Statically determinate structures

A statically determinate structure is the one in which reactions and internal forces can be determined solely from free-body diagrams and equations of equilibrium. These equations are:
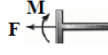
$\Sigma H = 0$, $\Sigma V = 0$ and $\Sigma M = 0$. It should be noted that the results of analysis are independent of the material from which the structure has been fabricated.

Support and Connection Types

Structural systems transfer their loading through a combination of elements. This is accomplished by joining the elements at their intersections, which would form the connections. Each connection is designed so that it can transfer, or support, a certain type of load. To perform the structural analysis, it is necessary to be aware of the types of forces that can be resisted, and transferred, at each support throughout the structure. The support conditions have a great impact on the behavior of the elements which make up each structural system.

The most common types of connections are: roller; pinned; and fixed. The supports can be located anywhere along a structural element. They can be formed at the ends, at midpoints, or at any other intermediate points. The type of support connection determines the type of load that the support can resist. The diagrams shown in Table 1, illustrates the various representations of each type of support.

Table 1

Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(1) Light cable			One unknown. The reaction is a force that acts in the direction of the cable or link.
(2) rollers / rockers		F	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(3)		F	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.
(4)		F	One unknown. The reaction is a force that acts perpendicular to the surface at the point of contact.

| Type of Connection	Idealized Symbol	Reaction	Number of Unknowns
(5) Smooth pin or hinge		F_y, F_x	Two unknowns. The reactions are two force components.
(6) slider / fixed-connected collar		M, F	Two unknowns. The reactions are a force and moment.
(7) fixed support		M, F_y, F_x	Three unknowns. The reactions are the moment and the two force components.

Principle of Superposition

For the principle of superposition to apply two requirements must be satisfied:

1. The material must behave in a linear-elastic manner, so that Hook's law is valid, and therefore the load is proportional to displacement.

2. The geometry of the structure must not undergo significant change due to load application, i.e., small displacement theory applies.
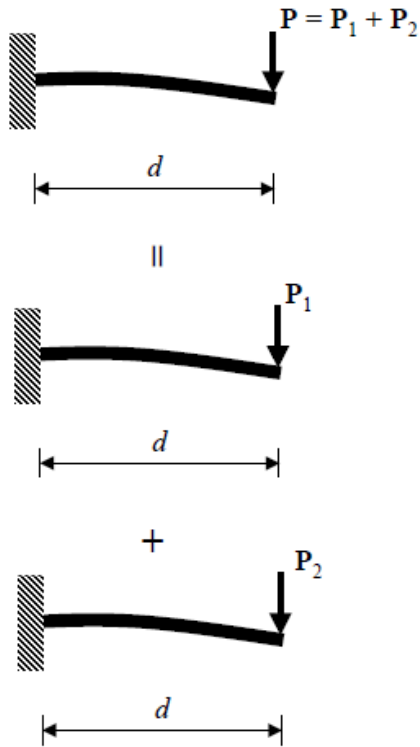
Equations of Equilibrium

For a structure in equilibrium, the following must be satisfied at any point P:

$$\sum F = 0 \quad and \quad \sum M_P = 0$$

Using the xyz coordinate system, the equations can be represented in the following forms:

$$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$$

$$\Sigma M_x = 0 \qquad \Sigma M_y = 0 \qquad \Sigma M_z = 0$$

Determinacy and Stability

A stable structure remains stable for any imaginable system of loads. Therefore, the types of loads, their number and their points of application are not considered

3

when deciding the stability or determinacy of the structure. A given structure considered externally determinate if the total number of reaction components is equal to the equations of equilibrium available. In other words:

Determinacy:

$r = 3n$, statically determinate

$r > 3n$, statically indeterminate

n = the total parts of structure members.
r = the total number of unknown reactive force and moment components
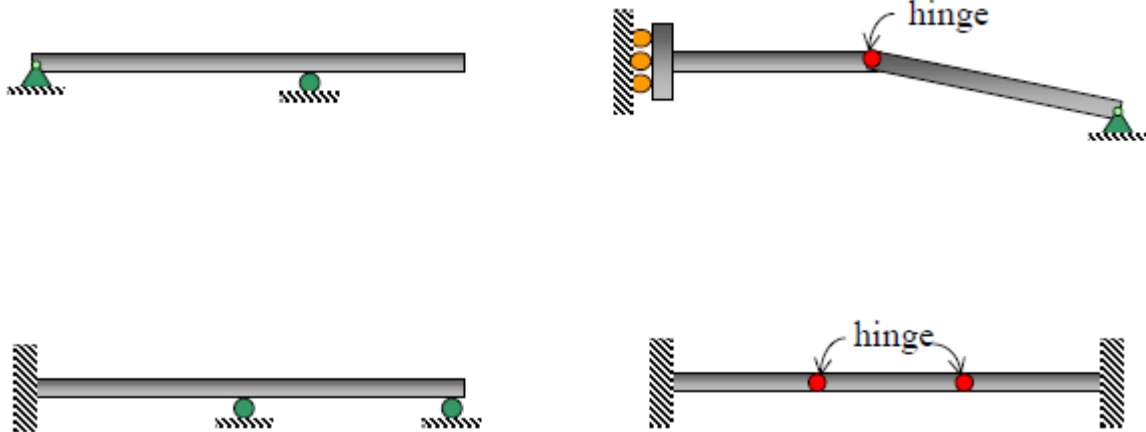
Stability:

$r < 3n$, unstable

$r \geqslant 3n$, unstable if member reactions are concurrent or parallel or some of the components form a collapsible mechanism

Example 1:

Classify if each of the following beams is determinate or indeterminate. If statically indeterminate, what is the number of degree of indeterminacy?

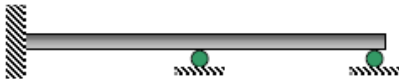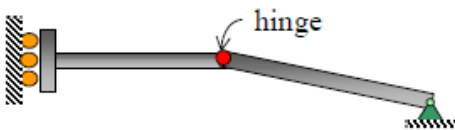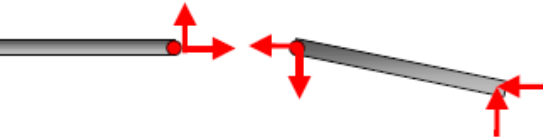### SOLUTION

$r = 3, n = 1, 3 = 3(1)$ Statically **determinate**

$r = 5, n = 1, 5 - 3(1) = 2$ Statically **indeterminate** to the second degree
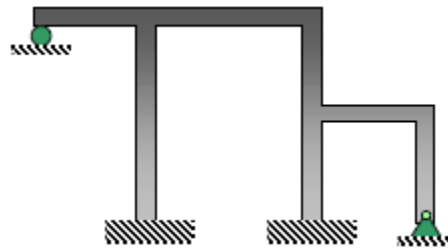

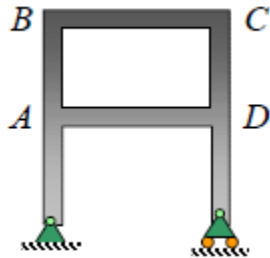
$r = 6, n = 2, 6 = 3(2)$ Statically **determinate**

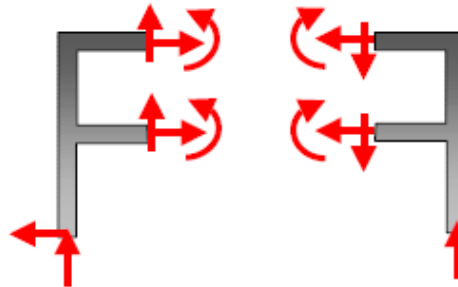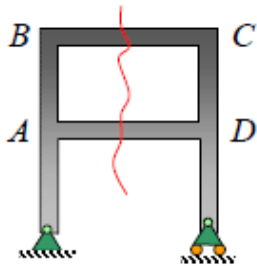$r = 10, n = 3, 10 - 3(3) = 1$ Statically **indeterminate** to the **first** degree

5

Example 2:

Classify if each of the following frames is determinate or indeterminate. If statically indeterminate, what is the number of degree of indeterminacy?
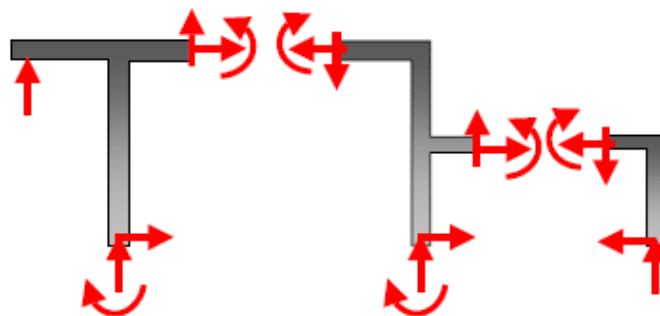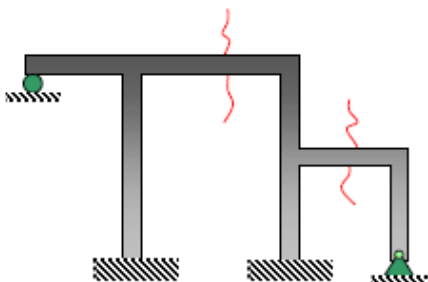

SOLUTION

$r = 9, n = 2, 9 - 6 = 3$

Statically **indeterminate** to the **third** degree

$r = 15, n = 3, 15 - 9 = 6$

Statically **indeterminate** to the **sixth** degree

Truss Structures

A truss can be defined as a structure that is composed of links or bars, assumed to be connected by frictionless pins at the joints, and arranged so that the area enclosed within the boundaries of the structure is subdivided by the bars into geometrical figures which are normally triangles.

Internal determinacy is a type of indeterminacy that is associated to trusses. The basic form of the truss is a triangle. To make the truss, add two members and one joint, and repeat.
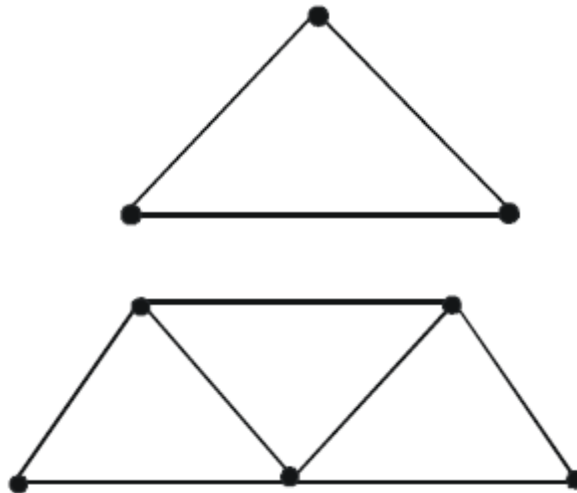
Assume:

j=total number of joints, i=total number of links, r= minimum number of reaction required for external determinacy/stability.

$i + r = 2j$

Total number of unknown = total number of equations available (at joints)

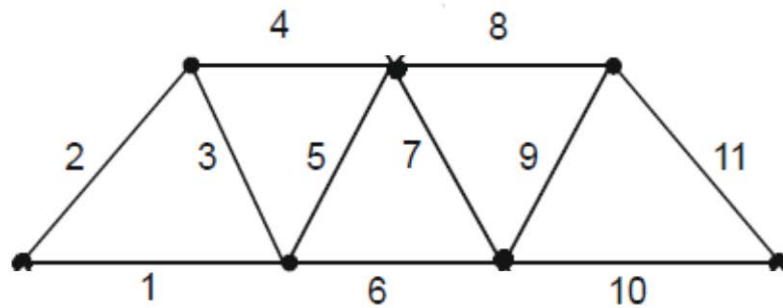If $i + r = 2j$ stable and internally determinate

If $i + r > 2j$ stable and internally indeterminate

If $i + r < 2j$ unstable

It should be noted that the structure is said to have determinacy and indeterminacy **ONLY** if the structure is stable.

Problem Examples 1: Assume r = 3

For the truss shown, determine if the truss is

I-Stable II-Indeterminate III- Determinate
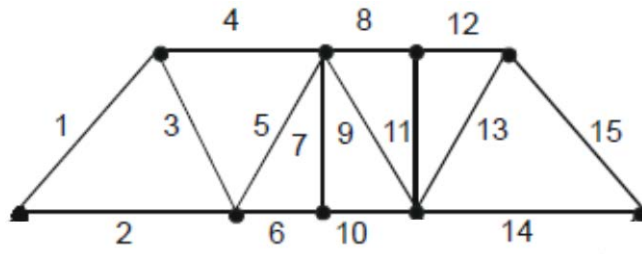
A-I ONLY

B-II ONLY

C-I and III

D-I and II

$i = 11$,
$r = 3$,
$j = 7$,
$i + r = 11+3 = 14$, $2j = 2\times7 = 14$.

 The truss is **stable and internally determinate**. Answer is C

Problem Examples 2: Assume r=3

For the truss shown, determine if the truss is

I-Stable II-Indeterminate III- Determinate

A-I ONLY

B-II ONLY

C-I and III

D-I and II

Solution

i = 15,
r = 3,
j = 9,
i + r = 15+3 = 18,
 2j = 2x9 = 18.
The truss is stable and internally determinate. Answer is C

Problem Examples 3:

For the truss shown, determine if the truss is

I-Stable II-Indeterminate III- Determinate

A-I ONLY

B-II ONLY
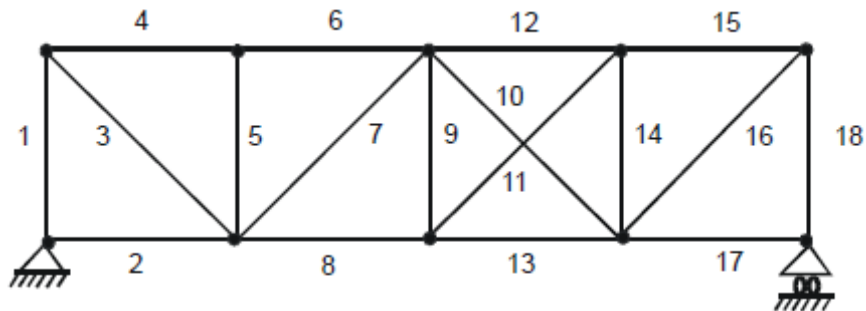
C-I and III

D-I and II

Solution

$i = 18$,

$r = 3$,

$j = 10$,

$i + r = 18+3 = 21$,

$2j = 2 \times 10 = 20$.

The truss is stable and internally indeterminate. The answer is D