Civil Engineering Design (1)

Prestressed Concrete

2006/7

Dr. Colin Caprani,

Chartered Engineer

Contents

1. Introduction

1.1 Background

The idea of prestressed concrete has been around since the latter decades of the 19th century, but its use was limited by the quality of the materials at the time. It took until the 1920s and ‘30s for its materials development to progress to a level where prestressed concrete could be used with confidence. Freyssinet in France, Magnel in Belgium and Hoyer in Germany were the principle developers.

The idea of prestressing has also been applied to many other forms, such as:

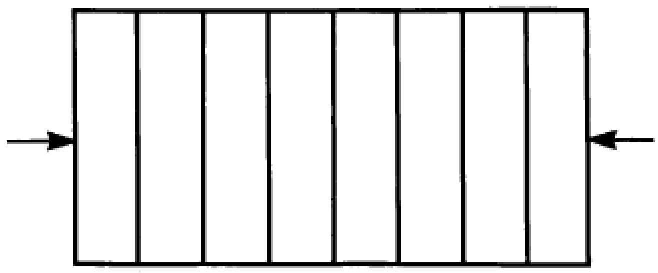
- Wagon wheels;
- Riveting;
- Barrels, i.e. the coopers trade;

In these cases heated metal is made to just fit an object. When the metal cools it contracts inducing prestress into the object.

1.2 Basic Principle of Prestressing

Basic Example

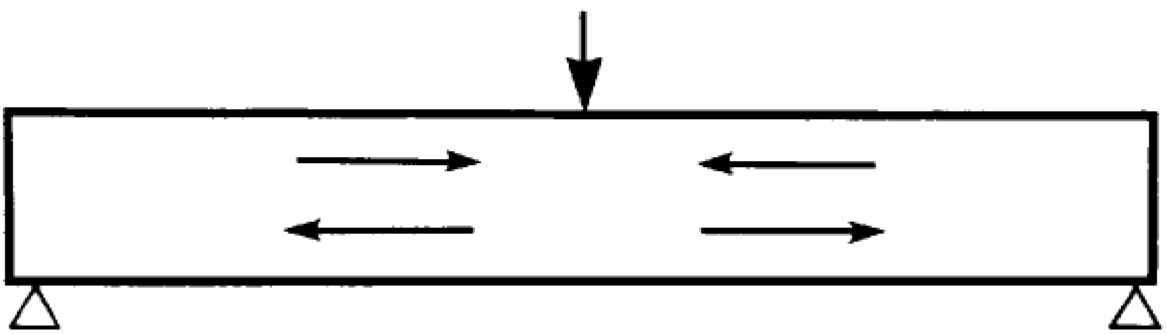
The classic everyday example of prestressing is this: a row of books can be lifted by squeezing the ends together:



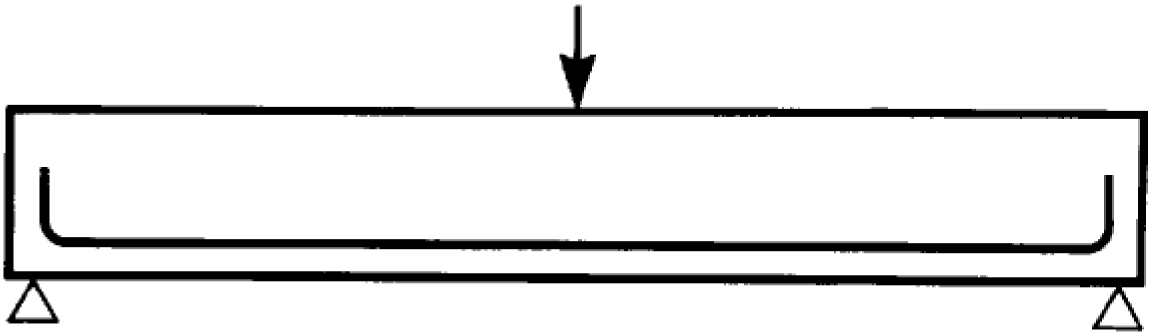
The structural explanation is that the row of books has zero tensile capacity. Therefore the 'beam' of books cannot even carry its self weight. To overcome this we provide an external initial stress (the prestress) which compresses the books together. Now they can only separate if the tensile stress induced by the self weight of the books is greater than the compressive prestress introduced.

Concrete

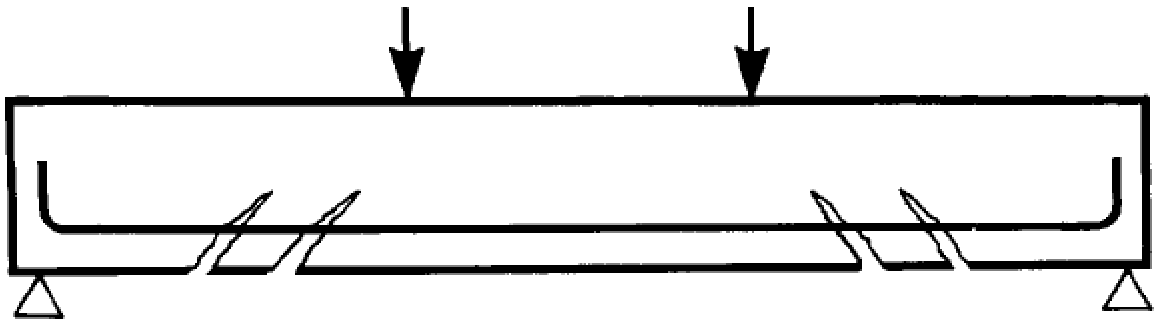
Concrete is very strong in compression but weak in tension. In an ordinary concrete beam the tensile stress at the bottom:



are taken by standard steel reinforcement:



But we still get cracking, which is due to both bending and shear:



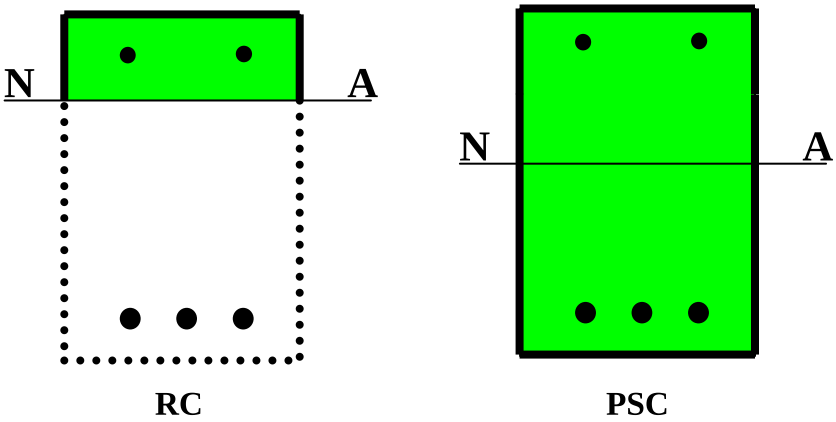
In prestressed concrete, because the prestressing keeps the concrete in compression, no cracking occurs. This is often preferable where durability is a concern.

1.3 Advantages of Prestressed Concrete

The main advantages of prestressed concrete (PSC) are:

Smaller Section Sizes

Since PSC uses the whole concrete section, the second moment of area is bigger and so the section is stiffer:



Smaller Deflections

The larger second moment of area greatly reduces deflections for a given section size.

Increased Spans

The smaller section size reduces self weight. Hence a given section can span further with prestressed concrete than it can with ordinary reinforced concrete.

Durability

Since the entire section remains in compression, no cracking of the concrete can occur and hence there is little penetration of the cover. This greatly improves the long-term durability of structures, especially bridges and also means that concrete tanks can be made as watertight as steel tanks, with far greater durability.

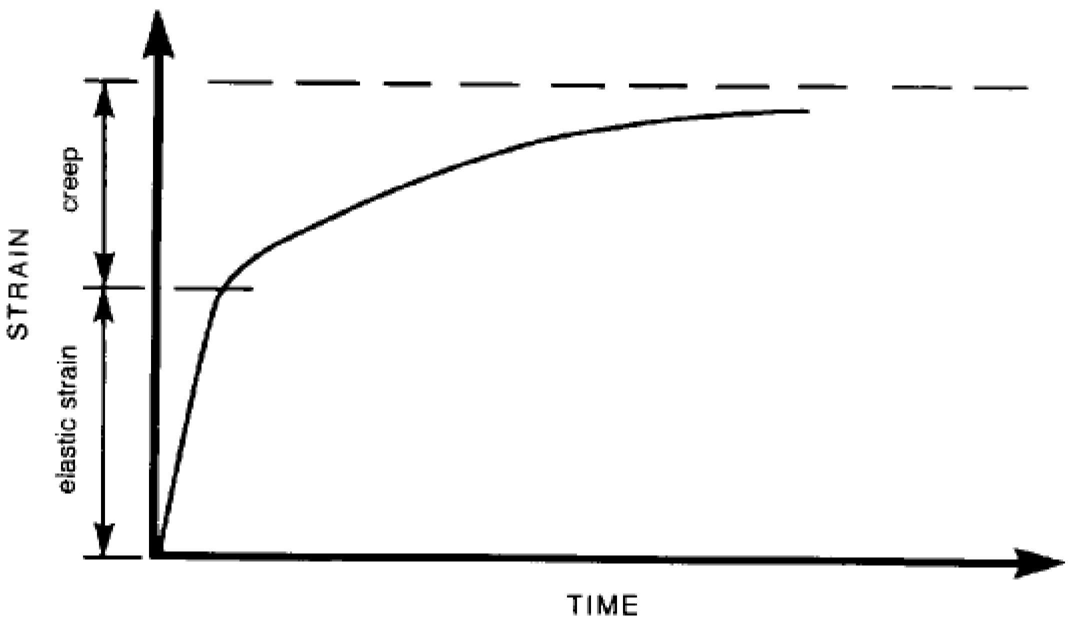
1.4 Materials

Concrete

The main factors for concrete used in PSC are:

- Ordinary portland cement-based concrete is used but strength usually greater than 50 N/mm^2;
- A high early strength is required to enable quicker application of prestress;
- A larger elastic modulus is needed to reduce the shortening of the member;
- A mix that reduces creep of the concrete to minimize losses of prestress;

You can see the importance creep has in PSC from this graph:

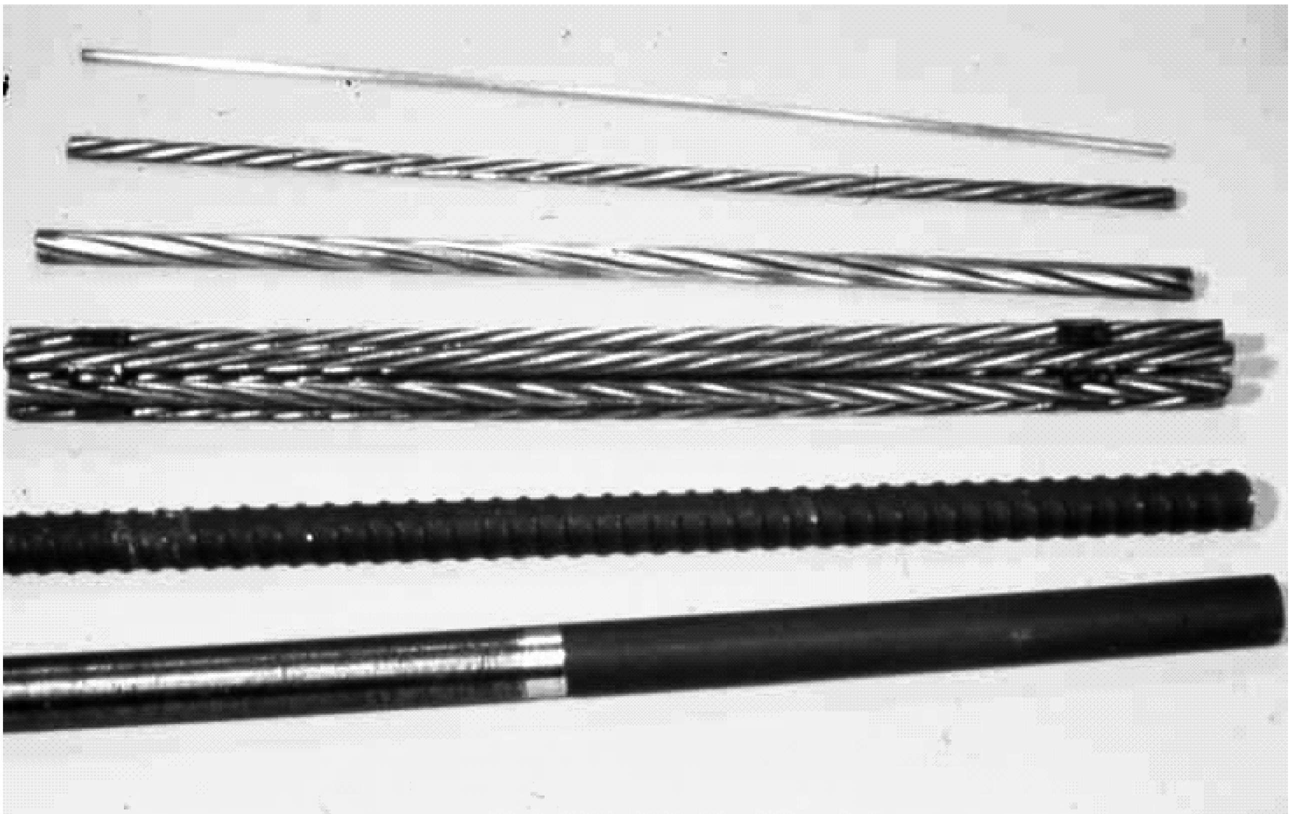


Steel

The steel used for prestressing has a nominal yield strength of between 1550 to 1800 N/mm^2. The different forms the steel may take are:

- Wires: individually drawn wires of 7 mm diameter;
- Strands: a collection of wires (usually 7) wound together and thus having a diameter that is different to its area;
- Tendon: A collection of strands encased in a duct – only used in post-tensioning;
- Bar: a specially formed bar of high strength steel of greater than 20 mm diameter.

Prestressed concrete bridge beams typically use 15.7 mm diameter (but with an area of 150 mm^2)7-wire super strand which has a breaking load of 265 kN.



		Nominal diameter (mm)	Nominal area (mm^2)	Nominal mass (kg/m)	Yield strength (N/mm^2)	Tensile strength (N/mm^2)	Minimum breaking load (kN)	Modulus of elasticity (kN/mm^2)	Relaxation[1] (class 2 or low relaxation)
7-wire strand low-relaxation									
13 mm (0·5″)	Euronorm 138–79, or BS 5896: 1980, Super	12·9	100	0·785	1580	1860	186	195	2·5%
	Euronorm 138–79, or BS 5896: 1980, Standard	12·5	93	0·73	1500	1770	164	195	2·5%
	Euronorm 138–79, or BS 5896: 1980, Drawn	12·7	112	0·89	1580	1860	209	195	2·5%
	ASTM A416-85, Grade 270	12·7	98·7	0·775	1670	1860	183·7	195	2·5%
15 mm (0·6″)	Euronorm 138–79, or BS 5896: 1980, Super	15·7	150	1·18	1500	1770	265	195	2·5%
	Euronorm 138–79, or BS 5896: 1980, Standard	15·2	139	1·09	1420	1670	232	195	2·5%
	Euronorm 138–79, or BS 5896: 1980, Drawn	15·2	165	1·295	1550	1820	300	195	2·5%
	ASTM A416-85, Grade 270	15·2	140	1·10	1670	1860	260·7	195	2·5%
Stress bars									
20 mm	BS 4486: 1980	20	314	2·39	835	1030	323	170/205	3·5%
25 mm	BS 4486: 1980	25	491	3·9	835	1030	505	170/205	3·5%
32 mm	BS 4486: 1980	32	804	6·66	835	1030	828	170/205	3·5%
40 mm	BS 4486: 1980	40	1257	10	835	1030	1300	170/205	3·5%
50 mm	BS 4486: 1980	50	1963	16·02	835	1030	2022	170/205	3·5%
Cold-drawn wire									
7 mm	BS 5896: 1980	7	38·5	302	1300	1570	60·4	205	2·5%
	BS 5896: 1980				1390	1670	64·3	205	2·5%
5 mm	BS 5896: 1980	5	19·6	154	1390	1670	32·7	205	2·5%
	BS 5896: 1980				1470	1770	34·7	205	2·5%

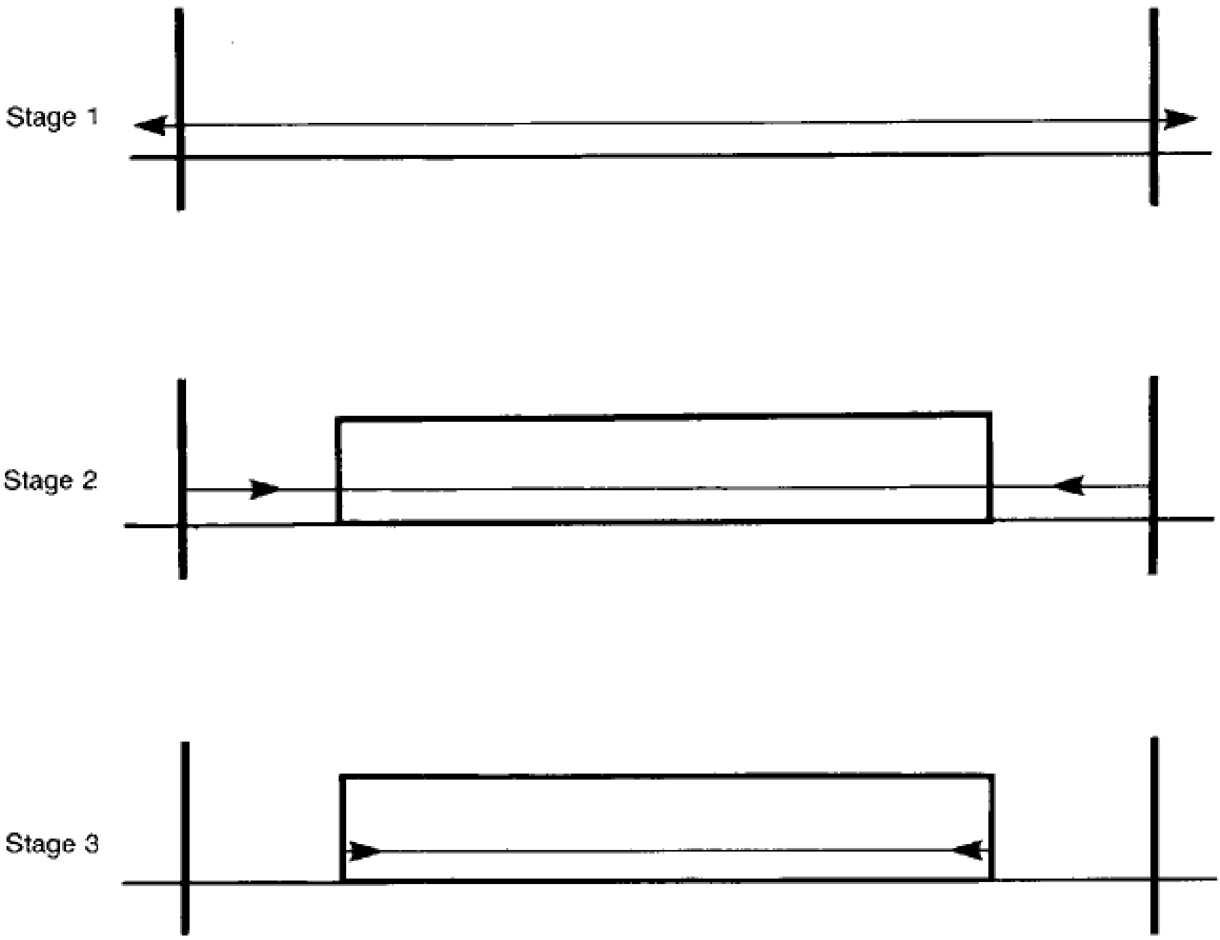
1.5 Methods of Prestressing

There are two methods of prestressing:

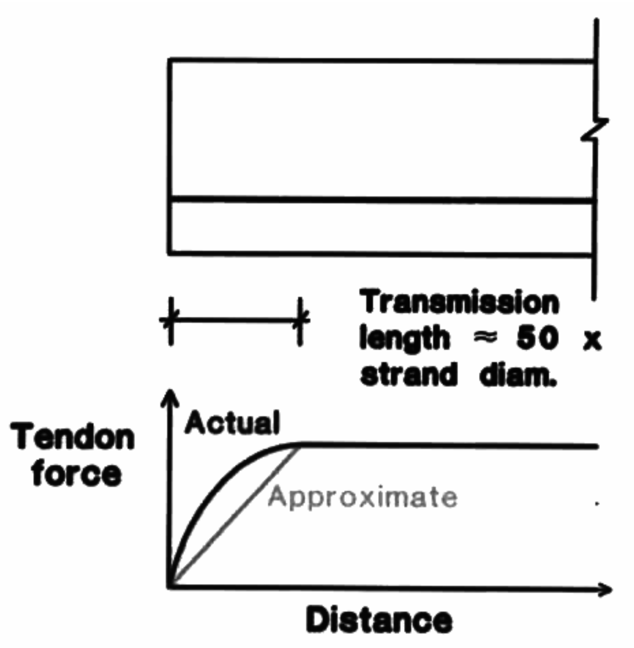
- Pre-tensioning: Apply prestress to steel strands *before* casting concrete;
- Post-tensioning: Apply prestress to steel tendons *after* casting concrete.

Pre-tensioning

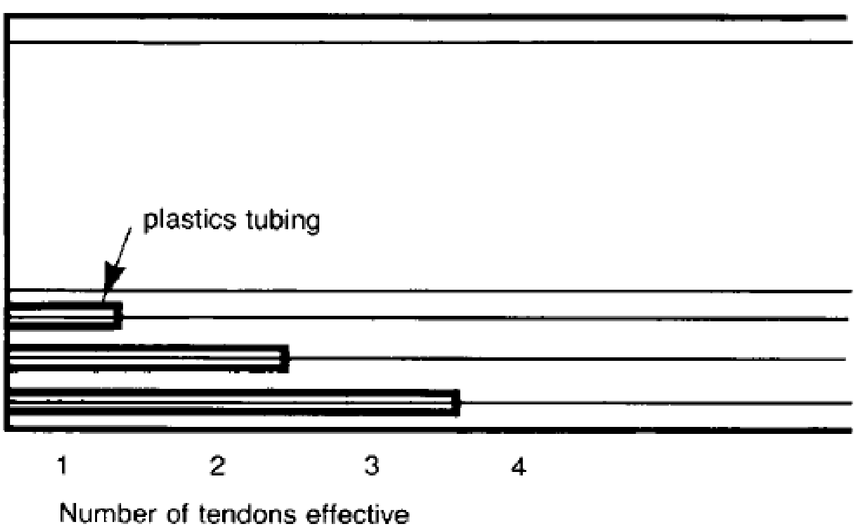
This is the most common form for precast sections. In Stage 1 the wires or strands are stressed; in Stage 2 the concrete is cast around the stressed wires/strands; and in Stage 3 the prestressed in *transferred* from the external anchorages to the concrete, once it has sufficient strength:



In pre-tensioned members, the strand is directly bonded to the concrete cast around it. Therefore, at the ends of the member, there is a transmission length where the strand force is transferred to the concrete through the bond:

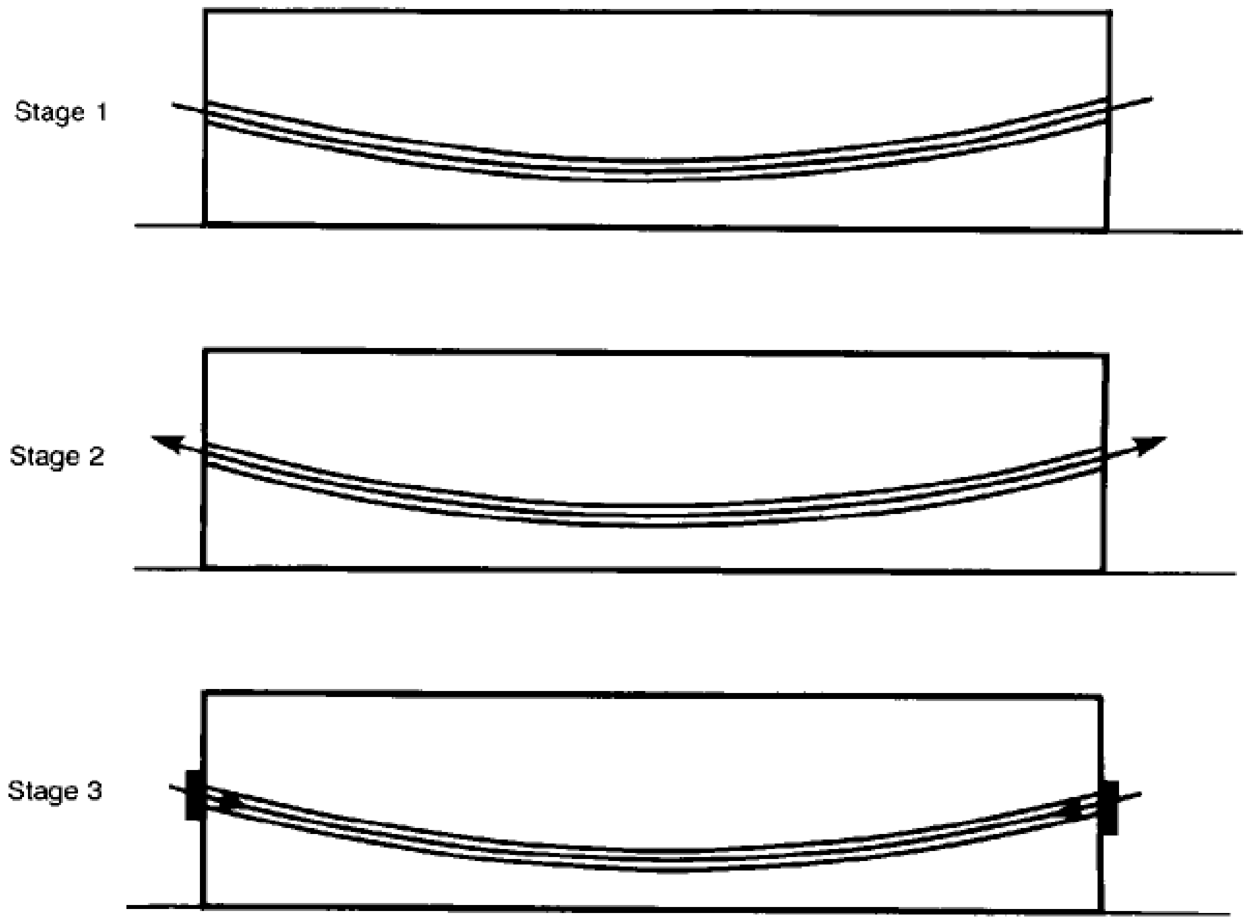


At the ends of pre-tensioned members it is sometimes necessary to debond the strand from the concrete. This is to keep the stresses within allowable limits where there is little stress induced by self with or other loads:



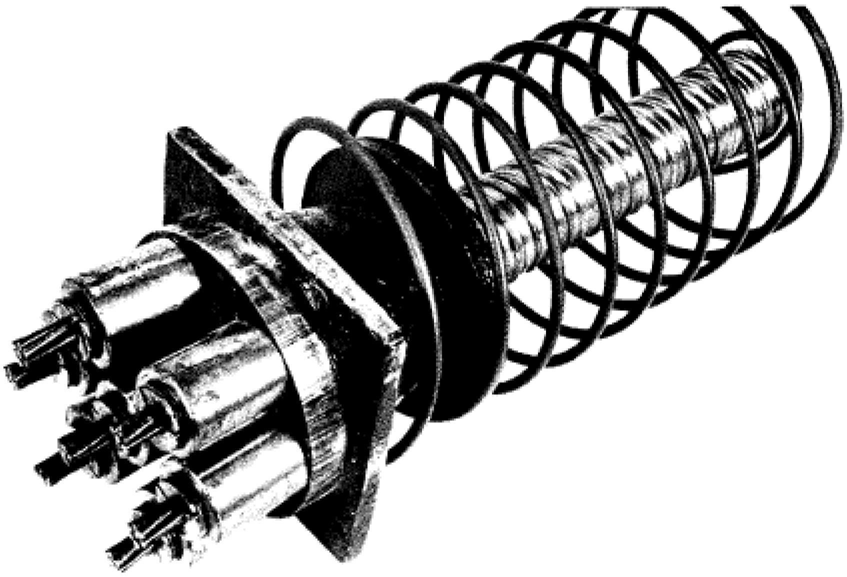
Post-tensioned

In this method, the concrete has already set but has ducts cast into it. The strands or tendons are fed through the ducts (Stage 1) then tensioned (Stage 2) and then anchored to the concrete (Stage 3):

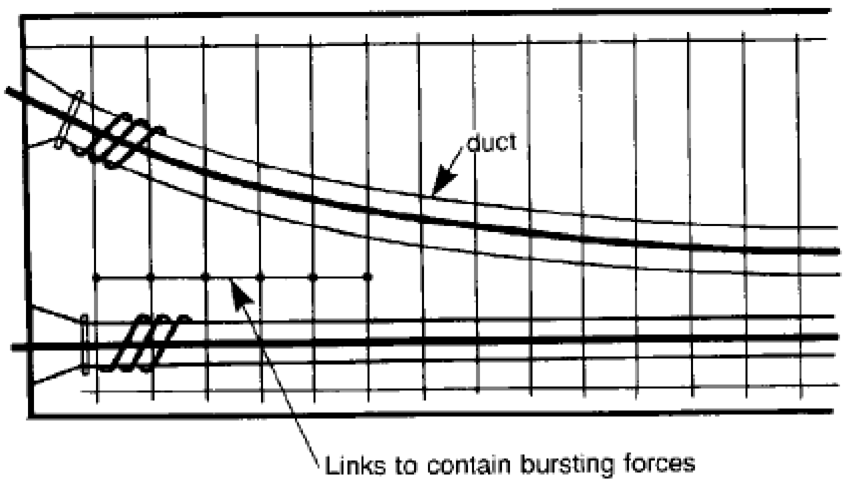


The anchorages to post-tensioned members must distribute a large load to the concrete, and must resist bursting forces as a result. A lot of ordinary reinforcement is often necessary.

A typical tendon anchorage is:



And the end of a post-tensioned member has reinforcement such as:



Losses

From the time the prestress is applied, the prestress force gradually reduces over time to an equilibrium level. The sources of these losses depend on the method by which prestressing is applied.

In both methods:

- The member shortens due to the force and this relieves some of the prestress;
- The concrete shrinks as it further cures;
- The steel 'relaxes', that is, the steel stress reduces over time;
- The concrete creeps, that is, continues to strain over time.

In post-tensioning, there are also losses due to the anchorage (which can 'draw in' an amount) and to the friction between the tendons and the duct and also initial imperfections in the duct setting out.

For now, losses will just be considered as a percentage of the initial prestress.

1.6 Uses of Prestressed Concrete

There are a huge number of uses:

- Railway Sleepers;
- Communications poles;
- Pre-tensioned precast “hollowcore” slabs;
- Pre-tensioned Precast Double T units - for very long spans (e.g., 16 m span for car parks);
- Pre-tensioned precast inverted T beam for short-span bridges;
- Pre-tensioned precast PSC piles;
- Pre-tensioned precast portal frame units;
- Post-tensioned ribbed slab;
- In-situ balanced cantilever construction - post-tensioned PSC;
- This is “glued segmental” construction;
- Precast segments are joined by post-tensioning;
- PSC tank - precast segments post-tensioned together on site. Tendons around circumference of tank;
- Barges;
- And many more.

2. Stresses in Prestressed Members

2.1 Background

The codes of practice limit the allowable stresses in prestressed concrete. Most of the work of PSC design involves ensuring that the stresses in the concrete are within the permissible limits.

Since we deal with allowable stresses, only service loading is used, i.e. the SLS case. For the SLS case, at any section in a member, there are two checks required:

At Transfer

This is when the concrete first feels the prestress. The concrete is less strong but the situation is temporary and the stresses are only due to prestress and self weight.

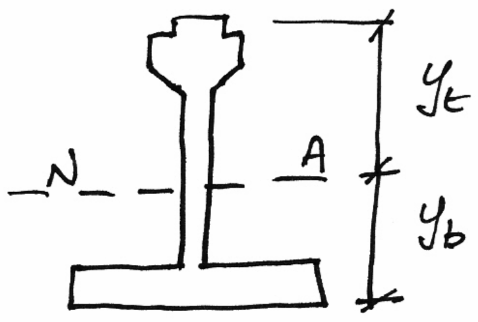
At Service

The stresses induced by the SLS loading, in addition to the prestress and self weight, must be checked. At service stage, the concrete has its full strength but losses will have occurred and so the prestress force is reduced.

The ultimate capacity at ULS of the PSC section (as for RC) must also be checked. If there is insufficient capacity, you can add non-prestressed reinforcement. This often does not govern.

Notation

For a typical prestressed section:



We have:

Z_t Section modulus, top fibre $= I/y_t$;

Z_b Section modulus, bottom fibre $= -I/y_b$ (taken to be negative);

f_{tt} Allowable tensile stress at transfer;

f_{tc} Allowable compressive stress at transfer;

f_{st} Allowable tensile stress in service;

f_{sc} Allowable compressive stress ain service;

M_t The applied moment at transfer;

M_s The applied moment in service;

α The ratio of prestress after losses (service) to prestress before losses, (transfer).

Allowable Stresses

Concrete does have a small tensile strength and this can be recognized by the designer. In BS 8110, there are 3 classes of prestressed concrete which depend on the level of tensile stresses and/or cracking allowed:

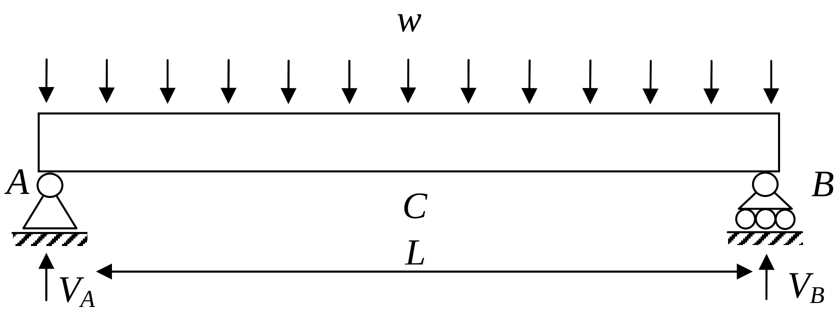
<table>
<tr><th colspan="2">Stresses</th><th>Class 1</th><th>Class 2</th><th>Class 3</th></tr>
<tr><td rowspan="2">At transfer</td><td>Tension: f_{tt}</td><td>1 N/mm^2</td><td colspan="2">$0.45\sqrt{f_{ci}}$ for pre-tensioned members
$0.36\sqrt{f_{ci}}$ for post-tensioned members</td></tr>
<tr><td>Compression: f_{tc}</td><td colspan="3">$0.5 f_{ci}$ *</td></tr>
<tr><td rowspan="2">In service</td><td>Tension: f_{st}</td><td>0 N/mm^2</td><td>$0.45\sqrt{f_{ci}}$ (pre)
$0.36\sqrt{f_{ci}}$ (post)</td><td>See code table</td></tr>
<tr><td>Compression: f_{sc}</td><td colspan="3">$0.33 f_{cu}$</td></tr>
</table>

* there are other requirements for unusual cases – see code

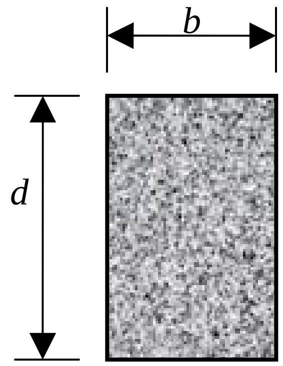
2.2 Basic Principle of Prestressed Concrete

Theoretical Example

Consider the basic case of a simply-supported beam subjected to a UDL of *w* kN/m:



In this case, we have the mid-span moment as:



$$M_C = \frac{wL^2}{8}$$

Also, if we assume a rectangular section as shown, we have the following section properties:

$$A = bd \qquad I = \frac{bd^3}{12}$$

$$Z_t = \frac{bd^2}{6} \qquad Z_b = \frac{bd^2}{6}$$

Therefore the stresses at *C* are:

$$\sigma_t = \frac{M_C}{Z_t} \qquad \sigma_b = \frac{M_C}{Z_b}$$

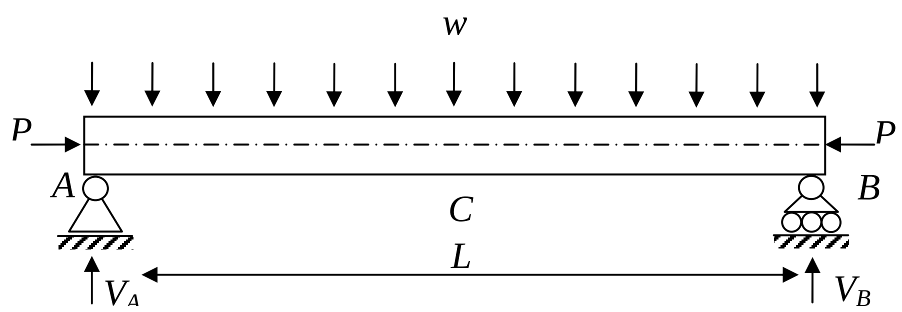
Case I

If we take the beam to be constructed of plain concrete (no reinforcement) and we neglect the (small) tensile strength of concrete ($f_t = 0$), then, as no tensile stress can occur, no load can be taken:

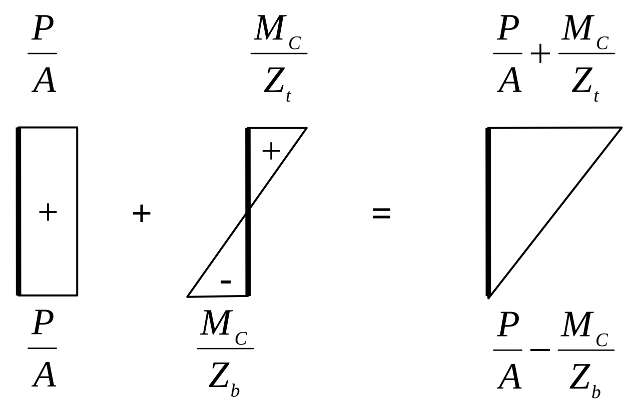
$$w_I = 0$$

Case II

We consider the same beam, but with **centroidal** axial prestress as shown:



Now we have two separate sources of stress:



For failure to occur, the moment caused by the load must induce a tensile stress greater than $\frac{P}{A}$. Hence, just prior to failure, we have:

$$\frac{M_C}{Z_b} = \frac{P}{A} = \frac{wL^2}{8Z_b}$$

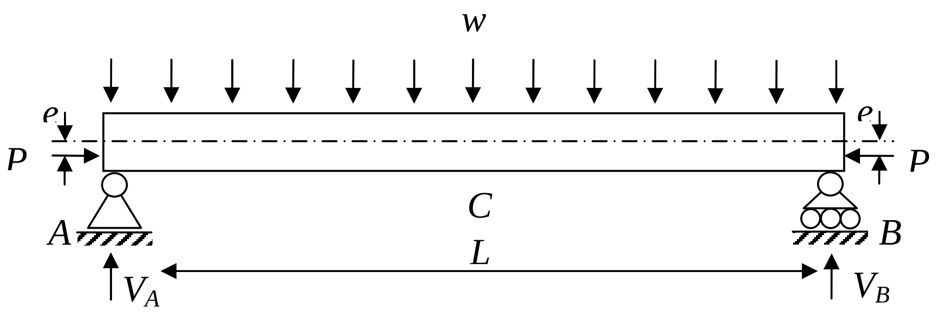
$$w_{II} = \frac{8Z_b}{L^2}\frac{P}{A}$$

Note that we take **Compression as positive** and **tension as negative**.

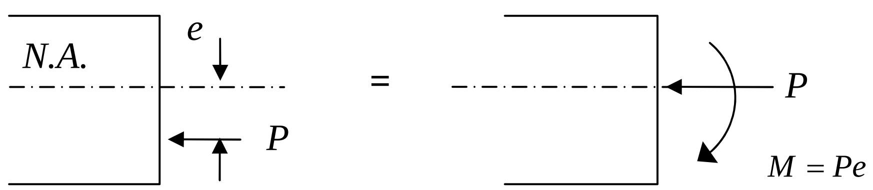
Also, we will normally **take Z_b to be negative** to simplify the signs.

Case III

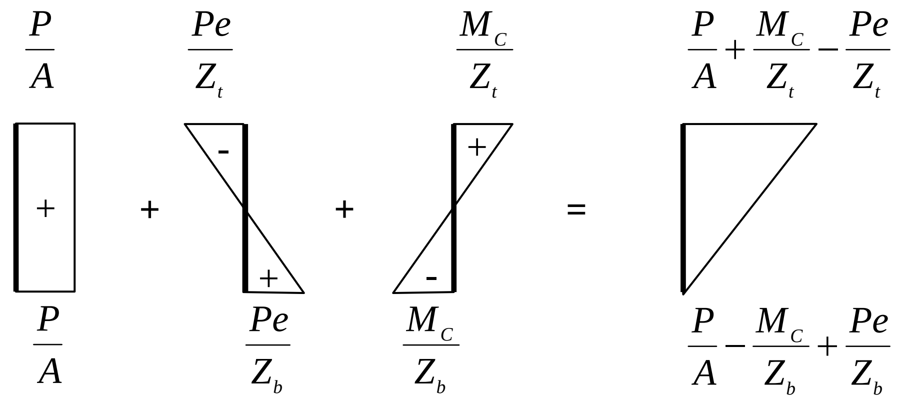
In this case we place the prestress force at an eccentricity:



Using an equilibrium set of forces as shown, we now have three stresses acting on the section:



Thus the stresses are:



Hence, for failure we now have:

$$\frac{M_C}{Z_b}=\frac{P}{A}+\frac{Pe}{Z_b}$$

$$w_{III}=\frac{8Z_b}{L^2}\left(\frac{P}{A}+\frac{Pe}{Z_b}\right)$$

If, for example, we take $e=\frac{d}{6}$, then:

$$w_{III}=\frac{8Z_b}{L^2}\left(\frac{P}{A}+\frac{P\,d/6}{bd^2/6}\right)=\frac{16Z_b}{L^2}\frac{P}{A}=2\times w_{II}$$

So the introduction of a small eccentricity has *doubled* the allowable service load.

Numerical Example – No Eccentricity

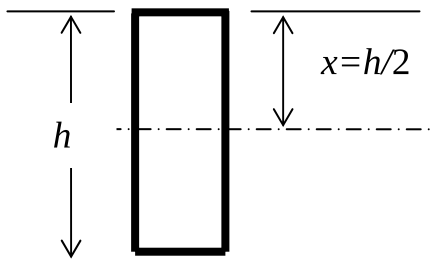
Prestress force (at transfer), P = 2500 kN. Losses between transfer and SLS = 20%. Check stresses. Permissible stresses are:

f_{tt} = -1 N/mm^2 f_{tc} = 18 N/mm^2

f_{st} = 0 N/mm^2 f_{sc} = 22 N/mm^2

The section is rectangular, 300 wide and 650 mm deep. It is simply supported spanning 12 m with dead load equal to self weight and a live load of 6 kN/m (unfactored). The prestress force is applied at the centroid.

First calculate the section properties for a 300×650 beam:



$$A = 300\times650 = 195\,000 \text{ mm}^2$$

Second moment of area, I, is $bh^3/12$:

$$I = 300\times650^3/12 = 6866\times10^6 \text{ mm}^4$$

Section modulus for the top fibre, Z_t, is I/x. For a rectangular section 650 mm deep, the centroid is at the centre and this is:

$$Z_t = 6866\times10^6/325 = 21.12\times10^6 \text{ mm}^3$$

(some people use the formula for rectangular sections, $Z_t = bh^2/6$ which gives the same answer).

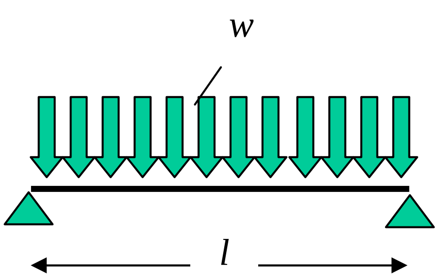
Similarly, Z_b = -21.12×10^6 mm^3 (sign convention: Z_b is always negative as the measurement to the bottom fibre is negative).

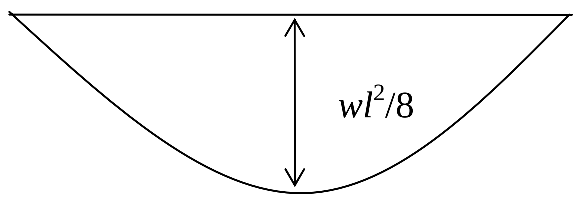
The only applied loading at transfer is the self weight which is (density of concrete) × (area). Hence:

$$\text{self weight} = 25(0.3 \times 0.65) = 4.88 \text{ kN/m}$$

The maximum moment due to this loading is:

$$\text{transfer moment, } M_t = 4.88(12)^2/8 = 87.8 \text{ kNm}$$

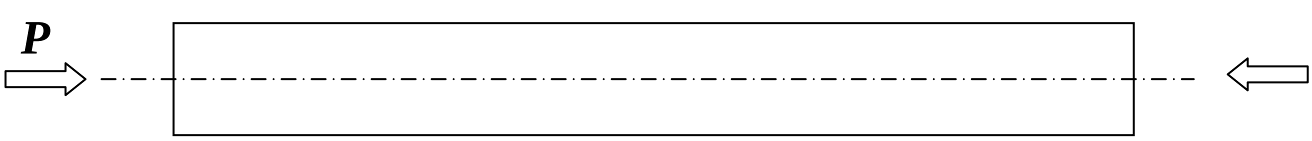


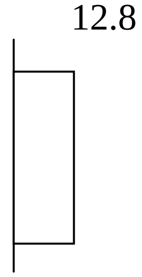


The total loading at SLS is this plus the imposed loading, i.e.:

$$\text{SLS moment, } M_s = (4.88 + 6)(12)^2/8$$
$$= 195.8 \text{ kNm}$$

The prestress causes an axial stress of P/A = 2500×10^3 / 195 000 = 12.8 N/mm^2:

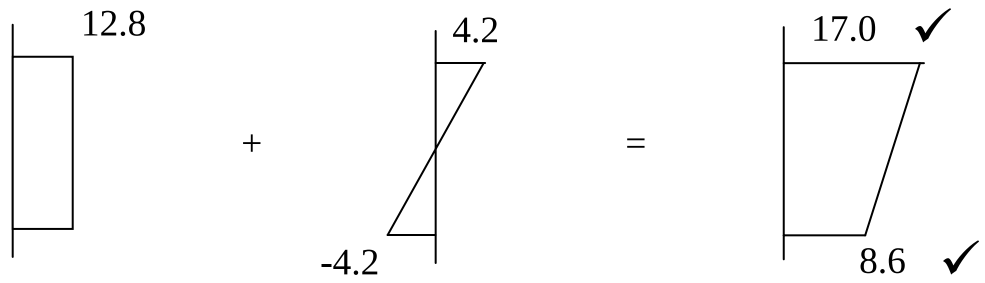




At transfer, the stress due to prestress applies and, after the beam is lifted, the stress due to self weight. The self weight moment at the centre generates a top stress of:

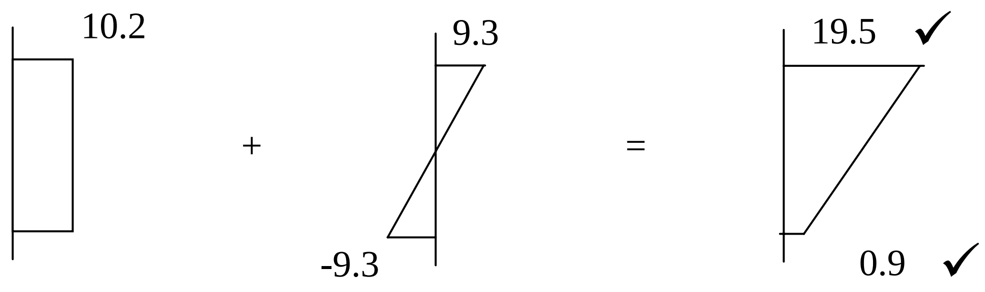
$M_t/Z_t = 87.8\times10^6 / 21.12\times10^6 = 4.2$ N/mm^2.

Hence the transfer check at the centre is:



Prestress Self Weight Total

At SLS, the prestress has reduced by 20%. The top and bottom stresses due to applied load (M_s) are $\pm195.8\times10^6/21.12\times10^6 = \pm9.3$ N/mm^2. Hence the SLS check is:

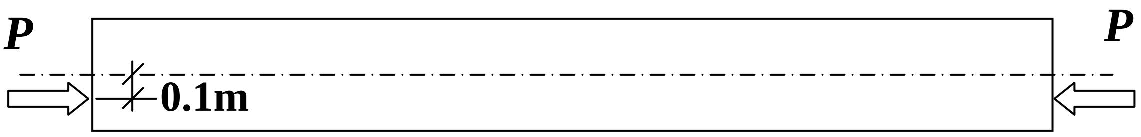


Prestress Self Weight Total

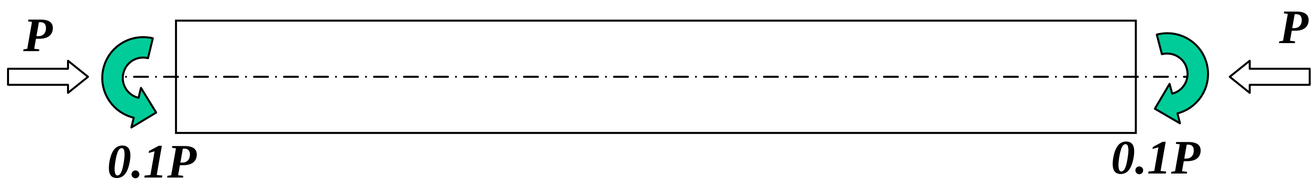
Numerical Example – With Eccentricity

As per the previous example, but the prestress force is P = 1500 kN at 100 mm below the centroid.

An eccentric force is equivalent to a force at the centroid plus a moment of force × eccentricity:



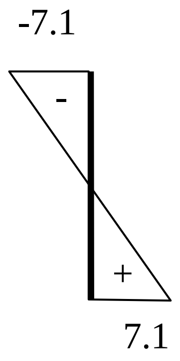
This is equivalent to:



Hence the distribution of stress due to prestress at transfer is made up of 2 components:

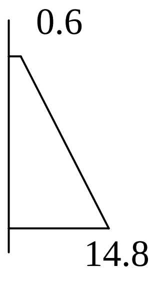
$P/A = 1500\times10^3 / 195\ 000 = 7.7$ N/mm^2

And + *Pe/Z*. At top fibre, this is $-\dfrac{(1500\times10^{3})(100)}{21.12\times10^{6}}$ = -7.1

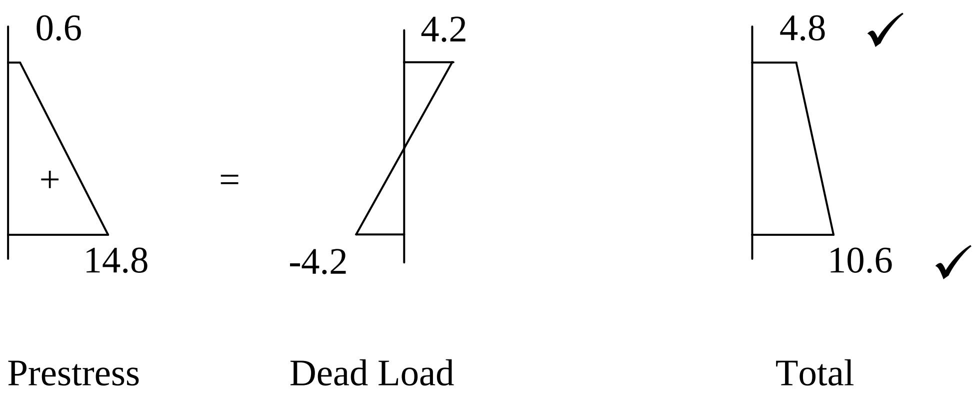


As the moment is hog, the stress at the top is tension, i.e., negative. Similarly the stress at the bottom fibre is +7.1

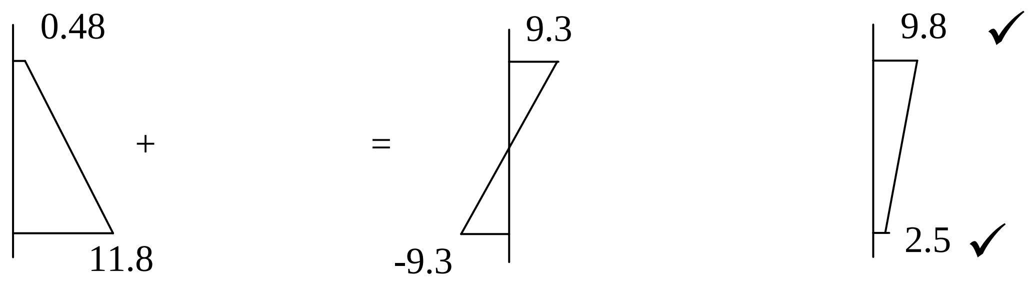
Hence the total distribution of stress due to PS is:



Hence the transfer check is:



At SLS, the prestress has reduced by 20% (both the *P/A* and the *Pe/Z* components are reduced by 20% as *P* has reduced by that amount). The stress distribution due to applied load is as for Example 1. Hence the SLS check is:



Prestress Applied load Total

3. Design of PSC Members

3.1 Basis

Sign Convention

In order to derive equations that enable design, we maintain a rigid sign convention:

- Moment:

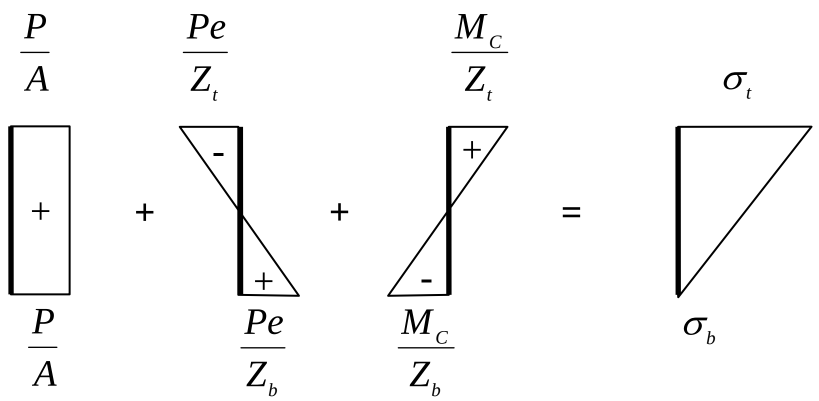
  Positive sag;
- Eccentricity of prestress:

  Positive *above* centroid;
- Section Modulus:

  Negative for the bottom of the member;
- Stress:

  Positive compression.

With this sign convention, we now have:



Thus the final stresses are numerically given by:

Top fibre: $$\sigma_t = \frac{P}{A} + \frac{Pe}{Z_t} + \frac{M_C}{Z_t}$$

Bottom fibre: $$\sigma_b = \frac{P}{A} + \frac{Pe}{Z_b} + \frac{M_C}{Z_b}$$

Note that the sign convention means that:

- the P/A terms is always positive;
- the M_C/Z term is positive or negative depending on whether it is Z_t or Z_b, and;
- the Pe/Z term is negative for Z_t since Z_t is positive and *e* is negative and the term is positive for Z_b since now both Z_b and *e* are negative.

These signs of course match the above diagrams, as they should.

Governing Inequalities

Given the rigid sign convention and the allowable stresses in the concrete, and noting that the losses are to be taken into account, the stresses are limited as:

Transfer

Top fibre – stress must be bigger than the minimum allowable tensile stress:

$$\begin{gathered} \sigma_t \geq f_{tt} \\ \frac{P}{A} + \frac{Pe}{Z_t} + \frac{M_t}{Z_t} \geq f_{tt} \end{gathered} \tag{1}$$

Bottom fibre – stress must be less than the maximum allowable compressive stress:

$$\sigma_b \le f_{tc}$$
$$\frac{P}{A} + \frac{Pe}{Z_b} + \frac{M_t}{Z_b} \le f_{tc} \tag{2}$$

Service

Top fibre – stress must be less than the maximum allowable compressive stress:

$$\sigma_t \le f_{sc}$$
$$\alpha\left(\frac{P}{A} + \frac{Pe}{Z_t}\right) + \frac{M_s}{Z_t} \le f_{sc} \tag{3}$$

Bottom fibre – stress must be bigger than the minimum allowable tensile stress:

$$\sigma_b \ge f_{st}$$
$$\alpha\left(\frac{P}{A} + \frac{Pe}{Z_b}\right) + \frac{M_s}{Z_b} \ge f_{st} \tag{4}$$

In these equations it must be remembered that numerically, any allowable tension is a negative quantity. Therefore all permissible stresses must be greater than this allowable tension, that is, ideally a positive number indicating the member is in compression at the fibre under consideration. Similarly, all stresses must be less than the allowable compressive stress.

3.2 Minimum Section Modulus

Given a blank piece of paper, it is difficult to check stresses. Therefore we use the governing inequalities to help us calculate minimum section modulii for the expected moments. This is the first step in the PSC design process.

Top Fibre

The top fibre stresses must meet the criteria of equations (1) and (3). Hence, from equation (1):

$$\frac{P}{A}+\frac{Pe}{Z_t}+\frac{M_t}{Z_t}\geq f_{tt}$$

$$\alpha\left(\frac{P}{A}+\frac{Pe}{Z_t}\right)+\alpha\frac{M_t}{Z_t}\geq\alpha f_{tt}$$

$$-\left[\alpha\left(\frac{P}{A}+\frac{Pe}{Z_t}\right)+\alpha\frac{M_t}{Z_t}\right]\leq-\alpha f_{tt}$$

If we now add this to equation (3):

$$\left[\alpha\left(\frac{P}{A}+\frac{Pe}{Z_t}\right)+\frac{M_s}{Z_t}\right]-\left[\alpha\left(\frac{P}{A}+\frac{Pe}{Z_t}\right)+\alpha\frac{M_t}{Z_t}\right]\leq f_{sc}-\alpha f_{tt}$$

$$\frac{M_s}{Z_t}-\alpha\frac{M_t}{Z_t}\leq f_{sc}-\alpha f_{tt}$$

Hence:

$$\boxed{Z_t\geq\frac{M_s-\alpha M_t}{f_{sc}-\alpha f_{tt}}} \qquad (5)$$

Bottom Fibre

The bottom fibre stresses must meet equations (2) and (4). Thus, from equation (2):

$$\frac{P}{A}+\frac{Pe}{Z_b}+\frac{M_t}{Z_b}\le f_{tc}$$

$$\alpha\left(\frac{P}{A}+\frac{Pe}{Z_b}\right)+\alpha\frac{M_t}{Z_b}\le\alpha f_{tc}$$

$$-\left[\alpha\left(\frac{P}{A}+\frac{Pe}{Z_b}\right)+\alpha\frac{M_t}{Z_b}\right]\ge-\alpha f_{tc}$$

Adding equation (4) to this:

$$\left[\alpha\left(\frac{P}{A}+\frac{Pe}{Z_b}\right)+\frac{M_s}{Z_b}\right]-\left[\alpha\left(\frac{P}{A}+\frac{Pe}{Z_b}\right)+\alpha\frac{M_t}{Z_b}\right]\ge f_{st}-\alpha f_{tc}$$

$$\frac{M_s}{Z_b}-\alpha\frac{M_t}{Z_b}\ge f_{st}-\alpha f_{tc}$$

Hence:

$$\boxed{Z_b\ge\frac{M_s-\alpha M_t}{\alpha f_{tc}-f_{st}} \qquad (6)}$$

Note that in these developments the transfer moment is required. However, this is a function of the self weight of the section which is unknown at this point. Therefore a trial section or a reasonable self weight must be assumed initially and then checked once a section has been decided upon giving the actual Z_t and Z_b values.

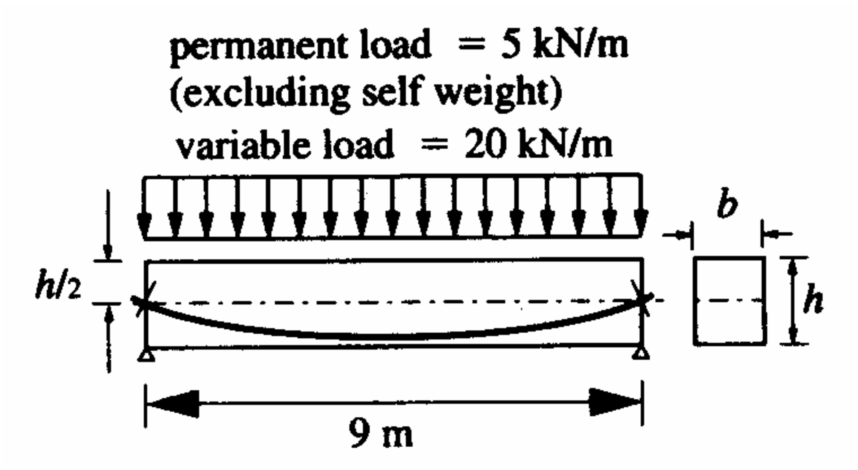
Example

Problem

The single-span, simply-supported beam shown below carried the loads as shown. Taking the losses to be 25% and:

- permissible tensile stresses are 2.5 N/mm^2 at transfer and 2.0 N/mm^2 in service;
- permissible compressive stresses are 20 N/mm^2 at transfer and at service.

Determine an appropriate rectangular section for the member taking the density of prestressed concrete to be 25 kN/m^3.



Solution

We will check the requirements at the critical section which is at mid-span. To determine an approximate initial transfer moment, M_t, we estimate the section depth based on a span depth ratio of, say, 15:

$$d_{trial} = \frac{9000}{15} = 600 \text{ mm}$$

Try this with a width of 250 mm, which is a practical minimum. Hence, the sefl weight load is:

$$w_{sw} = 25(0.6 \times 0.25) = 3.75 \text{ kN/m}$$

At transfer only self weight is present. Therefore:

$$M_t = \frac{wL^2}{8} = \frac{3.75 \times 9^2}{8} = 38 \text{ kNm}$$

At service the moment is:

$$M_s = \frac{wL^2}{8} = \frac{[3.75 + 5 + 20] \times 9^2}{8} = 291 \text{ kNm}$$

Check the section modulus at the top fibre:

$$Z_t \geq \frac{M_s - \alpha M_t}{f_{sc} - \alpha f_{tt}}$$

$$Z_t \geq \frac{291 - 0.75 \times 38}{20 - 0.75(-2.5)}$$

$$Z_t \geq 12.0 \times 10^6 \text{ mm}^3$$

And at the bottom:

$$Z_b \geq \frac{M_s - \alpha M_t}{\alpha f_{tc} - f_{st}}$$

$$Z_b \geq \frac{291 - 0.75 \times 38}{0.75(20) - (-2.0)}$$

$$Z_b \geq 15.4 \times 10^6 \text{ mm}^3$$

If the section is to be rectangular, then $Z_b = Z_t$ and so the requirement for Z_b governs:

$$Z = \frac{bh^2}{6} \geq 15.4 \times 10^6$$

Keeping the 250 mm width:

$$\begin{aligned} &\frac{250h^2}{6} \geq 15.4 \times 10^6 \\ &h \geq \sqrt{\frac{6\left(15.4 \times 10^6\right)}{250}} \\ &h \geq 609 \text{ mm} \end{aligned}$$

Thus adopt a 250 mm × 650 mm section.

Note that this changes the self weight and so the calculations need to be performed again to verify that the section is adequate. However, the increase in self weight is offset by the larger section depth and hence larger section modulii which helps reduce stresses. These two effects just about cancel each other out.

Verify This

3.3 Prestressing Force & Eccentricity

Once the actual Z_t and Z_b have been determined, the next step is to determine what combination of prestress force, *P* and eccentricity, *e*, to use at that section. Taking each stress limit in turn:

Tensile Stress at Transfer

Taking the governing equation for tensile stress at transfer, equation (1), we have:

$$\frac{P}{A} + \frac{Pe}{Z_t} + \frac{M_t}{Z_t} \geq f_{tt}$$

$$P\left(\frac{1}{A} + \frac{e}{Z_t}\right) \geq f_{tt} - \frac{M_t}{Z_t}$$

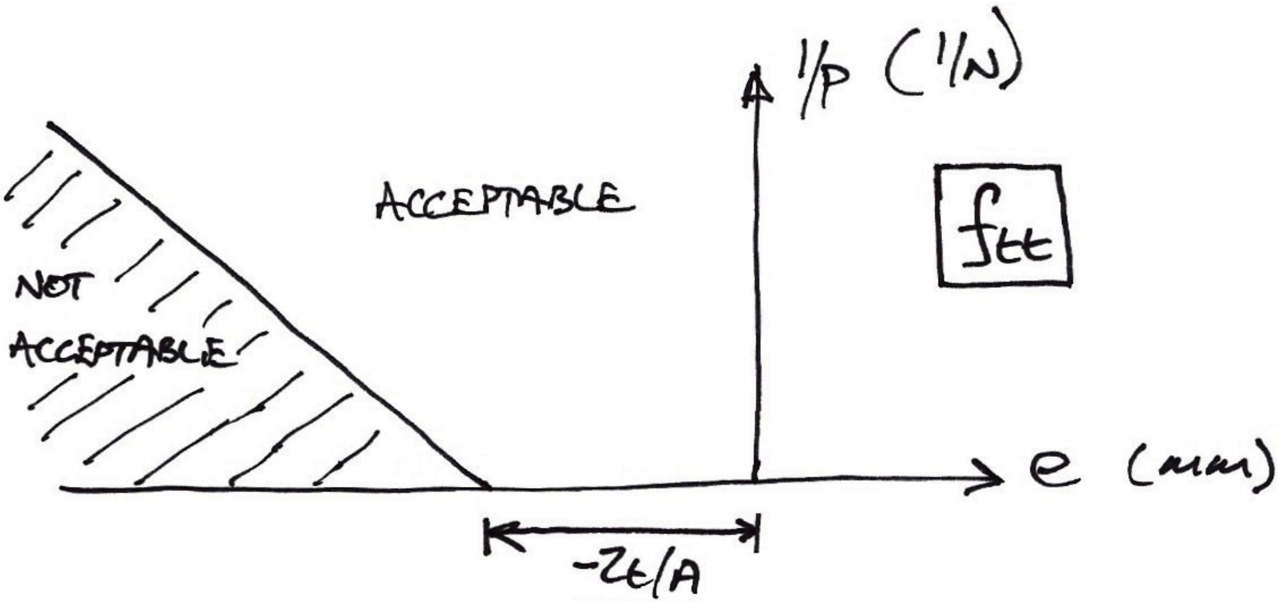
$$P \leq \frac{f_{tt} - M_t/Z_t}{1/A + e/Z_t} \quad \text{since } e/Z_t \text{ is negative}$$

$$\frac{1}{P} \geq \frac{1/A + e/Z_t}{f_{tt} - M_t/Z_t}$$

Hence:

$$\frac{1}{P} \geq \left[\frac{1/Z_t}{f_{tt} - M_t/Z_t}\right] e + \left[\frac{1/A}{f_{tt} - M_t/Z_t}\right] \qquad (7)$$

This is a linear equation in $1/P$ and *e*. Therefore a plot of these two quantities will give a region that is acceptable and a region that is not acceptable, according to the inequality.


1/P (1/N)
ACCEPTABLE
NOT
ACCEPTABLE
ftt
e (mm)
-Zt/A

Compressive Stress at Transfer

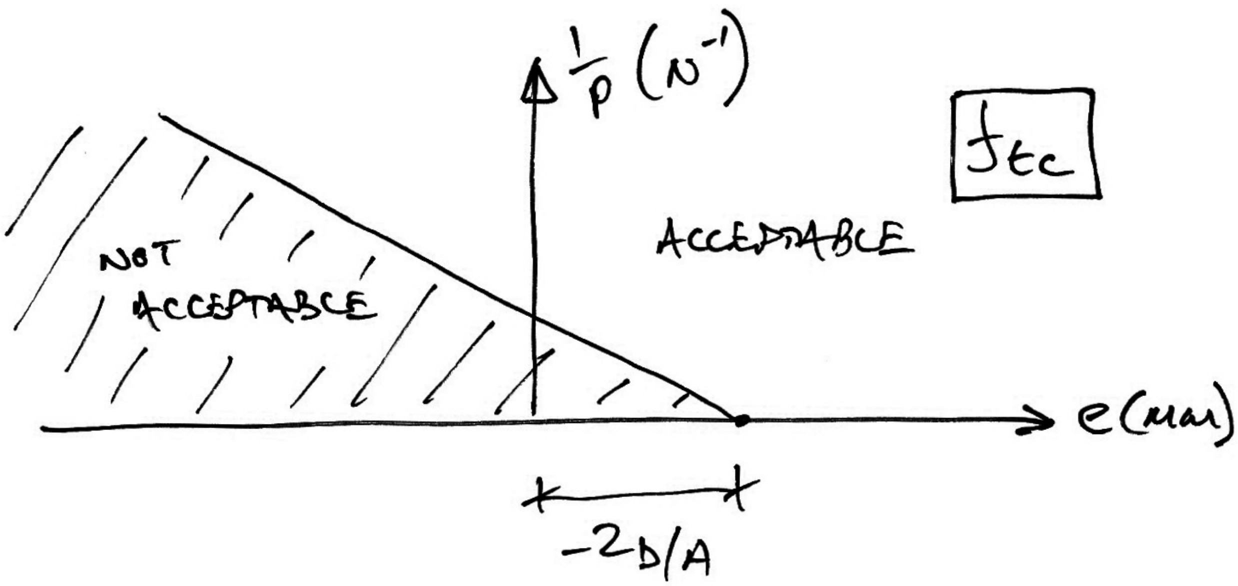
Based on equation (2) which governs for compression at transfer, we have:

$$P\left(\frac{1}{A}+\frac{e}{Z_b}\right) \le f_{tc} - \frac{M_t}{Z_b}$$

Which leads to:

$$\frac{1}{P} \ge \left[\frac{1/Z_b}{f_{tc} - M_t/Z_b}\right] e + \left[\frac{1/A}{f_{tc} - M_t/Z_b}\right] \tag{8}$$

Another linear equation which gives the feasible region graphed as:



Compressive Stress in Service

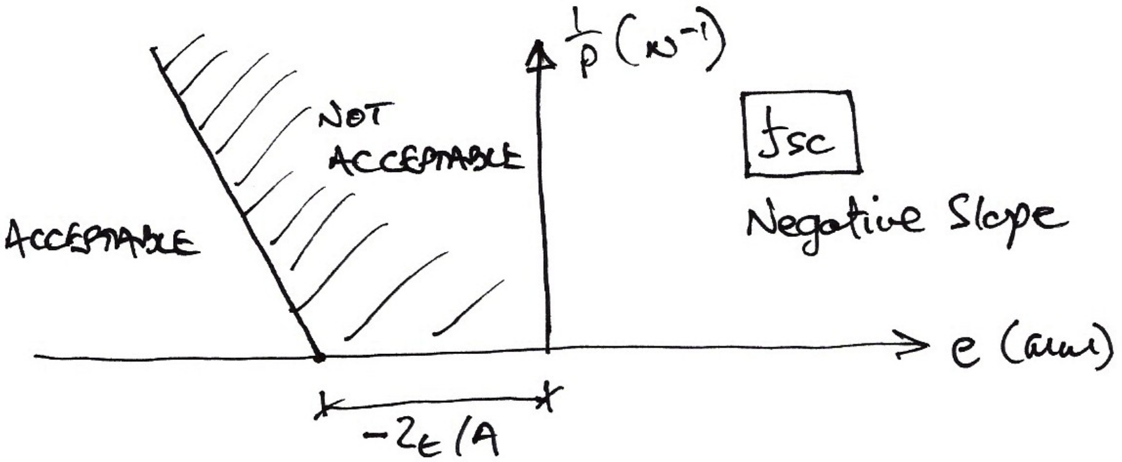
Equation (3) governs for compression in service and so we have:

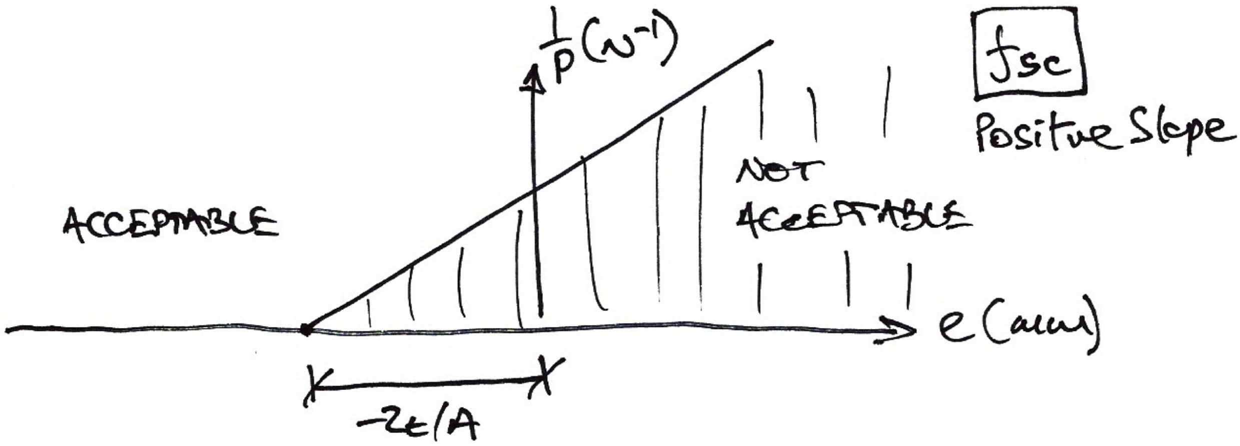
$$\alpha P\left(\frac{1}{A}+\frac{e}{Z_t}\right)\leq f_{sc}-\frac{M_s}{Z_t}$$

From which we get another linear equation in $1/P$ and *e*:

$$\frac{1}{P}\leq\left[\frac{\alpha/Z_t}{f_{sc}-M_s/Z_t}\right]e+\left[\frac{\alpha/A}{f_{sc}-M_s/Z_t}\right] \qquad (9)$$

This equation can again be graphed to show the feasible region. However, this line can have a positive or negative slope. When the slope is negative, $1/P$ must be under the line; when the slope is positive, $1/P$ must be over the line. A simple way to remember this is that the origin is always not feasible. Both possible graphs are:


1/P (N^-1)
fsc
Negative Slope
NOT
ACCEPTABLE
ACCEPTABLE
e (mm)
-Zt/A


1/P (N^-1)
fsc
Positive Slope
NOT
ACCEPTABLE
ACCEPTABLE
e (mm)
-Zt/A

Tensile Stress in Service

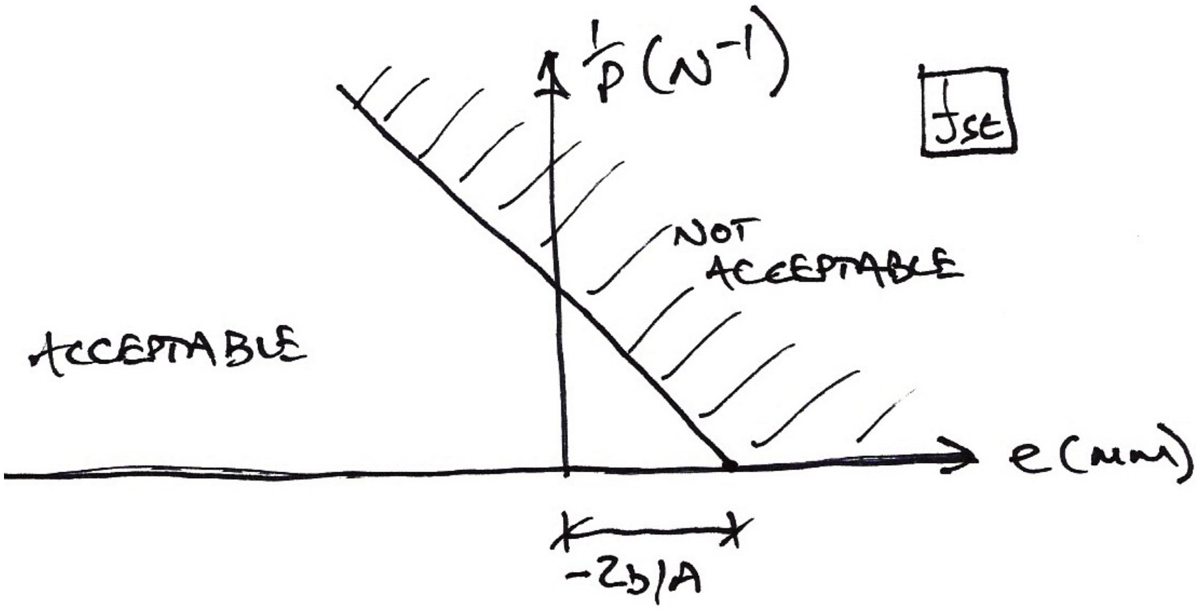
Lastly, we have equation (4) which governs for tension at the bottom fibre during service. This gives:

$$\alpha P\left(\frac{1}{A}+\frac{e}{Z_b}\right)\geq f_{st}-\frac{M_s}{Z_b}$$

Which gives:

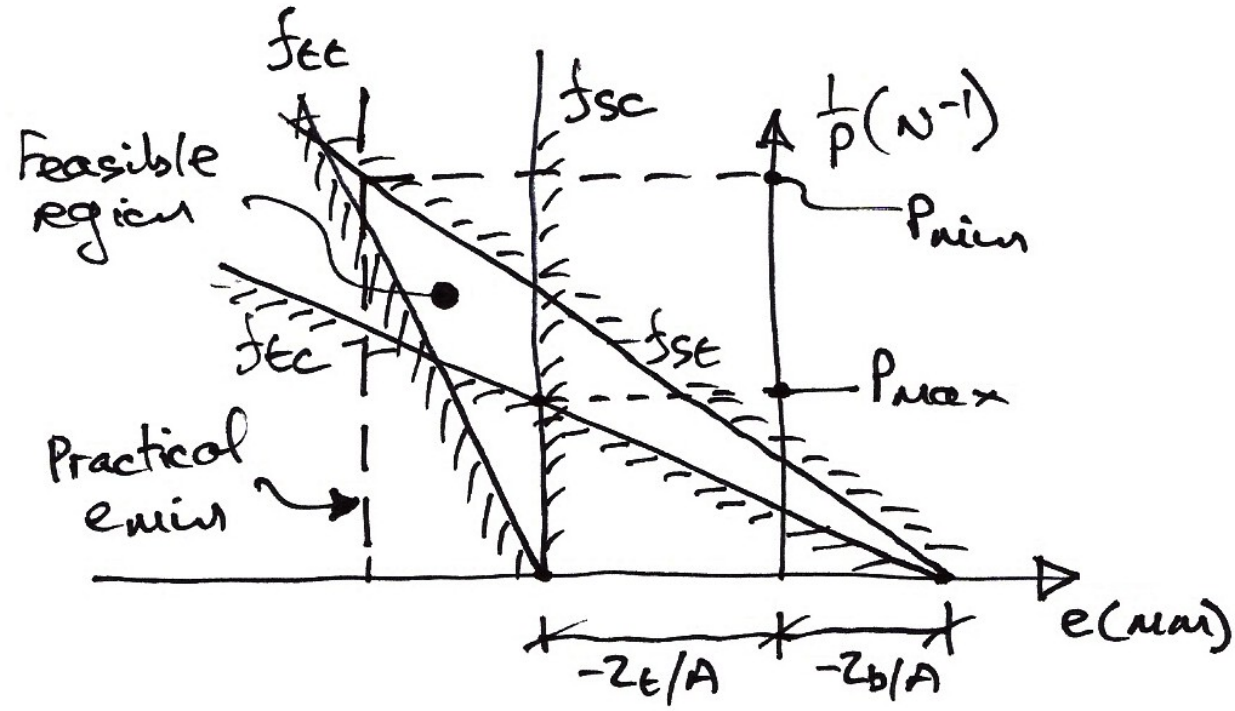
$$\frac{1}{P}\leq\left[\frac{\alpha/Z_b}{f_{st}-M_s/Z_b}\right]e+\left[\frac{\alpha/A}{f_{st}-M_s/Z_b}\right] \qquad (10)$$

And this is graphed to show the feasible region:



Magnel Diagram

A Magnel Diagram is a plot of the four lines associated with the limits on stress. As can be seen, when these four equations are plotted, a feasible region is found in which points of $1/P$ and e simultaneously satisfy all four equations. Any such point then satisfies all four stress limits.



Added to the basic diagram is the maximum possible eccentricity – governed by the depth of the section minus cover and ordinary reinforcement – along with the maximum and minimum allowable prestressing forces. For economy we usually try to use a prestressing force close to the minimum.

The geometric quantities Z_t/A and Z_b/A are known as the **upper and lower kerns** respectively. They will feature in laying out the tendons.

Example

Problem

A beam, 200 mm wide × 350 mm deep spans 10 m and carries 3 kN/m live load in addition to its dead weight. The concrete is grade 40 and the prestress is transferred at 32 N/mm^2 cube strength. Take prestress losses to be 20%. The member is to be a Class 1 member.

Draw the Magnel diagram for the mid-span section and determine an economic prestressing force and its corresponding eccentricity.

Solution

The self weight is:

$$w_{sw} = 25\left(0.2 \times 0.35\right) = 1.75 \text{ kN/m}$$

The section properties are:

$$A = bh = 200 \times 350 = 70 \times 10^3 \text{ mm}^2$$

$$Z_t = Z_b = \frac{bh^2}{6} = \frac{200 \times 350^2}{6} = 4.083 \times 10^6 \text{ mm}^3$$

At transfer only self weight is present. Therefore:

$$M_t = \frac{wL^2}{8} = \frac{1.75 \times 10^2}{8} = 21.9 \text{ kNm}$$

At service the moment is:

$$M_s = \frac{wL^2}{8} = \frac{[1.75+3]\times 10^2}{8} = 59.4 \text{ kNm}$$

The allowable stresses are, for a Class 1 member, from the previous table:

$$f_{tt} = -1 \text{ N/mm}^2 \qquad f_{tc} = 0.5\times 32 = 16 \text{ N/mm}^2$$
$$f_{st} = 0 \text{ N/mm}^2 \qquad f_{sc} = 0.33\times 40 = 13.33 \text{ N/mm}^2$$

Next we determine the equations of the four lines:

- f_{tt}: The denominator stress is:

$$\sigma_{tt} = f_{tt} - M_t/Z_t = -1 - 21.9/4.083 = -6.37 \text{ N/mm}^2$$

Hence:

$$\frac{1}{P} \geq \left[\frac{1/4.083\times 10^6}{-6.37}\right]e + \left[\frac{1/70\times 10^3}{-6.37}\right]$$
$$\frac{10^6}{P} \geq -0.0385e - 2.243$$

- f_{tc}: The denominator stress is:

$$\sigma_{tc} = f_{tc} - M_t/Z_b = 16 - 21.9/-4.083 = 21.37 \text{ N/mm}^2$$

Hence:

$$\frac{1}{P} \geq \left[\frac{1/-4.083\times10^{6}}{21.37}\right]e + \left[\frac{1/70\times10^{3}}{21.37}\right]$$
$$\frac{10^{6}}{P} \geq -0.0115e + 0.668$$

- f_{tc}: The denominator stress is:

$$\sigma_{sc} = f_{sc} - M_s/Z_t = 13.33 - 59.4/4.083 = -1.24 \text{ N/mm}^2$$

Hence:

$$\frac{1}{P} \leq \left[\frac{0.8/4.083\times10^{6}}{-1.24}\right]e + \left[\frac{0.8/70\times10^{3}}{-1.24}\right]$$
$$\frac{10^{6}}{P} \leq -0.158e - 9.21$$

- f_{st}: The denominator stress is:

$$\sigma_{st} = f_{st} - M_s/Z_b = 0 - 59.4/-4.083 = 14.57 \text{ N/mm}^2$$

Hence:

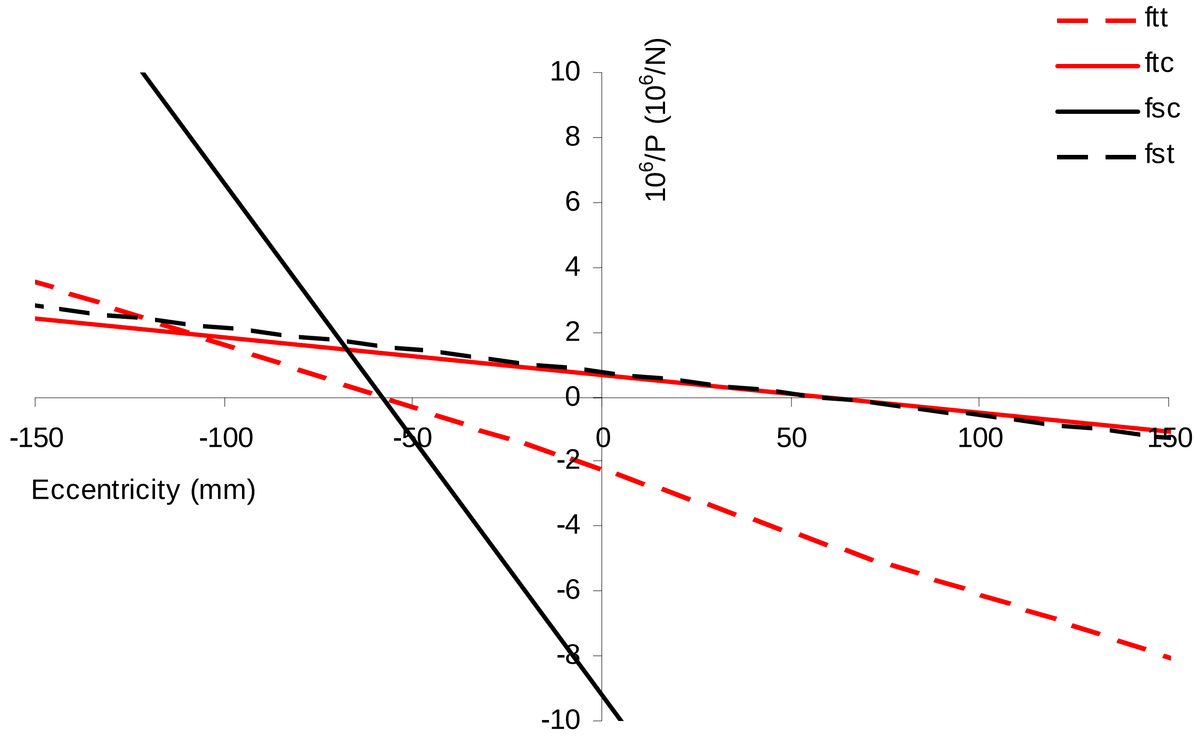
$$\frac{1}{P} \leq \left[\frac{0.8/-4.083\times10^{6}}{14.57}\right]e + \left[\frac{0.8/70\times10^{3}}{14.57}\right]$$
$$\frac{10^{6}}{P} \leq -0.00135e + 0.784$$

Notice that the 10^6 has been brought to the left so that we are working with larger numbers on the right hand sides. To plot these lines, note that the vertical intercepts are known and the *x*-axis values are also known to be:

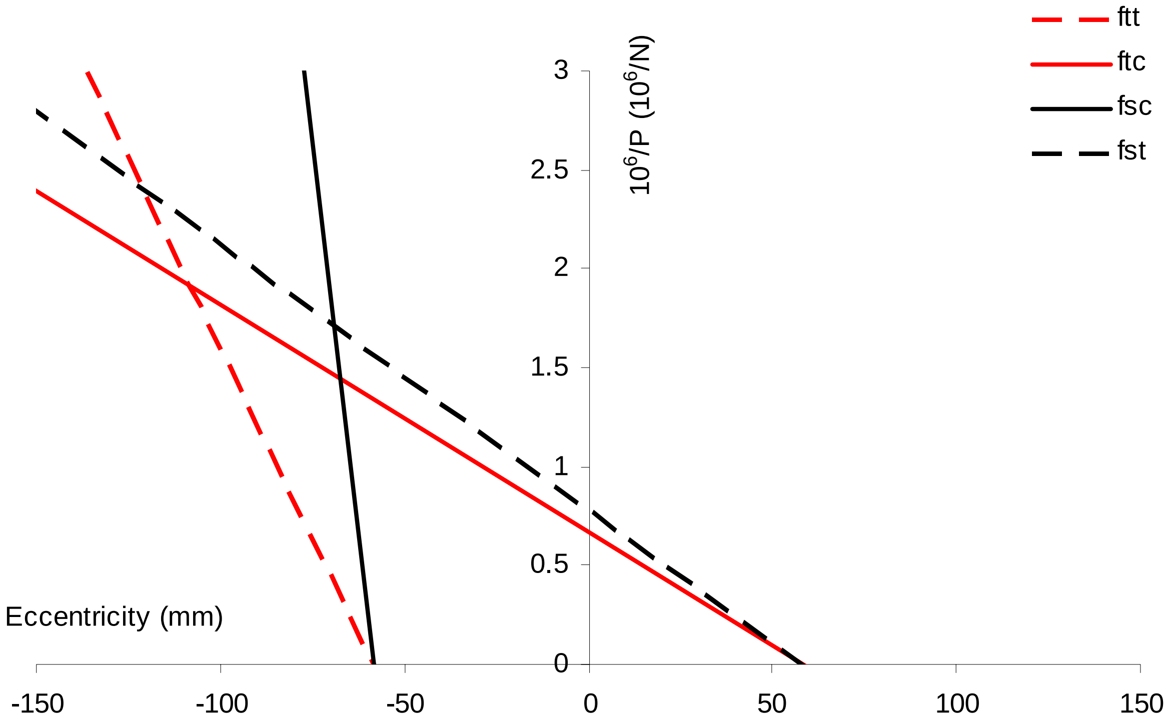
$$\frac{-Z_t}{A} = \frac{-4.083\times10^6}{70\times10^3} = -58.33 \text{ mm}$$

$$\frac{-Z_b}{A} = \frac{-\left(-4.083\times10^6\right)}{70\times10^3} = +58.33 \text{ mm}$$

We plot the lines using these sets of points on each of the axes:



For clarity in these notes, we zoom into the area of interest:



So from this figure, the minimum prestressing is the highest point in the region (or maximum *y*-axis value) permissible, which is about 2.4. Hence:

$$\frac{10^6}{P} = 2.4$$

$$P = \frac{10^6}{2.4}$$

$$= 416 \times 10^3 \text{ N}$$

$$= 416 \text{ kN}$$

The corresponding eccentricity is about 120 mm below the centroidal axis.

Obviously these values can be worked out algebraically, however such exactitude is not necessary as prestress can only be applied in multiples of tendon force and eccentricities are to the nearest 10 mm practically.

3.4 Eccentricity Limits and Tendon Profile

Up to now we have only considered the critical location as defined by the position of the maximum moment: the mid-span of the simply supported beam. However, the governing inequalities (i.e. the stress limits) must apply at every section along the beam.

Thus far we have established an appropriate section and chosen a value for P, which remains essentially constant along the length of the span. We have also found a value for the eccentricity at the critical location.

Since the moments change along the length of a beam, we must change some aspect of the prestress also. The only remaining variable is the eccentricity. As we wish to limit possible tensile stresses, we only examine equations (1) and (4) corresponding to tension on the top at transfer and tension on the bottom in service.

Tension on Top at Transfer

We determine an expression for the eccentricity in terms of the other knowns:

$$\frac{P}{A} + \frac{Pe}{Z_t} + \frac{M_t}{Z_t} \geq f_{tt}$$

$$e \geq \frac{Z_t f_{tt}}{P} - \left(\frac{Z_t}{A} + \frac{M_t}{P} \right)$$

Hence we have a lower limit for the eccentricity as:

$$\boxed{e_{lower} \geq -\frac{Z_t}{A} + \frac{Z_t f_{tt} - M_t}{P} \qquad (11)}$$

Tension on the Bottom in Service

Using equation (4):

$$\alpha\left(\frac{P}{A}+\frac{Pe}{Z_b}\right)+\frac{M_s}{Z_b}\geq f_{st}$$

$$e\geq\frac{Z_b f_{st}}{\alpha P}-\left(\frac{Z_b}{A}+\frac{M_s}{\alpha P}\right)$$

Hence we have an upper limit for the eccentricity as:

$$e_{upper}\leq-\frac{Z_b}{A}+\frac{Z_b f_{st}-M_s}{\alpha P} \tag{12}$$

Since the upper and lower kerns (the Z_t/A and Z_b/A quantities) are constant for constant geometry, and since P is also constant, it can be seen that the cable limits follow the same profile as the bending moment diagrams at transfer and in service.

Example

Problem

For the beam of the previous example, sketch the upper and lower eccentricities given a prestress of 450 kN.

Solution

In the previous example, the kerns were found to be

$$\frac{-Z_t}{A} = -58.33 \text{ mm}$$
$$\frac{-Z_b}{A} = +58.33 \text{ mm}$$

The ‘tension moments’ are:

$$M_{tt} = Z_t f_{tt} = \left(4.083\times10^6\right)\left(-1\right) = -4.083 \text{ kNm}$$
$$M_{st} = Z_b f_{st} = \left(-4.083\times10^6\right)\left(0\right) = 0 \text{ kNm}$$

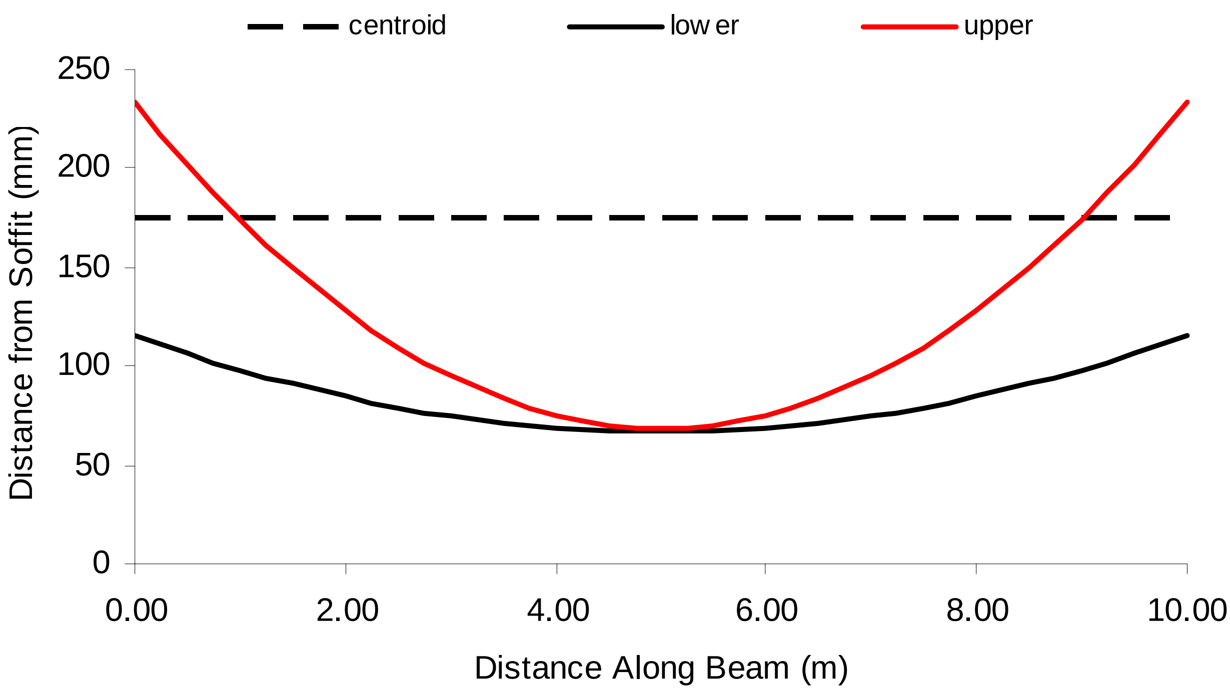
Hence for the lower limit we have:

$$\begin{aligned} e_{lower} &= -\frac{Z_t}{A} + \frac{Z_t f_{tt} - M_t}{P} \\ &= -58.33 + \frac{-4.083 - M_{t,x}}{450} \\ &= -58.33 - 1.02 - \frac{M_{t,x}}{450} \\ &= -59.35 - \frac{M_{t,x}}{450} \end{aligned}$$

And for the upper:

$$\begin{aligned} e_{upper} &= -\frac{Z_b}{A} + \frac{Z_b f_{st} - M_s}{\alpha P} \\ &= 58.33 + \frac{0 - M_{s,x}}{0.8 \times 450} \\ &= 58.33 - \frac{M_{s,x}}{360} \end{aligned}$$

Knowing the values of the bending moments at positions along the beam, we can now plot the eccentricity limits:

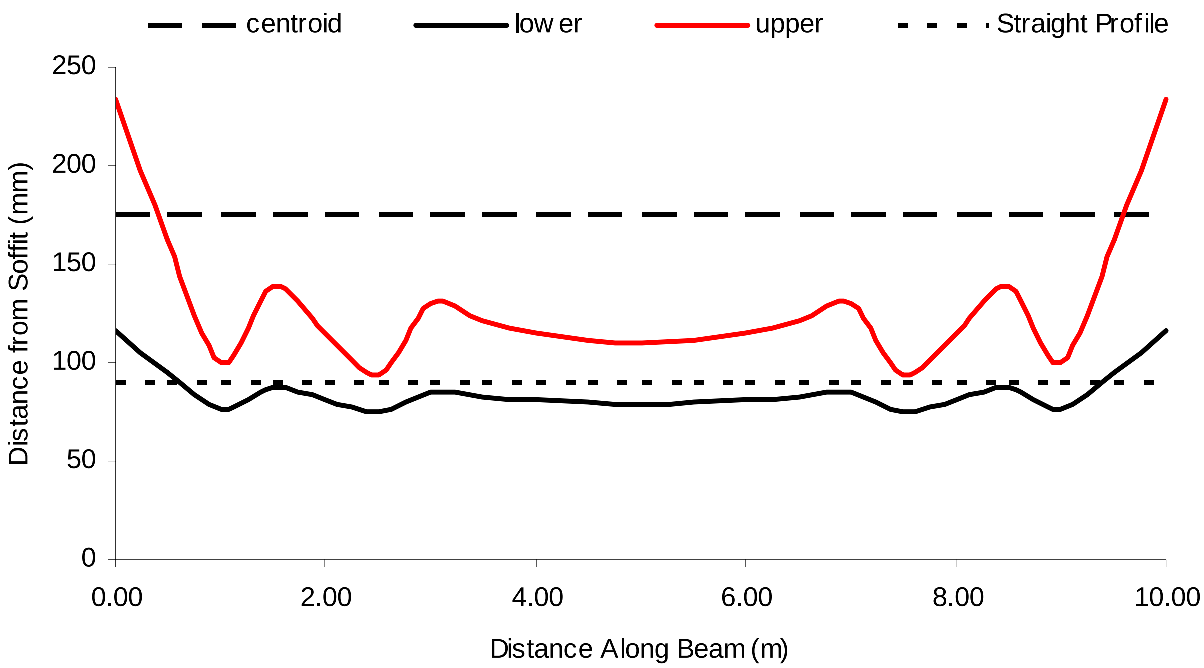


In this figure the upper and lower eccentricities are very close at the critical section. This is because the chosen prestress is very close to the minimum possible from the permissible region. A higher prestress would give 'looser' limits.

Debonding

Notice from the eccentricity limits of the previous example that a straight cable/tendon profile will not work. Yet we know from the discussion on factory precasting of prestressed sections that straight tendons are preferable.

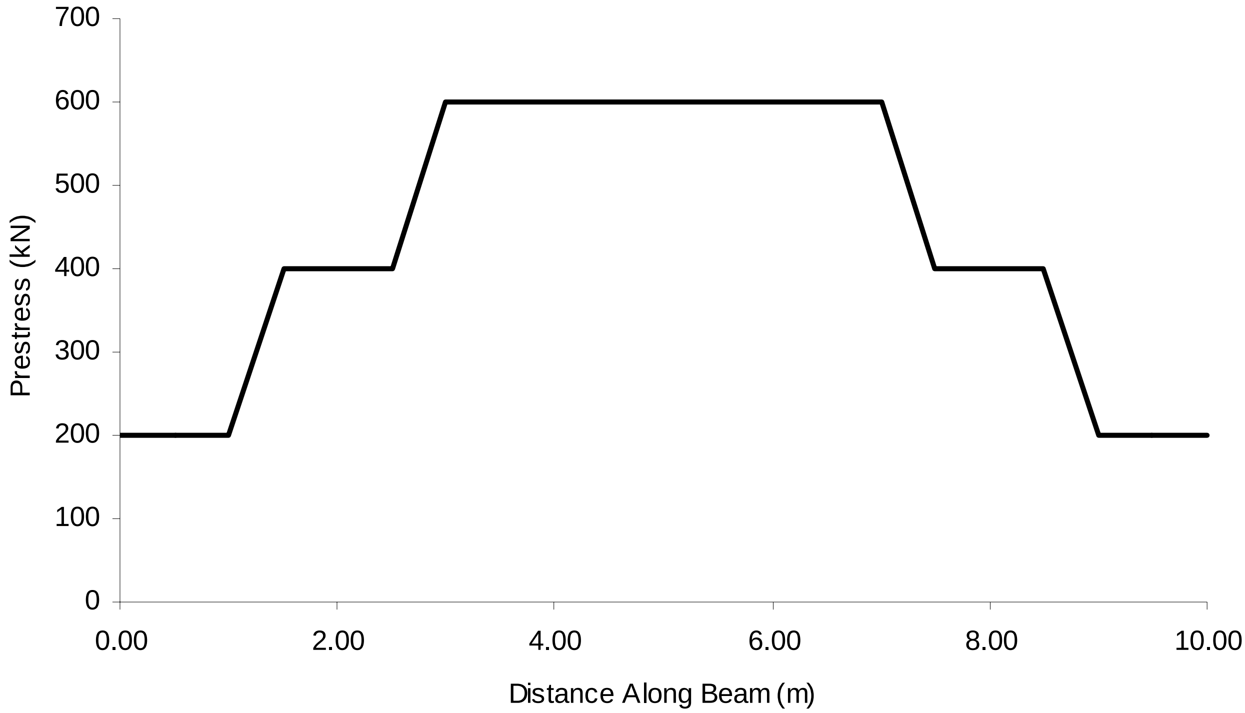
By changing the prestress force along the length of the beam, we can find a solution to this problem. From the previous example the new eccentricity limits are:



It can be seen that an eccentricity of 85 mm below the centroidal axis works.

Note that it appears that there remains a problem at the ends of the beam. However, the cable force gradually increases in reality due to the transmission zone (i.e. the bond between the tendon and concrete). Hence the stresses will be acceptable in this zone also.

The straight profile required the following variation of prestress:



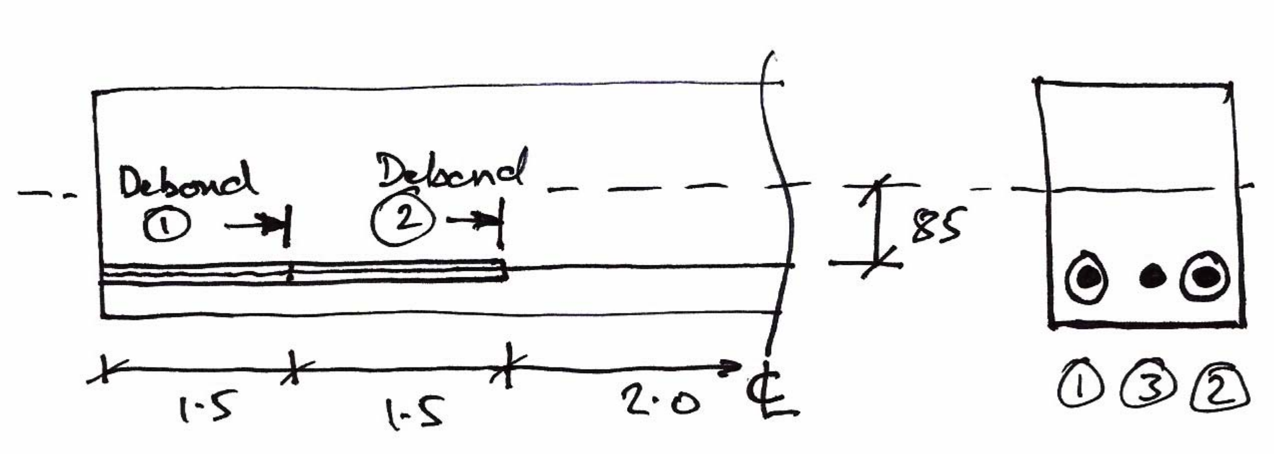
Note that the maximum prestress has been increased from 450 kN to 600 kN. Even thought he extra strand is an added expense; it is offset by the constructability of the member.

Note that the prestress steps up in units of 200 kN at two points. Imagine there to be 3 tendons acting, each of 200 kN prestress:

- From 0 to 1.5 m only 1 tendon is to act;
- From 1.5 to 3 m 2 tendons are to act;
- From 3 to 5 m all 3 tendons are to act

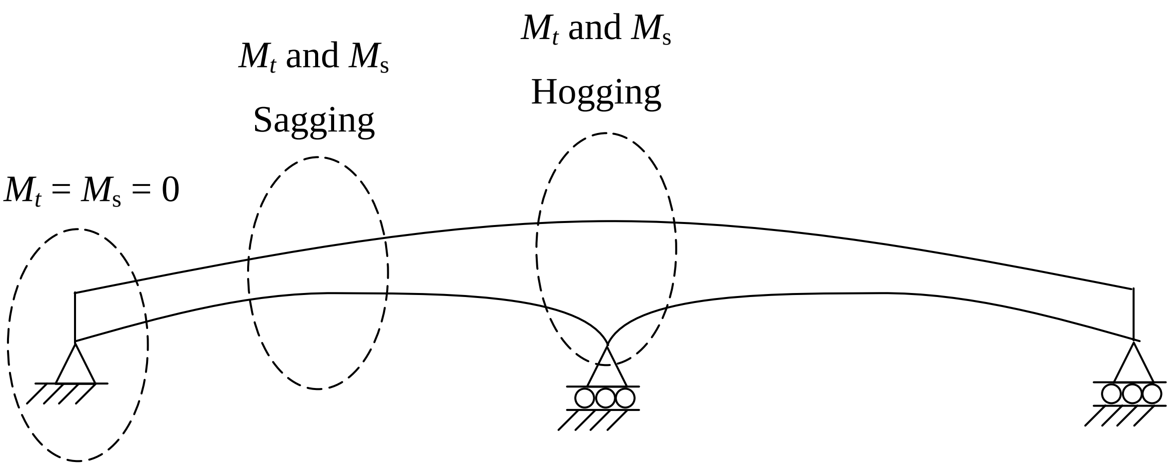
(Note that the prestress is symmetrical.)

In a factory setting, all 3 tendons will be present and yet some are not to act. Therefore the bond between the concrete and tendon is broken by using plastic tubing over the required length. Hence for our beam, the layout will look like:



(Note this is slightly unrealistic as vertical symmetry is maintained in reality.)

Briefly then for post-tensioned members in general, the section properties, the bending moments, and the prestress all may change along the length of a member:



In such beams, the governing stress limits must be checked at 'every' point along the length of the beam.

4. Prestressing Losses

4.1 Basis and Notation

Recall that transfer is the moment at which the concrete first feels the prestress. Losses can be before or after transfer:

- Before-transfer losses are the difference between what is applied by the hydraulic jack, P_{jack} and what the concrete feels at transfer, P_t;
- After-transfer losses are the difference between the transfer force, P and the force at SLS, $P_s = \alpha P_t$.

The calculation of losses is different for pre- and post-tensioned PSC.

Notation

f_p = prestress in strand or tendon;

A_p = cross-sectional area of prestressing tendon;

E_p = modulus of elasticity (Young's modulus) of prestressing steel;

E_c = modulus of elasticity (Young's modulus) of concrete;

A_g = gross cross-sectional area of concrete member;

I_g = gross second moment of area of concrete member;

P = prestress force at transfer;

P_{jack} = prestress force applied by jack (prior to before-transfer losses);

f_c = stress in concrete;

ε = strain;

ε_{sh} = strain in concrete due to shrinkage;

ε_{∞} = creep strain in concrete an infinite time after prestressing at SLS;

ϕ = creep strain per unit of sustained stress.

4.2 Losses in Pre-Tensioned PSC

1. Elastic Shortening Loss

In pre-tensioning, the strands are stressed before the concrete is cast. Therefore, after jacking (P_{jack}) has been applied, at a level *y*, the concrete is subjected to the stress:

$$f_{c,y} = \frac{P_{jack}}{A_g} + \frac{\left(P_{jack}e\right)y}{I_g} + \frac{M_o y}{I_g}$$

At the level of the strands, $y = e$. Hence, the stress in the concrete at that level is:

$$f_{c,e} = \frac{P_{jack}}{A_g} + \frac{P_{jack}e^2}{I_g} + \frac{M_o e}{I_g}$$

And the concrete at the level of the strands has a strain of:

$$\varepsilon_{c,e} = \frac{f_{c,e}}{E_c}$$

Since the strands are bonded to the concrete, they undergo a loss of strain of the same amount, $\varepsilon_{c,e}$, with associated stress and force losses of $E_p\varepsilon_{c,e}$ and $A_pE_p\varepsilon_{c,e}$. Hence:

$$\Delta P_\varepsilon = \frac{A_pE_p}{E_c}\left(\frac{P_{jack}}{A_g} + \frac{P_{jack}e^2}{I_g} + \frac{M_t e}{I_g}\right)$$

The concrete never feels the full jacking force since the strands shorten as the concrete strains. Hence, elastic shortening loss in pre-tensioning is 'before-transfer'.

2. Shrinkage Loss

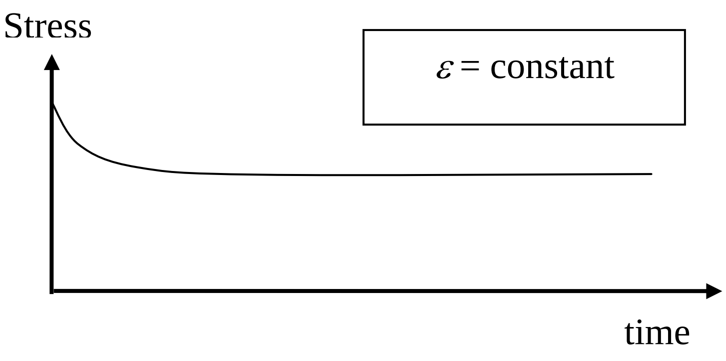
Concrete shrinks as it sets and for a period afterwards. The amount of shrinkage depends on the humidity and the surface area to volume ratio. Once again, as the strands are bonded to the concrete they undergo the same strain. Hence the loss of prestress is:

$$\Delta P_{sh} = A_p E_p \varepsilon_{sh}$$

Shrinkage loss is long-term. Hence it is 'after-transfer'.

3. Relaxation Loss

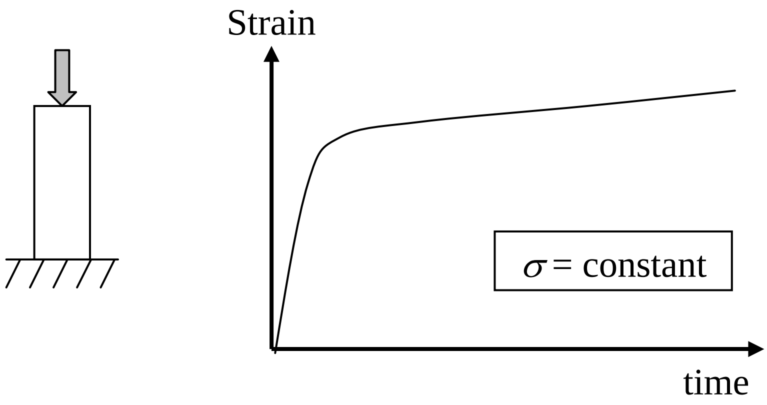
When maintained at a constant strain, prestressing strand gradually loses its stress with time (like a guitar going out of tune). It is due to a realignment of the steel fibres and is the same phenomenon as creep. Depending on the quality of the steel, relaxation losses can vary in the range of 3% to 8%.



Relaxation is 'after-transfer'.

4. Creep Loss

Creep is the ongoing increase in strain with time, when stress is kept constant:



Creep causes a strain in the concrete adjacent to the strands and over a long time is:

$$\varepsilon_\infty = \phi f_{c,e}$$

The creep factor ϕ depends on a number of things such as concrete quality and its age when loaded. Thus the strands slacken as before, and the loss of force is:

$$\begin{aligned} \Delta P_{creep} &= A_p E_p \varepsilon_\infty \\ &= A_p E_p \phi f_{c,e} \\ &= A_p E_p \phi \left[\frac{\alpha P_t}{A_g} + \frac{\alpha P_t e^2}{I_g} + \frac{M_{perm} e}{I_g} \right] \end{aligned}$$

M_{perm} is the moment due to the permanent loads such as the prestress, dead load and superimposed dead loads such as such as parapets and road pavement). As creep loss contributes to α, an exact calculation is iterative. Creep is long term, hence it is an 'after-transfer' loss.

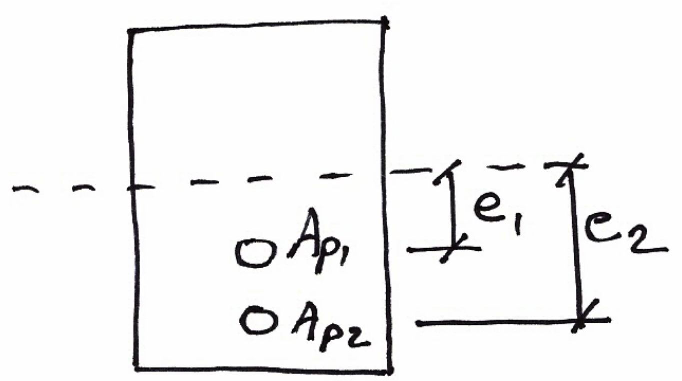
4.3 Losses in Post-tensioned PSC

1. Elastic Shortening Loss

This loss only occurs in post-tensioning when there is more than one tendon.

Two Tendons

As a simple example, take the following beam with two tendons:



Tendon 1 is stressed first. The operator applies prestress until the gauge indicates the required prestress force is applied. The beam shortens during this process but this is compensated for, as it is jacked until the required prestress force is applied. Hence there is no elastic shortening loss to be allowed for at this stage.

Tendon 1 is anchored and Tendon 2 is stressed. This causes strain at the level of Tendon 1, resulting in a prestress loss in Tendon 1. The stress due to Tendon 2 is:

$$f_{c,2} = \frac{P_2}{A_g} + \frac{(P_2 e_2) y}{I_g}$$

Note that the actual properties are approximated with the gross properties, A_g and I_g. The strain due to the stressing of Tendon 2, in the concrete adjacent to Tendon 1, is:

$$\varepsilon_{c,2} = \frac{1}{E_c}\left(\frac{P_2}{A_g} + \frac{(P_2 e_2)e_1}{I_g}\right)$$

This is the amount of strain lost in Tendon 1. Hence the loss of prestress force is:

$$\Delta P_2 = \frac{A_{p1}E_p}{E_c}\left(\frac{P_2}{A_g} + \frac{(P_2 e_2)e_1}{I_g}\right)$$

The stressing of Tendon 2 does not cause any losses in Tendon 2 – it only causes losses in tendons that have already been anchored.

Multiple Tendons

Similar calculations can be made for any arrangement of tendons. For 4 tendons:

1. stressing the first tendon does not cause any losses;
2. stressing the 2nd causes a loss in the 1st;
3. stressing the 3rd causes losses in the 1st and 2nd;
4. stressing the 4th causes losses in the 1st, 2nd and 3rd.

Hence the first tendon has losses due to the stressing of Tendons 2, 3 and 4, the second due to the stressing of 3 and 4 and the third due to the stressing of 4 giving:

$$\Delta P_{2-4} = \frac{A_{p1}E_p}{E_c}\left[\frac{P_2+P_3+P_4}{A_g} + \frac{(P_2e_2+P_3e_3+P_4e_4)e_1}{I_g}\right] + \frac{A_{p2}E_p}{E_c}\left[\frac{P_3+P_4}{A_g} + \frac{(P_3e_3+P_4e_4)e_2}{I_g}\right] + \frac{A_{p3}E_p}{E_c}\left(\frac{P_4}{A_g} + \frac{(P_4e_4)e_3}{I_g}\right)$$

Elastic shortening loss is 'before-transfer'.

2. Shrinkage Loss

Calculation is as for pre-tensioned PSC and is an ‘after-transfer’ loss.

3. Relaxation Loss

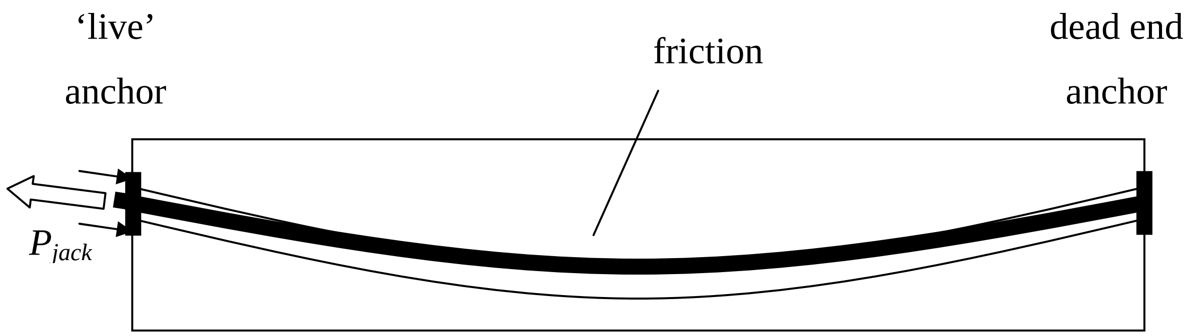
Calculation is as for pre-tensioned PSC and is an ‘after-transfer’ loss.

4. Creep Loss

Calculation is as for pre-tensioned PSC and is an ‘after-transfer’ loss.

5. Friction Curvature Loss

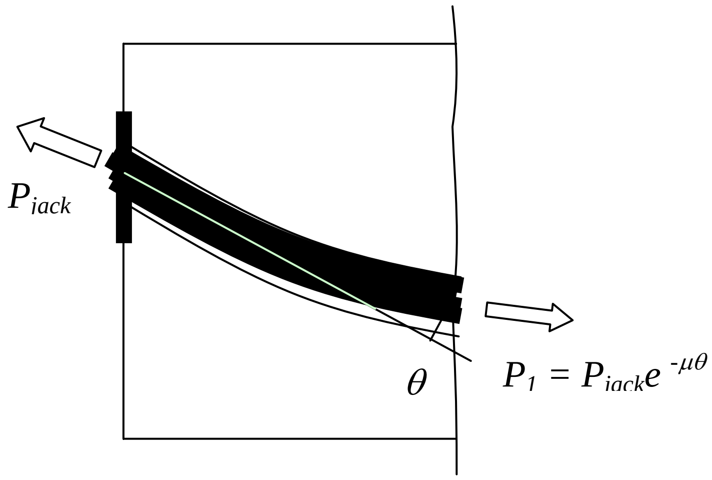
When the jacking force is applied, there is a friction between the tendon and the duct that prevents the interior parts of the beam/slab from feeling the full force:



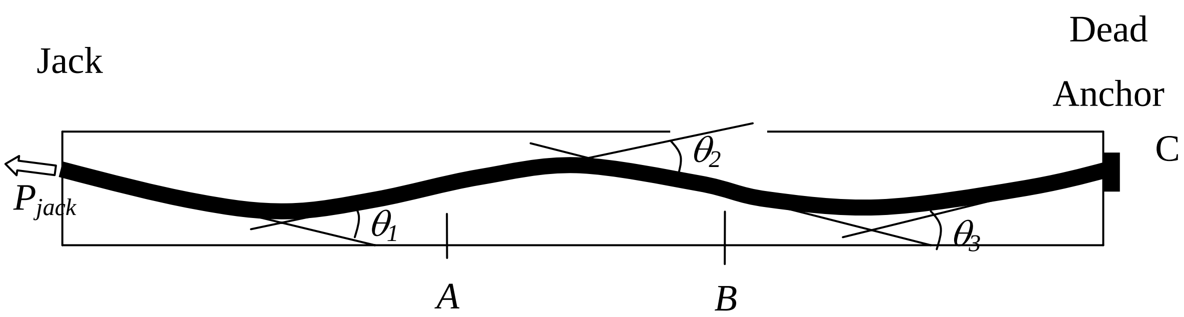
Examining a portion of the tendon, the loss is:

$$\begin{aligned}\Delta P_{fric} &= P_{jack} - P_l \\ &= P_{jack}\left(1 - e^{-\mu\theta}\right)\end{aligned}$$

where θ is the *aggregate* change in angle from the jack (in radians) (the sum of the absolute values of each change in angle) and μ is the roughness (friction) coefficient.



Example (jacked from left only)



At A: $loss = P_{jack}(1 - e^{-\mu\theta_1})$

At B: $loss = P_{jack}(1 - e^{-\mu(\theta_1+\theta_2)})$

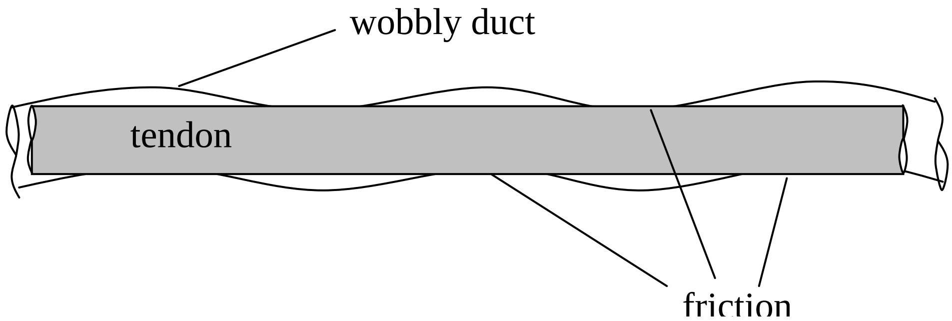
At C: $loss = P_{jack}(1 - e^{-\mu(\theta_1+\theta_2+\theta_3)})$

where $\theta_1 + \theta_2 + \theta_3$ is the aggregate change in angle from P_{jack}, regardless of sign.

Friction curvature loss happens between the jack and a-section. Therefore the *section* never feels the full jacking force and this loss is ‘before transfer’.

6. Friction Wobble Loss

This loss is due to ‘unintentional variation of the duct from the prescribed profile’:



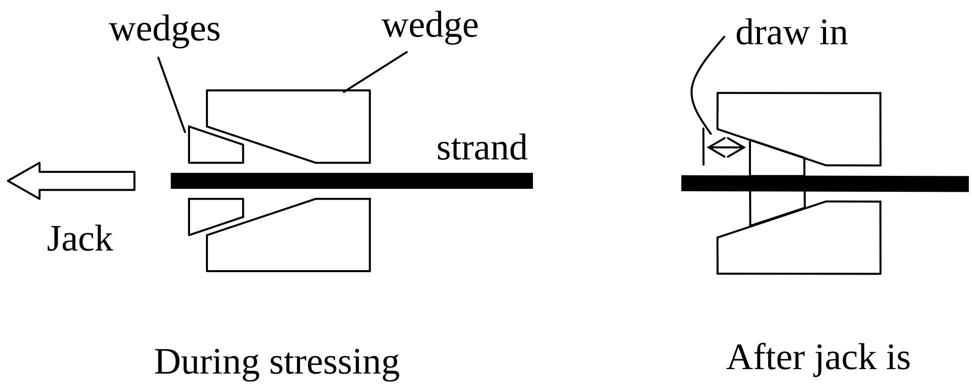
This loss increases with distance from the jack and is given by:

$$\Delta P_{wobble} = P_{jack}\left(1 - e^{-\mu kx}\right)$$

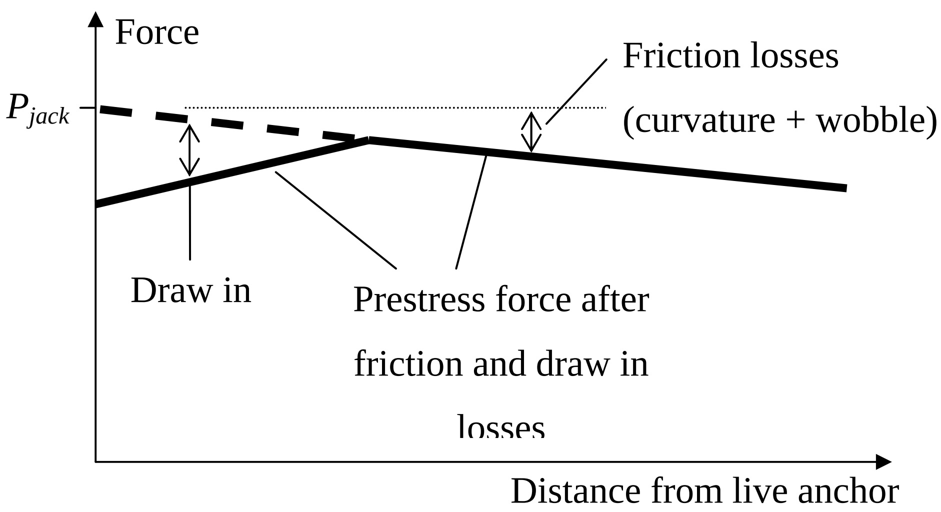
where x = distance from jack, and k is a wobble coefficient (function of workmanship, distance between duct supports, stiffness of duct, etc.). This is a ‘before transfer’ loss.

7. Draw-In Loss

Anchors consist of wedges that 'draw in' when the jack ceases to be applied:



The draw in of the wedges can be up to 10 mm but the loss is generally local:



Draw in losses are not transmitted more than about 5-10 m from the live anchor. Formulas are available which give the extent and magnitude of the draw in loss.

There is some debate about whether draw in should be considered to be 'before-transfer' or 'after-Transfer' but it is safer to take it to be an 'after-transfer' loss.

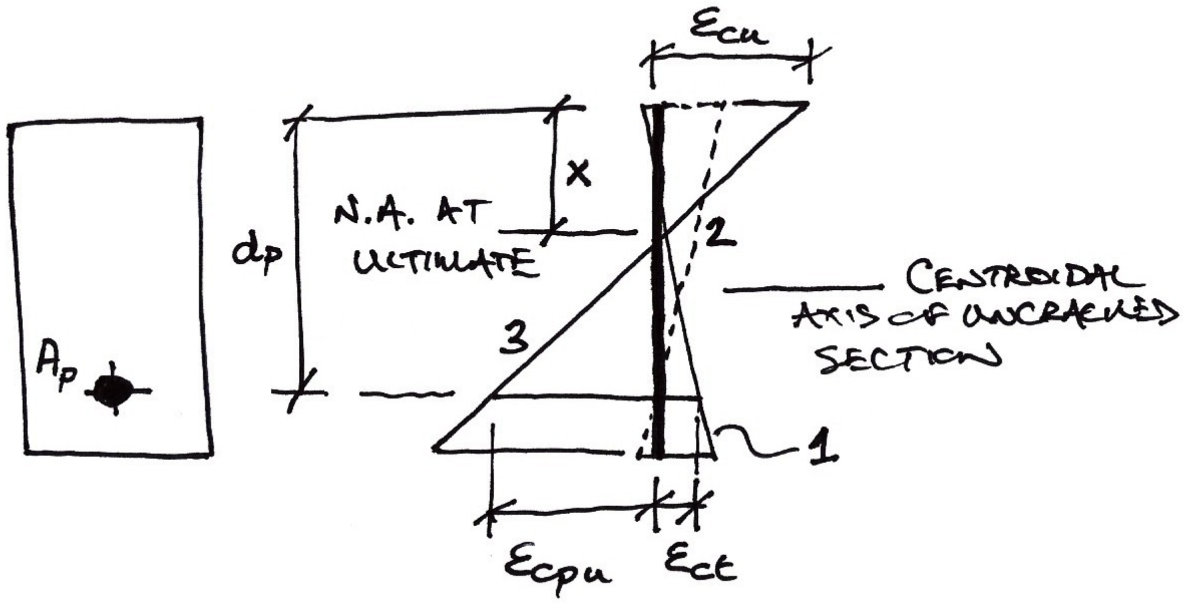
5. Ultimate Limit State Design of PSC

5.1 Ultimate Moment Capacity

The ULS moment capacity of a PSC section is calculated using the assumption that plane sections remain plane. This is similar to the approach for ordinary reinforced concrete. There is one difference:

- the strain in bonded prestress steel is equal to the strain caused by the initial prestress plus the change in strain in the concrete at the prestressing steel level.

The following diagram shows the strain diagram at three stages of loading:



Stage 1

This is the strain diagram at transfer. The strain in the concrete at steel level is compressive, with magnitude of:

$$\varepsilon_{ct} = \frac{1}{E_c}\left(\frac{P_t}{A} + \frac{Pe^2}{I}\right)$$

The stress and strain in the prestressing steel are:

$$f_{pt} = \frac{P_t}{A_p} \qquad \varepsilon_{pt} = \frac{f_{pt}}{E_p}$$

Stage 2

The applied moment is sufficient to decompress the concrete at the steel level. Provided there is a bond between the steel and concrete, the change in strain in the prestressing steel is equal to that of the concrete at the steel level. Hence the strain in the prestressing steel is now $\varepsilon_{pt} + \varepsilon_{ct}$.

Stage 3

This is the strain diagram at the ultimate load. The concrete strain at the steel level, ε_{cpu}, is related to the concrete strain at the top of the section by similar triangles:

$$\varepsilon_{cpu} = \varepsilon_{cu}\left(\frac{d_p - x}{x}\right)$$

The change in strain in the prestressing steel is, by compatibility, the same as ε_{cpu}.

Final

The final strain in the prestressing steel at ultimate load is thus:

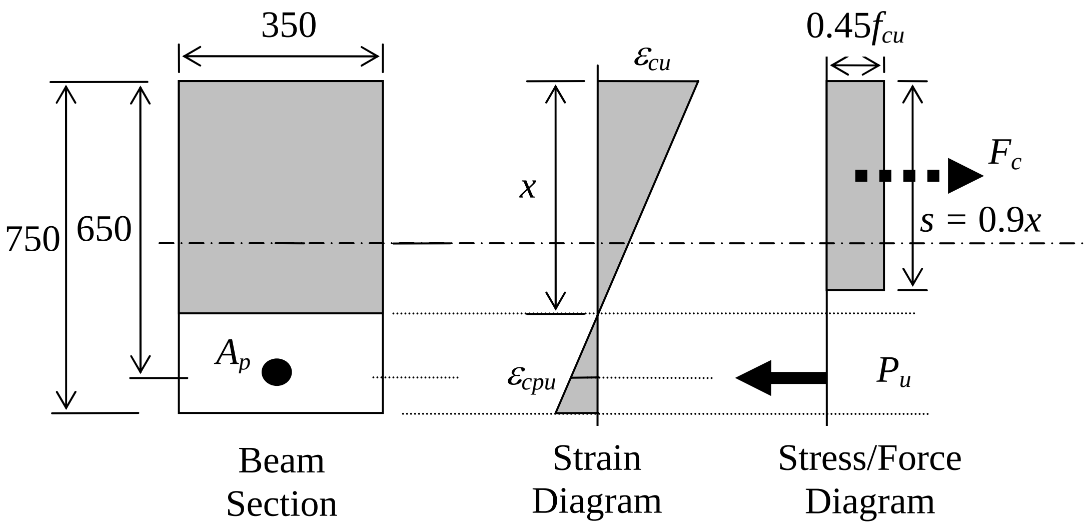
$$\varepsilon_{pu} = \varepsilon_{pt} + \varepsilon_{ct} + \varepsilon_{cpu}$$

In this equation, it is only ε_{cpu} that is not known. This can be obtained once a depth of neutral axis is found that balances horizontal forces.

Example

Problem

Determine the ultimate moment capacity of the section shown. The steel tendon has a transfer prestress of 1200 kN and the area of the strand is 1000 mm^2. The elastic modulus for the prestressing steel is 195 kN/mm^2 and the concrete is 45 N.



Solution

The initial strain in the tendon due to the prestress is:

$$\varepsilon_{pt} = \frac{P_t}{E_p A_p} = \frac{1200\times10^3}{\left(195\times10^3\right)\left(1000\right)} = 0.00615$$

The strain in the concrete caused by the initial prestress is:

$$\begin{aligned}
\varepsilon_{ct} &= \frac{1}{E_c}\left(\frac{P_t}{A} + \frac{Pe^2}{I}\right) \\
&= \frac{1}{29.8\times10^3}\left(\frac{1200\times10^3}{750\times350} + \frac{\left(1200\times10^3\right)\left(275\right)^2}{350\times750^3/12}\right) \\
&= 0.00040
\end{aligned}$$

At failure, the concrete strain at the level of the prestressing is:

$$\varepsilon_{cpu} = 0.0035\left(\frac{650 - x}{x}\right)$$

And so the final strain in the prestressing steel is:

$$\varepsilon_{pu} = 0.00655 + 0.0035\left(\frac{650 - x}{x}\right)$$

And the final force in the prestressing steel is:

$$\begin{aligned} P_u &= A_p E_p \varepsilon_{pu} \\ &= (1000)\left(195\times10^3\right)\left[0.00655 + 0.0035\left(\frac{650 - x}{x}\right)\right] \\ &= 195\times10^6\left[0.00655 + 0.0035\left(\frac{650 - x}{x}\right)\right] \end{aligned}$$

The force in the concrete is:

$$\begin{aligned} F_c &= (0.45\times45)(350\times0.9x) \\ &= 6379x \end{aligned}$$

At the correct depth to the neutral axis we will have horizontal equilibrium of forces, $P_u = F_c$. Hence we iteratively try to find this value of *x*:

x	F_c	P_u	Difference
350	2232563	1862250	370312.5
250	1594688	2369250	-774563
300	1913625	2073500	-159875
315	2009306	2003083	6222.917
314	2002928	2007568	-4640.97
314.4	2005479	2005771	-291.992

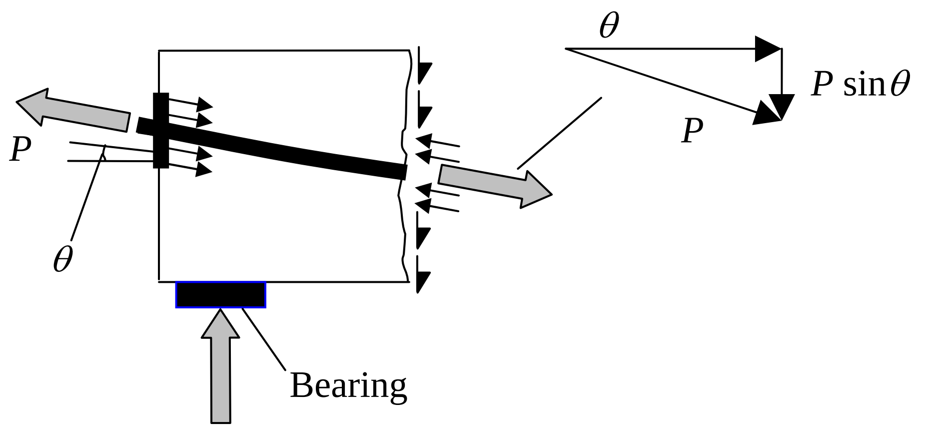
Hence the moment capacity is:

$$\begin{aligned} M_u &= P_u z \\ &= P_u \left(d_p - 0.45x \right) \\ &= 2005771 \times \left(650 - 0.45 \times 314.4 \right) \\ &= 1020 \text{ kNm} \end{aligned}$$

5.2 Ultimate Shear Design

Background

The shear design procedure for PSC is quite different from ordinary RC. It varies significantly from code to code. However, there is one issue with PSC that can have a significant influence on shear design. This is the vertical component of the prestress force which can have a significant *beneficial* effect.



Also, the shear capacity depends on whether or not the section has cracked under the ultimate flexural moments at the section.

Note that nominal links are required in PSC members, similarly to ordinary reinforced members.

Shear Capacity for Cracked Sections

A section is cracked in flexure if the moment is greater than a design cracking moment, M_0, defined below. The cracked shear capacity, V_{cr}, is empirically give by:

$$V_{cr} = \left(1 - 0.55\frac{f_{ps}}{f_{pu}}\right)v_c b_v d + \frac{M_o}{M}V \geq 0.1 b_v d\sqrt{f_{cu}}$$

In which:

M is the moment acting at the section;

V is the shear acting at the section;

M_0 is the moment required to remove 0.8 of the compressive stress at the level of the prestress:

$$M_0 = 0.8 f_{c,e}\frac{I}{e}$$

where:

$$f_{c,e} = \frac{P_s}{A_g} + \frac{P_s e^2}{I_g}$$

f_{ps} is the stress in the tendon at SLS;

f_{pu} is the ultimate tensile strength of the tendon;

b_v is the shear width of the web/section;

d is the effective depth;

v_c is the design shear strength of the concrete:

$$v_c = \frac{0.79}{1.25}\left(\frac{100A_s}{b_v d}\right)^{0.33}\left(\frac{400}{d}\right)^{0.25}\left(\frac{f_{cu}}{25}\right)^{0.33}$$

Shear Capacity for Uncracked Sections

A section is uncracked if the applied moment is less than M_0. In such section, the principal tensile stress in the web is limited to $f_t = 0.24\sqrt{f_{cu}}$. Based on a Mohr's circle analysis, the following equation is derived:

$$V_{co} = 0.67bh\sqrt{\left(f_t^{\,2} + 0.8 f_t f_{cp} \right)} + V_p$$

In which:

f_{cp} is the compressive stress due to prestress at the centroidal axis:

$$f_{cp} = \frac{P_s}{A}$$

V_p is the vertical component of prestress at the section, resisting the applied shear;

And the remaining variables have their previous meaning.

Shear Design

The design shear resistance at a section is:

$$\text{Uncracked:} \quad V_c = V_{co}$$
$$\text{Cracked:} \quad V_c = \min\left[V_{co}, V_{cr}\right]$$

The shear design is:

- $V \le 0.5V_c$: no links are required;
- $V \le V_c + 0.4b_v d$: Shear links are used:

$$\frac{A_{sv}}{s_v} = \frac{0.4b_v}{0.87 f_{yv}}$$

- $V > V_c + 0.4b_v d$: Shear links are used:

$$\frac{A_{sv}}{s_v} = \frac{V - V_c}{0.87 f_{yv} d_t}$$

Where:

A_{sv} is the link area;

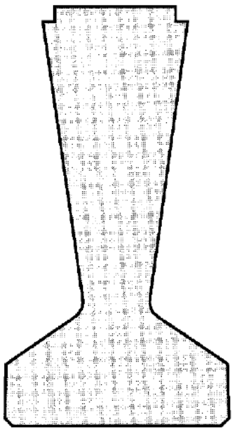
s_v is the link spacing;

f_{yv} is the characteristic strength of the links;

d_t is the depth to the furthest steel, ordinary or prestressed, from the compression face.

Example

Problem



A Y-beam, as shown, has rib width of 200 mm and a depth of 1000 mm. Its area is 310 × 10^3 mm^2 and its second moment of area is 36 × 10^9 mm^4. The area of prestressing steel is 1803 mm^2 which has strength of 1750 N/mm^2 and is stressed to 60% of its strength in service at an eccentricity of 290 mm. The concrete is grade 50.

Check the section for a shear of 400 kN and associated moment of 800 kNm. Note that the tendon is inclined by 3° at the section considered.

Solution

First determine the following:

$$P_t = A_p f_s = (1803)(0.6 \times 1750) = 1893 \text{ kN}$$

$$f_{cp} = \frac{P_s}{A} = \frac{1893 \times 10^3}{310 \times 10^3} = 6.1 \text{ N/mm}^2$$

$$f_t = 0.24\sqrt{f_{cu}} = 0.24\sqrt{50} = 1.7 \text{ N/mm}^2$$

Uncracked shear capacity:

$$\begin{aligned} V_{co} &= 0.67bh\sqrt{\left(f_t^2 + 0.8 f_t f_{cp}\right)} + V_p \\ &= 0.67(200 \times 1000)\sqrt{\left(1.7^2 + 0.8 \times 1.7 \times 6.1\right)} + 1893\sin 3^\circ \\ &= 448.2 + 99.1 \\ &= 547.3 \text{ kN} \end{aligned}$$

For the cracked shear capacity, we have the following inputs:

$$\frac{f_{ps}}{f_{pu}} = 0.6 \text{ given}$$

$$\begin{aligned}
f_{c,e} &= \frac{P_s}{A_g} + \frac{P_s e^2}{I_g} \\
&= \frac{1893\times10^3}{310\times10^3} + \frac{\left(1893\times10^3\right)\left(290\right)^2}{36\times10^9} \\
&= 10.52 \text{ N/mm}^2
\end{aligned}$$

$$\begin{aligned}
M_0 &= 0.8 f_{c,e} \frac{I}{e} \\
&= 0.8\left(10.52\right)\left(\frac{36\times10^9}{290}\right) \\
&= 1046 \text{ kNm}
\end{aligned}$$

$$\begin{aligned}
v_c &= \frac{0.79}{1.25}\left(\frac{100A_s}{b_v d}\right)^{0.33}\left(\frac{400}{d}\right)^{0.25}\left(\frac{f_{cu}}{25}\right)^{0.33} \\
&= \frac{0.79}{1.25}\left(\frac{100\times1803}{200\times790}\right)^{0.33}\left(\frac{400}{790}\right)^{0.25}\left(\frac{50}{25}\right)^{0.33} \\
&= 0.78 \text{ N/mm}^2
\end{aligned}$$

Hence:

$$\begin{aligned}
V_{cr} &= \left(1-0.55\frac{f_{ps}}{f_{pu}}\right)v_c b_v d + \frac{M_o}{M}V \geq 0.1 b_v d\sqrt{f_{cu}} \\
&= \left(1-0.55\times0.6\right)\left(0.78\times200\times790\right) + \frac{1046}{800}\left(400\times10^3\right) \geq 0.1\left(200\times790\right)\sqrt{50} \\
&= 605.4 \geq 111.7 \\
&= 605.4 \text{ kN}
\end{aligned}$$

The section is uncracked since $M \leq M_0$ and so the design shear strength is 547.3 kN which is greater than the applied 400 kN. Therefore, use nominal links.