dependent.

Steady one-dimensional flow is the simplest case, to which Darcy's law can be applied. This can be extended to cases of variable aquifer thickness and radial flow. The analysis of steady two-dimensional flow is more complex and results in flow nets.

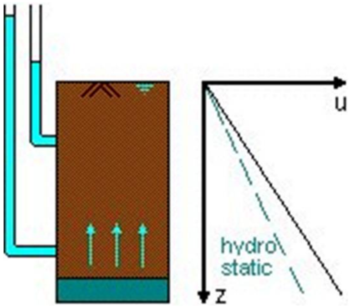
Analytical solutions

Steady one-dimensional flow

[Darcy's Law](#) indicates the link between flow rate and hydraulic gradient. For one-dimensional flow, constant flow rate implies constant hydraulic gradient.



Steady downward flow occurs when water is pumped from an underground aquifer. Pore pressures are then lower than hydrostatic pressures.

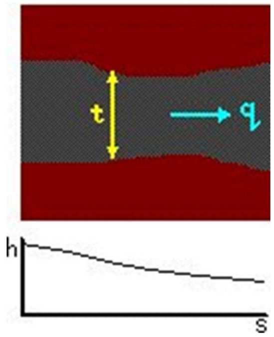


Steady upward flow occurs as a result of artesian pressure when a less permeable layer is underlain by a permeable layer which is connected through the ground to a water source providing pressures higher than local hydrostatic pressures.
The fountains of London were originally driven by artesian pressure in the aquifers trapped beneath the London clay. Pumping from aquifers over the centuries has lowered the water pressures below artesian levels.

[Analytical solutions](#)

Quasi-one-dimensional and radial flow

- [Cylindrical flow: confined aquifer](#)
- [Cylindrical flow: groundwater lowering](#)
- [Spherical flow](#)



Where flow occurs in a confined aquifer whose thickness varies gently with position the flow can be treated as being essentially one-dimensional.
The horizontal flow rate q is constant. For an aquifer of width B and varying thickness t, the discharge velocity

$$v = \frac{q}{Bt}$$

and Darcy's Law indicates that

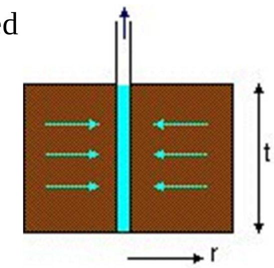
$$i = \frac{dh}{ds} = \frac{q}{Btk}$$

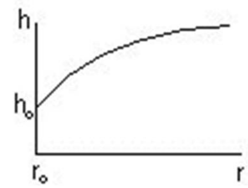
Hydraulic gradient varies inversely with aquifer thickness.

[Quasi-one-dimensional and radial flow](#)

Cylindrical flow: confined aquifer

Steady-state pumping to a well which extends the full thickness of a confined aquifer is a one-dimensional problem which can be analysed in cylindrical coordinates: pore pressure or head varies only with radius r.





Darcy's Law still applies, with hydraulic gradient dh/dr and area A varying with radius: A = 2πr.t

$$\frac{q}{A} = ki = \frac{dh}{dr}$$

$$dh = \frac{q}{2\pi tk}\frac{dr}{r} \qquad h - h_o = \frac{q}{2\pi tk}\ln(r/r_o)$$

where r_o is the radius of the borehole and h_0 the constant head in the borehole.

[Quasi-one-dimensional and radial flow]

Cylindrical flow: groundwater lowering

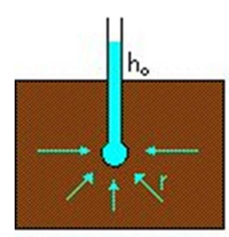
Pumping from a borehole can be used for deliberate groundwater lowering in order to facilitate excavation. This is an example of quasi-one-dimensional radial flow with flow thickness t=h. Then A=2πr.h and

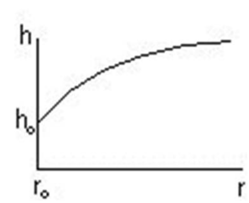
$$h.dh = \frac{q}{2\pi k}\frac{dr}{r} \qquad h^2 - h_o^2 = \frac{q}{\pi k}\ln(r/r_o)$$

[Quasi-one-dimensional and radial flow]

Spherical flow

Variation of pore pressure around a point source or side (for example, a piezometer being used for *in-situ* determination of permeability) is a one-dimensional problem which can be analysed in spherical coordinates: pore pressure or head varies only with radius r.





Darcy's Law still applies, with hydraulic gradient dh/dr and area A varying with radius: $A=4\pi r^2$

$$\frac{q}{A} = ki = \frac{dh}{dr}$$

$$dh = \frac{q}{4\pi k}\frac{dr}{r^2} \qquad h - h_o = \frac{q}{4\pi k}\left(\frac{1}{r_o} - \frac{1}{r}\right)$$

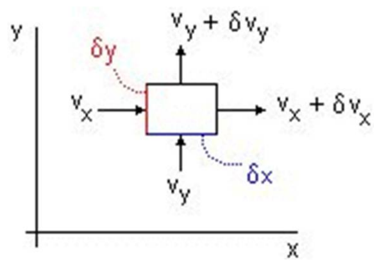
where r_0 is the radius of the piezometer and h_0 the constant head in the piezometer.

[Analytical solutions](#)

Two-dimensional flow, Laplace

- [Anisotropic soil](#)

Two-dimensional steady flow of the incompressible pore fluid is governed by Laplace's equation which indicates simply that any imbalance in flows into and out of an element in the x direction must be compensated by a corresponding opposite imbalance in the y direction. Laplace's equation can be solved graphically, analytically, numerically, or analogically.



For a rectangular element with dimensions δ_x, δ_y and unit thickness, in the x direction the velocity of flow into the element is

$$v_x = -k \frac{\partial h}{\partial x}$$

the negative sign being required because flow occurs **down** the hydraulic gradient. The velocity of flow out of the element is

$$v_x + \delta v_x = -k\left(\frac{\partial h}{\partial x} + \frac{\partial^2 h}{\partial x^2}\delta x\right)$$

Similar expressions can be written for the y direction. Balance of flow requires that

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

and this is Laplace's equation. In three dimensions, Laplace's equation becomes

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

[Two-dimensional flow, Laplace](#)

Anisotropic soil

For a soil with permeability k_x and k_y in the x and y directions respectively, Laplace's equation for two-dimensional seepage becomes

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial_y{}^2} = 0$$

This can be solved by applying a scale factor to the x dimensions so that transformed coordinates xt are used

$$x_t = x\sqrt{\frac{k_y}{k_x}}$$

In the transformed coordinates the equation regains its simple form

$$\frac{\partial^2 h}{\partial x_t{}^2} + \frac{\partial^2 h}{\partial y^2} = 0$$

and flownet generation can proceed as usual. Calculations of flow are made using an equivalent permeability

$$k_t = \sqrt{k_x k_y}$$

It may be preferable in some cases to transform the y coordinates using:

$$y_t = y\sqrt{\frac{k_x}{k_y}}$$

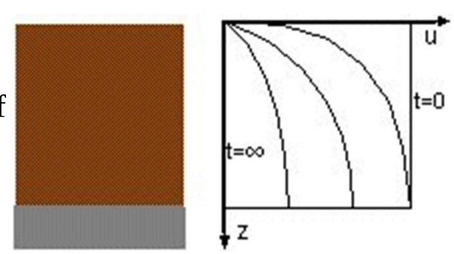
The equivalent permeability remains unchanged.

For many natural sedimentary soils seasonal variations in the depositional regime have resulted in horizontal macroscopic permeabilities significantly greater than vertical permeabilities. Transformation of coordinates lends itself to analysis of seepage in such situations.

Analytical solutions

Transient flow, consolidation

Since water may be regarded as being essentially incompressible, unsteady flow may arise when water is drawn into or expelled from pores as a result of changes in the size of pores. This can only occur as a result of changes volume associated with changes in effective stress.



The time-dependent transient change in pore pressure that occurs as a result of some perturbation, and associated change in effective stress is called **consolidation**.

One-dimensional compression tests in an oedometer define the relationship between vertical effective stress σ'_v and specific volume v or void ratio e from which a one-dimensional compliance mv can be defined

$$m_v = \frac{\delta v}{v \delta \sigma'_v}$$

Then, under conditions of constant total stress, consolidation is governed by a diffusion equation:

$$\frac{\partial u}{\partial t} = c_v \frac{\partial^2 u}{\partial z^2} \quad c_v = \frac{k}{m_v \gamma_w}$$

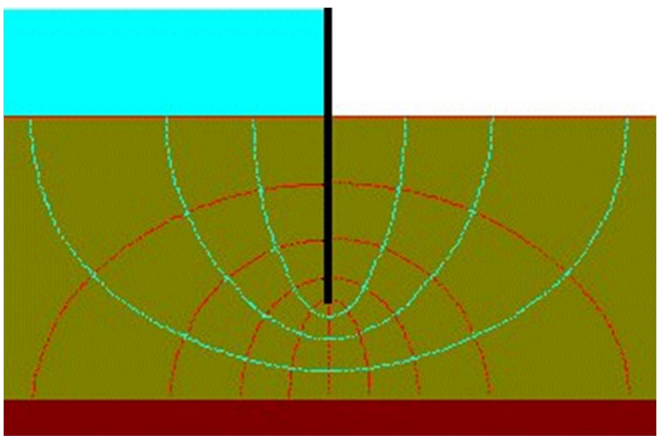
where cv is the coefficient of consolidation having dimensions (length²/time).

Solutions of the consolidation equation are typically presented as **isochrones**, i.e. variations of pore pressure with position at successive times, but can also be converted to curves linking settlement with time.

Groundwater

Flow nets

- Calculation of flow
- Calculation of total flow
- Boundary between layers
- Boundary conditions
- Flow through embankments



Solutions to Laplace's equation for two-dimensional seepage can be presented as flow nets. Two orthogonal sets of curves form a flow net:
equipotentials connecting points of equal total head h
flow lines indicating the direction of seepage down a hydraulic gradient

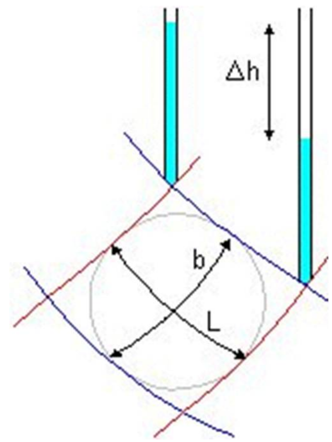
If standpipe piezometers were inserted into the ground with their tips on a single equipotential then the water would rise to the same level in each standpipe. (The pore pressures would be different because of their different elevations.)

There can be no flow along an equipotential, because there is no hydraulic gradient, so there can be no component of flow across a flow line. The flow lines define channels along which the volume flow rate is constant.

Flow nets

Calculation of flow

Consider an element from a flow channel of length L between equipotentials which indicate a fall in total head Δh and between flow lines b apart. The average hydraulic gradient is



$$i = \frac{\Delta h}{L}$$

and for unit width of flow net the volume flow rate is

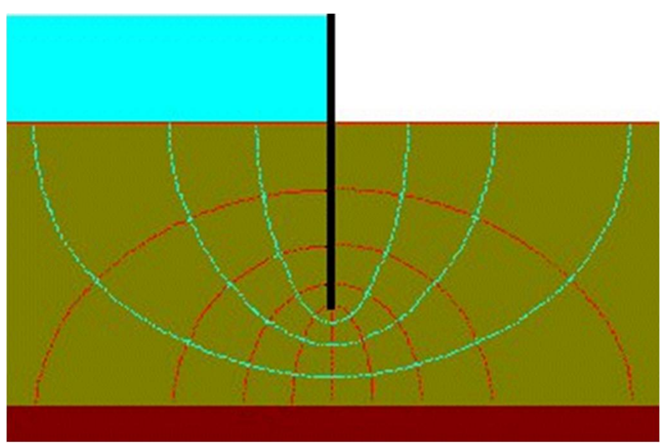
$$q = kb\frac{\Delta h}{L}$$

There is an advantage in displaying or sketching flownets in the form of curvilinear 'squares' so that a circle can be insrcibed within each four-sided figure bounded by two equipotentials and two flow lines. Then b = L and q = kΔh
so the flow rate through the flow channel is the permeability multiplied by the uniform interval Δh between equipotentials.

Flow nets

Calculation of total flow

For a complete problem, the flownet has been drawn with the overall head drop h divided into Nd equal intervals:



$\Delta h = h / N_d$

with N_f flow channels.

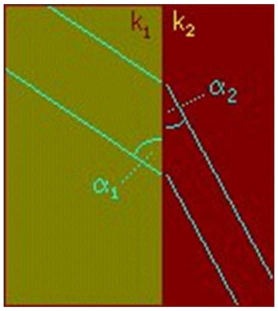
Then the total flow rate per unit width is

$$q = N_f \cdot \frac{k.h}{N_d} = (\frac{N_f}{N_d})kh$$

It is usually convenient in sketching flownets to make Nd an integer. The number of flow channels Nf will then generally not be an integer. In the example shown, of flow under a sheet pile wall
Nd ;= 10, Nf = 3.5 and q = 0.35kh per unit width.

[Flow nets]

Boundary between layers



Flow across a boundary between two layers of soil of different permeability produces a refraction effect.

Consideration of continuity of flow and of continuity of velocity normal to the interface shows that

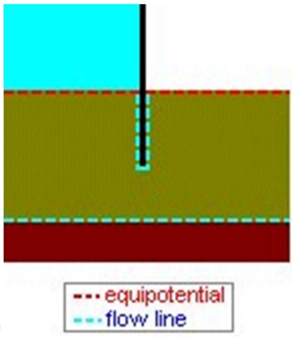
$$\frac{\tan\alpha_1}{\tan\alpha_2} = \frac{k_1}{k_2}$$

It is not possible to construct a flow net with curvilinear squares on both sides of the interface unless the head drop between equipotentials is changed in inverse proportion to the permeability ratio.

If the ratio of permeabilities is greater than about 10, e.g. at the boundary of a drainage layer then construction of the part of the flow net in the more permeable soil is unlikely to be necessary.

[Flow nets]

Boundary conditions



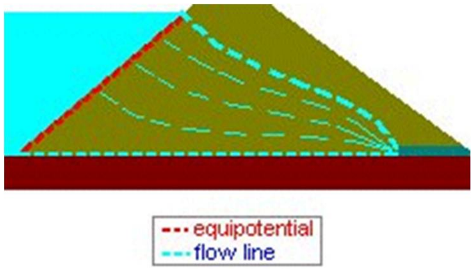
A surface on which the total head is fixed (for example, from the level of a river, pool, reservoir) is an [equipotential]. A surface across which there is no flow (for example, an impermeable soil layer or an impermeable wall) is a [flow line]

For the situation shown, with flow occurring under a sheet pile wall, the axis of symmetry must also be an equipotential.

[Flow nets]

Flow through embankments

Seepage through an embankment dam is an example of unconfined flow bounded at the upper surface by a phreatic surface which represents the top flow line and on which the pore pressure is everywhere zero (atmospheric).



Total head changes and elevation changes thus match and for equal head intervals Δh between equipotentials there must be equal vertical intervals between the points of intersection of equipotentials with the phreatic surface.

Groundwater

Quick condition and piping

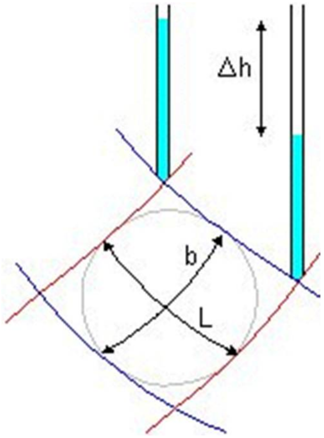
- Seepage force
- Critical hydraulic gradient

If the flow is upward then the water pressure tends to lift the soil element. If the upward water pressure is high enough the effective stresses in the soil disappear, no frictional strength can be mobilised and the soil behaves as a fluid. This is the **quick** or **quicksand** condition and is associated with piping instabilities around excavations and with liquefaction events in or following earthquakes.

Quick condition and piping

Seepage force

The viscous drag of water flowing through a soil imposes a **seepage force** on the soil in the direction of flow.



Consider the actual distribution of pore water pressure around an element length L and thickness b taken from a flownet, bounded by two equipotentials with fall in head Δh, and two flow lines.

These pore water pressures are partly supporting the weight $\gamma_w bL$ of water in the element and partly providing the seepage force. It is found

that the seepage force is
$J = i \gamma_w b L$
equivalent to a seepage pressure (force per unit volume) in the direction of flow
$j = i \gamma w$

Quick condition and piping

Critical hydraulic gradient

The quick condition occurs at a critical upward hydraulic gradient i_c, when the seepage force just balances the buoyant weight of an element of soil. (Shear stresses on the sides of the element are neglected.)

$$i_c \gamma_w V = (\gamma - \gamma_w) V = \gamma' V$$

$$i_c = \frac{\gamma'}{\gamma_w} = \frac{G_s - 1}{v}$$

The critical hydraulic gradient is typically around 1.0 for many soils. Fluidised beds in chemical engineering systems rely on deliberate generation of quick conditions to ensure that the chemical process can occur most efficiently.