

Stress in the ground

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Total stress

Unit weight ranges are:

dry soil $\square \gamma_d$ 14 - 20 kN/m³ (average 17kN/m³)

saturated soil $\square \gamma_g$ 18 - 23 kN/m³ (average 20kN/m³)

water $\square \gamma_w$ 9.81 kN/m³ (≈ 10 kN/m³)

Any change in vertical total stress (σ_v) may also result in a change in the **horizontal** total stress (σ_h) at the same point. The relationships between vertical and horizontal stress are complex.

Total stress in homogeneous soil

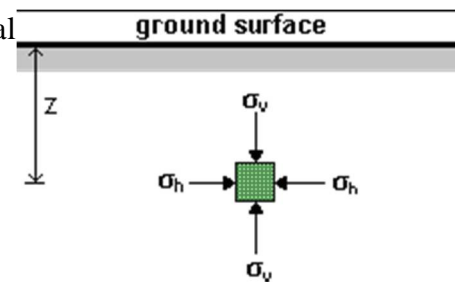
Total stress increases with depth and with unit weight: Vertical total stress at depth z ,

$$\sigma_v = \gamma \cdot z$$

The symbol for total stress may also be written σ_z , *i.e. related to depth z .*

The unit weight, γ , will vary with the water content of the soil.

$$\gamma_d \leq \gamma \leq \gamma_g$$



Total stress below a river or lake

The total stress is the sum of the weight of the soil up to the surface and the weight of water above this: Vertical total stress at depth z ,

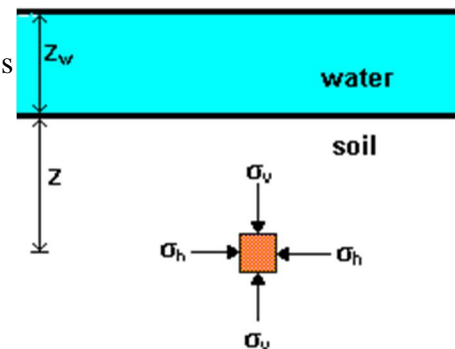
$$\sigma_v = \gamma \cdot z + \gamma_w \cdot z_w$$

where

γ = unit weight of the saturated soil,

i.e. the total weight of soil grains and water

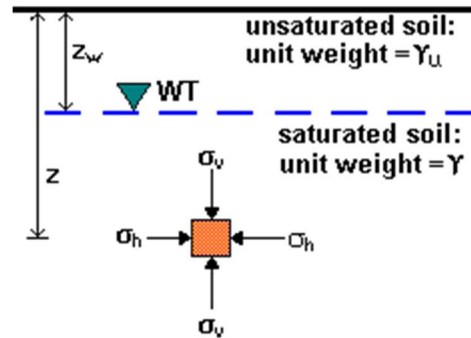
γ_w = unit weight of water



Total stress in unsaturated soil

Just above the water table the soil will remain saturated due to capillarity, but at some distance above the water table the soil will become unsaturated, with a consequent reduction in unit weight (unsaturated unit weight = γ_u)

$$\sigma_v = \gamma_w \cdot z_w + \gamma_g(z - z_w)$$

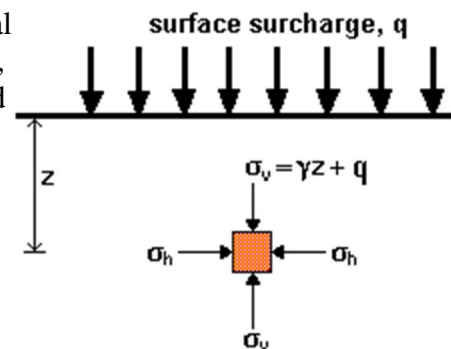


The height above the water table up to which the soil will remain saturated depends on the grain size.

See [Negative pore pressure \(suction\)](#).

Total stress with a surface surcharge load

The addition of a surface surcharge load will increase the total stresses below it. If the surcharge loading is extensively wide, the increase in vertical total stress below it may be considered constant with depth and equal to the magnitude of the surcharge.



Vertical total stress at depth z ,

For narrow surcharges, e.g. under strip and pad foundations, the induced vertical total stresses will decrease both with depth and horizontal distance from the load. In such cases, it is necessary to use a suitable stress distribution theory - an example is Boussinesq's theory.

Pore pressure

The water in the pores of a soil is called **porewater**. The pressure within this porewater is called **pore pressure (u)**. The magnitude of pore pressure depends on:

- the depth below the [water table](#)
- the conditions of seepage flow

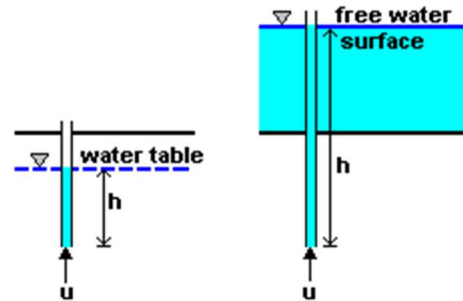
Groundwater and hydrostatic pressure

Under hydrostatic conditions (no water flow) the pore pressure at a given point is given by the **hydrostatic pressure**:

$$u = \gamma_w \cdot h_w$$

where

h_w = depth below water table or overlying water surface



It is convenient to think of pore pressure represented by the column of water in an imaginary standpipe; the pressure just outside being equal to that inside.

Water table, phreatic surface

The natural static level of water in the ground is called the **water table** or the **phreatic surface** (or sometimes the **groundwater level**). Under conditions of no seepage flow, the water table will be horizontal, as in the surface of a lake. The magnitude of the pore pressure at the water table is zero. Below the water table, pore pressures are positive.

$$u = \gamma \cdot h$$

In conditions of steady-state or variable seepage flow, the calculation of pore pressures becomes more complex.

See [Groundwater](#)

Negative pore pressure (suction)

Below the water table, pore pressures are **positive**. In dry soil, the pore pressure is **zero**. Above the water table, when the soil is saturated, pore pressure will be **negative**.

$$u = - \gamma \cdot h$$

The height above the water table to which the soil is saturated is called the **capillary rise**, and this depends on the grain size and type (and thus the size of pores):

- in coarse soils capillary rise is very small
- in silts it may be up to 2m
- in clays it can be over 20m

Pore water and pore air pressure

Between the ground surface and the top of the saturated zone, the soil will often be partially saturated, i.e. the pores contain a mixture of water and air. The pore pressure in a partially saturated soil consists of two components:

- porewater pressure = u
- pore-air pressure = u

Note that water is incompressible, but air is compressible. The combined effect is a complex relationship involving partial pressures and the degree of saturation of the soil. In Europe and

other temperate climate countries most design states are associated with saturated conditions, and the study of partially saturated soils is considered to be a specialist subject.

Pore pressure in steady state seepage conditions

In conditions of seepage in the ground there is a change in pore pressure. Consider seepage occurring between two points P and Q. The hydraulic gradient, i

Hydraulic gradient P-Q, $i = -\frac{\delta h}{\delta s}$

$$\delta s = \frac{\delta u}{\gamma_w}$$

$$\delta s = \frac{\delta u}{\gamma_w}$$

$$\gamma_w \delta u = i \cdot \gamma_w \cdot \delta s$$

But in **steady-state** seepage, $i = \text{constant}$

Therefore the change in pore pressure due to seepage alone, $\delta u_s = i \cdot \gamma_w \cdot s$

For seepage flow vertically downward, i is negative

For seepage flow vertically upward, i is positive.

Effective stress

Ground movements and instabilities can be caused by changes in total stress (such as loading due to foundations or unloading due to excavations), but they can also be caused by changes in pore pressures (slopes can fail after rainfall increases the pore pressures).

In fact, it is the combined effect of total stress and pore pressure that controls soil behaviour such as shear strength, compression and distortion. The difference between the total stress and the pore pressure is called the effective stress:

effective stress = total stress - pore pressure

$$\text{or } \sigma' = \sigma - u$$

Note that the prime (dash mark $'$) indicates effective stress.

Terzaghi's principle and equation

Karl Terzaghi was born in Vienna and subsequently became a professor of soil mechanics in the USA. He was the first person to propose the relationship for effective stress (in 1936):

All measurable effects of a change of stress, such as compression, distortion and a change of shearing resistance are due exclusively to changes in effective stress. The effective stress σ' is related to total stress and pore pressure by $\sigma' = \sigma - u$.

The adjective 'effective' is particularly apt, because it is *effective* stress that is effective in causing important changes: changes in strength, changes in volume, changes in shape. It does not represent the exact contact stress between particles but the distribution of load carried by the soil over the area considered.

Mohr circles for total and effective stress

Mohr circles can be drawn for both total and effective stress. The points E and T represent the total and effective stresses on the same plane. The two circles are displaced along the normal stress axis by the amount of pore pressure ($\sigma_n = \sigma_n' + u$), and their diameters are the same. The total and effective shear stresses are equal ($\tau' = \tau$).

The importance of effective stress

The principle of effective stress is fundamentally important in soil mechanics. It must be treated as the basic axiom, since soil behaviour is governed by it. Total and effective stresses must be distinguishable in all calculations: algebraically the *prime* should indicate effective stress, e.g. σ'

Changes in water level *below* ground (water table changes) result in changes in effective stresses below the water table. Changes in water level *above* ground (e.g. in lakes, rivers, etc.) **do not** cause changes in effective stresses in the ground below.

Changes in effective stress

In some analyses it is better to work in *changes* of quantity, rather than in absolute quantities; the effective stress expression then becomes:

$$\Delta\sigma' = \Delta\sigma - \Delta u$$

If both total stress and pore pressure change by the same amount, the effective stress remains constant. A change in effective stress will cause: a change in strength and a change in volume.

The of soil is proportional to the effective normal stress; thus, a change in effective stress brings about a change in strength.

Therefore, if the pore pressure in a soil slope increases, effective stresses will be reduced by $\Delta\sigma'$ and the critical strength of the soil will be reduced by $\Delta\tau$ - sometimes leading to failure.

A seaside sandcastle will remain intact while damp, because the pore pressure is negative; as it dries, this pore pressure suction is lost and it collapses. Note: Sometimes a sandcastle will remain intact even when nearly dry because salt deposited as seawater evaporates slightly and cements the grains together.

Immediately after the construction of a foundation on a fine soil, the pore pressure increases, but immediately begins to drop as drainage occurs.

The rate of change of effective stress under a loaded foundation, once it is constructed, will be the same as the rate of change of pore pressure, and this is controlled by the permeability of the soil.

Settlement occurs as the volume (and therefore thickness) of the soil layers change. Thus, settlement occurs rapidly in coarse soils with high permeabilities and slowly in fine soils with low permeabilities.

Calculating vertical stress in the ground

The worked examples here are designed to illustrate the principles and methods dealt with in *Pore pressure, effective stress and stresses in the ground*. The examples chosen are typical and simple.

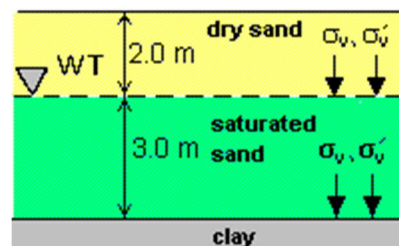
Simple total and effective stresses

The figure shows soil layers on a site.

Unit weights are:

$$\text{dry sand: } \gamma_d = 16 \text{ kN/m}^3$$

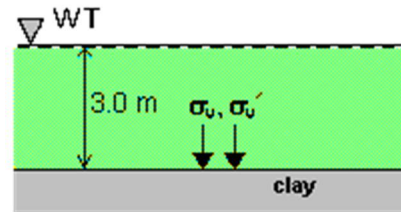
$$\text{saturated sand: } \gamma_g = 20 \text{ kN/m}^3$$



(a) At the top of saturated sand (z = 2.0 m) Vertical total stress $\sigma_v = 16.0 \times 2.0 = 32.0 \text{ kPa}$
Pore pressure $u = 0$ Vertical effective stress $\sigma'_v = \sigma_v - u = 32.0 \text{ kPa}$ **(b) At the top of the clay (z = 5.0 m)** Vertical total stress $\sigma_v = 32.0 + 20.0 \times 3.0 = 92.0 \text{ kPa}$ Pore pressure $u = 9.81 \times 3.0 = 29.4 \text{ kPa}$ Vertical effective stress $\sigma'_v = \sigma_v - u = 92.0 - 29.4 = 62.6 \text{ kPa}$

The figure shows soil layers on a site. The unit weight of the silty sand is 19.0 kN/m^3 both above and below the water

table. The water level is presently at the surface of the silty sand, it may drop or it may rise. The



$$\sigma_v = 19.0 \times 3 = 57.0 \text{ kPa}$$

$$\sigma'_v = 57.0 - 29.4 = 27.6 \text{ kPa}$$

following calculations show the effects of this:

From an initial state, the stresses under a foundation are first changed by excavation, i.e. vertical stresses are reduced. After construction the foundation loading increases stresses. Other changes could result if the water table level changed.

The figure shows the elevation of a foundation to be constructed in a homogeneous soil. The change in thickness of the clay layer is to be calculated and so the initial and final effective stresses are required at the mid-depth of the clay.

Unit weights: sand above WT = 16 kN/m³, sand below WT = 20 kN/m³, clay = 18 kN/m³.

Short-term and long-term stresses

The figure shows how an extensive layer of fill will be placed on a certain site.

The unit weights are:

clay and sand = 20kN/m³ ,

rolled fill 18kN/m³ ,

assume water = 10 kN/m³.

Calculations are made for the total and effective stress at the mid-depth of the sand and the mid-depth of the clay for the following conditions: initially, before construction; immediately after construction; many years after construction.

Initial stresses at mid-depth of clay (z = 2.0m)

Vertical total stress

$$\sigma_v = 20.0 \times 2.0 = \mathbf{40.0kPa}$$

Pore pressure

$$u = 10 \times 2.0 = 20.0kPa$$

Vertical effective stress

$$\sigma'_v = \sigma_v - u = \mathbf{20.0kPa}$$

Initial stresses at mid-depth of sand (z = 5.0 m)

Vertical total stress

$$\sigma_v = 20.0 \times 5.0 = \mathbf{100.0 \text{ kPa}}$$

Pore pressure

$$u = 10 \times 5.0 = 50.0 \text{ kPa}$$

Vertical effective stress

$$\sigma'_v = \sigma_v - u = \mathbf{50.0 \text{ kPa}}$$

The construction of the embankment applies a surface surcharge:

$$q = 18 \times 4 = 72.0 \text{ kPa.}$$

The sand is drained (either horizontally or into the rock below) and so there is no increase in pore pressure. The clay is undrained and the pore pressure increases by 72.0 kPa.

Initial stresses at mid-depth of clay (z = 2.0m)

Vertical total stress

$$\sigma_v = 20.0 \times 2.0 + 72.0 = \mathbf{112.0 \text{ kPa}}$$

Pore pressure

$$u = 10 \times 2.0 + 72.0 = 92.0 \text{ kPa}$$

Vertical effective stress

$$\sigma'_v = \sigma_v - u = \mathbf{20.0 \text{ kPa}}$$

(i.e. no change immediately)

Initial stresses at mid-depth of sand (z = 5.0m)

Vertical total stress

$$\sigma_v = 20.0 \times 5.0 + 72.0 = \mathbf{172.0 \text{ kPa}}$$

Pore pressure

$$u = 10 \times 5.0 = 50.0 \text{ kPa}$$

Vertical effective stress

$$\sigma'_v = \sigma_v - u = \mathbf{122.0 \text{ kPa}}$$

(i.e. an immediate increase)

After many years, the excess pore pressures in the clay will have dissipated. The pore pressures will now be the same as they were initially.

Initial stresses at mid-depth of clay (z = 2.0 m)

Vertical total stress

$$\sigma_v = 20.0 \times 2.0 + 72.0 = \mathbf{112.0 \text{ kPa}}$$

Pore pressure

$$u = 10 \times 2.0 = 20.0 \text{ kPa}$$

Vertical effective stress

$$\sigma'_v = \sigma_v - u = \mathbf{92.0 \text{ kPa}}$$

(i.e. a long-term increase)

Initial stresses at mid-depth of sand (z = 5.0 m)

Vertical total stress

$$\sigma_v = 20.0 \times 5.0 + 72.0 = \mathbf{172.0 \text{ kPa}}$$

Pore pressure

$$u = 10 \times 5.0 = 50.0 \text{ kPa}$$

Vertical effective stress

$$\sigma'_v = \sigma_v - u = \mathbf{122.0 \text{ kPa}}$$

(i.e. no further change)

The figure shows seepage occurring around embedded sheet piling.

In steady state, the hydraulic gradient,

$$i = \Delta\eta / \Delta\sigma = 4 / (7 + 3) = 0.4$$

Then the effective stresses are:

$$\sigma'_A = 20 \times 3 - 2 \times 10 + 0.4 \times 10 = 44 \text{ kPa}$$

$$\sigma'_B = 20 \times 3 - 2 \times 10 - 0.4 \times 10 = 36 \text{ kPa}$$