DESIGN EXAMPLES

Version 14.1

AMERICAN INSTITUTE
OF
STEEL CONSTRUCTION

The information presented in this publication has been prepared in accordance with recognized engineering principles and is for general information only. While it is believed to be accurate, this information should not be used or relied upon for any specific application without competent professional examination and verification of its accurac y, suitability, and applicability by a licensed professional engineer, designer, or architect. The publication of the material contained herein is n ot intended as a represen tation or warranty on the part of the Am erican Institute of Steel Construction or of any other person named herein, that this information is suitable for any general or particular use or of freedom from infringement of any patent or pat ents. Anyone making use o f this information assumes all liability arising from such use.

Caution must be exerci sed when relying upon other specifications and co des developed by other bodies and incorporated by reference herein since such material may be modified or amended from time to time subsequent to the printing of this edition. The Institute bears no responsibility for such material other than to refer to it and incorporate it by reference at the time of the initial publication of this edition.

Printed in the United States of America

Revision February 2013

PREFACE

The primary objective of these design examples is to provide illustrations of the use of the 2010 AISC *Specification for Structural Steel Buildings* (ANSI/AISC 360-10) and the 14 th Edition of the AISC *Steel Construction Manual*. The design examples provide coverage of all applicable limit states whether or not a particular limit state controls the design of the member or connection.

In addition to the examples which demonstrate the use of the *Manual* tables, design examples are provided for connection designs beyond the scope of the tables in the Manual. These design examples are intended to demonstrate an approach to the design, and are not intended to suggest that the approach presented is the only approach. The committee responsible for the development of these design examples recognizes that designers have alternate approaches that work best for them and their projects. Design approaches that differ from those presented in these examples are considered viable as long as the *Specification*, sound engineering, and project specific requirements are satisfied.

Part I of these examples is organized to correspond with the organization of the Specification. The Chapter titles match the corresponding chapters in the Specification.

Part II is devoted primarily to connection examples that draw on the tables from the Manual, recommended design procedures, and the breadth of the Specification. The chapters of Part II are labeled II-A, II-B, II-C, etc.

Part III addresses aspects of design that are linked to the performance of a building as a whole. This includes coverage of lateral stability and second order analysis, illustrated through a four-story braced-frame and moment-frame building.

The Design Examples are arranged with LRFD and ASD designs presented side by side, for consistency with the AISC *Manual*. Design with ASD and LRFD are based on the same nominal strength for each element so that the only differences between the approaches are which set of load combinations from ASCE/SEI 7-10 are used for design and whether the resistance factor for LRFD or the safety factor for ASD is used.

CONVENTIONS

The following conventions are used throughout these examples:

1. The 2010 AISC *Specification for Structural Steel Buildings* is referred to as the AISC *Specification* and the 14th Edition AISC *Steel Construction Manual,* is referred to as the AISC *Manual*.

2. The source of equations or tabulated values taken from the AISC *Specification* or AISC *Manual* is noted along the right-hand edge of the page.

3. When the design process differs between LRFD and ASD, the designs equations are presented side-by-side. This rarely occurs, except when the resistance factor, ϕ, and the safety factor, Ω, are applied.

4. The results of design equations are presented to three significant figures throughout these calculations.

ACKNOWLEDGMENTS

The AISC Committee on Manuals reviewed and approved V14.0 of the AISC *Design Examples*:

William A. Thornton, Chairman
Mark V. Holland, Vice Chairman
Abbas Aminmansour
Charles J. Carter
Harry A. Cole
Douglas B. Davis
Robert O. Disque
Bo Dowswell
Edward M. Egan
Marshall T. Ferrell

The AISC Committee on Manuals gratefully acknowledges the contributions of the following individuals who assisted in the development of this document: Leigh Arber, Eric Bolin, Janet Cummins, Thomas Dehlin, William Jacobs, Richard C. Kaehler, Margaret Matthew, Heath Mitchell, Thomas J. Schlafly, and Sriramulu Vinnakota.

TABLE OF CONTENTS

CHAPTER G DESIGN OF MEMBERS FOR SHEAR...G-1

CHAPTER H DESIGN OF MEMBERS FOR COMBINED FORCES AND TORSION H-1

Part I
Examples Based on the AISC *Specification*

This part contains design examples demonstrating select provisions of the AISC *Specification for Structural Steel Buildings.*

Chapter A
General Provisions

A1. SCOPE

These design examples are intended to illustrate the application of the 2010 AISC *Specification for Structural Steel Buildings* (ANSI/AISC 360-10) (AISC, 2010) and the AISC *Steel Construction Manual*, 14th Edition (AISC, 2011) in low-seismic applications. For information on design applications requiring seismic detailing, see the AISC *Seismic Design Manual*.

A2. REFERENCED SPECIFICATIONS, CODES AND STANDARDS

Section A2 includes a detailed list of the specifications, codes and standards referenced throughout the AISC *Specification*.

A3. MATERIAL

Section A3 includes a list of the steel materials that are approved for use with the AISC *Specification*. The complete ASTM standards for the most commonly used steel materials can be found in *Selected ASTM Standards for Structural Steel Fabrication* (ASTM, 2011).

A4. STRUCTURAL DESIGN DRAWINGS AND SPECIFICATIONS

Section A4 requires that structural design drawings and specifications meet the requirements in the AISC *Code of Standard Practice for Steel Buildings and Bridges* (AISC, 2010b).

CHAPTER A REFERENCES

AISC (2010a), *Specification for Structural Steel Buildings*, ANSI/AISC 360-10, American Institute for Steel Construction, Chicago, IL.

AISC (2010b), *Code of Standard Practice for Steel Buildings and Bridges*, American Institute for Steel Construction, Chicago, IL.

AISC (2011), *Steel Construction Manual*, 14th Ed., American Institute for Steel Construction, Chicago, IL.

ASTM (2011), *Selected ASTM Standards for Structural Steel Fabrication*, ASTM International, West Conshohocken, PA.

Chapter B
Design Requirements

B1. GENERAL PROVISIONS

B2. LOADS AND LOAD COMBINATIONS

In the absence of an applicable building code, the default load combinations to be used with this *Specification* are those from *Minimum Design Loads for Buildings and Other Structures* (ASCE/SEI 7-10) (ASCE, 2010).

B3. DESIGN BASIS

Chapter B of the AISC *Specification* and Part 2 of the AISC *Manual* describe the basis of design, for both LRFD and ASD.

This Section describes three basic types of connections: simple connections, fully restrained (FR) moment connections, and partially restrained (PR) moment connections. Several examples of the design of each of these types of connection are given in Part II of these design examples.

Information on the application of serviceability and ponding provisions may be found in AISC *Specification* Chapter L and AISC *Specification* Appendix 2, respectively, and their associated commentaries. Design examples and other useful information on this topic are given in AISC Design Guide 3, *Serviceability Design Considerations for Steel Buildings*, Second Edition (West et al., 2003).

Information on the application of fire design provisions may be found in AISC *Specification* Appendix 4 and its associated commentary. Design examples and other useful information on this topic are presented in AISC Design Guide 19, *Fire Resistance of Structural Steel Framing* (Ruddy et al., 2003).

Corrosion protection and fastener compatibility are discussed in Part 2 of the AISC *Manual*.

B4. MEMBER PROPERTIES

AISC *Specification* Tables B4.1a and B4.1b give the complete list of limiting width-to-thickness ratios for all compression and flexural members defined by the AISC *Specification*.

Except for one section, the W-shapes presented in the compression member selection tables as column sections meet the criteria as nonslender element sections. The W-shapes presented in the flexural member selection tables as beam sections meet the criteria for compact sections, except for 10 specific shapes. When noncompact or slender-element sections are tabulated in the design aids, local buckling criteria are accounted for in the tabulated design values.

The shapes listing and other member design tables in the AISC *Manual* also include footnoting to highlight sections that exceed local buckling limits in their most commonly available material grades. These footnotes include the following notations:

[c] Shape is slender for compression.
[f] Shape exceeds compact limit for flexure.
[g] The actual size, combination and orientation of fastener components should be compared with the geometry of the cross section to ensure compatibility.
[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.
[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1a.

CHAPTER B REFERENCES

ASCE (2010), *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10, American Society of Civil Engineers, Reston, VA.

West, M., Fisher, J. and Griffis, L.G. (2003), *Serviceability Design Considerations for Steel Buildings*, Design Guide 3, 2nd Ed., AISC, Chicago, IL.

Ruddy, J.L., Marlo, J.P., Ioannides, S.A. and Alfawakhiri, F. (2003), *Fire Resistance of Structural Steel Framing*, Design Guide 19, AISC, Chicago, IL.

Chapter C
Design for Stability

C1. GENERAL STABILITY REQUIREMENTS

The AISC *Specification* requires that the designer account for both the stability of the structural system as a whole, and the stability of individual elements. Thus, the lateral analysis used to assess stability must include consideration of the combined effect of gravity and lateral loads, as well as member inelasticity, out-of-plumbness, out-of-straightness and the resulting second-order effects, P-Δ and P-δ. The effects of "leaning columns" must also be considered, as illustrated in the examples in this chapter and in the four-story building design example in Part III of AISC *Design Examples*.

P-Δ and P-δ effects are illustrated in AISC *Specification Commentary* Figure C-C2.1. Methods for addressing stability, including P-Δ and P-δ effects, are provided in AISC *Specification* Section C2 and Appendix 7.

C2. CALCULATION OF REQUIRED STRENGTHS

The calculation of required strengths is illustrated in the examples in this chapter and in the four-story building design example in Part III of AISC *Design Examples*.

C3. CALCULATION OF AVAILABLE STRENGTHS

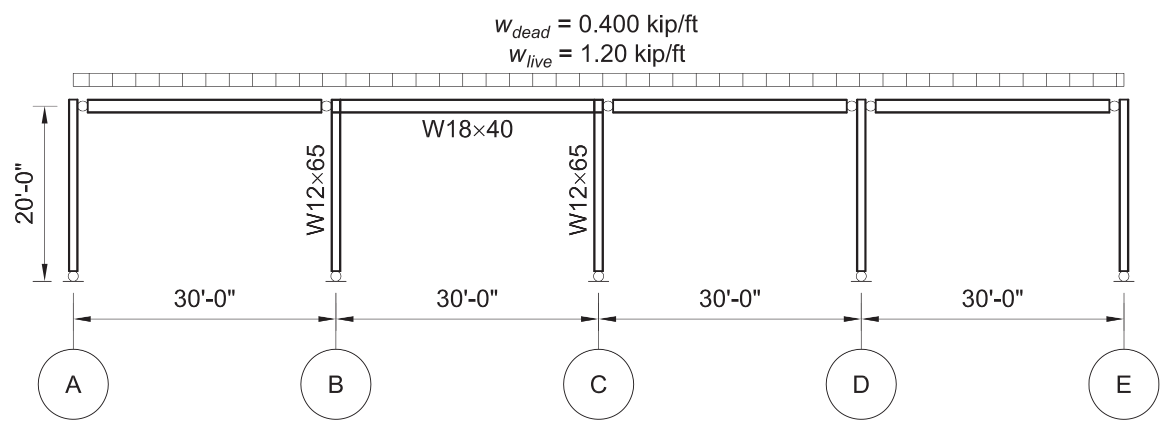
The calculation of available strengths is illustrated in the four-story building design example in Part III of AISC *Design Examples*.

EXAMPLE C.1A DESIGN OF A MOMENT FRAME BY THE DIRECT ANALYSIS METHOD

Given:

Determine the required strengths and effective length factors for the columns in the rigid frame shown below for the maximum gravity load combination, using LRFD and ASD. Use the direct analysis method. All members are ASTM A992 material.

Columns are unbraced between the footings and roof in the *x*- and *y*-axes and are assumed to have pinned bases.



Solution:

From *Manual* Table 1-1, the W12×65 has A = 19.1 in.^{2}

The beams from grid lines A to B, and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P-Δ effects to consider in these members and they may be designed using K=1.0.

The moment frame between grid lines B and C is the source of lateral stability and therefore must be designed using the provisions of Chapter C of the AISC *Specification*. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the analysis. For the analysis, the entire frame could be modeled or the model can be simplified as shown in the figure below, in which the stability loads from the three "leaning" columns are combined into a single column.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$ $= 2.40 \text{ kip/ft}$	$w_a = D + L$ $= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$ $= 1.60 \text{ kip/ft}$

Per AISC *Specification* Section C2.1, for LRFD perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis using 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame Analysis Gravity Loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

LRFD	ASD
$w_u' = 2.40$ kip/ft	$w_a' = 1.6(1.60 \text{ kip/ft})$ $= 2.56$ kip/ft

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P_u' = (15.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 36.0$ kips	$P_a' = 1.6(15.0 \text{ ft})(1.60 \text{ kip/ft})$ $= 38.4$ kips

Concentrated Gravity Loads on the Pseudo "Leaning" Column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but is not directly applied to it.

LRFD	ASD
$P_{uL}' = (60.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 144$ kips	$P_{aL}' = 1.6(60.0 \text{ ft})(1.60 \text{ kip/ft})$ $= 154$ kips

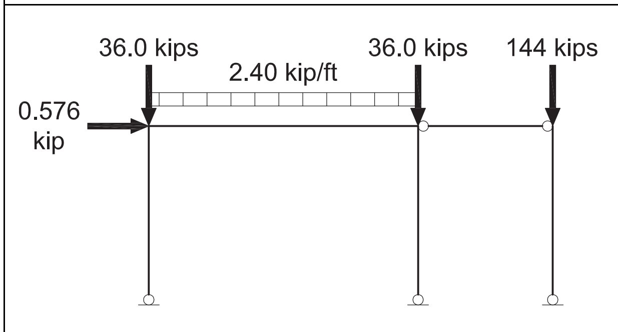
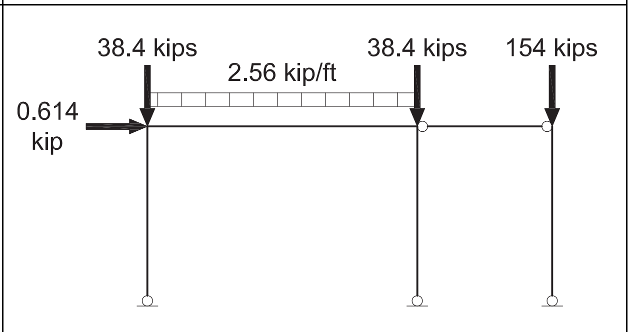
Frame Analysis Notional Loads

Per AISC *Specification* Section C2.2, frame out-of-plumbness must be accounted for either by explicit modeling of the assumed out-of-plumbness or by the application of notional loads. Use notional loads.

From AISC *Specification* Equation C2-1, the notional loads are:

LRFD	ASD
$\alpha = 1.0$ $Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$ $= 288$ kips $N_i = 0.002\alpha Y_i$ (*Spec.* Eq. C2-1) $= 0.002(1.0)(288 \text{ kips})$ $= 0.576$ kips	$\alpha = 1.6$ $Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$ $= 192$ kips $N_i = 0.002\alpha Y_i$ (*Spec.* Eq. C2-1) $= 0.002(1.6)(192 \text{ kips})$ $= 0.614$ kips

Summary of Applied Frame Loads

LRFD	ASD
 	

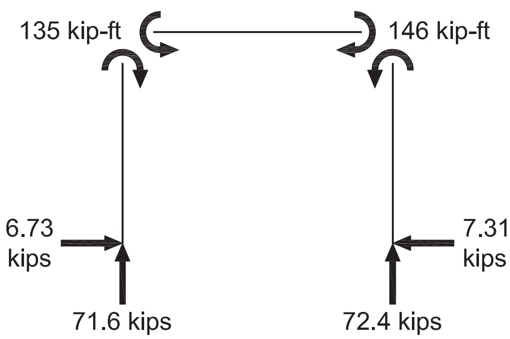
Per AISC *Specification* Section C2.3, conduct the analysis using 80% of the nominal stiffnesses to account for the effects of inelasticity. Assume, subject to verification, that $\alpha P_r/P_y$ is no greater than 0.5; therefore, no additional stiffness reduction is required.

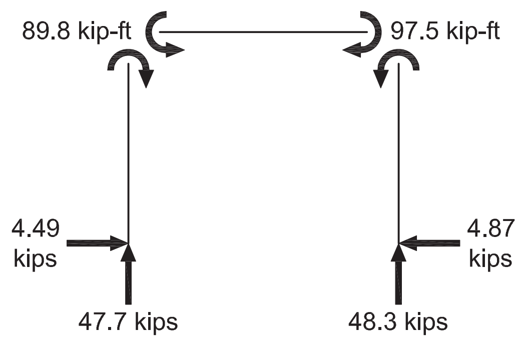
50% of the gravity load is carried by the columns of the moment resisting frame. Because the gravity load supported by the moment resisting frame columns exceeds one third of the total gravity load tributary to the frame, per AISC *Specification* Section C2.1, the effects of P-δ upon P-Δ must be included in the frame analysis. If the software used does not account for P-δ effects in the frame analysis, this may be accomplished by adding joints to the columns between the footing and beam.

Using analysis software that accounts for both P-Δ and P-δ effects, the following results are obtained:

First-order results

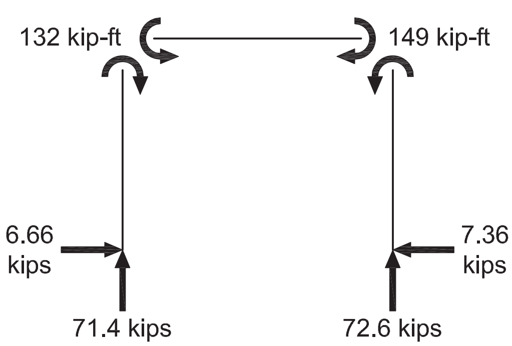
LRFD	ASD
$\Delta_{1st} = 0.149$ in.	$\Delta_{1st} = 0.159$ in. (prior to dividing by 1.6)

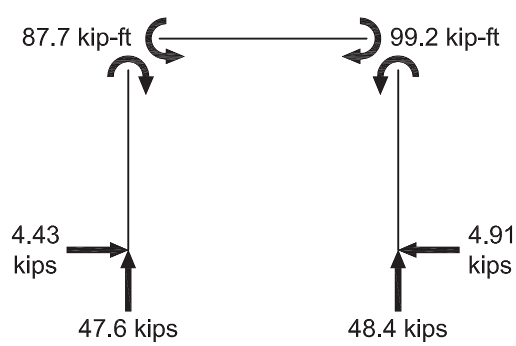




Second-order results

LRFD	ASD
$\Delta_{2nd} = 0.217$ in.	$\Delta_{2nd} = 0.239$ in. (prior to dividing by 1.6)
$\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.217\text{ in.}}{0.149\text{ in.}} = 1.46$	$\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.239\text{ in.}}{0.159\text{ in.}} = 1.50$





Check the assumption that $\alpha P_r/P_y \le 0.5$ and therefore, $\tau_b = 1.0$:

$P_y = F_yA_g$
$= 50 \text{ ksi}(19.1 \text{ in.}^2)$
$= 955 \text{ kips}$

LRFD	ASD
$\dfrac{\alpha P_r}{P_y} = \dfrac{1.0(72.6 \text{ kips})}{955 \text{ kips}}$ $= 0.0760 \le 0.5$ **o.k.**	$\dfrac{\alpha P_r}{P_y} = \dfrac{1.6(48.4 \text{ kips})}{955 \text{ kips}}$ $= 0.0811 \le 0.5$ **o.k.**

The stiffness assumption used in the analysis, $\tau_b = 1.0$, is verified.

Although the second-order sway multiplier is approximately 1.5, the change in bending moment is small because the only sway moments are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Verify the column strengths using the second-order forces shown above, using the following effective lengths (calculations not shown):

Columns:
Use $KL_x = 20.0$ ft
Use $KL_y = 20.0$ ft

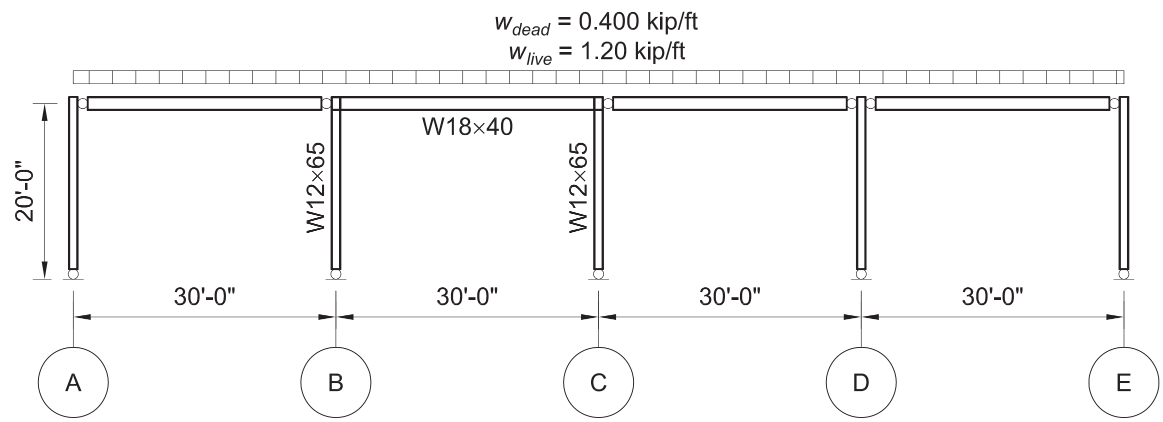
EXAMPLE C.1B DESIGN OF A MOMENT FRAME BY THE EFFECTIVE LENGTH METHOD

Repeat Example C.1A using the effective length method.

Given:

Determine the required strengths and effective length factors for the columns in the rigid frame shown below for the maximum gravity load combination, using LRFD and ASD. Use the effective length method.

Columns are unbraced between the footings and roof in the *x*- and *y*-axes and are assumed to have pinned bases.



Solution:

From *Manual* Table 1-1, the W12×65 has I_x = 533 in.^{4}

The beams from grid lines A to B, and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P-Δ effects to consider in these members and they may be designed using K=1.0.

The moment frame between grid lines B and C is the source of lateral stability and therefore must be designed using the provisions of Appendix 7 of the AISC *Specification*. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the analysis. For the analysis, the entire frame could be modeled or the model can be simplified as shown in the figure below, in which the stability loads from the three "leaning" columns are combined into a single column.

Check the limitations for the use of the effective length method given in Appendix 7, Section 7.2.1:
(1) The structure supports gravity loads through nominally vertical columns.
(2) The ratio of maximum second-order drift to the maximum first-order drift will be assumed to be no greater than 1.5, subject to verification following.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$ $= 2.40 \text{ kip/ft}$	$w_a = D + L$ $= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$ $= 1.60 \text{ kip/ft}$

Per AISC *Specification* Appendix 7, Section 7.2.1, the analysis must conform to the requirements of AISC *Specification* Section C2.1, with the exception of the stiffness reduction required by the provisions of Section C2.3.

Per AISC *Specification* Section C2.1, for LRFD perform a second-order analysis and member strength checks using the LRFD load combinations. For ASD, perform a second-order analysis at 1.6 times the ASD load combinations and divide the analysis results by 1.6 for the ASD member strength checks.

Frame Analysis Gravity Loads

The uniform gravity loads to be considered in a second-order analysis on the beam from B to C are:

LRFD	ASD
$w_u{}'$ = 2.40 kip/ft	$w_a{}'$ = 1.6(1.60 kip/ft) = 2.56 kip/ft

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P_u{}'$ = (15.0 ft)(2.40 kip/ft) = 36.0 kips	$P_a{}'$ = 1.6(15.0 ft)(1.60 kip/ft) = 38.4 kips

Concentrated Gravity Loads on the Pseudo "Leaning" Column

The load in this column accounts for all gravity loading that is stabilized by the moment frame, but is not directly applied to it.

LRFD	ASD
$P_{uL}{}'$ = (60.0 ft)(2.40 kip/ft) = 144 kips	$P_{aL}{}'$ = 1.6(60.0 ft)(1.60 kip/ft) = 154 kips

Frame Analysis Notional Loads

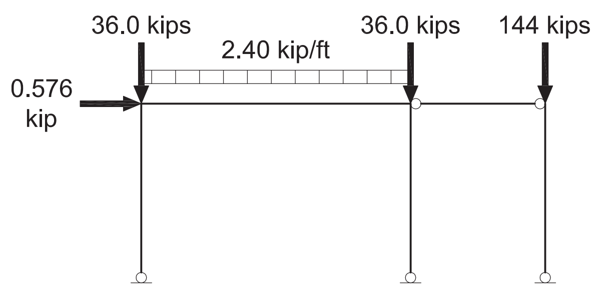
Per AISC *Specification* Appendix 7, Section 7.2.2, frame out-of-plumbness must be accounted for by the application of notional loads in accordance with AISC *Specification* Section C2.2b.

From AISC *Specification* Equation C2-1, the notional loads are:

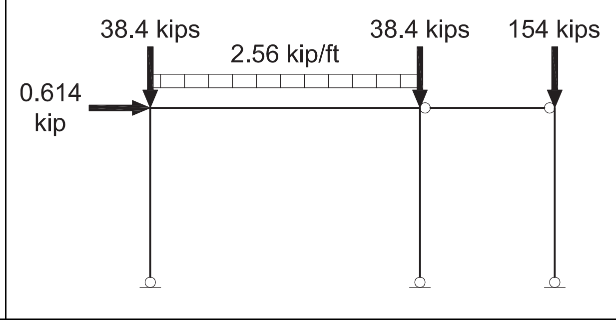
LRFD	ASD
α = 1.0 Y_i = (120 ft)(2.40 kip/ft) = 288 kips $N_i = 0.002\alpha Y_i$ (*Spec.* Eq. C2-1) = 0.002(1.0)(288 kips) = 0.576 kips	α = 1.6 Y_i = (120 ft)(1.60 kip/ft) = 192 kips $N_i = 0.002\alpha Y_i$ (*Spec.* Eq. C2-1) = 0.002(1.6)(192 kips) = 0.614 kips

Summary of Applied Frame Loads

LRFD



ASD



Per AISC *Specification* Appendix 7, Section 7.2.2, conduct the analysis using the full nominal stiffnesses.

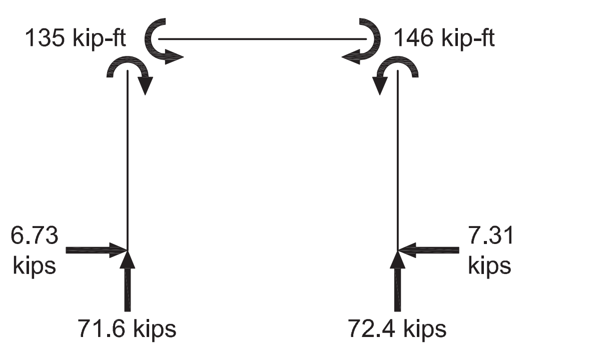
50% of the gravity load is carried by the columns of the moment resisting frame. Because the gravity load supported by the moment resisting frame columns exceeds one third of the total gravity load tributary to the frame, per AISC *Specification* Section C2.1, the effects of P-δ must be included in the frame analysis. If the software used does not account for P-δ effects in the frame analysis, this may be accomplished by adding joints to the columns between the footing and beam.

Using analysis software that accounts for both P-Δ and P-δ effects, the following results are obtained:

First-order results

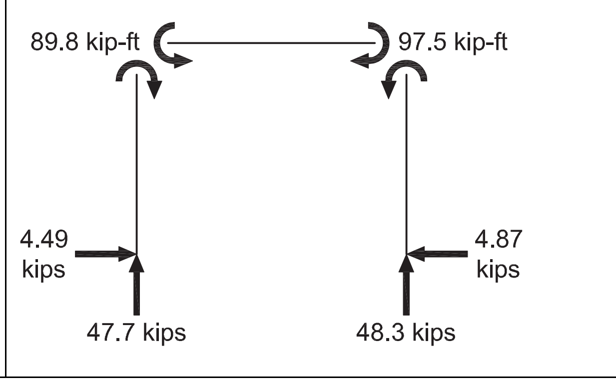
LRFD

Δ_{1st} = 0.119 in.

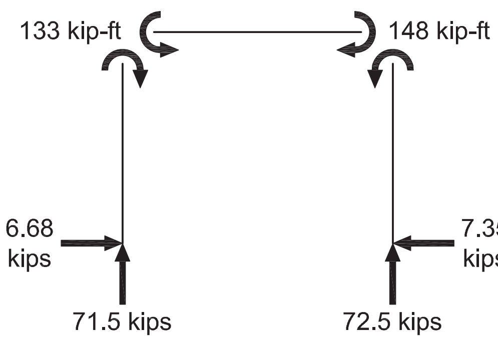
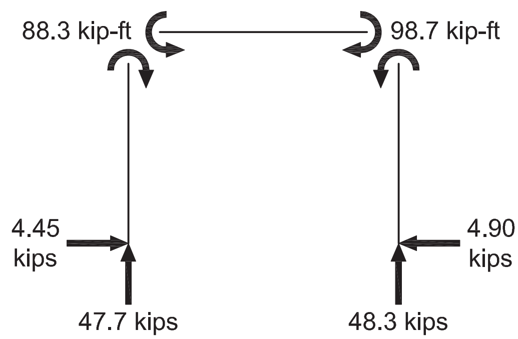


ASD

Δ_{1st} = 0.127 in. (prior to dividing by 1.6)



Second-order results

LRFD	ASD
$\Delta_{2nd} = 0.159 \text{ in.}$ $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.159 \text{ in.}}{0.119 \text{ in.}}$ $= 1.34$ 	$\Delta_{2nd} = 0.174 \text{ in. (prior to dividing by 1.6)}$ $\dfrac{\Delta_{2nd}}{\Delta_{1st}} = \dfrac{0.174 \text{ in.}}{0.127 \text{ in.}}$ $= 1.37$ 

The assumption that the ratio of the maximum second-order drift to the maximum first-order drift is no greater than 1.5 is verified; therefore, the effective length method is permitted.

Although the second-order sway multiplier is approximately 1.35, the change in bending moment is small because the only sway moments for this load combination are th ose produced by the small notional loads. For lo ad combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Calculate the in-plane effective length factor, K_x, using the "story stiffness method" and Equation C-A-7-5 presented in Commentary Appendix 7, Section 7.2. Take $K_x = K_2$

$$K_x = K_2 = \sqrt{\frac{\Sigma P_r}{\left(0.85+0.15R_L\right)P_r}\left(\frac{\pi^2 EI}{L^2}\right)\left(\frac{\Delta_H}{\Sigma HL}\right)} \geq \sqrt{\frac{\pi^2 EI}{L^2}\left(\frac{\Delta_H}{1.7HL}\right)} \qquad (Spec.\ \text{Eq. C-A-7-5})$$

Calculate the total load in all columns, ΣP_r

LRFD	ASD
$\Sigma P_r = 2.40 \text{ kip/ft}\left(120 \text{ ft}\right)$ $= 288 \text{ kips}$	$\Sigma P_r = 1.60 \text{ kip/ft}\left(120 \text{ ft}\right)$ $= 192 \text{ kips}$

Calculate the ratio of the leaning column loads to the total load, R_L

LRFD	ASD
$R_L = \dfrac{\Sigma P_r - \Sigma P_{r\ moment\ frame}}{\Sigma P_r}$ $= \dfrac{288 \text{ kips} - \left(71.5 \text{ kips} + 72.5 \text{ kips}\right)}{288 \text{ kips}}$ $= 0.500$	$R_L = \dfrac{\Sigma P_r - \Sigma P_{r\ moment\ frame}}{\Sigma P_r}$ $= \dfrac{192 \text{ kips} - \left(47.7 \text{ kips} + 48.3 \text{ kips}\right)}{192 \text{ kips}}$ $= 0.500$

Calculate the Euler buckling strength of an individual column.

$$\frac{\pi^2 EI}{L^2} = \frac{\pi^2(29{,}000 \text{ ksi})(533 \text{ in.}^4)}{(240 \text{ in.})^2}$$
$$= 2{,}650 \text{ kips}$$

Calculate the drift ratio using the first-order notional loading results.

LRFD	ASD
$\frac{\Delta_H}{L} = \frac{0.119 \text{ in.}}{240 \text{ in.}}$ $= 0.000496 \text{ in./in.}$	$\frac{\Delta_H}{L} = \frac{0.127 \text{ in.}}{240 \text{ in.}}$ $= 0.000529 \text{ in./in.}$

For the column at line C:

LRFD	ASD
$K_x = \sqrt{\frac{288 \text{ kips}}{[0.85 + 0.15(0.500)](72.4 \text{ kips})} \times (2{,}650 \text{ kips})\left(\frac{0.000496 \text{ in./in.}}{0.576 \text{ kips}}\right)}$ $\geq \sqrt{2{,}650 \text{ kips}\left(\frac{0.000496 \text{ in./in.}}{1.7(7.35 \text{ kips})}\right)}$ $= 3.13 \geq 0.324$ Use $K_x = 3.13$	$K_x = \sqrt{\frac{1.6(192 \text{ kips})}{[0.85 + 0.15(0.500)](1.6)(48.3 \text{ kips})} \times (2{,}650 \text{ kips})\left(\frac{0.000529 \text{ in./in.}}{0.614 \text{ kips}}\right)}$ $\geq \sqrt{2{,}650 \text{ kips}\left(\frac{0.000529 \text{ in./in.}}{1.7(1.6)(4.90 \text{ kips})}\right)}$ $= 3.13 \geq 0.324$ Use $K_x = 3.13$

Note that it is necessary to multiply the column loads by 1.6 for ASD in the expression above.

Verify the column strengths using the second-order forces shown above, using the following effective lengths (calculations not shown):

Columns:

Use $K_x L_x = 3.13(20.0 \text{ ft}) = 62.6 \text{ ft}$

Use $K_y L_y = 20.0$ ft

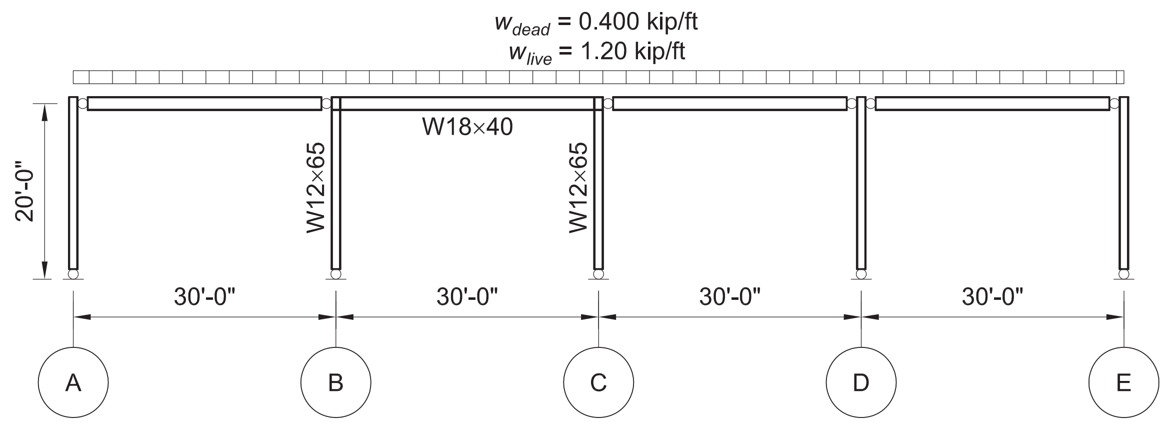
EXAMPLE C.1C DESIGN OF A MOMENT FRAME BY THE FIRST-ORDER METHOD

Repeat Example C.1A using the first-order analysis method.

Given:

Determine the required strengths and effective length factors for the columns in the rigid frame shown below for the maximum gravity load combination, using LRFD and ASD. Use the first-order analysis method.

Columns are unbraced between the footings and roof in the *x*- and *y*-axes and are assumed to have pinned bases.



Solution:

From *Manual* Table 1-1, the W12×65 has A = 19.1 in.^{2}

The beams from grid lines A to B, and C to E and the columns at A, D and E are pinned at both ends and do not contribute to the lateral stability of the frame. There are no P-Δ effects to consider in these members and they may be designed using K=1.0.

The moment frame between grid lines B and C is the source of lateral stability and therefore must be designed using the provisions of Appendix 7 of the AISC *Specification*. Although the columns at grid lines A, D and E do not contribute to lateral stability, the forces required to stabilize them must be considered in the analysis. These members need not be included in the analysis model, except that the forces in the “leaning” columns must be included in the calculation of notional loads.

Check the limitations for the use of the first-order analysis method given in Appendix 7, Section 7.3.1:

(1) The structure supports gravity loads through nominally vertical columns.
(2) The ratio of maximum second-order drift to the maximum first-order drift will be assumed to be no greater than 1.5, subject to verification.
(3) The required axial strength of the members in the moment frame will be assumed to be no more than 50% of the axial yield strength, subject to verification.

From Chapter 2 of ASCE/SEI 7, the maximum gravity load combinations are:

LRFD	ASD
$w_u = 1.2D + 1.6L$ $= 1.2(0.400 \text{ kip/ft}) + 1.6(1.20 \text{ kip/ft})$ $= 2.40$ kip/ft	$w_a = D + L$ $= 0.400 \text{ kip/ft} + 1.20 \text{ kip/ft}$ $= 1.60$ kip/ft

Per AISC *Specification* Appendix 7, Section 7.3.2, the required strengths are determined from a first-order analysis using notional loads determined below along with a B_1 multiplier as determined from Appendix 8.

For ASD, do not multiply loads or divide results by 1.6.

Frame Analysis Gravity Loads

The uniform gravity loads to be considered in the first-order analysis on the beam from B to C are:

LRFD	ASD
$w_u' = 2.40$ kip/ft	$w_a' = 1.60$ kip/ft

Concentrated gravity loads to be considered in a second-order analysis on the columns at B and C contributed by adjacent beams are:

LRFD	ASD
$P_u' = (15.0 \text{ ft})(2.40 \text{ kip/ft})$ $= 36.0$ kips	$P_a' = (15.0 \text{ ft})(1.60 \text{ kip/ft})$ $= 24.0$ kips

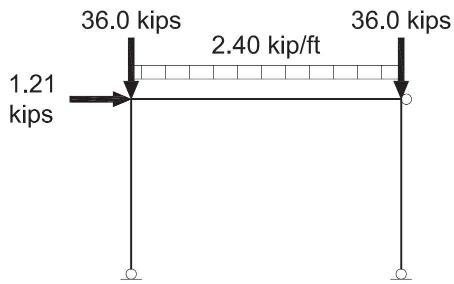
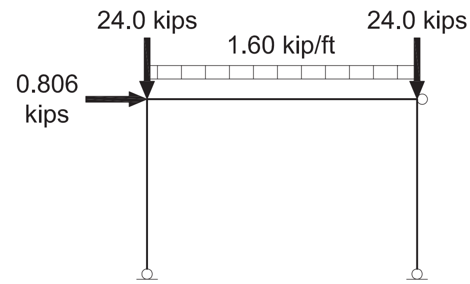
Frame Analysis Notional Loads

Per AISC *Specification* Appendix 7, Section 7.3.2, frame out-of-plumbness must be accounted for by the application of notional loads.

From AISC *Specification* Appendix Equation A-7-2, the required notional loads are:

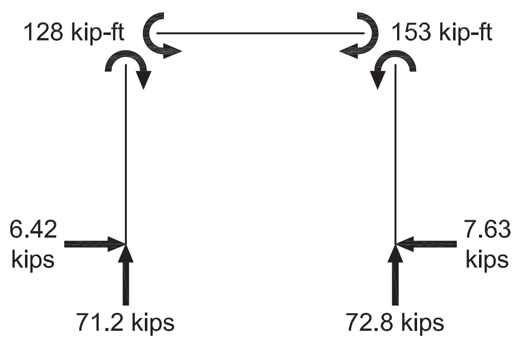
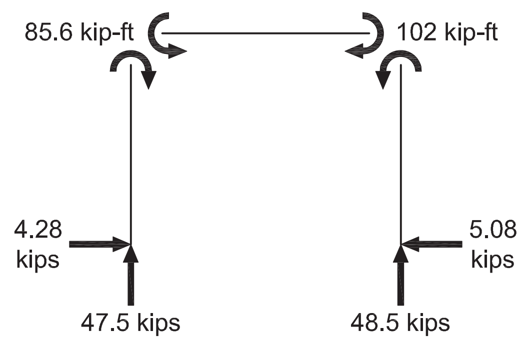
LRFD	ASD
$\alpha = 1.0$ $Y_i = (120 \text{ ft})(2.40 \text{ kip/ft})$ $= 288$ kips $\Delta = 0.0$ in. (no drift in this load combination) $L = 240$ in. $N_i = 2.1\alpha(\Delta/L)Y_i \geq 0.0042Y_i$ $= 2.1(1.0)(0.0 \text{ in.}/240 \text{ in.})(288 \text{ kips})$ $\geq 0.0042(288 \text{ kips})$ $= 0.0 \text{ kips} \geq 1.21 \text{ kips}$ Use $N_i = 1.21$ kips	$\alpha = 1.6$ $Y_i = (120 \text{ ft})(1.60 \text{ kip/ft})$ $= 192$ kips $\Delta = 0.0$ in. (no drift in this load combination) $L = 240$ in. $N_i = 2.1\alpha(\Delta/L)Y_i \geq 0.0042Y_i$ $= 2.1(1.6)(0.0 \text{ in.}/240 \text{ in.})(192 \text{ kips})$ $\geq 0.0042(192 \text{ kips})$ $= 0.0 \text{ kips} \geq 0.806 \text{ kips}$ Use $N_i = 0.806$ kips

Summary of Applied Frame Loads

LRFD	ASD
	

Per AISC *Specification* Appendix 7, Section 7.2.2, conduct the analysis using the full nominal stiffnesses.

Using analysis software, the following first-order results are obtained:

LRFD	ASD
Δ_{1st} = 0.250 in. 	Δ_{1st} = 0.167 in. 

Check the assumption that the ratio of the second-order drift to the first-order drift does not exceed 1.5. B_2 can be used to check this limit. Calculate B_2 per the provisions of Section 8.2.2 of Appendix 8 using the results of the first-order analysis.

LRFD	ASD
P_{mf} = 71.2 kips + 72.8 kips = 144 kips P_{story} = 144 kips + 4(36 kips) = 288 kips $R_M = 1 - 0.15\left(P_{mf}/P_{story}\right)$ (*Spec*. Eq. A-8-8) $= 1 - 0.15\left(144\text{ kips}/288\text{ kips}\right)$ $= 0.925$ Δ_H = 0.250 in. H = 1.21 kips L = 240 in.	P_{mf} = 47.5 kips + 48.5 kips = 96 kips P_{story} = 96 kips + 4(24 kips) = 192 kips $R_M = 1 - 0.15\left(P_{mf}/P_{story}\right)$ (*Spec*. Eq. A-8-8) $= 1 - 0.15\left(96\text{ kips}/192\text{ kips}\right)$ $= 0.925$ Δ_H = 0.167 in. H = 0.806 kips L = 240 in.

LRFD	ASD
$P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec.* Eq. A-8-7)	$P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec.* Eq. A-8-7)
$= 0.925 \dfrac{1.21 \text{ kips}(240 \text{ in.})}{0.250 \text{ in.}}$	$= 0.925 \dfrac{0.806 \text{ kips}(240 \text{ in.})}{0.167 \text{ in.}}$
$= 1{,}070$ kips	$= 1{,}070$ kips
$\alpha = 1.00$	$\alpha = 1.60$
$B_2 = \dfrac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec.* Eq. A-8-6)	$B_2 = \dfrac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec.* Eq. A-8-6)
$= \dfrac{1}{1-\dfrac{1.00(288 \text{ kips})}{1{,}070 \text{ kips}}} \geq 1$	$= \dfrac{1}{1-\dfrac{1.60(192 \text{ kips})}{1{,}070 \text{ kips}}} \geq 1$
$= 1.37$	$= 1.40$

The assumption that the ratio of the maximum second-order drift to the maximum first-order drift is no greater than 1.5 is correct; therefore, the first-order analysis method is permitted.

Check the assumption that $\alpha P_r \leq 0.5P_y$ and therefore, the first-order analysis method is permitted.

$$0.5P_y = 0.5F_yA_g$$
$$= 0.5(50 \text{ ksi})(19.1 \text{ in.}^2)$$
$$= 478 \text{ kips}$$

LRFD	ASD
$\alpha P_r = 1.0(72.8 \text{ kips})$	$\alpha P_r = 1.6(48.5 \text{ kips})$
$= 72.8 \text{ kips} < 478 \text{ kips}$ **o.k.**	$= 77.6 \text{ kips} < 478 \text{ kips}$ **o.k.**

The assumption that the first-order analysis method can be used is verified.

Although the second-order sway multiplier is approximately 1.4, the change in bending moment is small because the only sway moments are those produced by the small notional loads. For load combinations with significant gravity and lateral loadings, the increase in bending moments is larger.

Verify the column strengths using the second-order forces, using the following effective lengths (calculations not shown):

Columns:

Use $KL_x = 20.0$ ft
Use $KL_y = 20.0$ ft

Chapter D
Design of Members for Tension

D1. SLENDERNESS LIMITATIONS

Section D1 does not establish a slenderness limit for tension members, but recommends limiting L/r to a maximum of 300. This is not an absolute requirement. Rods and hangers are specifically excluded from this recommendation.

D2. TENSILE STRENGTH

Both tensile yielding strength and tensile rupture strength must be considered for the design of tension members. It is not unusual for tensile rupture strength to govern the design of a tension member, particularly for small members with holes or heavier sections with multiple rows of holes.

For preliminary design, tables are provided in Part 5 of the AISC *Manual* for W-shapes, L-shapes, WT-shapes, rectangular HSS, square HSS, round HSS, Pipe and 2L-shapes. The calculations in these tables for available tensile rupture strength assume an effective area, A_e, of $0.75A_g$. If the actual effective area is greater than $0.75A_g$, the tabulated values will be conservative and calculations can be performed to obtain higher available strengths. If the actual effective area is less than $0.75A_g$, the tabulated values will be unconservative and calculations are necessary to determine the available strength.

D3. EFFECTIVE NET AREA

The gross area, A_g, is the total cross-sectional area of the member.

In computing net area, A_n, AISC *Specification* Section B4.3 requires that an extra 1/16 in. be added to the bolt hole diameter.

A computation of the effective area for a chain of holes is presented in Example D.9.

Unless all elements of the cross section are connected, $A_e = A_nU$, where U is a reduction factor to account for shear lag. The appropriate values of U can be obtained from Table D3.1 of the AISC *Specification*.

D4. BUILT-UP MEMBERS

The limitations for connections of built-up members are discussed in Section D4 of the AISC *Specification*.

D5. PIN-CONNECTED MEMBERS

An example of a pin-connected member is given in Example D.7.

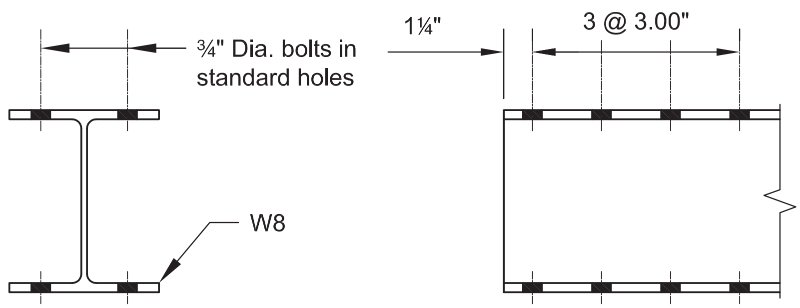
D6. EYEBARS

An example of an eyebar is given in Example D.8. The strength of an eyebar meeting the dimensional requirements of AISC *Specification* Section D6 is governed by tensile yielding of the body.

EXAMPLE D.1 W-SHAPE TENSION MEMBER

Given:

Select an 8-in. W-shape, ASTM A992, to carry a dead load of 30 kips and a live load of 90 kips in tension. The member is 25 ft long. Verify the member strength by both LRFD and ASD with the bolted end connection shown. Verify that the member satisfies the recommended slenderness limit. Assume that connection limit states do not govern.



Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_u = 1.2(30 kips) + 1.6(90 kips) = 180 kips	P_a = 30 kips + 90 kips = 120 kips

From AISC *Manual* Table 5-1, try a W8×21.

From AISC *Manual* Table 2-4, the material properties are as follows:

W8×21
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Tables 1-1 and 1-8, the geometric properties are as follows:

W8×21
A_g = 6.16 in.^{2}
b_f = 5.27 in.
t_f = 0.400 in.
d = 8.28 in.
r_y = 1.26 in.

WT4×10.5
$\bar{y}$ = 0.831 in.

Tensile Yielding

From AISC *Manual* Table 5-1, the tensile yielding strength is:

LRFD	ASD
277 kips > 180 kips **o.k.**	184 kips > 120 kips **o.k.**

Tensile Rupture

Verify the table assumption that $A_e/A_g \geq 0.75$ for this connection.

Calculate the shear lag factor, U, as the larger of the values from AISC *Specification* Section D3, Table D3.1 case 2 and case 7.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$\begin{aligned} U &= \frac{2b_f t_f}{A_g} \\ &= \frac{2(5.27\text{ in.})(0.400\text{ in.})}{6.16\text{ in.}^2} \\ &= 0.684 \end{aligned}$$

Case 2: Check as two WT-shapes per AISC *Specification* Commentary Figure C-D3.1, with $\bar{x} = \bar{y} = 0.831$ in.

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{0.831\text{ in.}}{9.00\text{ in.}} \\ &= 0.908 \end{aligned}$$

Case 7:

$b_f = 5.27$ in.
$d = 8.28$ in.
$b_f < \tfrac{2}{3}d$
$U = 0.85$

Use $U = 0.908$.

Calculate A_n using AISC *Specification* Section B4.3.

$$\begin{aligned} A_n &= A_g - 4(d_h + \tfrac{1}{16}\text{ in.})t_f \\ &= 6.16\text{ in.}^2 - 4(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(0.400\text{ in.}) \\ &= 4.76\text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U \\ &= 4.76\text{ in.}^2(0.908) \\ &= 4.32\text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

$$\begin{aligned} \frac{A_e}{A_g} &= \frac{4.32\text{ in.}^2}{6.16\text{ in.}^2} \\ &= 0.701 < 0.75\text{; therefore, table values for rupture are not valid.} \end{aligned}$$

The available tensile rupture strength is,

$$P_n = F_u A_e$$
$$= 65 \text{ ksi}(4.32 \text{ in.}^2)$$
$$= 281 \text{ kips}$$
(*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(281 \text{ kips})$ $= 211 \text{ kips}$ 211 kips > 180 kips **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{281 \text{ kips}}{2.00}$ $= 141 \text{ kips}$ 141 kips > 120 kips **o.k.**

Check Recommended Slenderness Limit

$$\frac{L}{r} = \left(\frac{25.0 \text{ ft}}{1.26 \text{ in.}}\right)\left(\frac{12.0 \text{ in.}}{\text{ft}}\right)$$
$= 238 < 300$ from AISC *Specification* Section D1 **o.k.**

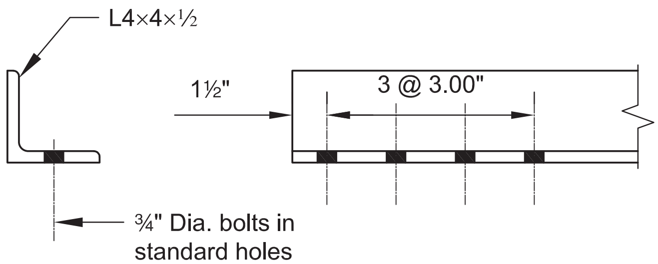
The W8×21 available tensile strength is governed by the tensile rupture limit state at the end connection.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.2 SINGLE ANGLE TENSION MEMBER

Given:

Verify, by both ASD and LRFD, the tensile strength of an L4×4×½, ASTM A36, with one line of (4) ¾-in.-diameter bolts in standard holes. The member carries a dead load of 20 kips and a live load of 60 kips in tension. Calculate at what length this tension member would cease to satisfy the recommended slenderness limit. Assume that connection limit states do not govern.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

L4×4×½
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×½
A_g = 3.75 in.^{2}
r_z = 0.776 in.
$\bar{y}$ = 1.18 in. = $\bar{x}$

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_u = 1.2(20 kips) + 1.6(60 kips) = 120 kips	P_a = 20 kips + 60 kips = 80.0 kips

Tensile Yielding

$$P_n = F_y A_g \qquad (Spec.\ \text{Eq. D2-1})$$
$$= 36 \text{ ksi}(3.75 \text{ in.}^2)$$
$$= 135 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile yielding strength is:

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(135 \text{ kips})$ = 122 kips	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{135 \text{ kips}}{1.67}$ = 80.8 kips

Tensile Rupture

Calculate U as the larger of the values from AISC *Specification* Section D3, Table D3.1 Case 2 and Case 8.

From AISC *Specification* Section D3, for open cross sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area, therefore,

$$U = 0.500$$

Case 2:

$$U = 1 - \frac{\bar{x}}{l}$$
$$= 1 - \frac{1.18 \text{ in.}}{9.00 \text{ in.}}$$
$$= 0.869$$

Case 8, with 4 or more fasteners per line in the direction of loading:

$$U = 0.80$$

Use $U = 0.869$.

Calculate A_n using AISC *Specification* Section B4.3.

$$A_n = A_g - (d_h + 1/16)t$$
$$= 3.75 \text{ in.}^2 - (13/16 \text{ in.} + 1/16 \text{ in.})(1/2 \text{ in.})$$
$$= 3.31 \text{ in.}^2$$

Calculate A_e using AISC *Specification* Section D3.

$$A_e = A_n U \qquad (\textit{Spec}. \text{ Eq. D3-1})$$
$$= 3.31 \text{ in.}^2(0.869)$$
$$= 2.88 \text{ in.}^2$$

$$P_n = F_u A_e \qquad (\textit{Spec}. \text{ Eq. D2-2})$$
$$= 58 \text{ ksi}(2.88 \text{ in.}^2)$$
$$= 167 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(167 \text{ kips})$ $= 125 \text{ kips}$	$\Omega_t = 2.00$ $\frac{P_n}{\Omega_t} = \frac{167 \text{ kips}}{2.00}$ $= 83.5 \text{ kips}$

The L4×4×½ available tensile strength is governed by the tensile yielding limit state.

LRFD	ASD
$\phi_t P_n = 122 \text{ kips}$ $122 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\frac{P_n}{\Omega_t} = 80.8 \text{ kips}$ $80.8 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Recommended L_{max}

Using AISC *Specification* Section D1:

$$L_{max} = 300r_z$$
$$= (300)(0.776\text{ in.})\left(\frac{\text{ft}}{12.0\text{ in.}}\right)$$
$$= 19.4\text{ ft}$$

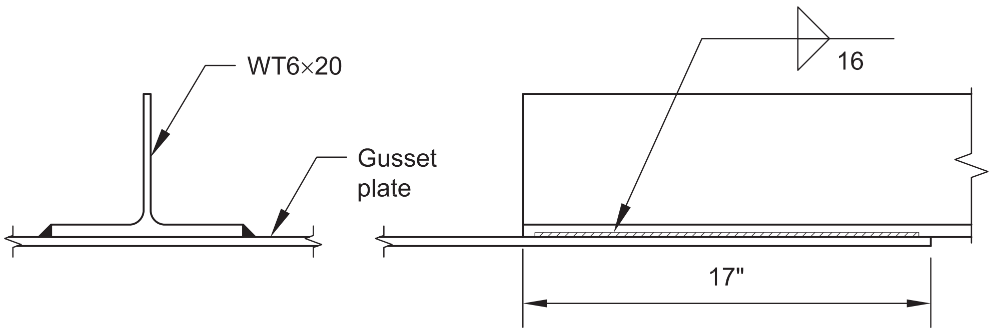
Note: The *L/r* limit is a recommendation, not a requirement.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.3 WT-SHAPE TENSION MEMBER

Given:

A WT6×20, ASTM A992 member has a length of 30 ft and carries a dead load of 40 kips and a live load of 120 kips in tension. The end connection is fillet welded on each side for 16 in. Verify the member tensile strength by both LRFD and ASD. Assume that the gusset plate and the weld are satisfactory.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

WT6×20
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT6×20
A_g = 5.84 in.^{2}
b_f = 8.01 in.
t_f = 0.515 in.
r_x = 1.57 in.
$\bar{y}$ = 1.09 in. = $\bar{x}$ (in equation for U)

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_u = 1.2(40 kips) + 1.6(120 kips) = 240 kips	P_a = 40 kips + 120 kips = 160 kips

Tensile Yielding

Check tensile yielding limit state using AISC *Manual* Table 5-3.

LRFD	ASD
$\phi_t P_n$ = 263 kips > 240 kips **o.k.**	$\dfrac{P_n}{\Omega_t}$ = 175 kips > 160 kips **o.k.**

Tensile Rupture

Check tensile rupture limit state using AISC *Manual* Table 5-3.

LRFD	ASD
$\phi_t P_n = 214 \text{ kips} < 240 \text{ kips}$ **n.g.**	$\dfrac{P_n}{\Omega_t} = 142 \text{ kips} < 160 \text{ kips}$ **n.g.**

The tabulated available rupture strengths may be conservative for this case; therefore, calculate the exact solution.

Calculate U as the larger of the values from AISC *Specification* Section D3 and Table D3.1 case 2.

From AISC *Specification* Section D3, for open cross-sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$
\begin{aligned}
U &= \frac{b_f t_f}{A_g} \\
&= \frac{8.01 \text{ in.}(0.515 \text{ in.})}{5.84 \text{ in.}^2} \\
&= 0.706
\end{aligned}
$$

Case 2:

$$
\begin{aligned}
U &= 1 - \frac{\bar{x}}{l} \\
&= 1 - \frac{1.09 \text{ in.}}{16.0 \text{ in.}} \\
&= 0.932
\end{aligned}
$$

Use $U = 0.932$.

Calculate A_n using AISC *Specification* Section B4.3.

$$
\begin{aligned}
A_n &= A_g \text{ (because there are no reductions due to holes or notches)} \\
&= 5.84 \text{ in.}^2
\end{aligned}
$$

Calculate A_e using AISC *Specification* Section D3.

$$
\begin{aligned}
A_e &= A_n U \\
&= 5.84 \text{ in.}^2(0.932) \\
&= 5.44 \text{ in.}^2
\end{aligned}
$$
(*Spec*. Eq. D3-1)

Calculate P_n.

$$
\begin{aligned}
P_n &= F_u A_e \\
&= 65 \text{ ksi}(5.44 \text{ in.}^2) \\
&= 354 \text{ kips}
\end{aligned}
$$
(*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(354\text{ kips})$ $= 266\text{ kips}$ 266 kips > 240 kips **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{354\text{ kips}}{2.00}$ $= 177\text{ kips}$ 177 kips > 160 kips **o.k.**

Alternately, the available tensile rupture strengths can be determined by modifying the tabulated values. The available tensile rupture strengths published in the tension member selection tables are based on the assumption that $A_e = 0.75A_g$. The actual available strengths can be determined by adjusting the values from AISC *Manual* Table 5-3 as follows:

LRFD	ASD
$\phi_t P_n = 214\text{ kips}\left(\dfrac{A_e}{0.75A_g}\right)$ $= 214\text{ kips}\left(\dfrac{5.44\text{ in.}^2}{0.75\left(5.84\text{ in.}^2\right)}\right)$ $= 266\text{ kips}$	$\dfrac{P_n}{\Omega_t} = 142\text{ kips}\left(\dfrac{A_e}{0.75A_g}\right)$ $= 142\text{ kips}\left(\dfrac{5.44\text{ in.}^2}{0.75\left(5.84\text{ in.}^2\right)}\right)$ $= 176\text{ kips}$

The WT6×20 available tensile strength is governed by the tensile yielding limit state.

LRFD	ASD
$\phi_t P_n = 263\text{ kips}$ 263 kips > 240 kips **o.k.**	$\dfrac{P_n}{\Omega_t} = 175\text{ kips}$ 175 kips > 160 kips **o.k.**

Recommended Slenderness Limit

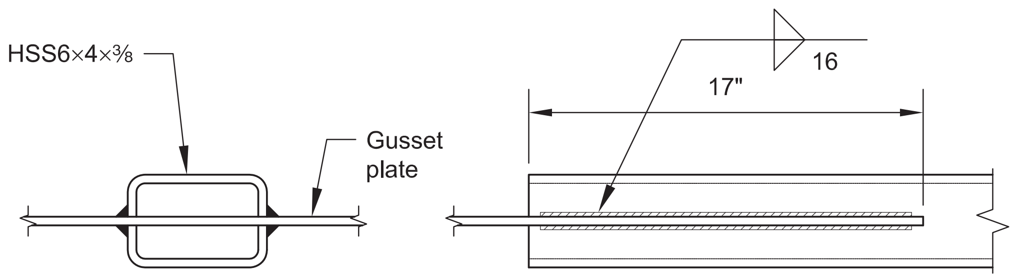
$$\frac{L}{r} = \left(\frac{30.0\text{ ft}}{1.57\text{ in.}}\right)\left(\frac{12.0\text{ in.}}{\text{ft}}\right)$$
$$= 229 < 300 \text{ from AISC } \textit{Specification} \text{ Section D1} \quad \textbf{o.k.}$$

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.4 RECTANGULAR HSS TENSION MEMBER

Given:

Verify the tensile strength of an HSS6×4×⅜, ASTM A500 Grade B, with a length of 30 ft. The member is carrying a dead load of 35 kips and a live load of 105 kips in tension. The end connection is a fillet welded ½-in.-thick single concentric gusset plate with a weld length of 16 in. Assume that the gusset plate and weld are satisfactory.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS6×4×⅜
A_g = 6.18 in.^{2}
r_y = 1.55 in.
t = 0.349 in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_u = 1.2(35 kips) + 1.6(105 kips) = 210 kips	P_a = 35 kips + 105 kips = 140 kips

Tensile Yielding

Check tensile yielding limit state using AISC *Manual* Table 5-4.

LRFD	ASD
$\phi_t P_n$ = 256 kips > 210 kips **o.k.**	$\dfrac{P_n}{\Omega_t}$ = 170 kips > 140 kips **o.k.**

Tensile Rupture

Check tensile rupture limit state using AISC *Manual* Table 5-4.

LRFD	ASD
$\phi_t P_n = 201\text{ kips} < 210\text{ kips}$ **n.g.**	$\dfrac{P_n}{\Omega_t} = 134\text{ kips} < 140\text{ kips}$ **n.g.**

The tabulated available rupture strengths may be conservative in this case; therefore, calculate the exact solution.

Calculate U from AISC *Specification* Table D3.1 case 6.

$$\begin{aligned}\bar{x} &= \frac{B^2 + 2BH}{4(B+H)} \\ &= \frac{(4.00\text{ in.})^2 + 2(4.00\text{ in.})(6.00\text{ in.})}{4(4.00\text{ in.} + 6.00\text{ in.})} \\ &= 1.60\text{ in.}\end{aligned}$$

$$\begin{aligned}U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.60\text{ in.}}{16.0\text{ in.}} \\ &= 0.900\end{aligned}$$

Allowing for a $\frac{1}{16}$-in. gap in fit-up between the HSS and the gusset plate:

$$\begin{aligned}A_n &= A_g - 2(t_p + \tfrac{1}{16}\text{ in.})t \\ &= 6.18\text{ in.}^2 - 2(\tfrac{1}{2}\text{ in.} + \tfrac{1}{16}\text{ in.})(0.349\text{ in.}) \\ &= 5.79\text{ in.}^2\end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned}A_e &= A_n U \\ &= 5.79\text{ in.}^2(0.900) \\ &= 5.21\text{ in.}^2\end{aligned}$$ (*Spec*. Eq. D3-1)

Calculate P_n.

$$\begin{aligned}P_n &= F_u A_e \\ &= 58\text{ ksi}(5.21\text{ in}^2) \\ &= 302\text{ kips}\end{aligned}$$ (*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(302\text{ kips})$ $= 227\text{ kips}$ $227\text{ kips} > 210\text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{302\text{ kips}}{2.00}$ $= 151\text{ kips}$ $151\text{ kips} > 140\text{ kips}$ **o.k.**

The HSS available tensile strength is governed by the tensile rupture limit state.

Recommended Slenderness Limit

$$\frac{L}{r} = \left(\frac{30.0 \text{ ft}}{1.55 \text{ in.}}\right)\left(\frac{12.0 \text{ in.}}{\text{ft}}\right)$$

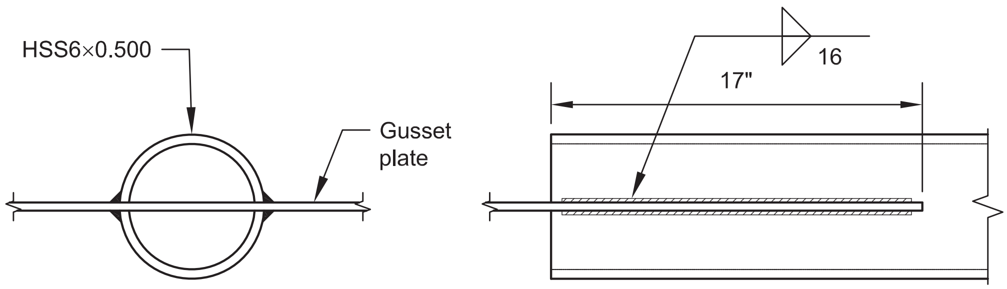
$= 232 < 300$ from AISC *Specification* Section D1 **o.k.**

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.5 ROUND HSS TENSION MEMBER

Given:

Verify the tensile strength of an HSS6×0.500, ASTM A500 Grade B, with a length of 30 ft. The member carries a dead load of 40 kips and a live load of 120 kips in tension. Assume the end connection is a fillet welded ½-in.-thick single concentric gusset plate with a weld length of 16 in. Assume that the gusset plate and weld are satisfactory.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
F_y = 42 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-13, the geometric properties are as follows:

HSS6×0.500
A_g = 8.09 in.^{2}
r = 1.96 in.
t = 0.465 in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_u = 1.2(40 kips) + 1.6(120 kips) = 240 kips	P_a = 40 kips + 120 kips = 160 kips

Tensile Yielding

Check tensile yielding limit state using AISC *Manual* Table 5-6.

LRFD	ASD
$\phi_t P_n$ = 306 kips > 240 kips **o.k.**	$\dfrac{P_n}{\Omega_t}$ = 203 kips > 160 kips **o.k.**

Tensile Rupture

Check tensile rupture limit state using AISC *Manual* Table 5-6.

LRFD	ASD
$\phi_t P_n = 264 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 176 \text{ kips} > 160 \text{ kips}$ **o.k.**

Check that $A_e/A_g \geq 0.75$ as assumed in table.

Determine U from AISC *Specification* Table D3.1 Case 5.

$$L = 16.0 \text{ in.}$$
$$D = 6.00 \text{ in.}$$
$$\frac{L}{D} = \frac{16.0 \text{ in.}}{6.00 \text{ in.}}$$
$$= 2.67 \geq 1.3, \text{ therefore } U = 1.0$$

Allowing for a ¹⁄₁₆-in. gap in fit-up between the HSS and the gusset plate,

$$A_n = A_g - 2(t_p + \tfrac{1}{16} \text{ in.})t$$
$$= 8.09 \text{ in.}^2 - 2(0.500 \text{ in.} + \tfrac{1}{16} \text{ in.})(0.465 \text{ in.})$$
$$= 7.57 \text{ in.}^2$$

Calculate A_e using AISC *Specification* Section D3.

$$A_e = A_n U \qquad (\textit{Spec}. \text{ Eq. D3-1})$$
$$= 7.57 \text{ in.}^2 (1.0)$$
$$= 7.57 \text{ in.}^2$$

$$\frac{A_e}{A_g} = \frac{7.57 \text{ in.}^2}{8.09 \text{ in.}^2}$$
$$= 0.936 \geq 0.75$$ **o.k., but conservative**

Calculate P_n.

$$P_n = F_u A_e \qquad (\textit{Spec}. \text{ Eq. D2-2})$$
$$= (58 \text{ ksi})(7.57 \text{ in.}^2)$$
$$= 439 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(439 \text{ kips})$ $= 329 \text{ kips}$ 329 kips > 240 kips **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{439 \text{ kips}}{2.00}$ $= 220 \text{ kips}$ 220 kips > 160 kips **o.k.**

Recommended Slenderness Limit

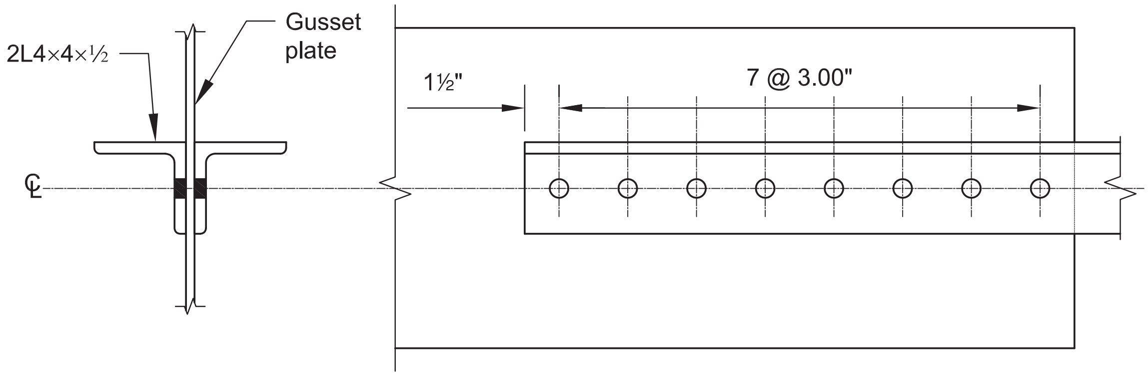
$$\frac{L}{r} = \left(\frac{30.0 \text{ ft}}{1.96 \text{ in.}}\right)\left(\frac{12.0 \text{ in.}}{\text{ft}}\right)$$
$$= 184 < 300$$ from AISC *Specification* Section D1 **o.k.**

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.6 DOUBLE ANGLE TENSION MEMBER

Given:

A 2L4×4×½ (⅜-in. separation), ASTM A36, has one line of (8) ¾-in.-diameter bolts in standard holes and is 25 ft in length. The double angle is carrying a dead load of 40 kips and a live load of 120 kips in tension. Verify the member tensile strength. Assume that the gusset plate and bolts are satisfactory.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L4×4×½
A_g = 3.75 in.^{2}
$\bar{x}$ = 1.18 in.

2L4×4×½ (s = ⅜ in.)
r_y = 1.83 in.
r_x = 1.21 in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_n = 1.2(40 kips) + 1.6(120 kips) = 240 kips	P_n = 40 kips + 120 kips = 160 kips

Tensile Yielding

$P_n = F_yA_g$ (*Spec*. Eq. D2-1)
= 36 ksi(2)(3.75 in.^{2})
= 270 kips

From AISC *Specification* Section D2, the available tensile yielding strength is:

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(270 \text{ kips})$ $= 243 \text{ kips}$	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{270 \text{ kips}}{1.67}$ $= 162 \text{ kips}$

Tensile Rupture

Calculate U as the larger of the values from AISC *Specification* Section D3, Table D3.1 case 2 and case 8.

From AISC *Specification* Section D3, for open cross-sections, U need not be less than the ratio of the gross area of the connected element(s) to the member gross area.

$$U = 0.500$$

Case 2:

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.18 \text{ in.}}{21.0 \text{ in.}} \\ &= 0.944 \end{aligned}$$

Case 8, with 4 or more fasteners per line in the direction of loading:

$$U = 0.80$$

Use $U = 0.944$.

Calculate A_n using AISC *Specification* Section B4.3.

$$\begin{aligned} A_n &= A_g - 2(d_h + \tfrac{1}{16} \text{ in.})t \\ &= 2(3.75 \text{ in.}^2) - 2(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.}) \\ &= 6.63 \text{ in.}^2 \end{aligned}$$

Calculate A_e using AISC *Specification* Section D3.

$$\begin{aligned} A_e &= A_n U \\ &= 6.63 \text{ in.}^2(0.944) \\ &= 6.26 \text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

Calculate P_n.

$$\begin{aligned} P_n &= F_u A_e \\ &= 58 \text{ ksi}(6.26 \text{ in.}^2) \\ &= 363 \text{ kips} \end{aligned}$$ (*Spec*. Eq. D2-2)

From AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(363 \text{ kips})$ $= 272 \text{ kips}$	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{363 \text{ kips}}{2.00}$ $= 182 \text{ kips}$

The double angle available tensile strength is governed by the tensile yielding limit state.

LRFD	ASD
243 kips > 240 kips **o.k.**	162 kips > 160 kips **o.k.**

Recommended Slenderness Limit

$$\frac{L}{r_x} = \left(\frac{25.0 \text{ ft}}{1.21 \text{ in.}}\right)\left(\frac{12.0 \text{ in.}}{\text{ft}}\right)$$
$= 248 < 300$ from AISC *Specification* Section D1 **o.k.**

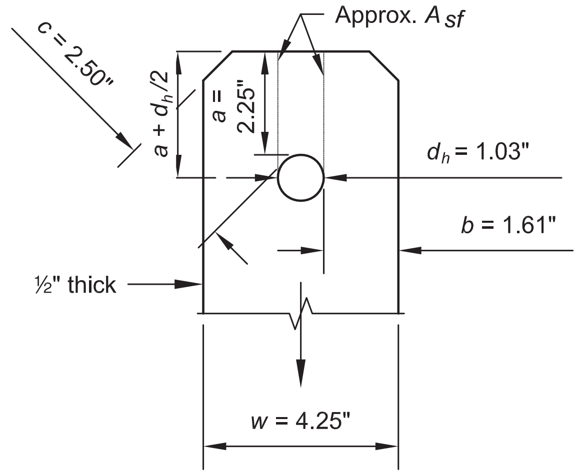
Note: From AISC *Specification* Section D4, the longitudinal spacing of connectors between components of built-up members should preferably limit the slenderness ratio in any component between the connectors to a maximum of 300.

See Chapter J for illustrations of connection limit state checks.

EXAMPLE D.7 PIN-CONNECTED TENSION MEMBER

Given:

An ASTM A36 pin-connected tension member with the dimensions shown as follows carries a dead load of 4 kips and a live load of 12 kips in tension. The diameter of the pin is 1 inch, in a 1⁄32-in. oversized hole. Assume that the pin itself is adequate. Verify the member tensile strength.



Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The geometric properties are as follows:

w = 4.25 in.
t = 0.500 in.
d = 1.00 in.
a = 2.25 in.
c = 2.50 in.
d_h = 1.03 in.

Check dimensional requirements using AISC *Specification* Section D5.2.

1. $b_e = 2t + 0.63$ in.
   $= 2(0.500\text{ in.}) + 0.63$ in.
   $= 1.63\text{ in.} \leq 1.61$ in.
   $b_e = 1.61$ in. **controls**

2. $a \geq 1.33b_e$
   $2.25\text{ in.} \geq 1.33(1.61\text{ in.})$
   $= 2.14$ in. **o.k.**

3. $w \geq 2b_e + d$
   4.25 in. $\geq$ 2(1.61 in.) + 1.00 in.
   = 4.22 in. **o.k.**

4. $c \geq a$
   2.50 in. $\geq$ 2.25 in. **o.k.**

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_u = 1.2(4 kips) + 1.6(12 kips) = 24.0 kips	P_a = 4 kips + 12 kips = 16.0 kips

Tensile Rupture

Calculate the available tensile rupture strength on the effective net area.

$$P_n = F_u(2tb_e) \qquad (Spec.\ \text{Eq. D5-1})$$
$$= 58 \text{ ksi } (2)(0.500 \text{ in.})(1.61 \text{ in.})$$
$$= 93.4 \text{ kips}$$

From AISC *Specification* Section D5.1, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n$ = 0.75(93.4 kips) = 70.1 kips	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{93.4 \text{ kips}}{2.00}$ = 46.7 kips

Shear Rupture

$$A_{sf} = 2t(a + d/2)$$
$$= 2(0.500 \text{ in.})[2.25 \text{ in.} + (1.00 \text{ in.}/2)]$$
$$= 2.75 \text{ in.}^2$$

$$P_n = 0.6F_uA_{sf} \qquad (Spec.\ \text{Eq. D5-2})$$
$$= 0.6(58 \text{ ksi})(2.75 \text{ in.}^2)$$
$$= 95.7 \text{ kips}$$

From AISC *Specification* Section D5.1, the available shear rupture strength is:

LRFD	ASD
$\phi_{sf} = 0.75$ $\phi_{sf} P_n$ = 0.75(95.7 kips) = 71.8 kips	$\Omega_{sf} = 2.00$ $\dfrac{P_n}{\Omega_{sf}} = \dfrac{95.7 \text{ kips}}{2.00}$ = 47.9 kips

Bearing

$$A_{pb} = 0.500 \text{ in.}(1.00 \text{ in.})$$
$$= 0.500 \text{ in.}^2$$

$R_n = 1.8F_yA_{pb}$ (*Spec*. Eq. J7-1)
$= 1.8(36 \text{ ksi})(0.500 \text{ in.}^2)$
$= 32.4 \text{ kips}$

From AISC *Specification* Section J7, the available bearing strength is:

LRFD	ASD
$\phi = 0.75$ $\phi P_n = 0.75(32.4 \text{ kips})$ $= 24.3 \text{ kips}$	$\Omega = 2.00$ $\dfrac{P_n}{\Omega} = \dfrac{32.4 \text{ kips}}{2.00}$ $= 16.2 \text{ kips}$

Tensile Yielding

$A_g = wt$
$= 4.25 \text{ in.} (0.500 \text{ in.})$
$= 2.13 \text{ in.}^2$

$P_n = F_yA_g$ (*Spec*. Eq. D2-1)
$= 36 \text{ ksi} (2.13 \text{ in.}^2)$
$= 76.7 \text{ kips}$

From AISC *Specification* Section D2, the available tensile yielding strength is:

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(76.7 \text{ kips})$ $= 69.0 \text{ kips}$	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{76.7 \text{ kips}}{1.67}$ $= 45.9 \text{ kips}$

The available tensile strength is governed by the bearing strength limit state.

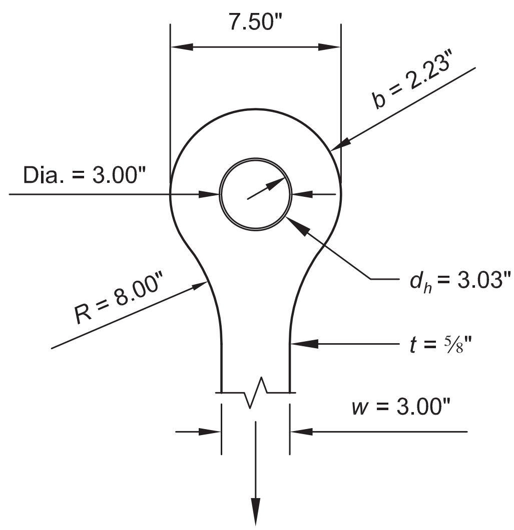
LRFD	ASD
$\phi P_n = 24.3 \text{ kips}$ $24.3 \text{ kips} > 24.0 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega} = 16.2 \text{ kips}$ $16.2 \text{ kips} > 16.0 \text{ kips}$ **o.k.**

See Example J.6 for an illustration of the limit state calculations for a pin in a drilled hole.

EXAMPLE D.8 EYEBAR TENSION MEMBER

Given:

A ⅝-in.-thick, ASTM A36 eyebar member as shown, carries a dead load of 25 kips and a live load of 15 kips in tension. The pin diameter, d, is 3 in. Verify the member tensile strength.



Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The geometric properties are as follows:

w = 3.00 in.
b = 2.23 in.
t = ⅝ in.
d_{head} = 7.50 in.
d = 3.00 in.
d_h = 3.03 in.
R = 8.00 in.

Check dimensional requirements using AISC *Specification* Section D6.1 and D6.2.

1. $t \geq$ ½ in.
   ⅝ in. $\geq$ ½ in. **o.k.**

2. $w \leq 8t$
   3.00 in. $\leq$ 8(⅝ in.)
   = 5.00 in. **o.k.**

3. $d \geq \frac{7}{8} w$
   3.00 in. $\geq \frac{7}{8}(3.00 \text{ in.})$
   $= 2.63$ in. **o.k.**

4. $d_h \leq d + \frac{1}{32}$ in.
   3.03 in. $\leq 3.00 \text{ in.} + (\frac{1}{32} \text{ in.})$
   $= 3.03$ in. **o.k.**

5. $R \geq d_{head}$
   8.00 in. ≥ 7.50 in. **o.k.**

6. $\frac{2}{3} w < b \leq \frac{3}{4} w$
   $\frac{2}{3}(3.00 \text{ in.}) < 2.23 \text{ in.} \leq \frac{3}{4}(3.00 \text{ in.})$
   $2.00 \text{ in.} < 2.23 \text{ in.} \leq 2.25 \text{ in.}$ **o.k.**

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(25.0 \text{ kips}) + 1.6(15.0 \text{ kips})$ $= 54.0 \text{ kips}$	$P_a = 25.0 \text{ kips} + 15.0 \text{ kips}$ $= 40.0 \text{ kips}$

Tensile Yielding

Calculate the available tensile yielding strength at the eyebar body (at *w*).

$$A_g = wt$$
$$= 3.00 \text{ in.}(\tfrac{5}{8} \text{ in.})$$
$$= 1.88 \text{ in.}^2$$

$$P_n = F_y A_g \qquad (Spec. \text{ Eq. D2-1})$$
$$= 36 \text{ ksi}(1.88 \text{ in.}^2)$$
$$= 67.7 \text{ kips}$$

From AISC *Specification* Section D2, the available tensile yielding strength is:

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(67.7 \text{ kips})$ $= 60.9 \text{ kips}$ 60.9 kips > 54.0 kips **o.k.**	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega_t} = \dfrac{67.7 \text{ kips}}{1.67}$ $= 40.5 \text{ kips}$ 40.5 kips > 40.0 kips **o.k.**

The eyebar tension member available strength is governed by the tensile yielding limit state.

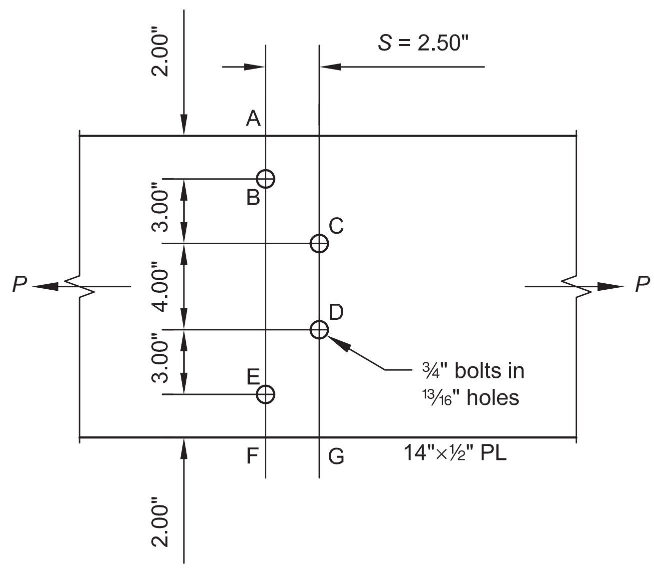
Note: The eyebar detailing limitations ensure that the tensile yielding limit state at the eyebar body will control the strength of the eyebar itself. The pin should also be checked for shear yielding, and, if the material strength is less than that of the eyebar, bearing.

See Example J.6 for an illustration of the limit state calculations for a pin in a drilled hole.

EXAMPLE D.9 PLATE WITH STAGGERED BOLTS

Given:

Compute A_n and A_e for a 14-in.-wide and ½-in.-thick plate subject to tensile loading with staggered holes as shown.



Solution:

Calculate net hole diameter using AISC *Specification* Section B4.3.

$$
\begin{aligned}
d_{net} &= d_h + \tfrac{1}{16}\text{ in.} \\
&= \tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.} \\
&= 0.875\text{ in.}
\end{aligned}
$$

Compute the net width for all possible paths across the plate. Because of symmetry, many of the net widths are identical and need not be calculated.

$$w = 14.0 - \Sigma d_{net} + \Sigma \frac{s^2}{4g} \quad \text{from AISC } \textit{Specification} \text{ Section B4.3.}$$

Line A-B-E-F:

$$
\begin{aligned}
w &= 14.0\text{ in.} - 2(0.875\text{ in.}) \\
&= 12.3\text{ in.}
\end{aligned}
$$

Line A-B-C-D-E-F:

$$
\begin{aligned}
w &= 14.0\text{ in.} - 4(0.875\text{ in.}) + \frac{(2.50\text{ in.})^2}{4(3.00\text{ in.})} + \frac{(2.50\text{ in.})^2}{4(3.00\text{ in.})} \\
&= 11.5\text{ in.} \qquad \textbf{controls}
\end{aligned}
$$

Line A-B-C-D-G:

$$
\begin{aligned}
w &= 14.0\text{ in.} - 3(0.875\text{ in.}) + \frac{(2.50\text{ in.})^2}{4(3.00\text{ in.})} \\
&= 11.9\text{ in.}
\end{aligned}
$$

Line A-B-D-E-F:

$$
\begin{aligned}
w &= 14.0\text{ in.} - 3(0.875\text{ in.}) + \frac{(2.50\text{ in.})^2}{4(7.00\text{ in.})} + \frac{(2.50\text{ in.})^2}{4(3.00\text{ in.})} \\
&= 12.1\text{ in.}
\end{aligned}
$$

Therefore, $A_n = 11.5\text{ in.}(0.500\text{ in.})$

$= 5.75 \text{ in.}^2$

Calculate *U*.

From AISC *Specification* Table D3.1 case 1, because tension load is transmitted to all elements by the fasteners,

$U = 1.0$

$$\begin{aligned} A_e &= A_n U \\ &= 5.75 \text{ in.}^2(1.0) \\ &= 5.75 \text{ in.}^2 \end{aligned}$$ (*Spec*. Eq. D3-1)

Chapter E
Design of Members for Compression

This chapter covers the design of compression members, the most common of which are columns. The AISC *Manual* includes design tables for the following compression member types in their most commonly available grades.

- wide-flange column shapes
- HSS
- double angles
- single angles

LRFD and ASD information is presented side-by-side for quick selection, design or verification. All of the tables account for the reduced strength of sections with slender elements.

The design and selection method for both LRFD and ASD designs is similar to that of previous AISC *Specifications*, and will provide similar designs. In this AISC *Specification*, ASD and LRFD will provide identical designs when the live load is approximately three times the dead load.

The design of built-up shapes with slender elements can be tedious and time consuming, and it is recommended that standard rolled shapes be used, when possible.

E1. GENERAL PROVISIONS

The design compressive strength, $\phi_c P_n$, and the allowable compressive strength, P_n/Ω_c, are determined as follows:

P_n = nominal compressive strength based on the controlling buckling mode

$\phi_c = 0.90$ (LRFD) $\quad\quad \Omega_c = 1.67$ (ASD)

Because F_{cr} is used extensively in calculations for compression members, it has been tabulated in AISC *Manual* Table 4-22 for all of the common steel yield strengths.

E2. EFFECTIVE LENGTH

In the AISC *Specification*, there is no limit on slenderness, KL/r. Per the AISC *Specification* Commentary, it is recommended that KL/r not exceed 200, as a practical limit based on professional judgment and construction economics.

Although there is no restriction on the unbraced length of columns, the tables of the AISC *Manual* are stopped at common or practical lengths for ordinary usage. For example, a double L3×3×¼, with a ⅜-in. separation has an r_y of 1.38 in. At a KL/r of 200, this strut would be 23’-0” long. This is thought to be a reasonable limit based on fabrication and handling requirements.

Throughout the AISC *Manual*, shapes that contain slender elements when supplied in their most common material grade are footnoted with the letter “c”. For example, see a W14×22[c].

E3. FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

Nonslender sections, including nonslender built-up I-shaped columns and nonslender HSS columns, are governed by these provisions. The general design curve for critical stress versus KL/r is shown in Figure E-1.

The term L is used throughout this chapter to describe the length between points that are braced against lateral and/or rotational displacement.

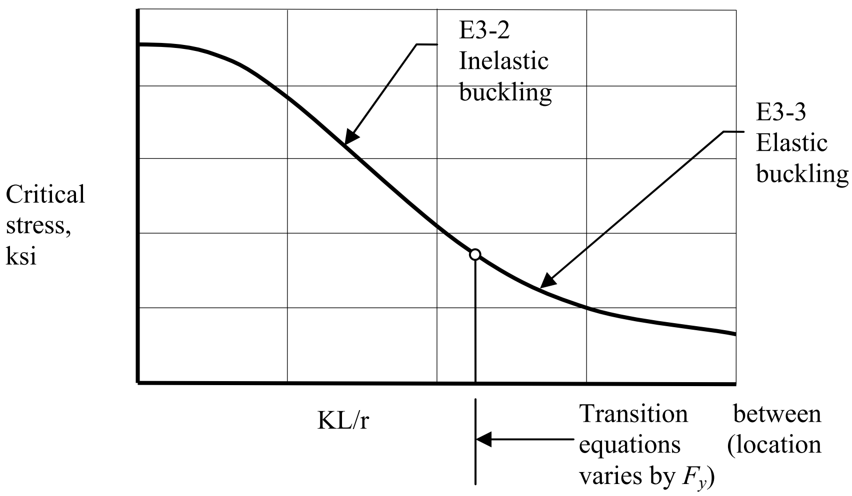


Fig. E-1. Standard column curve.

TRANSITION POINT LIMITING VALUES OF ***KL/r***		
F_y , ksi	**Limiting *KL/r***	**$0.44F_y$, ksi**
36	134	15.8
50	113	22.0
60	104	26.4
70	96	30.8

E4. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

This section is most commonly applicable to double angles and WT sections, which are singly-symmetric shapes subject to torsional and flexural-torsional buckling. The available strengths in axial compression of these shapes are tabulated in Part 4 of the AISC *Manual* and examples on the use of these tables have been included in this chapter for the shapes with $KL_z = KL_y$.

E5. SINGLE ANGLE COMPRESSION MEMBERS

The available strength of single angle compression members is tabulated in Part 4 of the AISC *Manual*.

E6. BUILT-UP MEMBERS

There are no tables for built-up shapes in the AISC *Manual*, due to the number of possible geometries. This section suggests the selection of built-up members without slender elements, thereby making the analysis relatively straightforward.

E7. MEMBERS WITH SLENDER ELEMENTS

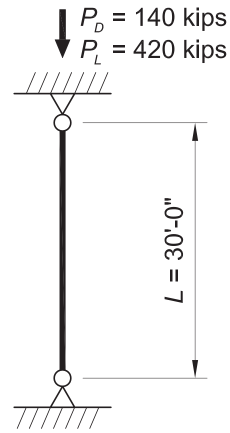
The design of these members is similar to members without slender elements except that the formulas are modified by a reduction factor for slender elements, Q. Note the similarity of Equation E7-2 with Equation E3-2, and the similarity of Equation E7-3 with Equation E3-3.

The tables of Part 4 of the AISC *Manual* incorporate the appropriate reductions in available strength to account for slender elements.

Design examples have been included in this Chapter for built-up I-shaped members with slender webs and slender flanges. Examples have also been included for a double angle, WT and an HSS shape with slender elements.

EXAMPLE E.1A W-SHAPE COLUMN DESIGN WITH PINNED ENDS

Given:



Select an ASTM A992 (F_y = 50 ksi) W-shape column to carry an axial dead load of 140 kips and live load of 420 kips. The column is 30 ft long and is pinned top and bottom in both axes. Limit the column size to a nominal 14-in. shape.

Solution:

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
P_u = 1.2(140 kips) + 1.6(420 kips) = 840 kips	P_a = 140 kips + 420 kips = 560 kips

Column Selection

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced length is the same in both the x-x and y-y directions and r_x exceeds r_y for all W-shapes, y-y axis bucking will govern.

Enter the table with an effective length, KL_y, of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×132.

Table 4-1 (continued)

Available Strength in Axial Compression, kips

W-Shapes

F_y = 50 ksi

W14

Shape	W14×											
lb/ft	145		132		120		109		99		90	
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1190
6	1250	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120
10	1200	1800	1080	1620	980	1470	888	1340	807	1210	735	1100
11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1090
12	1160	1750	1040	1570	948	1430	859	1290	780	1170	710	1070
13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050
14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030
15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000
16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979
17	1060	1590	937	1410	850	1280	770	1160	698	1050	635	955
18	1030	1550	913	1370	828	1240	750	1130	680	1020	618	929
19	1010	1510	888	1330	805	1210	729	1100	661	994	601	903
20	980	1470	862	1300	782	1180	708	1060	642	964	583	877
22	927	1390	810	1220	734	1100	664	998	602	904	547	822
24	872	1310	756	1140	685	1030	620	931	561	843	509	766
26	816	1230	702	1060	635	955	574	863	519	781	472	709
28	759	1140	648	974	586	880	529	796	478	719	434	653
30	703	1060	594	893	537	807	485	729	438	658	397	597

(Row labels: length, KL (ft), with respect to least radius of gyration, r_y)

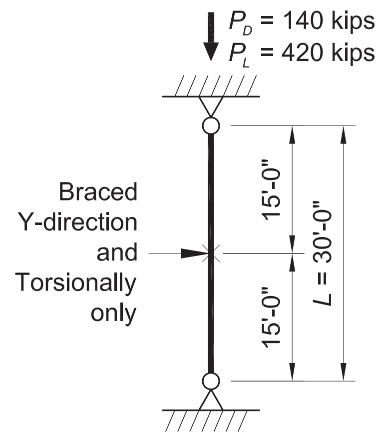
From AISC *Manual* Table 4-1, the available strength for a *y*-*y* axis effective length of 30 ft is:

LRFD	ASD
$\phi_c P_n$ = 893 kips > 840 kips **o.k.**	$\dfrac{P_n}{\Omega_c}$ = 594 kips > 560 kips **o.k.**

EXAMPLE E.1B W-SHAPE COLUMN DESIGN WITH INTERMEDIATE BRACING

Given:

Redesign the column from Example E.1A assuming the column is laterally braced about the *y-y* axis and torsionally braced at the midpoint.



Solution:

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(140\text{ kips}) + 1.6(420\text{ kips})$ $= 840\text{ kips}$	$P_a = 140\text{ kips} + 420\text{ kips}$ $= 560\text{ kips}$

Column Selection

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, $K = 1.0$.

Because the unbraced lengths differ in the two axes, select the member using the *y-y* axis then verify the strength in the *x-x* axis.

Enter AISC *Manual* Table 4-1 with a *y-y* axis effective length, KL_y, of 15 ft and proceed across the table until reaching a shape with an available strength that equals or exceeds the required strength. Try a W14×90. A 15 ft long W14×90 provides an available strength in the *y-y* direction of:

LRFD	ASD
$\phi_c P_n = 1{,}000\text{ kips}$	$\dfrac{P_n}{\Omega_c} = 667\text{ kips}$

The r_x/r_y ratio for this column, shown at the bottom of AISC *Manual* Table 4-1, is 1.66. The equivalent *y-y* axis effective length for strong axis buckling is computed as:

$$KL = \frac{30.0\text{ ft}}{1.66}$$
$$= 18.1\text{ ft}$$

Return to Table of Contents

F_y = 50 ksi

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

W14

Shape	W14×											
lb/ft	145		132		120		109		99		90	
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
length, KL (ft), with respect to least radius of gyration, r_y												
0	1280	1920	1160	1750	1060	1590	958	1440	871	1310	793	1190
6	1250	1880	1130	1700	1030	1550	932	1400	848	1270	772	1160
7	1240	1860	1120	1680	1020	1530	923	1390	839	1260	764	1150
8	1230	1840	1110	1660	1010	1510	913	1370	830	1250	755	1140
9	1210	1820	1090	1640	994	1490	901	1350	819	1230	745	1120
10	1200	1800	1080	1620	980	1470	888	1340	807	1210	735	1100
11	1180	1770	1060	1600	965	1450	874	1310	794	1190	723	1090
12	1160	1750	1040	1570	948	1430	859	1290	780	1170	710	1070
13	1140	1720	1020	1540	931	1400	843	1270	766	1150	697	1050
14	1120	1690	1000	1510	912	1370	826	1240	750	1130	682	1030
15	1100	1650	982	1480	892	1340	808	1210	733	1100	667	1000
16	1080	1620	960	1440	872	1310	789	1190	716	1080	652	979
17	1060	1590	937	1410	850	1280	770	1160	698	1050	635	955
18	1030	1550	913	1370	828	1240	750	1130	680	1020	618	929
19	1010	1510	888	1330	805	1210	729	1100	661	994	601	903
20	980	1470	862	1300	782	1180	708	1060	642	964	583	877
22	927	1390	810	1220	734	1100	664	998	602	904	547	822
24	872	1310	756	1140	685	1030	620	931	561	843	509	766
26	816	1230	702	1060	635	955	574	863	519	781	472	709
28	759	1140	648	974	586	880	529	796	478	719	434	653
30	703	1060	594	893	537	807	485	729	438	658	397	597

From AISC *Manual* Table 4-1, the available strength of a W14×90 with an effective length of 18 ft is:

LRFD	ASD
$\phi_c P_n = 929\text{ kips} > 840\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 618\text{ kips} > 560\text{ kips}$ **o.k.**

The available compressive strength is governed by the *x-x* axis flexural buckling limit state.

The available strengths of the columns described in Examples E.1A and E.1B are easily selected directly from the AISC *Manual* Tables. The available strengths can also be verified by hand calculations, as shown in the following Examples E.1C and E.1D.

EXAMPLE E.1C W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a W14×132 column with unbraced lengths of 30 ft in both axes. The material properties and loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×132
A_g = 38.8 in.^{2}
r_x = 6.28 in.
r_y = 3.76 in.

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced length is the same for both axes, the y-y axis will govern.

$$\frac{K_y L_y}{r_y} = \left(\frac{1.0(30.0\text{ ft})}{3.76\text{ in.}}\right)\left(\frac{12.0\text{ in}}{\text{ft}}\right)$$
$$= 95.7$$

For F_y = 50 ksi, the available critical stresses, $\phi_c F_{cr}$ and F_{cr}/Ω_c for KL/r = 95.7 are interpolated from AISC *Manual* Table 4-22 as follows:

LRFD	ASD
$\phi_c F_{cr} = 23.0\text{ ksi}$ $\phi_c P_n = 38.8\text{ in.}^2(23.0\text{ ksi})$ $= 892\text{ kips} > 840\text{ kips}$ **o.k.**	$\dfrac{F_{cr}}{\Omega_c} = 15.4\text{ ksi}$ $\dfrac{P_n}{\Omega_c} = 38.8\text{ in.}^2(15.4\text{ ksi})$ $= 598\text{ kips} > 560\text{ kips}$ **o.k.**

Note that the calculated values are approximately equal to the tabulated values.

EXAMPLE E.1D W-SHAPE AVAILABLE STRENGTH CALCULATION

Given:

Calculate the available strength of a W14×90 with a strong axis unbraced length of 30.0 ft and weak axis and torsional unbraced lengths of 15.0 ft. The material properties and loads are as given in Example E.1A.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×90
A_g = 26.5 in.^{2}
r_x = 6.14 in.
r_y = 3.70 in.

Slenderness Check

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

$$\frac{KL_x}{r_x} = \frac{1.0(30.0\text{ ft})}{6.14\text{ in.}}\left(\frac{12\text{ in.}}{\text{ft}}\right)$$
$$= 58.6 \qquad \textbf{governs}$$
$$\frac{KL_y}{r_y} = \frac{1.0(15.0\text{ ft})}{3.70\text{ in.}}\left(\frac{12\text{ in.}}{\text{ft}}\right)$$
$$= 48.6$$

Critical Stresses

The available critical stresses may be interpolated from AISC *Manual* Table 4-22 or calculated directly as follows:

Calculate the elastic critical buckling stress, F_e.

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$ (*Spec*. Eq. E3-4)
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(58.6)^2}$$
$$= 83.3\text{ ksi}$$

Calculate the flexural buckling stress, F_{cr}.

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 113$$

Because $\dfrac{KL}{r} = 58.6 \le 113$,

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \qquad (\textit{Spec}.\ \text{Eq. E3-2})$$

$$= \left[0.658^{\frac{50.0\text{ ksi}}{83.3\text{ ksi}}}\right] 50.0\text{ ksi}$$

$$= 38.9\text{ ksi}$$

Nominal Compressive Strength

$$P_n = F_{cr}A_g \qquad (\textit{Spec}.\ \text{Eq. E3-1})$$

$$= 38.9\text{ ksi}\left(26.5\text{ in.}^2\right)$$

$$= 1{,}030\text{ kips}$$

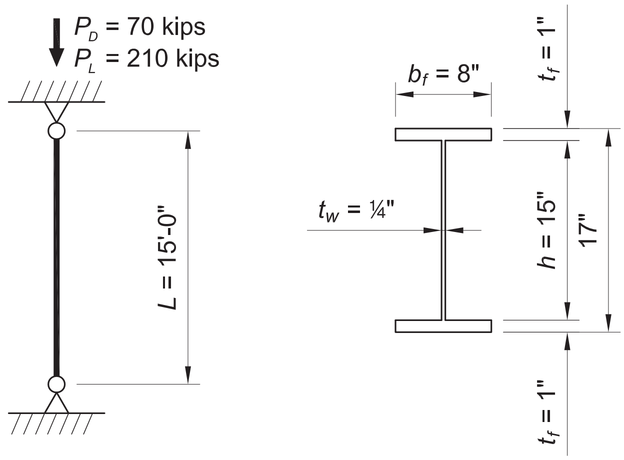
From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(1{,}030\text{ kips})$ $= 927\text{ kips} > 840\text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{1{,}030\text{ kips}}{1.67}$ $= 617\text{ kips} > 560\text{ kips}$ **o.k.**

EXAMPLE E.2 BUILT-UP COLUMN WITH A SLENDER WEB

Given:

Verify that a built-up, ASTM A572 Grade 50 column with PL1 in. × 8 in. flanges and a PL¼ in. × 15 in. web is sufficient to carry a dead load of 70 kips and live load of 210 kips in axial compression. The column length is 15 ft and the ends are pinned in both axes.



Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Built-Up Column
ASTM A572 Grade 50
F_y = 50 ksi
F_u = 65 ksi

The geometric properties are as follows:

Built-Up Column
d = 17.0 in.
b_f = 8.00 in.
t_f = 1.00 in.
h = 15.0 in.
t_w = ¼ in.

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(70\text{ kips}) + 1.6(210\text{ kips})$ $= 420\text{ kips}$	$P_a = 70\text{ kips} + 210\text{ kips}$ $= 280\text{ kips}$

Built-Up Section Properties (ignoring fillet welds)

$$A = 2(8.00\text{ in.})(1.00\text{ in.}) + 15.0\text{ in.}(\tfrac{1}{4}\text{ in.})$$
$$= 19.8\text{ in.}^2$$

$$I_y = \frac{2(1.00\text{ in.})(8.00\text{ in.})^3}{12} + \frac{15.0\text{ in.}(\tfrac{1}{4}\text{ in.})^3}{12}$$

$$= 85.4 \text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}}$$
$$= \sqrt{\frac{85.4 \text{ in.}^4}{19.8 \text{ in.}^2}}$$
$$= 2.08 \text{ in.}$$

$$I_x = \sum Ad^2 + \sum \frac{bh^3}{12}$$
$$= 2\left(8.00 \text{ in.}^2\right)\left(8.00 \text{ in.}\right)^2 + \frac{\left(\tfrac{1}{4} \text{ in.}\right)\left(15.00 \text{ in.}\right)^3}{12} + \frac{2\left(8.00 \text{ in.}\right)\left(1.00 \text{ in.}\right)^3}{12}$$
$$= 1{,}100 \text{ in.}^4$$

Elastic Flexural Buckling Stress

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced length is the same for both axes, the y-y axis will govern by inspection.

$$\frac{KL_y}{r_y} = \frac{1.0\left(15.0 \text{ ft}\right)}{2.08 \text{ in.}}\left(\frac{12.0 \text{ in.}}{\text{ft}}\right)$$
$$= 86.5$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \quad \text{(\textit{Spec}. Eq. E3-4)}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{\left(86.5\right)^2}$$
$$= 38.3 \text{ ksi}$$

Elastic Critical Torsional Buckling Stress

Note: Torsional buckling generally will not govern if $KL_y \geq KL_z$; however, the check is included here to illustrate the calculation.

From the User Note in AISC *Specification* Section E4,

$$C_w = \frac{I_y h_o^2}{4}$$
$$= \frac{85.4 \text{ in.}^4 (16.0 \text{ in.})^2}{4}$$
$$= 5{,}470 \text{ in.}^6$$

From AISC Design Guide 9, Equation 3.4,

$$J = \sum \frac{bt^3}{3}$$
$$= \frac{2(8.00 \text{ in.})(1.00 \text{ in.})^3 + (15.0 \text{ in.})(\tfrac{1}{4} \text{ in.})^3}{3}$$
$$= 5.41 \text{ in}^4$$

$$F_e = \left[\frac{\pi^2 EC_w}{(K_z L)^2} + GJ\right]\frac{1}{I_x + I_y} \quad \text{(\textit{Spec}. Eq. E4-4)}$$
$$= \left[\frac{\pi^2(29{,}000 \text{ ksi})(5{,}470 \text{ in.}^6)}{[(1.0)(15 \text{ ft})(12)]^2} + (11{,}200 \text{ ksi})(5.41 \text{ in.}^4)\right]\left(\frac{1}{1{,}100 \text{ in.}^4 + 85.4 \text{ in.}^4}\right)$$
$$= 91.9 \text{ ksi} > 38.3 \text{ ksi}$$

Therefore, the flexural buckling limit state controls.

Use F_e = 38.3 ksi.

Slenderness

Check for slender flanges using AISC *Specification* Table B4.1a, then determine Q_s, the unstiffened element (flange) reduction factor using AISC *Specification* Section E7.1.

Calculate k_c using AISC *Specification* Table B4.1b note [a].

$$k_c = \frac{4}{\sqrt{h/t_w}}$$
$$= \frac{4}{\sqrt{15.0 \text{ in.}/\tfrac{1}{4} \text{ in.}}}$$
$$= 0.516, \text{ which is between 0.35 and 0.76}$$

For the flanges,

$$\lambda = \frac{b}{t}$$
$$= \frac{4.00 \text{ in.}}{1.00 \text{ in.}}$$
$$= 4.00$$

Determine the flange limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a case 2

$$\lambda_r = 0.64\sqrt{\frac{k_c E}{F_y}}$$
$$= 0.64\sqrt{\frac{0.516(29{,}000 \text{ ksi})}{50 \text{ ksi}}}$$
$$= 11.1$$

$\lambda < \lambda_r$; therefore, the flange is not slender and Q_s = 1.0.

Check for a slender web, then determine Q_a, the stiffened element (web) reduction factor using AISC *Specification* Section E7.2.

$$\lambda = \frac{h}{t}$$
$$= \frac{15.0 \text{ in.}}{\frac{1}{4} \text{ in.}}$$
$$= 60.0$$

Determine the slender web limit from AISC *Specification* Table B4.1a case 5

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$
$$= 1.49\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 35.9$$

$\lambda > \lambda_r$; therefore, the web is slender

$$Q_a = \frac{A_e}{A_g} \qquad \text{(Spec. Eq. E7-16)}$$

where A_e = effective area based on the reduced effective width, b_e

For AISC *Specification* Equation E7-17, take f as F_{cr} with F_{cr} calculated based on Q = 1.0.

Select between AISC *Specification* Equations E7-2 and E7-3 based on KL/r_y.

KL/r = 86.5 as previously calculated

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{1.0(50 \text{ ksi})}}$$
$$= 113 > 86.5$$

Because $\dfrac{KL}{r} \leq 4.71\sqrt{\dfrac{E}{QF_y}}$,

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y \qquad \text{(Spec. Eq. E7-2)}$$
$$= 1.0\left[0.658^{\frac{1.0(50 \text{ ksi})}{38.3 \text{ ksi}}}\right](50 \text{ ksi})$$
$$= 29.0 \text{ ksi}$$

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.34}{(b/t)}\sqrt{\frac{E}{f}}\right] \leq b, \text{ where } b = h \qquad \text{(Spec. Eq. E7-17)}$$
$$= 1.92(\tfrac{1}{4} \text{ in.})\sqrt{\frac{29{,}000 \text{ ksi}}{29.0 \text{ ksi}}}\left[1 - \frac{0.34}{(15.0 \text{ in.}/\tfrac{1}{4} \text{ in.})}\sqrt{\frac{29{,}000 \text{ ksi}}{29.0 \text{ ksi}}}\right] \leq 15.0 \text{ in.}$$

= 12.5 in. ≤ 15.0 in.; therefore, compute A_e with reduced effective web width

$$A_e = b_e t_w + 2b_f t_f$$
$$= 12.5 \text{ in.}(\tfrac{1}{4} \text{ in.}) + 2(8.00 \text{ in.})(1.00 \text{ in.})$$
$$= 19.1 \text{ in.}^2$$

$$Q_a = \frac{A_e}{A}$$ (*Spec*. Eq. E7-16)

$$= \frac{19.1\text{ in.}^2}{19.8\text{ in.}^2}$$

$$= 0.965$$

$Q = Q_sQ_a$ from AISC *Specification* Section E7
$= 1.00(0.965)$
$= 0.965$

Flexural Buckling Stress

Determine whether AISC *Specification* Equation E7-2 or E7-3 applies.

KL/r = 86.5 as previously calculated

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{0.965(50\text{ ksi})}}$$

$$= 115 > 86.5$$

Therefore, AISC *Specification* Equation E7-2 applies.

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y$$ (*Spec*. Eq. E7-2)

$$= 0.965\left[0.658^{\frac{0.965(50\text{ ksi})}{38.3\text{ ksi}}}\right](50\text{ ksi})$$

$$= 28.5\text{ ksi}$$

Nominal Compressive Strength

$P_n = F_{cr}A_g$ (*Spec*. Eq. E7-1)
$= 28.5\text{ ksi}(19.8\text{ in.}^2)$
$= 564\text{ kips}$

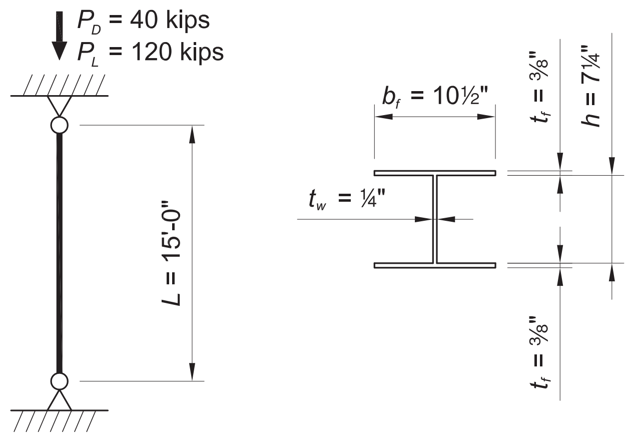
From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(564\text{ kips})$ $= 508\text{ kips} > 420\text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{564\text{ kips}}{1.67}$ $= 338\text{ kips} > 280\text{ kips}$ **o.k.**

EXAMPLE E.3 BUILT-UP COLUMN WITH SLENDER FLANGES

Given:

Determine if a built-up, ASTM A572 Grade 50 column with PL⅜ in. × 10½ in. flanges and a PL¼ in. × 7¼ in. web has sufficient available strength to carry a dead load of 40 kips and a live load of 120 kips in axial compression. The column's unbraced length is 15.0 ft in both axes and the ends are pinned.



Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Built-Up Column
ASTM A572 Grade 50
F_y = 50 ksi
F_u = 65 ksi

The geometric properties are as follows:

Built-Up Column
d = 8.00 in.
b_f = 10½ in.
t_f = ⅜ in.
h = 7¼ in.
t_w = ¼ in.

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
P_u = 1.2(40 kips) + 1.6(120 kips) = 240 kips	P_a = 40 kips + 120 kips = 160 kips

Built-Up Section Properties (ignoring fillet welds)

A_g = 2(10½ in.)(⅜ in.) + 7¼ in.(¼ in.)
= 9.69 in.^{2}

Because the unbraced length is the same for both axes, the weak axis will govern.

$$I_y = 2\left[\frac{(3/8\text{ in.})(10\tfrac{1}{2}\text{ in.})^3}{12}\right] + \frac{(7\tfrac{1}{4}\text{ in.})(1/4\text{ in.})^3}{12}$$
$$= 72.4\text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}}$$
$$= \sqrt{\frac{72.4\text{ in.}^4}{9.69\text{ in.}^2}}$$
$$= 2.73\text{ in.}$$

$$I_x = 2(10\tfrac{1}{2}\text{ in.})(3/8\text{ in.})(3.81\text{ in.})^2 + \frac{(1/4\text{ in.})(7\tfrac{1}{4}\text{ in.})^3}{12} + \frac{2(10\tfrac{1}{2}\text{ in.})(3/8\text{ in.})^3}{12}$$
$$= 122\text{ in.}^4$$

Web Slenderness

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a case 5:

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$
$$= 1.49\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 35.9$$

$$\lambda = \frac{h}{t_w}$$
$$= \frac{7\tfrac{1}{4}\text{ in.}}{1/4\text{ in.}}$$
$$= 29.0$$

$\lambda < \lambda_r$; therefore, the web is not slender.

Note that the fillet welds are ignored in the calculation of h for built up sections.

Flange Slenderness

Calculate k_c.

$$k_c = \frac{4}{\sqrt{h/t_w}} \text{ from AISC } \textit{Specification} \text{ Table B4.1b note [a]}$$
$$= \frac{4}{\sqrt{7\tfrac{1}{4}\text{ in.}/\tfrac{1}{4}\text{ in.}}}$$
$$= 0.743\text{, where } 0.35 \le k_c \le 0.76 \quad \textbf{o.k.}$$

Use $k_c = 0.743$

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a case 2.

$$\lambda_r = 0.64\sqrt{\frac{k_c E}{F_y}}$$
$$= 0.64\sqrt{\frac{29,000 \text{ ksi}(0.743)}{50 \text{ ksi}}}$$
$$= 13.3$$

$$\lambda = \frac{b}{t}$$
$$= \frac{5.25 \text{ in.}}{\frac{3}{8} \text{ in.}}$$
$$= 14.0$$

$\lambda > \lambda_r$; therefore, the flanges are slender

For compression members with slender elements, Section E7 of the AISC *Specification* applies. The nominal compressive strength, P_n, shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling. Depending on the slenderness of the column, AISC *Specification* Equation E7-2 or E7-3 applies. F_e is used in both equations and is calculated as the lesser of AISC *Specification* Equations E3-4 and E4-4.

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because the unbraced length is the same for both axes, the weak axis will govern.

$$\frac{K_y L_y}{r_y} = \frac{1.0(15.0 \text{ ft})}{2.73 \text{ in.}}\left(\frac{12 \text{ in.}}{\text{ft}}\right)$$
$$= 65.9$$

Elastic Critical Stress, F_e, for Flexural Buckling

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad (\textit{Spec}.\ \text{Eq. E3-4})$$
$$= \frac{\pi^2 (29,000 \text{ ksi})}{(65.9)^2}$$
$$= 65.9 \text{ ksi}$$

Elastic Critical Stress, F_e, for Torsional Buckling

Note: This limit state is not likely to govern, but the check is included here for completeness.

From the User Note in AISC *Specification* Section E4,

$$C_w = \frac{I_y h_o^2}{4}$$
$$= \frac{72.4 \text{ in.}^4 (7.63 \text{ in.})^2}{4}$$
$$= 1,050 \text{ in.}^6$$

From AISC Design Guide 9, Equation 3.4,

$$J = \sum \frac{bt^3}{3}$$
$$= \frac{2(10\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.})^3 + 7\tfrac{1}{4}\text{ in.}(\tfrac{1}{4}\text{ in.})^3}{3}$$
$$= 0.407\text{ in.}^4$$

$$F_e = \left[\frac{\pi^2 E C_w}{(K_z L)^2} + GJ\right]\frac{1}{I_x + I_y} \quad \text{(\textit{Spec}. Eq. E4-4)}$$
$$= \left[\frac{\pi^2(29{,}000\text{ ksi})(1{,}050\text{ in.}^6)}{(180\text{ in.})^2} + (11{,}200\text{ ksi})(0.407\text{ in.}^4)\right]\left(\frac{1}{122\text{ in.}^4 + 72.4\text{ in.}^4}\right)$$
$$= 71.2\text{ ksi} > 65.9\text{ ksi}$$

Therefore, use F_e = 65.9 ksi.

Slenderness Reduction Factor, Q

$Q = Q_sQ_a$ from AISC *Specification* Section E7, where Q_a = 1.0 because the web is not slender.

Calculate Q_s, the unstiffened element (flange) reduction factor from AISC *Specification* Section E7.1(b).

Determine the proper equation for Q_s by checking limits for AISC *Specification* Equations E7-7 to E7-9.

$$\frac{b}{t} = 14.0 \text{ as previously calculated}$$
$$0.64\sqrt{\frac{Ek_c}{F_y}} = 0.64\sqrt{\frac{29{,}000\text{ ksi}(0.743)}{50\text{ ksi}}}$$
$$= 13.3$$

$$1.17\sqrt{\frac{Ek_c}{F_y}} = 1.17\sqrt{\frac{29{,}000\text{ ksi}(0.743)}{50\text{ ksi}}}$$
$$= 24.3$$

$0.64\sqrt{\frac{Ek_c}{F_y}} < \frac{b}{t} \le 1.17\sqrt{\frac{Ek_c}{F_y}}$ therefore, AISC *Specification* Equation E7-8 applies

$$Q_s = 1.415 - 0.65\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{Ek_c}} \quad \text{(\textit{Spec}. Eq. E7-8)}$$
$$= 1.415 - 0.65(14.0)\sqrt{\frac{50\text{ ksi}}{(29{,}000\text{ ksi})(0.743)}}$$
$$= 0.977$$

$$Q = Q_sQ_a$$
$$= 0.977(1.0)$$
$$= 0.977$$

Nominal Compressive Strength

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{0.977\left(50 \text{ ksi}\right)}}$$

= 115 > 65.9, therefore, AISC *Specification* Equation E7-2 applies

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y \qquad \text{(}Spec\text{. Eq. E7-2)}$$

$$= 0.977\left[0.658^{\frac{0.977(50 \text{ ksi})}{65.9 \text{ ksi}}}\right]\left(50 \text{ ksi}\right)$$

$$= 35.8 \text{ ksi}$$

$$P_n = F_{cr}A_g \qquad \text{(}Spec\text{. Eq. E7-1)}$$
$$= 35.8 \text{ ksi}(9.69 \text{ in.}^2)$$
$$= 347 \text{ kips}$$

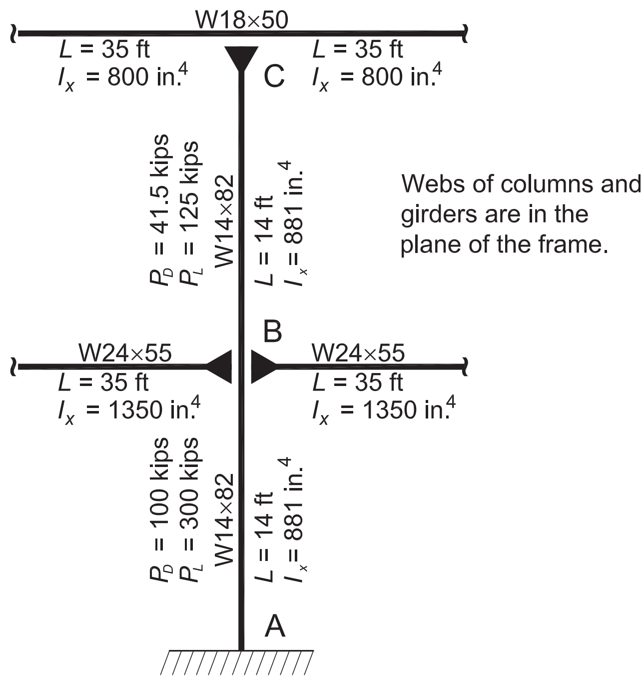
From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90\left(347 \text{ kips}\right)$ $= 312 \text{ kips} > 240 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{347 \text{ kips}}{1.67}$ $= 208 \text{ kips} > 160 \text{ kips}$ **o.k.**

Note: Built-up sections are generally more expensive than standard rolled shapes; therefore, a standard compact shape, such as a W8×35 might be a better choice even if the weight is somewhat higher. This selection could be taken directly from AISC *Manual* Table 4-1.

EXAMPLE E.4A W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

This example is primarily intended to illustrate the use of the alignment chart for sidesway uninhibited columns in conjunction with the effective length method.



Given:

The member sizes shown for the moment frame illustrated here (sidesway uninhibited in the plane of the frame) have been determined to be adequate for lateral loads. The material for both the column and the girders is ASTM A992. The loads shown at each level are the accumulated dead loads and live loads at that story. The column is fixed at the base about the *x-x* axis of the column.

Determine if the column is adequate to support the gravity loads shown. Assume the column is continuously supported in the transverse direction (the *y-y* axis of the column).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
I_x = 800 in.^{4}

W24×55
I_x = 1,350 in.^{4}

W14×82
A_g = 24.0 in.^{2}
I_x = 881 in.^{4}

Column B-C

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the roof and floor is:

LRFD	ASD
$P_u = 1.2(41.5 \text{ kips}) + 1.6(125 \text{ kips})$ $= 250 \text{ kips}$	$P_a = 41.5 + 125$ $= 167 \text{ kips}$

Effective Length Factor

Calculate the stiffness reduction parameter, τ_b, using AISC *Manual* Table 4-21.

LRFD	ASD
$\dfrac{P_u}{A_g} = \dfrac{250 \text{ kips}}{24.0 \text{ in.}^2}$ $= 10.4 \text{ ksi}$ $\tau_b = 1.00$	$\dfrac{P_a}{A_g} = \dfrac{167 \text{ kips}}{24.0 \text{ in.}^2}$ $= 6.96 \text{ ksi}$ $\tau_b = 1.00$

Therefore, no reduction in stiffness for inelastic buckling will be required.

Determine G_{top} and G_{bottom}.

$$G_{top} = \tau \frac{\sum (E_c I_c / L_c)}{\sum (E_g I_g / L_g)} \qquad \text{(from } \textit{Spec.} \text{ Comm. Eq. C-A-7-3)}$$

$$= (1.00) \frac{29{,}000 \text{ ksi} \left(\dfrac{881 \text{ in.}^4}{14.0 \text{ ft}} \right)}{2(29{,}000 \text{ ksi}) \left(\dfrac{800 \text{ in.}^4}{35.0 \text{ ft}} \right)}$$

$$= 1.38$$

$$G_{bottom} = \tau \frac{\sum (E_c I_c / L_c)}{\sum (E_g I_g / L_g)} \qquad \text{(from } \textit{Spec.} \text{ Comm. Eq. C-A-7-3)}$$

$$= (1.00) \frac{2(29{,}000 \text{ ksi}) \left(\dfrac{881 \text{ in.}^4}{14.0 \text{ ft}} \right)}{2(29{,}000 \text{ ksi}) \left(\dfrac{1{,}350 \text{ in.}^4}{35.0 \text{ ft}} \right)}$$

$$= 1.63$$

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is slightly less than 1.5; therefore use K = 1.5. Because the column available strength tables are based on the KL about the y-y axis, the equivalent effective column length of the upper segment for use in the table is:

$$KL = \frac{(KL)_x}{\left(\dfrac{r_x}{r_y} \right)}$$

$$= \frac{1.5(14.0 \text{ ft})}{2.44}$$

$$= 8.61 \text{ ft}$$

Take the available strength of the W14×82 from AISC *Manual* Table 4-1.

At KL = 9 ft, the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 250 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 626 \text{ kips} > 167 \text{ kips}$ **o.k.**

Column A-B

From Chapter 2 of ASCE/SEI 7, the required compressive strength for the column between the floor and the foundation is:

LRFD	ASD
$P_u = 1.2(100 \text{ kips}) + 1.6(300 \text{ kips})$ $= 600 \text{ kips}$	$P_a = 100 \text{ kips} + 300 \text{ kips}$ $= 400 \text{ kips}$

Effective Length Factor

Calculate the stiffness reduction parameter, τ_b, using AISC *Manual* Table 4-21.

LRFD	ASD
$\dfrac{P_u}{A_g} = \dfrac{600 \text{ kips}}{24.0 \text{ in.}^2}$ $= 25.0 \text{ ksi}$ $\tau_b = 1.00$	$\dfrac{P_a}{A_g} = \dfrac{400 \text{ kips}}{24.0 \text{ in.}^2}$ $= 16.7 \text{ ksi}$ $\tau_b = 0.994$

Determine G_{top} and G_{bottom} accounting for column inelasticity by replacing E_cI_c with $\tau_b(E_cI_c)$. Use $\tau_b = 0.994$.

$$G_{top} = \tau \frac{\Sigma\left(E_c I_c / L_c\right)}{\Sigma\left(E_g I_g / L_g\right)} \qquad \text{(from } \textit{Spec.} \text{ Comm. Eq. C-A-7-3)}$$

$$= \left(0.994\right) \frac{2\left(\dfrac{29{,}000 \text{ ksi}\left(881 \text{ in.}^4\right)}{14.0 \text{ ft}}\right)}{2\left(\dfrac{29{,}000 \text{ ksi}\left(1{,}350 \text{ in.}^4\right)}{35.0 \text{ ft}}\right)}$$

$$= 1.62$$

$G_{bottom} = 1.0 \text{ (fixed)}$ from AISC *Specification* Commentary Appendix 7, Section 7.2

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately 1.40. Because the column available strength tables are based on the KL about the y-y axis, the effective column length of the lower segment for use in the table is:

$$KL = \frac{(KL)_x}{\left(\dfrac{r_x}{r_y}\right)}$$

$$= \frac{1.40\left(14.0 \text{ ft}\right)}{2.44}$$

$$= 8.03 \text{ ft}$$

Take the available strength of the W14×82 from AISC *Manual* Table 4-1.

At KL = 9 ft, (conservative) the available strength in axial compression is:

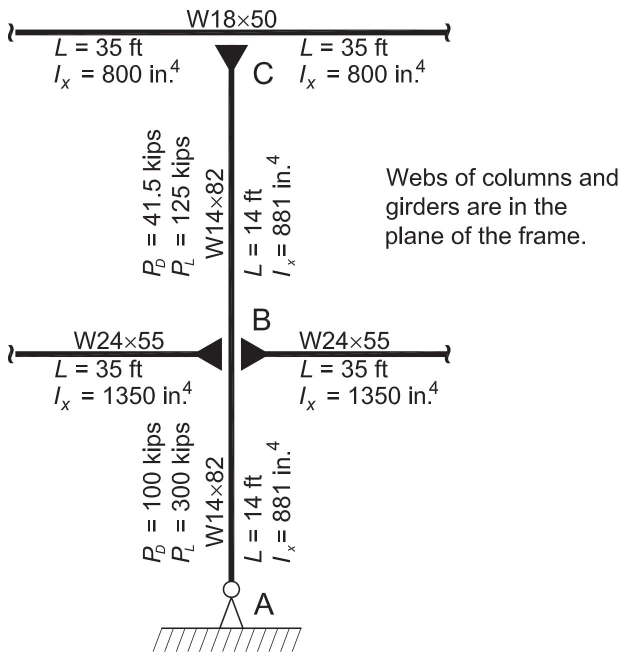
LRFD	ASD
$\phi_c P_n = 940 \text{ kips} > 600 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 626 \text{ kips} > 400 \text{ kips}$ **o.k.**

A more accurate strength could be determined by interpolation from AISC *Manual* Table 4-1.

EXAMPLE E.4B W-SHAPE COMPRESSION MEMBER (MOMENT FRAME)

Given:

Using the effective length method, determine the available strength of the column shown subject to the same gravity loads shown in Example E.4A with the column pinned at the base about the *x-x* axis. All other assumptions remain the same.



Solution:

As determined in Example E.4A, for the column segment B-C between the roof and the floor, the column strength is adequate.

As determined in Example E.4A, for the column segment A-B between the floor and the foundation,

$G_{top} = 1.62$

At the base,

$G_{bottom} = 10$ (pinned) from AISC *Specification* Commentary Appendix 7, Section 7.2

Note: this is the only change in the analysis.

From the alignment chart, AISC *Specification* Commentary Figure C-A-7.2, K is approximately equal to 2.00. Because the column available strength tables are based on the effective length, KL, about the y-y axis, the effective column length of the segment A-B for use in the table is:

$$KL = \frac{(KL)_x}{\left(\dfrac{r_x}{r_y}\right)}$$

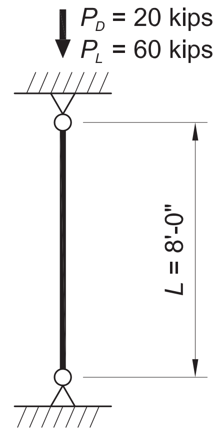
$$= \frac{2.00(14.0 \text{ ft})}{2.44}$$

$$= 11.5 \text{ ft}$$

Interpolate the available strength of the W14×82 from AISC *Manual* Table 4-1.

LRFD	ASD
$\phi_c P_n = 861 \text{ kips} > 600 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 573 \text{ kips} > 400 \text{ kips}$ **o.k.**

EXAMPLE E.5 DOUBLE ANGLE COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS



Given:

Verify the strength of a 2L4×3½×⅜ LLBB (¾-in. separation) strut, ASTM A36, with a length of 8 ft and pinned ends carrying an axial dead load of 20 kips and live load of 60 kips. Also, calculate the required number of pretensioned bolted or welded intermediate connectors required.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L4×3½×⅜ LLBB
r_z = 0.719 in.

2L4×3½×⅜ LLBB
r_x = 1.25 in.
r_y = 1.55 in. for ⅜-in. separation
r_y = 1.69 in. for ¾-in. separation

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
P_u = 1.2(20 kips) + 1.6(60 kips) = 120 kips	P_a = 20 kips + 60 kips = 80.0 kips

Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

For $(KL)_x$ = 8 ft, the available strength in axial compression is taken from the upper (X-X) portion of AISC *Manual* Table 4-9 as:

LRFD	ASD
$\phi_c P_n = 127 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 84.7 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

For buckling about the y-y axis, the values are tabulated for a separation of ⅜ in.

To adjust to a spacing of ¾ in., $(KL)_y$ is multiplied by the ratio of the r_y for a ⅜-in. separation to the r_y for a ¾-in. separation. Thus,

$$(KL)_y = 1.0(8.00 \text{ ft})\left(\frac{1.55 \text{ in.}}{1.69 \text{ in.}}\right)$$
$$= 7.34 \text{ ft}$$

The calculation of the equivalent $(KL)_y$ in the preceding text is a simplified approximation of AISC *Specification* Section E6.1. To ensure a conservative adjustment for a ¾-in. separation, take $(KL)_y$ = 8 ft.

The available strength in axial compression is taken from the lower (*Y-Y*) portion of AISC *Manual* Table 4-9 as:

LRFD	ASD
$\phi_c P_n = 130 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 86.3 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

Therefore, *x-x* axis flexural buckling governs.

Intermediate Connectors

From AISC *Manual* Table 4-9, at least two welded or pretensioned bolted intermediate connectors are required. This can be verified as follows:

$$\begin{aligned} a &= \text{distance between connectors} \\ &= \frac{8.00 \text{ ft}\left(12 \text{ in./ft}\right)}{3 \text{ spaces}} \\ &= 32.0 \text{ in.} \end{aligned}$$

From AISC *Specification* Section E6.2, the effective slenderness ratio of the individual components of the built-up member based upon the distance between intermediate connectors, *a*, must not exceed three-fourths of the governing slenderness ratio of the built-up member.

Therefore, $\dfrac{Ka}{r_i} \le \dfrac{3}{4}\left(\dfrac{Kl}{r}\right)_{max}$

Solving for *a* gives, $a \le \dfrac{3r_i\left(\dfrac{KL}{r}\right)_{max}}{4K}$

$$\begin{aligned} \frac{KL}{r_x} &= \frac{1.0\left(8.00 \text{ ft}\right)\left(12 \text{ in./ft}\right)}{1.25 \text{ in.}} \\ &= 76.8 \qquad \textbf{controls} \end{aligned}$$

$$\begin{aligned} \frac{KL}{r_y} &= \frac{1.0\left(8.00 \text{ ft}\right)\left(12 \text{ in./ft}\right)}{1.69 \text{ in.}} \\ &= 56.8 \end{aligned}$$

Thus, $a \le \dfrac{3r_z\left(\dfrac{KL}{r}\right)_{max}}{4K}$

$$= \frac{3(0.719\text{ in.})(76.8)}{4(1.0)}$$

$= 41.4$ in. > 32.0 in. **o.k.**

Note that one connector would not be adequate as 48.0 in. > 41.4 in. The available strength can be easily determined by using the tables of the AISC *Manual*. Available strength values can be verified by hand calculations, as follows:

Calculation Solution

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L4×3½×⅜
$J = 0.132$ in.[4]
$r_y = 1.05$ in.
$\bar{x} = 0.947$ in.

2L4×3½×⅜ LLBB (¾ in. separation)
$A_g = 5.36$ in.[2]
$r_y = 1.69$ in.
$\bar{r}_o = 2.33$ in.
$H = 0.813$

Slenderness Check

$$\lambda = \frac{b}{t}$$
$$= \frac{4.00\text{ in.}}{3/8\text{ in.}}$$
$$= 10.7$$

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a Case 3

$$\lambda_r = 0.45\sqrt{E/F_y}$$
$$= 0.45\sqrt{29{,}000\text{ ksi}/36\text{ ksi}}$$
$$= 12.8$$

$\lambda < \lambda_r$; therefore, there are no slender elements.

For compression members without slender elements, AISC *Specification* Sections E3 and E4 apply.

The nominal compressive strength, P_n, shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling.

Flexural Buckling about the *x-x* Axis

$$\frac{KL}{r_x} = \frac{1.0(8.00\text{ ft})(12\text{ in./ft})}{1.25\text{ in.}}$$

$= 76.8$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad (\textit{Spec}\text{. Eq. E3-4})$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(76.8)^2}$$

$= 48.5$ ksi

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$

$= 134 > 76.8$, therefore

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y \qquad (\textit{Spec}\text{. Eq. E3-2})$$

$$= \left[0.658^{\frac{36 \text{ ksi}}{48.5 \text{ ksi}}}\right](36 \text{ ksi})$$

$= 26.4$ ksi **controls**

Torsional and Flexural-Torsional Buckling

For nonslender double angle compression members, AISC *Specification* Equation E4-2 applies.

F_{cry} is taken as F_{cr}, for flexural buckling about the *y-y* axis from AISC *Specification* Equation E3-2 or E3-3 as applicable.

Using AISC *Specification* Section E6, compute the modified KL/r_y for built up members with pretensioned bolted or welded connectors. Assume two connectors are required.

$a = 96.0$ in./3
$= 32.0$ in.

$r_i = r_z$ (single angle)
$= 0.719$ in.

$$\frac{a}{r_i} = \frac{32 \text{ in.}}{0.719 \text{ in.}}$$

$= 44.5 > 40$, therefore

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} \text{ where } K_i = 0.50 \text{ for angles back-to-back} \qquad (\textit{Spec}\text{. Eq. E6-2b})$$

$$= \sqrt{(56.8)^2 + \left(\frac{0.50(32.0 \text{ in.})}{0.719 \text{ in.}}\right)^2}$$

$= 61.0 \leq 134$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$ (*Spec*. Eq. E3-4)

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(61.0)^2}$$

$$= 76.9 \text{ ksi}$$

$$F_{cry} = \left[0.658^{\frac{F_y}{F_e}}\right] F_y$$ (*Spec*. Eq. E3-2)

$$= \left[0.658^{\frac{36 \text{ ksi}}{76.9 \text{ ksi}}}\right](36 \text{ ksi})$$

$$= 29.6 \text{ ksi}$$

$$F_{crz} = \frac{GJ}{A_g \bar{r}_o^2}$$ (*Spec*. Eq. E4-3)

$$= \frac{(11{,}200 \text{ ksi})(2 \text{ angles})(0.132 \text{ in.}^4)}{(5.36 \text{ in.}^2)(2.33 \text{ in.})^2}$$

$$= 102 \text{ ksi}$$

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right]$$ (*Spec*. Eq. E4-2)

$$= \left(\frac{29.6 \text{ ksi} + 102 \text{ ksi}}{2(0.813)}\right)\left[1 - \sqrt{1 - \frac{4(29.6 \text{ ksi})(102 \text{ ksi})(0.813)}{(29.6 \text{ ksi} + 102 \text{ ksi})^2}}\right]$$

$= 27.7$ ksi **does not control**

Nominal Compressive Strength

$$P_n = F_{cr} A_g$$ (*Spec*. Eq. E4-1)

$$= 26.4 \text{ ksi}(5.36 \text{ in.}^2)$$

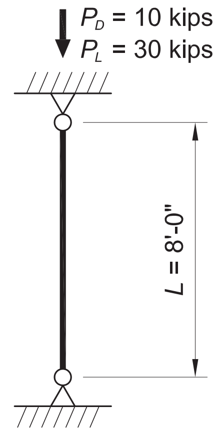
$$= 142 \text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(142 \text{ kips})$ $= 128 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{142 \text{ kips}}{1.67}$ $= 85.0 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

EXAMPLE E.6 DOUBLE ANGLE COMPRESSION MEMBER WITH SLENDER ELEMENTS

Given:



Determine if a 2L5×3×¼ LLBB (¾-in. separation) strut, ASTM A36, with a length of 8 ft and pinned ends has sufficient available strength to support a dead load of 10 kips and live load of 30 kips in axial compression. Also, calculate the required number of pretensioned bolted or welded intermediate connectors.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

L5×3×¼
r_z = 0.652 in.

2L5×3×¼ LLBB
r_x = 1.62 in.
r_y = 1.19 in. for ⅜-in. separation
r_y = 1.33 in. for ¾-in. separation

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(10\text{ kips}) + 1.6(30\text{ kips})$ $= 60.0\text{ kips}$	$P_a = 10\text{ kips} + 30\text{ kips}$ $= 40.0\text{ kips}$

Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

From the upper portion of AISC *Manual* Table 4-9, the available strength for buckling about the *x-x* axis, with $(KL)_x$ = 8 ft is:

LRFD	ASD
$\phi_c P_{nx} = 87.1\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\dfrac{P_{nx}}{\Omega_c} = 58.0\text{ kips} > 40.0\text{ kips}$ **o.k.**

For buckling about the y-y axis, the tabulated values are based on a separation of ⅜ in. To adjust for a spacing of ¾ in., $(KL)_y$ is multiplied by the ratio of r_y for a ⅜-in. separation to r_y for a ¾-in. separation.

$$\begin{aligned}(KL)_y &= 1.0(8.0\text{ ft})\left(\frac{1.19\text{ in.}}{1.33\text{ in.}}\right)\\ &= 7.16\text{ ft}\end{aligned}$$

This calculation of the equivalent $(KL)_y$ does not completely take into account the effect of AISC *Specification* Section E6.1 and is slightly unconservative.

From the lower portion of AISC *Manual* Table 4-9, interpolate for a value at $(KL)_y$ = 7.16 ft.

The available strength in compression is:

LRFD	ASD
$\phi_c P_{ny} = 65.2\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\dfrac{P_{ny}}{\Omega_c} = 43.3\text{ kips} > 40.0\text{ kips}$ **o.k.**

These strengths are approximate due to the linear interpolation from the table and the approximate value of the equivalent $(KL)_y$ noted in the preceding text. These can be compared to the more accurate values calculated in detail as follows:

Intermediate Connectors

From AISC *Manual* Table 4-9, it is determined that at least two welded or pretensioned bolted intermediate connectors are required. This can be confirmed by calculation, as follows:

$$\begin{aligned}a &= \text{distance between connectors}\\ &= \frac{(8.00\text{ ft})(12\text{ in./ft})}{3\text{ spaces}}\\ &= 32.0\text{ in.}\end{aligned}$$

From AISC *Specification* Section E6.2, the effective slenderness ratio of the individual components of the built-up member based upon the distance between intermediate connectors, a, must not exceed three-fourths of the governing slenderness ratio of the built-up member.

Therefore, $\dfrac{Ka}{r_i} \le \dfrac{3}{4}\left(\dfrac{Kl}{r}\right)_{max}$

Solving for a gives, $a \le \dfrac{3r_i\left(\dfrac{KL}{r}\right)_{max}}{4K}$

$$r_i = r_z = 0.652\text{ in.}$$

$$\begin{aligned}\frac{KL_x}{r_x} &= \frac{1.0(8.0\text{ ft})(12.0\text{ in./ft})}{1.62\text{ in.}}\\ &= 59.3\end{aligned}$$

$$\begin{aligned}\frac{KL_y}{r_y} &= \frac{1.0(8.0\text{ ft})(12.0\text{ in./ft})}{1.33\text{ in.}}\\ &= 72.2 \quad \textbf{controls}\end{aligned}$$

$$\text{Thus, } a \le \frac{3r_z\left(\dfrac{KL}{r}\right)_{max}}{4K}$$
$$= \frac{3(0.652\text{ in.})(72.2)}{4(1.0)}$$
$$= 35.3\text{ in.} > 32.0\text{ in.} \qquad \textbf{o.k.}$$

The governing slenderness ratio used in the calculations of the AISC *Manual* tables includes the effects of the provisions of Section E6.1 and is slightly higher as a result. See the following for these calculations. As a result, the maximum connector spacing calculated here is slightly conservative.

Available strength values can be verified by hand calculations, as follows.

Calculation Solution

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows.

L5×3×¼
$J = 0.0438\text{ in.}^4$
$r_y = 0.853\text{ in.}$
$\bar{x} = 0.648\text{ in.}$

2L5×3×¼ LLBB
$A_g = 3.88\text{ in.}^2$
$r_y = 1.33\text{ in.}$
$\bar{r}_o = 2.59\text{ in.}$
$H = 0.657$

Slenderness Check

$$\lambda = \frac{b}{t}$$
$$= \frac{5.00\text{ in.}}{¼\text{ in.}}$$
$$= 20.0$$

Calculate the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a Case 3.

$$\lambda_r = 0.45\sqrt{\frac{E}{F_y}}$$
$$= 0.45\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}}$$
$$= 12.8$$

$\lambda > \lambda_r$; therefore, the angle has a slender element

For a double angle compression member with slender elements, AISC *Specification* Section E7 applies. The nominal compressive strength, P_n, shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling. F_{cr} will be determined by AISC *Specification* Equation E7-2 or E7-3.

Calculate the slenderness reduction factor, Q.

$Q = Q_sQ_a$ from AISC *Specification* Section E7.

Calculate Q_s for the angles individually using AISC *Specification* Section E7.1c.

$$0.45\sqrt{\frac{E}{F_y}} = 12.8 < 20.0$$

$$\begin{aligned} 0.91\sqrt{\frac{E}{F_y}} &= 0.91\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}} \\ &= 25.8 \geq 20.0 \end{aligned}$$

Therefore, AISC *Specification* Equation E7-11 applies.

$$\begin{aligned} Q_s &= 1.34 - 0.76\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}} \\ &= 1.34 - 0.76(20.0)\sqrt{\frac{36 \text{ ksi}}{29{,}000 \text{ ksi}}} \\ &= 0.804 \end{aligned}$$ (*Spec*. Eq. E7-11)

$Q_a = 1.0$ (no stiffened elements)

Therefore, $Q = Q_sQ_a$

$$\begin{aligned} &= 0.804(1.0) \\ &= 0.804 \end{aligned}$$

Critical Stress, F_{cr}

From the preceding text, $K = 1.0$.

AISC *Specification* Equations E7-2 and E7-3 require the computation of F_e. For singly symmetric members, AISC *Specification* Equations E3-4 and E4-5 apply.

Flexural Buckling about the *x-x* Axis

$$\begin{aligned} \frac{K_xL}{r_x} &= \frac{1.0(8.0 \text{ ft})(12.0 \text{ in./ft})}{1.62 \text{ in.}} \\ &= 59.3 \end{aligned}$$

$$\begin{aligned} F_e &= \frac{\pi^2 E}{\left(\frac{K_xL}{r_x}\right)^2} \\ &= \frac{\pi^2 (29{,}000 \text{ ksi})}{(59.3)^2} \\ &= 81.4 \text{ ksi} \end{aligned}$$ (*Spec*. Eq. E3-4)

does not control

Torsional and Flexural-Torsional Buckling

$$\frac{K_yL}{r_y} = \frac{1.0(8.0 \text{ ft})(12.0 \text{ in./ft})}{1.33 \text{ in.}}$$

$= 72.2$

Using AISC *Specification* Section E6, compute the modified KL/r_y for built-up members with pretensioned bolted or welded connectors.

$a = 96.0 \text{ in.}/3$
$= 32.0 \text{ in.}$

$r_i = r_z$ (single angle)
$= 0.652 \text{ in.}$

$$\frac{a}{r_i} = \frac{32 \text{ in.}}{0.652 \text{ in.}}$$
$= 49.1 > 40$, therefore,

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} \text{ where } K_i = 0.50 \text{ for angles back-to-back} \quad (Spec. \text{ Eq. E6-2b})$$

$$= \sqrt{(72.2)^2 + \left(\frac{0.50(32.0 \text{ in.})}{0.652 \text{ in.}}\right)^2}$$
$$= 76.3$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{K_y L}{r_y}\right)_m^2} \quad (\text{from } Spec. \text{ Eq. E4-8})$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(76.3)^2}$$
$$= 49.2 \text{ ksi}$$

$$F_{ez} = \left(\frac{\pi^2 E C_w}{(K_z L)^2} + GJ\right)\frac{1}{A_g \bar{r}_o^2} \quad (Spec. \text{ Eq. E4-9})$$

For double angles, omit term with C_w per the User Note at the end of AISC *Specification* Section E4.

$$F_{ez} = \frac{GJ}{A_g \bar{r}_o^2}$$
$$= \frac{(11{,}200 \text{ ksi})(2 \text{ angles})(0.0438 \text{ in.}^4)}{(3.88 \text{ in.}^2)(2.59 \text{ in.})^2}$$
$$= 37.7 \text{ ksi}$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4 F_{ey} F_{ez} H}{(F_{ey} + F_{ez})^2}}\right] \quad (Spec. \text{ Eq. E4-5})$$

$$= \left(\frac{49.2 \text{ ksi} + 37.7 \text{ ksi}}{2(0.657)}\right)\left[1 - \sqrt{1 - \frac{4(49.2 \text{ ksi})(37.7 \text{ ksi})(0.657)}{(49.2 \text{ ksi} + 37.7 \text{ ksi})^2}}\right]$$

$= 26.8\text{ ksi}$ **controls**

Use the limits based on F_e to determine whether to apply *Specification* Equation E7-2 or E7-3.

$$\frac{QF_y}{2.25} = \frac{0.804(36\text{ ksi})}{2.25}$$
$= 12.9\text{ ksi} \le 26.8\text{ ksi}$, therefore AISC *Specification* Equation E7-2 applies

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y \qquad \text{(Spec. Eq. E7-2)}$$
$$= 0.804\left[0.658^{\frac{(0.804)(36\text{ ksi})}{26.8\text{ ksi}}}\right](36\text{ ksi})$$
$$= 18.4\text{ ksi}$$

Nominal Compressive Strength

$$P_n = F_{cr}A_g \qquad \text{(Spec. Eq. E7-1)}$$
$$= 18.4\text{ ksi}(3.88\text{ in.}^2)$$
$$= 71.4\text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

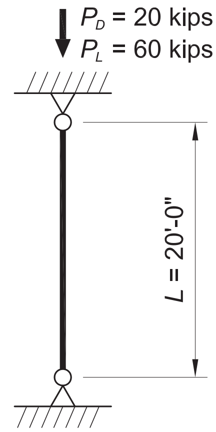
LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(71.4\text{ kips})$ $= 64.3\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\frac{P_n}{\Omega_c} = \frac{71.4\text{ kips}}{1.67}$ $= 42.8\text{ kips} > 40.0\text{ kips}$ **o.k.**

Return to Table of Contents

EXAMPLE E.7 WT COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:

Select an ASTM A992 nonslender WT-shape compression member with a length of 20 ft to support a dead load of 20 kips and live load of 60 kips in axial compression. The ends are pinned.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(20\text{ kips}) + 1.6(60\text{ kips})$ $= 120\text{ kips}$	$P_a = 20\text{ kips} + 60\text{ kips}$ $= 80.0\text{ kips}$

Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Therefore, $(KL)_x = (KL)_y$ = 20.0 ft.

Select the lightest nonslender member from AISC *Manual* Table 4-7 with sufficient available strength about both the x-x axis (upper portion of the table) and the y-y axis (lower portion of the table) to support the required strength.

Try a WT7×34.

The available strength in compression is:

LRFD	ASD
$\phi_c P_{nx} = 128\text{ kips} > 120\text{ kips}$ **controls** **o.k.** $\phi_c P_{ny} = 221\text{ kips} > 120\text{ kips}$ **o.k.**	$\dfrac{P_{nx}}{\Omega_c} = 85.5\text{ kips} > 80.0\text{ kips}$ **controls** **o.k.** $\dfrac{P_{ny}}{\Omega_c} = 147\text{ kips} > 80.0\text{ kips}$ **o.k.**

The available strength can be easily determined by using the tables of the AISC *Manual*. Available strength values can be verified by hand calculations, as follows.

Calculation Solution

From AISC *Manual* Table 1-8, the geometric properties are as follows.

WT7×34
A_g = 10.0 in.^{2}
r_x = 1.81 in.
r_y = 2.46 in.

J = 1.50 in.^{4}
$\bar{y}$ = 1.29 in.
I_x = 32.6 in.^{4}
I_y = 60.7 in.^{4}
d = 7.02 in.
t_w = 0.415 in.
b_f = 10.0 in.
t_f = 0.720 in.

Stem Slenderness Check

$$\lambda = \frac{d}{t_w}$$
$$= \frac{7.02\text{ in.}}{0.415\text{ in.}}$$
$$= 16.9$$

Determine the stem limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a Case 4

$$\lambda_r = 0.75\sqrt{\frac{E}{F_y}}$$
$$= 0.75\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 18.1$$

$\lambda < \lambda_r$; therefore, the stem is not slender

Flange Slenderness Check

$$\lambda = \frac{b_f}{2t_f}$$
$$= \frac{10\text{ in.}}{2(0.720\text{ in.})}$$
$$= 6.94$$

Determine the flange limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a Case 1

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$
$$= 0.56\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 13.5$$

$\lambda < \lambda_r$; therefore, the flange is not slender

There are no slender elements.

For compression members without slender elements, AISC *Specification* Sections E3 and E4 apply. The nominal compressive strength, P_n, shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling.

Flexural Buckling About the *x-x* Axis

$$\frac{KL}{r_x} = \frac{1.0(20.0\text{ ft})(12\text{ in./ft})}{1.81\text{ in.}}$$
$$= 133$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 113 < 133\text{, therefore, AISC } \textit{Specification} \text{ Equation E3-3 applies}$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad (\textit{Spec}.\text{ Eq. E3-4})$$
$$= \frac{\pi^2 (29{,}000\text{ ksi})}{(133)^2}$$
$$= 16.2\text{ ksi}$$

$$F_{cr} = 0.877F_e \qquad (\textit{Spec}.\text{ Eq. E3-3})$$
$$= 0.877(16.2\text{ ksi})$$
$$= 14.2\text{ ksi} \quad \textbf{controls}$$

Torsional and Flexural-Torsional Buckling

Because the WT7×34 section does not have any slender elements, AISC *Specification* Section E4 will be applicable for torsional and flexural-torsional buckling. F_{cr} will be calculated using AISC *Specification* Equation E4-2.

Calculate F_{cry}.

F_{cry} is taken as F_{cr} from AISC *Specification* Section E3, where $KL/r = KL/r_y$.

$$\frac{KL}{r_y} = \frac{1.0(20.0\text{ ft})(12\text{ in./ft})}{2.46\text{ in.}}$$
$$= 97.6 \leq 113\text{, therefore, AISC } \textit{Specification} \text{ Equation E3-2 applies}$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad (\textit{Spec}.\text{ Eq. E3-4})$$
$$= \frac{\pi^2 (29{,}000\text{ ksi})}{(97.6)^2}$$
$$= 30.0\text{ ksi}$$

$$F_{cry} = F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y \qquad (\textit{Spec}.\text{ Eq. E3-2})$$
$$= \left[0.658^{\frac{50.0\text{ ksi}}{30.0\text{ ksi}}}\right]50.0\text{ ksi}$$
$$= 24.9\text{ ksi}$$

The shear center for a T-shaped section is located on the axis of symmetry at the mid-depth of the flange.

$$x_o = 0.0 \text{ in.}$$

$$\begin{aligned} y_o &= \bar{y} - \frac{t_f}{2} \\ &= 1.29 \text{ in.} - \frac{0.720 \text{ in.}}{2} \\ &= 0.930 \text{ in.} \end{aligned}$$

$$\bar{r}_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_g} \qquad \text{(Spec. Eq. E4-11)}$$

$$\begin{aligned} &= (0.0 \text{ in.})^2 + (0.930 \text{ in.})^2 + \frac{32.6 \text{ in.}^4 + 60.7 \text{ in.}^4}{10.0 \text{ in.}^2} \\ &= 10.2 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} \bar{r}_o &= \sqrt{\bar{r}_o^2} \\ &= \sqrt{10.2 \text{ in.}^2} \\ &= 3.19 \text{ in.} \end{aligned}$$

$$H = 1 - \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \qquad \text{(Spec. Eq. E4-10)}$$

$$\begin{aligned} &= 1 - \frac{(0.0 \text{ in.})^2 + (0.930 \text{ in.})^2}{10.2 \text{ in.}^2} \\ &= 0.915 \end{aligned}$$

$$F_{crz} = \frac{GJ}{A_g \bar{r}_o^2} \qquad \text{(Spec. Eq. E4-3)}$$

$$\begin{aligned} &= \frac{(11{,}200 \text{ ksi})(1.50 \text{ in.}^4)}{(10.0 \text{ in.}^2)(10.2 \text{ in.}^2)} \\ &= 165 \text{ ksi} \end{aligned}$$

$$F_{cr} = \left(\frac{F_{cry} + F_{crz}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{cry}F_{crz}H}{(F_{cry} + F_{crz})^2}}\right] \qquad \text{(Spec. Eq. E4-2)}$$

$$= \left(\frac{24.9 \text{ ksi} + 165 \text{ ksi}}{2(0.915)}\right)\left[1 - \sqrt{1 - \frac{4(24.9 \text{ ksi})(165 \text{ ksi})(0.915)}{(24.9 \text{ ksi} + 165 \text{ ksi})^2}}\right]$$

$= 24.5$ ksi **does not control**

x-x axis flexural buckling governs, therefore,

$$P_n = F_{cr} A_g \qquad \text{(Spec. Eq. E3-1)}$$

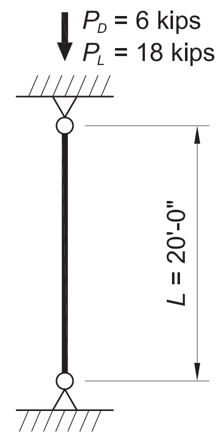
$$\begin{aligned} &= 14.2 \text{ ksi}(10.0 \text{ in.}^2) \\ &= 142 \text{ kips} \end{aligned}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c P_n = 0.90(142\text{ kips})$ $= 128\text{ kips} > 120\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = \dfrac{142\text{ kips}}{1.67}$ $= 85.0\text{ kips} > 80.0\text{ kips}$ **o.k.**

EXAMPLE E.8 WT COMPRESSION MEMBER WITH SLENDER ELEMENTS

Given:



Select an ASTM A992 WT-shape compression member with a length of 20 ft to support a dead load of 6 kips and live load of 18 kips in axial compression. The ends are pinned.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(6 \text{ kips}) + 1.6(18 \text{ kips})$ $= 36.0 \text{ kips}$	$P_a = 6 \text{ kips} + 18 \text{ kips}$ $= 24.0 \text{ kips}$

Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Therefore, $(KL)_x = (KL)_y$ = 20.0 ft.

Select the lightest member from AISC *Manual* Table 4-7 with sufficient available strength about the both the x-x axis (upper portion of the table) and the y-y axis (lower portion of the table) to support the required strength.

Try a WT7×15.

The available strength in axial compression from AISC *Manual* Table 4-7 is:

LRFD	ASD
$\phi_c P_{nx} = 66.7 \text{ kips} > 36.0 \text{ kips}$ **o.k.** $\phi_c P_{ny} = 36.6 \text{ kips} > 36.0 \text{ kips}$ **controls** **o.k.**	$\dfrac{P_{nx}}{\Omega_c} = 44.4 \text{ kips} > 24.0 \text{ kips}$ **o.k.** $\dfrac{P_{ny}}{\Omega_c} = 24.4 \text{ kips} > 24.0 \text{ kips}$ **controls** **o.k.**

The available strength can be easily determined by using the tables of the AISC *Manual*. Available strength values can be verified by hand calculations, as follows.

Calculation Solution

From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT7×15
A_g = 4.42 in.^{2}
r_x = 2.07 in.
r_y = 1.49 in.
J = 0.190 in.^{4}
Q_s = 0.611

$\bar{y} = 1.58$ in.
$I_x = 19.0$ in.^{4}
$I_y = 9.79$ in.^{4}
$d = 6.92$ in.
$t_w = 0.270$ in.
$b_f = 6.73$ in.
$t_f = 0.385$ in.

Stem Slenderness Check

$$\lambda = \frac{d}{t_w}$$
$$= \frac{6.92 \text{ in.}}{0.270 \text{ in.}}$$
$$= 25.6$$

Determine stem limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a case 4

$$\lambda_r = 0.75\sqrt{\frac{E}{F_y}}$$
$$= 0.75\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 18.1$$

$\lambda > \lambda_r$; therefore, the web is slender

Flange Slenderness Check

$$\lambda = \frac{b_f}{2t_f}$$
$$= \frac{6.73 \text{ in.}}{2(0.385 \text{ in.})}$$
$$= 8.74$$

Determine flange limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a Case 1

$$\lambda_r = 0.56\sqrt{\frac{E}{F_y}}$$
$$= 0.56\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 13.5$$

$\lambda < \lambda_r$; therefore, the flange is not slender

Because this WT7×15 has a slender web, AISC *Specification* Section E7 is applicable. The nominal compressive strength, P_n, shall be determined based on the limit states of flexural, torsional and flexural-torsional buckling.

x-x Axis Critical Elastic Flexural Buckling Stress

$$\frac{K_x L}{r_x} = \frac{1.0(20.0 \text{ ft})(12 \text{ in./ft})}{2.07 \text{ in.}}$$
$$= 116$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(}Spec\text{. Eq. E3-4)}$$

$$= \frac{\pi^2(29{,}000 \text{ ksi})}{(116)^2}$$

$$= 21.3 \text{ ksi}$$

Critical Elastic Torsional and Flexural-Torsional Buckling Stress

$$\frac{K_y L}{r_y} = \frac{1.0(20.0 \text{ ft})(12 \text{ in./ft})}{1.49 \text{ in.}}$$

$$= 161$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{K_y L}{r_y}\right)^2} \qquad \text{(}Spec\text{. Eq. E4-8)}$$

$$= \frac{\pi^2(29{,}000 \text{ ksi})}{(161)^2}$$

$$= 11.0 \text{ ksi}$$

Torsional Parameters

The shear center for a T-shaped section is located on the axis of symmetry at the mid-depth of the flange.

$$x_o = 0.0 \text{ in.}$$

$$y_o = \bar{y} - \frac{t_f}{2}$$

$$= 1.58 \text{ in.} - \frac{0.385 \text{ in.}}{2}$$

$$= 1.39 \text{ in.}$$

$$\bar{r}_o^2 = x_o^2 + y_o^2 + \frac{I_x + I_y}{A_g} \qquad \text{(}Spec\text{. Eq. E4-11)}$$

$$= (0.0 \text{ in.})^2 + (1.39 \text{ in.})^2 + \frac{19.0 \text{ in.}^4 + 9.79 \text{ in.}^4}{4.42 \text{ in.}^2}$$

$$= 8.45 \text{ in.}^2$$

$$\bar{r}_o = \sqrt{\bar{r}_o^2}$$

$$= \sqrt{8.45 \text{ in.}^2}$$

$$= 2.91 \text{ in.}$$

$$H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2} \qquad \text{(}Spec\text{. Eq. E4-10)}$$

$$= 1 - \frac{(0.0 \text{ in.})^2 + (1.39 \text{ in.})^2}{8.45 \text{ in.}^2}$$

$$= 0.771$$

$$F_{ez} = \left(\frac{\pi^2 E C_w}{(K_z L)^2} + GJ \right) \frac{1}{A_g \bar{r}_o^{\,2}}$$ (*Spec*. Eq. E4-9)

Omit term with C_w per User Note at end of AISC *Specification* Section E4.

$$\begin{aligned} F_{ez} &= \frac{GJ}{A_g \bar{r}_o^{\,2}} \\ &= \frac{11{,}200 \text{ ksi}(0.190 \text{ in.}^4)}{4.42 \text{ in.}^2 (8.45 \text{ in.}^2)} \\ &= 57.0 \text{ ksi} \end{aligned}$$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H} \right) \left[1 - \sqrt{1 - \frac{4 F_{ey} F_{ez} H}{(F_{ey} + F_{ez})^2}} \right]$$ (*Spec*. Eq. E4-5)

$$\begin{aligned} &= \left(\frac{11.0 \text{ ksi} + 57.0 \text{ ksi}}{2(0.771)} \right) \left[1 - \sqrt{1 - \frac{4(11.0 \text{ ksi})(57.0 \text{ ksi})(0.771)}{(11.0 \text{ ksi} + 57.0 \text{ ksi})^2}} \right] \\ &= 10.5 \text{ ksi} \quad \textbf{controls} \end{aligned}$$

Check limit for the applicable equation.

$$\begin{aligned} \frac{QF_y}{2.25} &= \frac{(0.611)(50 \text{ ksi})}{2.25} \\ &= 13.6 \text{ ksi} > 10.5 \text{ ksi, therefore, AISC } \textit{Specification} \text{ Equation E7-3 applies} \end{aligned}$$

$$\begin{aligned} F_{cr} &= 0.877 F_e \\ &= 0.877(10.5 \text{ ksi}) \\ &= 9.21 \text{ ksi} \end{aligned}$$ (*Spec*. Eq. E7-3)

$$\begin{aligned} P_n &= F_{cr} A_g \\ &= 9.21 \text{ ksi}(4.42 \text{ in.}^2) \\ &= 40.7 \text{ kips} \end{aligned}$$ (*Spec*. Eq. E7-1)

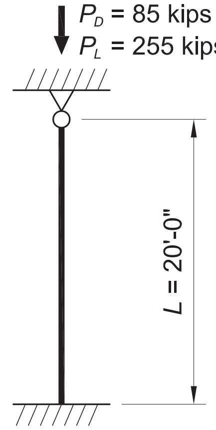
From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(40.7\text{ kips})$ $= 36.6\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{40.7\text{ kips}}{1.67}$ $= 24.4\text{ kips} > 24.0\text{ kips}$ **o.k.**

EXAMPLE E.9 RECTANGULAR HSS COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

Given:

Select an ASTM A500 Grade B rectangular HSS compression member, with a length of 20 ft, to support a dead load of 85 kips and live load of 255 kips in axial compression. The base is fixed and the top is pinned.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(85 \text{ kips}) + 1.6(255 \text{ kips})$ $= 510 \text{ kips}$	$P_a = 85 \text{ kips} + 255 \text{ kips}$ $= 340 \text{ kips}$

Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a fixed-pinned condition, K = 0.8.

$(KL)_x = (KL)_y$ = 0.8(20.0 ft) = 16.0 ft

Enter AISC *Manual* Table 4-3 for rectangular sections or AISC *Manual* Table 4-4 for square sections.

Try an HSS12×10×⅜.

From AISC *Manual* Table 4-3, the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 518 \text{ kips} > 510 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 345 \text{ kips} > 340 \text{ kips}$ **o.k.**

The available strength can be easily determined by using the tables of the AISC *Manual*. Available strength values can be verified by hand calculations, as follows.

Calculation Solution

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS12×10×⅜
A_g = 14.6 in.^{2}
r_x = 4.61 in.
r_y = 4.01 in.
t_{des} = 0.349 in.

Slenderness Check

Note: According to AISC *Specification* Section B4.1b, if the corner radius is not known, b and h shall be taken as the outside dimension minus three times the design wall thickness. This is generally a conservative assumption.

Calculate b/t of the most slender wall.

$$\lambda = \frac{h}{t}$$
$$= \frac{12.0\text{ in.} - 3(0.349\text{ in.})}{0.349\text{ in.}}$$
$$= 31.4$$

Determine the wall limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a Case 6

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$
$$= 1.40\sqrt{\frac{29{,}000\text{ ksi}}{46\text{ ksi}}}$$
$$= 35.2$$

$\lambda < \lambda_r$; therefore, the section does not contain slender elements.

Because $r_y < r_x$ and $(KL)_x = (KL)_y$, r_y will govern the available strength.

Determine the applicable equation.

$$\frac{K_y L}{r_y} = \frac{0.8(20.0\text{ ft})(12\text{ in./ft})}{4.01\text{ in.}}$$
$$= 47.9$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{46\text{ ksi}}}$$
$$= 118 \geq 47.9\text{, therefore, use AISC } \textit{Specification} \text{ Equation E3-2}$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad (\textit{Spec}.\text{ Eq. E3-4})$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(47.9)^2}$$
$$= 125\text{ ksi}$$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y \qquad (\textit{Spec}.\text{ Eq. E3-2})$$
$$= \left(0.658^{\frac{46\text{ ksi}}{125\text{ ksi}}}\right)(46\text{ ksi})$$
$$= 39.4\text{ ksi}$$

$$P_n = F_{cr}A_g \qquad (\textit{Spec}.\text{ Eq. E3-1})$$
$$= 39.4\text{ ksi}\left(14.6\text{ in.}^2\right)$$

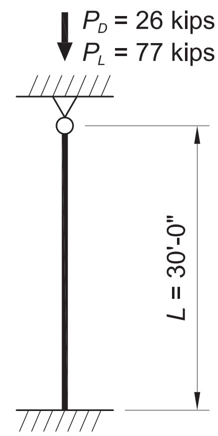
= 575 kips

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(575\text{ kips})$ $= 518\text{ kips} > 510\text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{575\text{ kips}}{1.67}$ $= 344\text{ kips} > 340\text{ kips}$ **o.k.**

Return to Table of Contents

EXAMPLE E.10 RECTANGULAR HSS COMPRESSION MEMBER WITH SLENDER ELEMENTS



Given:

Select an ASTM A500 Grade B rectangular HSS12×8 compression member with a length of 30 ft, to support an axial dead load of 26 kips and live load of 77 kips. The base is fixed and the top is pinned.

A column with slender elements has been selected to demonstrate the design of such a member.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(26\text{ kips}) + 1.6(77\text{ kips})$ $= 154\text{ kips}$	$P_a = 26\text{ kips} + 77\text{ kips}$ $= 103\text{ kips}$

Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a fixed-pinned condition, $K = 0.8$.

$(KL)_x = (KL)_y = 0.8(30.0\text{ ft}) = 24.0\text{ ft}$

Enter AISC *Manual* Table 4-3, for the HSS12×8 section and proceed to the lightest section with an available strength that is equal to or greater than the required strength, in this case an HSS 12×8×3⁄16.

From AISC *Manual* Table 4-3, the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 156\text{ kips} > 154\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 103\text{ kips} \geq 103\text{ kips}$ **o.k.**

The available strength can be easily determined by using the tables of the AISC *Manual*. Available strength values can be verified by hand calculations, as follows, including adjustments for slender elements.

Calculation Solution

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS12×8×3/16
A_g = 6.76 in.^{2}
r_x = 4.56 in.
r_y = 3.35 in.

$$\frac{b}{t} = 43.0$$

$$\frac{h}{t} = 66.0$$

t_{des} = 0.174 in.

Slenderness Check

Calculate the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a case 6 for walls of HSS.

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$

$$= 1.40\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}$$

= 35.2 < 43.0 and 35.2 < 66.0, therefore both the 8-in. and 12-in. walls are slender elements

Note that for determining the width-to-thickness ratio, b is taken as the outside dimension minus three times the design wall thickness per AISC *Specification* Section B4.1b(d).

For the selected shape,

b = 8.0 in. – 3(0.174 in.)
= 7.48 in.

h = 12.0 in. – 3(0.174 in.)
= 11.5 in.

AISC *Specification* Section E7 is used for an HSS member with slender elements. The nominal compressive strength, P_n, is determined based upon the limit states of flexural buckling. Torsional buckling will not govern for HSS unless the torsional unbraced length greatly exceeds the controlling flexural unbraced length.

Effective Area, A_e

$$Q_a = \frac{A_e}{A_g} \quad \text{(}\textit{Spec}\text{. Eq. E7-16)}$$

where A_e = summation of the effective areas of the cross section based on the reduced effective widths, b_e

For flanges of square and rectangular slender-element sections of uniform thickness,

$$b_e = 1.92t\sqrt{\frac{E}{f}}\left[1 - \frac{0.38}{(b/t)}\sqrt{\frac{E}{f}}\right] \le b \quad \text{(}\textit{Spec}\text{. Eq. E7-18)}$$

where $f = P_n / A_e$, but can conservatively be taken as F_y according to the User Note in *Specification* Section E7.2.

For the 8-in. walls,

$$b_e = 1.92t\sqrt{\frac{E}{F_y}}\left[1-\frac{0.38}{(b/t)}\sqrt{\frac{E}{F_y}}\right] \qquad (Spec.\ \text{Eq. E7-18})$$

$$= 1.92(0.174\text{ in.})\sqrt{\frac{29{,}000\text{ ksi}}{46\text{ ksi}}}\left[1-\frac{0.38}{(43.0)}\sqrt{\frac{29{,}000\text{ ksi}}{46\text{ ksi}}}\right]$$

$$= 6.53\text{ in.} \leq 7.48\text{ in.}$$

Length that is ineffective $= b - b_e$
$= 7.48\text{ in.} - 6.53\text{ in.}$
$= 0.950\text{ in.}$

For the 12-in. walls,

$$b_e = 1.92t\sqrt{\frac{E}{F_y}}\left[1-\frac{0.38}{(b/t)}\sqrt{\frac{E}{F_y}}\right] \qquad (Spec.\ \text{Eq. E7-18})$$

$$= 1.92(0.174\text{ in.})\sqrt{\frac{29{,}000\text{ ksi}}{46\text{ ksi}}}\left[1-\frac{0.38}{(66.0)}\sqrt{\frac{29{,}000\text{ ksi}}{46\text{ ksi}}}\right]$$

$$= 7.18\text{ in.} \leq 11.5\text{ in.}$$

Length that is ineffective $= b - b_e$
$= 11.5\text{ in.} - 7.18\text{ in.}$
$= 4.32\text{ in.}$

$$A_e = 6.76\text{ in.}^2 - 2(0.174\text{ in.})(0.950\text{ in.}) - 2(0.174\text{ in.})(4.32\text{ in.})$$
$$= 4.93\text{ in.}^2$$

For cross sections composed of only stiffened slender elements, $Q = Q_a$ ($Q_s = 1.0$).

$$Q = \frac{A_e}{A_g} \qquad (Spec.\ \text{Eq. E7-16})$$

$$= \frac{4.93\text{ in.}^2}{6.76\text{ in.}^2}$$

$$= 0.729$$

Critical Stress, F_{cr}

$$\frac{K_yL}{r_y} = \frac{0.8(30.0\text{ ft})(12\text{ in./ ft})}{3.35\text{ in.}}$$
$$= 86.0$$

$$4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{0.729(46\text{ ksi})}}$$

$= 139 \geq 86.0$, therefore AISC *Specification* Equation E7-2 applies

For the limit state of flexural buckling.

$$F_e = \frac{\pi^2 E}{\left(\dfrac{KL}{r}\right)^2} \qquad \text{(\textit{Spec}. Eq. E3-4)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(86.0)^2}$$

$$= 38.7 \text{ ksi}$$

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y \qquad \text{(\textit{Spec}. Eq. E7-2)}$$

$$= 0.729\left[0.658^{\frac{0.729(46 \text{ ksi})}{38.7 \text{ ksi}}}\right]46 \text{ ksi}$$

$$= 23.3 \text{ ksi}$$

Nominal Compressive Strength

$$P_n = F_{cr}A_g \qquad \text{(\textit{Spec}. Eq. E7-1)}$$
$$= 23.3 \text{ ksi}(6.76 \text{ in.}^2)$$
$$= 158 \text{ kips}$$

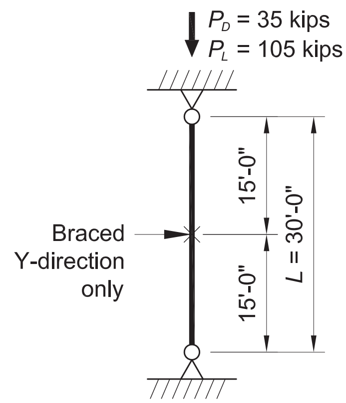
From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(158 \text{ kips})$ $= 142 \text{ kips} < 154 \text{ kips}$ **See following note.**	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{158 \text{ kips}}{1.67}$ $= 94.6 \text{ kips} < 103 \text{ kips}$ **See following note.**

Note: A smaller available strength is calculated here because a conservative initial assumption ($f = F_y$) was made in applying AISC *Specification* Equation E7-18. A more exact solution is obtained by iterating from the effective area, A_e, step using $f = P_n/A_e$ until the value of f converges. The HSS column strength tables in the AISC *Manual* were calculated using this iterative procedure.

EXAMPLE E.11 PIPE COMPRESSION MEMBER

Given:



Select an ASTM A53 Grade B Pipe compression member with a length of 30 ft to support a dead load of 35 kips and live load of 105 kips in axial compression. The column is pin-connected at the ends in both axes and braced at the midpoint in the *y*-*y* direction.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A53 Grade B
F_y = 35 ksi
F_u = 60 ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_u = 1.2(35\text{ kips}) + 1.6(105\text{ kips})$ $= 210\text{ kips}$	$P_a = 35\text{ kips} + 105\text{ kips}$ $= 140\text{ kips}$

Table Solution

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Therefore, $(KL)_x$ = 30.0 ft and $(KL)_y$ = 15.0 ft. Buckling about the *x*-*x* axis controls.

Enter AISC *Manual* Table 4-6 with a *KL* of 30 ft and proceed across the table until reaching the lightest section with sufficient available strength to support the required strength.

Try a 10-in. Standard Pipe.

From AISC *Manual* Table 4-6, the available strength in axial compression is:

LRFD	ASD
$\phi_c P_n = 222\text{ kips} > 210\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 148\text{ kips} > 140\text{ kips}$ **o.k.**

The available strength can be easily determined by using the tables of the AISC *Manual*. Available strength values can be verified by hand calculations, as follows.

Calculation Solution

From AISC *Manual* Table 1-14, the geometric properties are as follows:

Pipe 10 Std.
A_g = 11.5 in.^{2}
r = 3.68 in.
$$\lambda = \frac{D}{t} = 31.6$$

No Pipes shown in AISC *Manual* Table 4-6 are slender at 35 ksi, so no local buckling check is required; however, some round HSS are slender at higher steel strengths. The following calculations illustrate the required check.

Limiting Width-to-Thickness Ratio

$\lambda_r = 0.11E/F_y$ from AISC *Specification* Table B4.1a case 9
$$= 0.11(29{,}000\text{ ksi}/35\text{ ksi})$$
$$= 91.1$$

$\lambda < \lambda_r$; therefore, the pipe is not slender

Critical Stress, F_{cr}

$$\frac{KL}{r} = \frac{(30.0\text{ ft})(12\text{ in./ft})}{3.68\text{ in.}}$$
$$= 97.8$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000\text{ ksi}}{35\text{ ksi}}}$$
= 136 ≥ 97.8, therefore AISC *Specification* Equation E3-2 applies

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$ (*Spec*. Eq. E3-4)
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(97.8)^2}$$
$$= 29.9\text{ ksi}$$

$$F_{cr} = \left(0.658^{\frac{F_y}{F_e}}\right)F_y$$ (*Spec*. Eq. E3-2)
$$= \left(0.658^{\frac{35\text{ ksi}}{29.9\text{ ksi}}}\right)(35\text{ ksi})$$
$$= 21.4\text{ ksi}$$

Nominal Compressive Strength

$$P_n = F_{cr}A_g$$ (*Spec*. Eq. E3-1)
$$= 21.4\text{ ksi}(11.5\text{ in.}^2)$$
$$= 246\text{ kips}$$

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$	$\Omega_c = 1.67$
$\phi_c P_n = 0.90(246 \text{ kips})$	$\dfrac{P_n}{\Omega_c} = \dfrac{246 \text{ kips}}{1.67}$
$= 221 \text{ kips} > 210 \text{ kips}$ **o.k.**	$= 147 \text{ kips} > 140 \text{ kips}$ **o.k.**

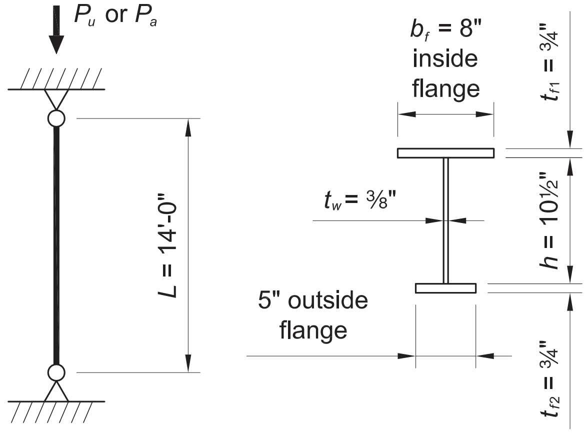
Note that the design procedure would be similar for a round HSS column.

Return to Table of Contents

EXAMPLE E.12 BUILT-UP I-SHAPED MEMBER WITH DIFFERENT FLANGE SIZES

Given:

Compute the available strength of a built-up compression member with a length of 14 ft. The ends are pinned. The outside flange is PL¾-in.×5-in., the inside flange is PL¾-in.×8-in., and the web is PL⅜-in.×10½-in. Material is ASTM A572 Grade 50.



Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A572 Grade 50
$F_y = 50$ ksi
$F_u = 65$ ksi

There are no tables for special built-up shapes; therefore the available strength is calculated as follows.

Slenderness Check

Check outside flange slenderness.

Calculate k_c.

$$k_c = \frac{4}{\sqrt{h/t_w}} \text{ from AISC \textit{Specification} Table B4.1b note [a]}$$

$$= \frac{4}{\sqrt{10\tfrac{1}{2}\text{ in.}/\tfrac{3}{8}\text{ in.}}}$$

$$= 0.756,\ 0.35 \le k_c \le 0.76 \qquad \textbf{o.k.}$$

For the outside flange, the slenderness ratio is,

$$\lambda = \frac{b}{t}$$

$$= \frac{2.50\text{ in.}}{\tfrac{3}{4}\text{ in.}}$$

$$= 3.33$$

Determine the limiting slenderness ratio, λ_r, from AISC *Specification* Table B4.1a case 2

$$\lambda_r = 0.64\sqrt{\frac{k_c E}{F_y}}$$
$$= 0.64\sqrt{\frac{0.756(29{,}000\text{ ksi})}{50\text{ ksi}}}$$
$$= 13.4$$

$\lambda \le \lambda_r$; therefore, the outside flange is not slender

Check inside flange slenderness.

$$\lambda = \frac{b}{t}$$
$$= \frac{4.0\text{ in.}}{\frac{3}{4}\text{ in.}}$$
$$= 5.33$$

$\lambda \le \lambda_r$; therefore, the inside flange is not slender

Check web slenderness.

$$\lambda = \frac{h}{t}$$
$$= \frac{10\frac{1}{2}\text{ in.}}{\frac{3}{8}\text{ in.}}$$
$$= 28.0$$

Determine the limiting slenderness ratio, λ_r, for the web from AISC *Specification* Table B4.1a Case 8

$$\lambda_r = 1.49\sqrt{\frac{E}{F_y}}$$
$$= 1.49\sqrt{\frac{29{,}000\text{ ksi}}{50\text{ ksi}}}$$
$$= 35.9$$

$\lambda \le \lambda_r$; therefore, the web is not slender

Section Properties (ignoring welds)

$$A_g = b_{f1}t_{f1} + ht_w + b_{f2}t_{f2}$$
$$= (8.00\text{ in.})(\tfrac{3}{4}\text{ in.}) + (10\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.}) + (5.00\text{ in.})(\tfrac{3}{4}\text{ in.})$$
$$= 13.7\text{ in.}^2$$

$$\bar{y} = \frac{\Sigma A_i y_i}{\Sigma A_i}$$
$$= \frac{(6.00\text{ in.}^2)(11.6\text{ in.}) + (3.94\text{ in.}^2)(6.00\text{ in.}) + (3.75\text{ in.}^2)(0.375\text{ in.})}{(6.00\text{ in.}^2) + (3.94\text{ in.}^2) + (3.75\text{ in.}^2)}$$
$$= 6.91\text{ in.}$$

Note that the center of gravity about the x-axis is measured from the bottom of the outside flange.

$$I_x = \left[\frac{(8.00\text{ in.})(3/4\text{ in.})^3}{12} + (8.00\text{ in.})(3/4\text{ in.})(4.72\text{ in.})^2\right] +$$
$$\left[\frac{(3/8\text{ in.})(10\tfrac{1}{2}\text{ in.})^3}{12} + (3/8\text{ in.})(10\tfrac{1}{2}\text{ in.})(0.910\text{ in.})^2\right] +$$
$$\left[\frac{(5.00\text{ in.})(3/4\text{ in.})^3}{12} + (5.00\text{ in.})(3/4\text{ in.})(6.54\text{ in.})^2\right]$$
$$= 334\text{ in.}^4$$

$$r_x = \sqrt{\frac{I_x}{A}}$$
$$= \sqrt{\frac{334\text{ in.}^4}{13.7\text{ in.}^2}}$$
$$= 4.94\text{ in.}$$

$$I_y = \left[\frac{(3/4\text{ in.})(8.00\text{ in.})^3}{12}\right] + \left[\frac{(10\tfrac{1}{2}\text{ in.})(3/8\text{ in.})^3}{12}\right] + \left[\frac{(3/4\text{ in.})(5.00\text{ in.})^3}{12}\right]$$
$$= 39.9\text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}}$$
$$= \sqrt{\frac{39.9\text{ in.}^4}{13.7\text{ in.}^2}}$$
$$= 1.71\text{ in.}$$

x-x Axis Flexural Elastic Critical Buckling Stress, F_e

$$\frac{K_x L}{r_x} = \frac{1.0(14.0\text{ ft})(12\text{ in./ft})}{4.94\text{ in.}}$$
$$= 34.0$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(Spec. Eq. E3-4)}$$
$$= \frac{\pi^2(29{,}000\text{ ksi})}{(34.0)^2}$$
$$= 248\text{ ksi} \quad \textbf{does not control}$$

Flexural-Torsional Critical Elastic Buckling Stress

Calculate torsional constant, J.

$$J = \Sigma\left(\frac{bt^3}{3}\right) \text{ from AISC Design Guide 9}$$

$$= \frac{(8.00\text{ in.})(\tfrac{3}{4}\text{ in.})^3}{3} + \frac{(10\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.})^3}{3} + \frac{(5.00\text{ in.})(\tfrac{3}{4}\text{ in.})^3}{3}$$

$$= 2.01\text{ in.}^4$$

Distance between flange centroids:

$$h_o = d - \frac{t_{f1}}{2} - \frac{t_{f2}}{2}$$

$$= 12.0\text{ in.} - \frac{\tfrac{3}{4}\text{ in.}}{2} - \frac{\tfrac{3}{4}\text{ in.}}{2}$$

$$= 11.3\text{ in.}$$

Warping constant:

$$C_w = \frac{t_f h_o^2}{12}\left(\frac{b_1^3 b_2^3}{b_1^3 + b_2^3}\right)$$

$$= \frac{(\tfrac{3}{4}\text{ in.})(11.3\text{ in.})^2}{12}\left(\frac{(8.00\text{ in.})^3(5.00\text{ in.})^3}{(8.00\text{ in.})^3 + (5.00\text{ in.})^3}\right)$$

$$= 802\text{ in.}^6$$

Due to symmetry, both the centroid and the shear center lie on the *y*-axis. Therefore $x_o = 0$. The distance from the center of the outside flange to the shear center is:

$$e = h_o\left(\frac{b_1^3}{b_1^3 + b_2^3}\right)$$

$$= 11.3\text{ in.}\left(\frac{(8.00\text{ in.})^3}{(8.00\text{ in.})^3 + (5.00\text{ in.})^3}\right)$$

$$= 9.08\text{ in.}$$

Add one-half the flange thickness to determine the shear center location measured from the bottom of the outside flange.

$$e + \frac{t_f}{2} = 9.08\text{ in.} + \frac{\tfrac{3}{4}\text{ in.}}{2}$$

$$= 9.46\text{ in.}$$

$$y_o = \left(e + \frac{t_f}{2}\right) - \bar{y}$$

$$= 9.46\text{ in.} - 6.91\text{ in.}$$

$$= 2.55\text{ in.}$$

$$\bar{r}_0^2 = x_0^2 + y_0^2 + \frac{I_x + I_y}{A_g} \qquad \text{(}\textit{Spec}\text{. Eq. E4-11)}$$

$$= 0.0 + (2.55\text{ in.})^2 + \frac{334\text{ in.}^4 + 39.9\text{ in.}^4}{13.7\text{ in.}^2}$$

$= 33.8\text{ in.}^2$

$$H = 1 - \frac{x_0^2 + y_0^2}{\bar{r}_0^2} \qquad \text{(\textit{Spec}. Eq. E4-10)}$$

$$= 1 - \frac{0.0 + (2.55\text{ in.})^2}{33.8\text{ in.}^2}$$

$= 0.808$

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

$$\frac{KL}{r_y} = \frac{1.0(14.0\text{ ft})(12.0\text{ in./ft})}{1.71\text{ in.}}$$

$= 98.2$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{K_y L}{r_y}\right)^2} \qquad \text{(\textit{Spec}. Eq. E4-8)}$$

$$= \frac{\pi^2(29{,}000\text{ ksi})}{(98.2)^2}$$

$= 29.7\text{ ksi}$

$$F_{ez} = \left[\frac{\pi^2 E C_w}{(K_z L)^2} + GJ\right]\left(\frac{1}{A_g \bar{r}_o^2}\right) \qquad \text{(\textit{Spec}. Eq. E4-9)}$$

$$= \left[\frac{\pi^2(29{,}000\text{ ksi})(802\text{ in.}^6)}{\left[(1.0)(14.0\text{ ft})(12\text{ in./ft})\right]^2} + (11{,}200\text{ ksi})(2.01\text{ in.}^4)\right]\left(\frac{1}{(13.7\text{ in.}^2)(33.8\text{ in.}^2)}\right)$$

$= 66.2\text{ ksi}$

$$F_e = \left(\frac{F_{ey} + F_{ez}}{2H}\right)\left[1 - \sqrt{1 - \frac{4F_{ey}F_{ez}H}{(F_{ey} + F_{ez})^2}}\right] \qquad \text{(\textit{Spec}. Eq. E4-5)}$$

$$= \left(\frac{29.7\text{ ksi} + 66.2\text{ ksi}}{2(0.808)}\right)\left[1 - \sqrt{1 - \frac{4(29.7\text{ ksi})(66.2\text{ ksi})(0.808)}{(29.7\text{ ksi} + 66.2\text{ ksi})^2}}\right]$$

$= 26.4\text{ ksi}$ **controls**

Torsional and flexural-torsional buckling governs.

$$\frac{F_y}{2.25} = \frac{50\text{ ksi}}{2.25}$$

$= 22.2\text{ ksi} \le 26.4\text{ ksi}$, therefore, AISC *Specification* Equation E3-2 applies

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y \qquad \text{(\textit{Spec}. Eq. E3-2)}$$

$$= \left[0.658^{\frac{50\text{ ksi}}{26.4\text{ ksi}}}\right](50\text{ ksi})$$

$= 22.6 \text{ ksi}$

$$P_n = F_{cr} A_g \qquad (Spec. \text{ Eq. E3-1})$$
$$= 22.6 \text{ ksi}\left(13.7 \text{ in.}^2\right)$$
$$= 310 \text{ kips}$$

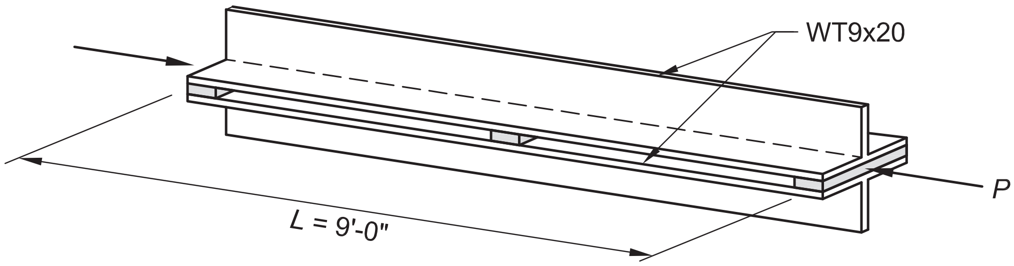
From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90\left(310 \text{ kips}\right)$ $= 279 \text{ kips}$	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{310 \text{ kips}}{1.67}$ $= 186 \text{ kips}$

EXAMPLE E.13 DOUBLE-WT COMPRESSION MEMBER

Given:

Determine the available compressive strength for the double-WT9×20 compression member shown below. Assume that ½-in.-thick connectors are welded in position at the ends and at equal intervals "a" along the length. Use the minimum number of intermediate connectors needed to force the two WT-shapes to act as a single built-up compressive section.



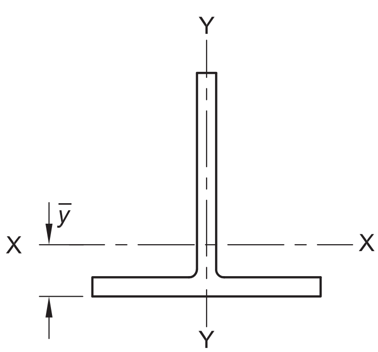
Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Tee
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

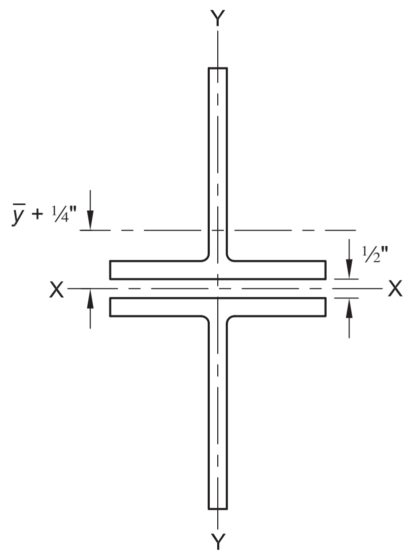
From AISC *Manual* Table 1-8 the geometric properties for a single WT9×20 are as follows:

A = 5.88 in.^{2}
I_x = 44.8 in.^{4}
I_y = 9.55 in.^{4}
r_x = 2.76 in.
r_y = 1.27 in.
$\bar{y}$ = 2.29 in.
J = 0.404 in.^{4}
C_w = 0.788 in.^{6}
Q_s = 0.496



From mechanics of materials, the combined section properties for two *WT9*×*20*'s, flange-to-flange, spaced 0.50 in. apart, are as follows:

$$A = \Sigma A_{single\ tee}$$
$$= 2(5.88\text{ in.}^2)$$
$$= 11.8\text{ in.}^2$$
$$I_x = \Sigma(I_x + A\bar{y}^2)$$
$$= 2\left[44.8\text{ in.}^4 + (5.88\text{ in.}^2)(2.29\text{ in.} + \tfrac{1}{4}\text{ in.})^2\right]$$
$$= 165\text{ in.}^4$$
$$r_x = \sqrt{\frac{I_x}{A}}$$
$$= \sqrt{\frac{165\text{ in.}^4}{11.8\text{ in.}^2}}$$
$$= 3.74\text{ in.}$$
$$I_y = \Sigma I_{y\ single\ tee}$$
$$= 2(9.55\text{ in.}^4)$$
$$= 19.1\text{ in.}^4$$
$$r_y = \sqrt{\frac{I_y}{A}}$$
$$= \sqrt{\frac{19.1\text{ in.}^4}{11.8\text{ in.}^2}}$$
$$= 1.27\text{ in.}$$
$$J = \Sigma J_{single\ tee}$$
$$= 2(0.404\text{ in.}^4)$$
$$= 0.808\text{ in.}^4$$



For the double-WT (cruciform) shape it is reasonable to take $C_w = 0$ and ignore any warping contribution to column strength.

The y-axis of the combined section is the same as the y-axis of the single section. When buckling occurs about the y-axis, there is no relative slip between the two WTs. For buckling about the x-axis of the combined section, the WT's will slip relative to each other unless restrained by welded or slip-critical end connections.

Intermediate Connectors

Determine the minimum adequate number of intermediate connectors.

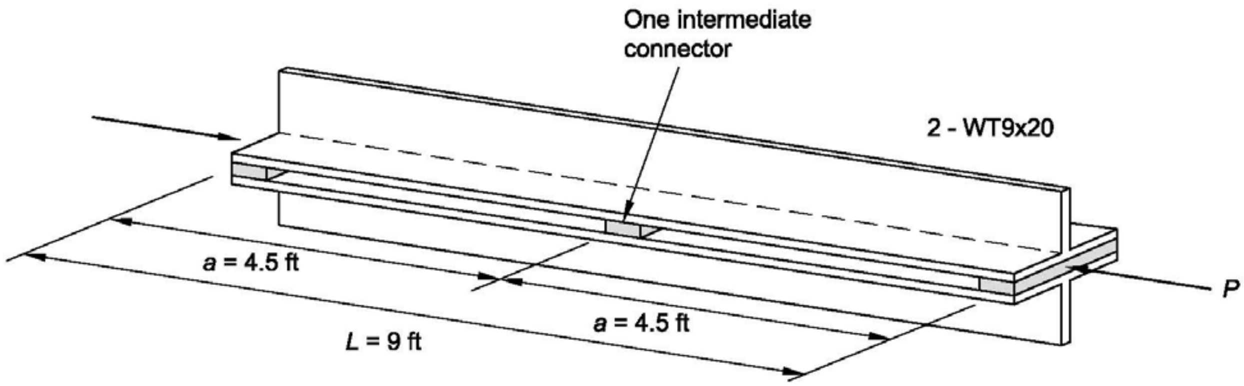
From AISC Specification Section E6.2, the maximum slenderness ratio of each tee may not exceed three-quarters of the maximum slenderness ratio of the double-tee built-up section. For a *WT9*×*20*, the minimum radius of gyration, $r_i = r_y = 1.27$ in.

Use K = 1.0 for both the single tee and the double tee:

$$\left(\frac{Ka}{r_i}\right)_{single\ tee} \le 0.75\left(\frac{KL}{r_{min}}\right)_{double\ tee}$$

$$a \le 0.75\frac{\left(r_y\right)_{single\ tee}}{\left(r_y\right)_{double\ tee}}\left(\frac{K_{double\ tee}L}{K_{single\ tee}}\right)$$
$$= 0.75\left(\frac{1.27\text{ in.}}{1.27\text{ in.}}\right)\frac{(1.0)(9.00\text{ ft})(12\text{ in./ft})}{1.0}$$
$$= 81.0\text{ in.}$$

Thus, one intermediate connector at mid-length (a = 4.5 ft = 54 in.) satisfies AISC Specification Section E6.2.



Flexural Buckling and Torsional Buckling Strength

The nominal compressive strength, P_n, is computed using

$$P_n = F_{cr}A_g \qquad \text{(Spec. Eq. E3-1 or Eq. E7-1)}$$

where the critical stress is determined using AISC Specification equations in Sections E3, E4 or E7, as appropriate.

For the WT9×20, the stem is slender because $d/t_w = 28.4 > 0.75\sqrt{29{,}000\text{ ksi}/50\text{ksi}} = 18.1$. Therefore, the member is a slender element member and the provisions of Section E7 must be followed. Determine the elastic buckling stress for flexural buckling about the y- and x-axes, and torsional buckling. Then, using Q_s, determine the critical buckling stress and the nominal strength.

Flexural buckling about the y-axis:

$$r_y = 1.27\text{ in.}$$
$$\frac{KL}{r_y} = \frac{1.0(9.0\text{ ft})(12\text{ in./ft})}{1.27\text{ in.}}$$
$$= 85.0$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} \qquad \text{(Spec. Eq. E3-4)}$$

$$= \frac{\pi^2(29{,}000 \text{ ksi})}{(85.0)^2}$$
$$= 39.6 \text{ ksi}$$

Flexural buckling about the x-axis:

Flexural buckling about the x-axis is determined using the modified slenderness ratio to account for shear deformation of the intermediate connectors.

Note that the provisions of AISC *Specification* Section E6.1, which require that KL/r be replaced with $(KL/r)_m$, apply if "the buckling mode involves relative deformations that produce shear forces in the connectors between individual shapes…". Relative slip between the two sections occurs for buckling about the x-axis so the provisions of the section apply only to buckling about the x-axis.

The connectors are welded at the ends and the intermediate point. The modified slenderness is calculated using the spacing between intermediate connectors:

$$a = 4.5 \text{ ft}(12.0 \text{ in./ft}) = 54.0 \text{ in.}$$

$$r_{ib} = r_y$$
$$= 1.27 \text{ in.}$$

$$\frac{a}{r_{ib}} = \frac{54.0 \text{ in.}}{1.27 \text{ in.}}$$
$$= 42.5$$

Because $a/r_{ib} > 40$, use AISC *Specification* Equation E6-2b.

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + \left(\frac{K_i a}{r_i}\right)^2} \qquad \text{(Spec. Eq. E6-2b)}$$

where

$$\left(\frac{KL}{r}\right)_o = \left(\frac{1.0(9.00 \text{ ft.})(12.0 \text{ in./ft})}{3.74 \text{ in.}}\right) = 28.9$$
$$\left(\frac{K_i a}{r_i}\right) = \left(\frac{0.86(4.50 \text{ ft.})(12.0 \text{ in./ft})}{1.27 \text{ in.}}\right) = 36.6$$

Thus,

$$\left(\frac{KL}{r}\right)_m = \sqrt{(28.9)^2 + (36.6)^2} = 46.6$$

and

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r_x}\right)_m^2} \qquad \text{(from Spec. Eq. E3-4)}$$

$$= \frac{\pi^2 (29{,}000 \text{ ksi})}{(46.6)^2}$$
$$= 132 \text{ ksi}$$

Torsional buckling:

$$F_e = \left[\frac{\pi^2 E C_w}{(K_z L)^2} + GJ \right] \frac{1}{I_x + I_y} \qquad (Spec.\ Eq.\ E4\text{-}4)$$

The cruciform section made up of two back-to-back WT's has virtually no warping resistance, thus the warping contribution is ignored and Equation E4-4 becomes

$$F_e = \frac{GJ}{I_x + I_y}$$
$$= \frac{(11{,}200 \text{ ksi})(0.808 \text{ in.}^4)}{(165 \text{ in.}^4 + 19.1 \text{ in.}^4)}$$
$$= 49.2 \text{ ksi}$$

Use the smallest elastic buckling stress, F_e, from the limit states considered above to determine F_{cr} by AISC Specification Equation E7-2 or Equation E7-3, as follows:

$Q_s = 0.496$

$F_e = F_{e(smallest)}$

$= 39.6$ ksi (y-axis flexural buckling)

$$\frac{QF_y}{F_e} = \frac{0.496(50 \text{ ksi})}{39.6 \text{ ksi}} = 0.626 < 2.25$$

Therefore use Equation E7-2,

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y$$
$$= 0.496\left[0.658^{0.626}\right](50 \text{ ksi})$$
$$= 19.1 \text{ ksi}$$

Determine the nominal compressive strength, P_n:

$$P_n = F_{cr} A_g \qquad (Spec.\ \text{Eq. E7-1})$$
$$= (19.1 \text{ ksi})(11.8 \text{ in.}^2)$$
$$= 225 \text{ kips}$$

Determine the available compressive strength:

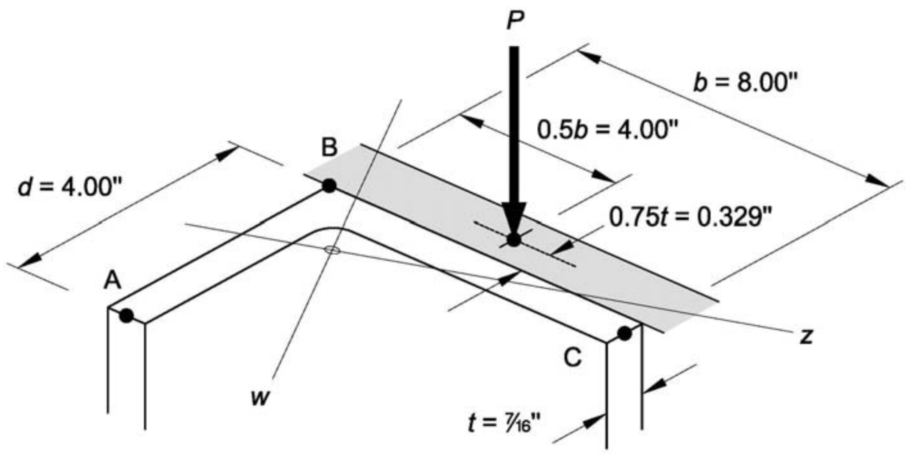
LRFD	ASD

$\phi_c = 0.90$ $\phi_c P_n = 0.90(225\text{ kips})$ $= 203\text{ kips}$	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{225\text{ kips}}{1.67}$ $= 135\text{ kips}$

EXAMPLE E.14 ECCENTRICALLY LOADED SINGLE-ANGLE COMPRESSION MEMBER (LONG LEG ATTACHED)

Given:

Determine the available strength of an eccentrically loaded ASTM A36 L8×4×7⁄16 single angle, as shown, with an effective length of 5 ft. The long leg of the angle is the attached leg, and the eccentric load is applied at 0.75*t* as shown. Use the provisions of the AISC *Specification* and compare the results to the available strength found in AISC *Manual* Table 4-12.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-7:

L8x4x7⁄16
$b = 8.00$ in.
$d = 4.00$ in.
$t = 7/16$ in.
$\bar{x} = 0.829$ in.
$\bar{y} = 2.81$ in.
$A = 5.11$ in.^{2}
$I_x = 34.2$ in.^{4}
$I_y = 6.03$ in.^{4}
$I_z = 3.84$ in.^{4}
$r_z = 0.867$ in.
$\tan\alpha = 0.268$

From AISC Shapes Database V14.0:

$I_w = 36.4 \text{ in.}^4$

$S_{wA} = 11.0 \text{ in.}^3$

$S_{wB} = 14.6 \text{ in.}^3$

$S_{wC} = 7.04 \text{ in.}^3$

$S_{zA} = 1.61 \text{ in.}^3$

$S_{zB} = 2.51 \text{ in.}^3$

$S_{zC} = 5.09 \text{ in.}^3$

From geometry, distances between points A, B, C and the principal *w-w* and *z-z* axes, as shown in Figure E.14-1, are determined as follows:

$$\begin{aligned}
\alpha &= \tan^{-1}(0.268) \\
&= 15.0^\circ
\end{aligned}$$

$$\begin{aligned}
w_A &= (d - \bar{x})\cos\alpha - (\bar{y} - \frac{t}{2})\sin\alpha \\
&= (4.00 \text{ in.} - 0.829 \text{ in.})\cos 15.0^\circ - \left(2.81 \text{ in.} - \frac{7/16 \text{ in.}}{2}\right)\sin 15.0^\circ \\
&= 2.39 \text{ in.}
\end{aligned}$$

$$\begin{aligned}
w_B &= \bar{x}\cos\alpha + \bar{y}\sin\alpha \\
&= 0.829 \text{ in.}(\cos 15.0^\circ) + 2.81 \text{ in.}(\sin 15.0^\circ) \\
&= 1.53 \text{ in.}
\end{aligned}$$

$$\begin{aligned}
w_C &= (b - \bar{y})\sin\alpha - (\bar{x} - \frac{t}{2})\cos\alpha \\
&= (8.00 \text{ in.} - 2.81 \text{ in.})\sin 15.0^\circ - \left(0.829 \text{ in.} - \frac{7/16 \text{ in.}}{2}\right)\cos 15.0^\circ \\
&= 0.754 \text{ in.}
\end{aligned}$$

$$\begin{aligned}
z_A &= \left[(d - \bar{x}) - (\bar{y} - \tfrac{t}{2})\tan\alpha\right]\sin\alpha + \frac{\bar{y} - \frac{t}{2}}{\cos\alpha} \\
&= \left[(4.00 \text{ in.} - 0.829 \text{ in.}) - \left(2.81 \text{ in.} - \frac{7/16 \text{ in.}}{2}\right)(0.268)\right]\sin 15.0^\circ + \frac{2.81 \text{ in.} - \frac{7/16 \text{ in.}}{2}}{\cos 15.0^\circ} \\
&= 3.32 \text{ in.}
\end{aligned}$$

$$\begin{aligned}
z_B &= \bar{y}\cos\alpha - \bar{x}\sin\alpha \\
&= 2.81 \text{ in.}(\cos 15.0^\circ) - 0.829 \text{ in.}(\sin 15.0^\circ) \\
&= 2.50 \text{ in.}
\end{aligned}$$

$$\begin{aligned}
z_C &= (b - \bar{y})\cos\alpha + \left(\bar{x} - \frac{t}{2}\right)\sin\alpha \\
&= (8.00 \text{ in.} - 2.81 \text{ in.})\cos 15.0^\circ + \left(0.829 \text{ in.} - \frac{7/16 \text{ in.}}{2}\right)\sin 15.0^\circ \\
&= 5.17 \text{ in.}
\end{aligned}$$

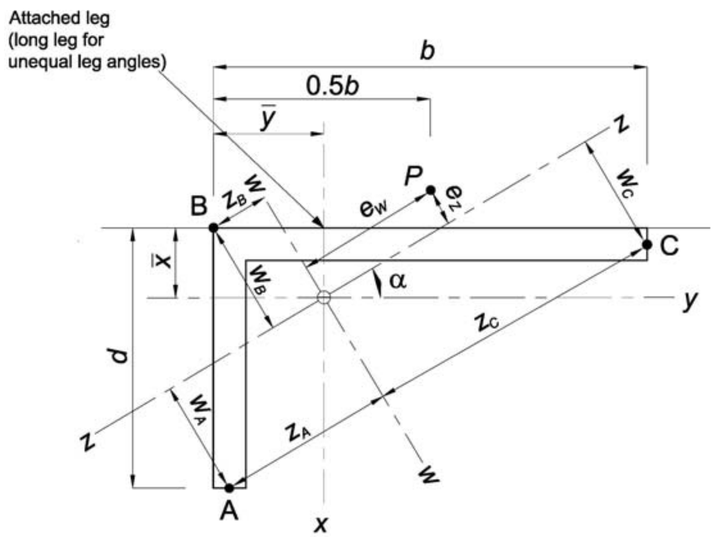


Fig. E.14-1. Geometry about principal axes.

The load is applied at the location shown in Figure E.14-1. Determine the eccentricities about the major (*w*-*w* axis) and minor (*z*-*z* axis) principal axes for the load, *P*. From AISC *Manual* Table 1-7, the angle of the principal axes is found to be $\alpha = \tan^{-1}(0.268) = 15.0°$.

Using the geometry shown in Figures E.14-1 and E.14-2:

$$\begin{aligned} e_w &= \left[\left(\bar{x} + 0.75t\right) - \left(0.5b - \bar{y}\right)\tan\alpha\right]\sin\alpha + \left(\frac{0.5b - \bar{y}}{\cos\alpha}\right) \\ &= \left\{\left[0.829 \text{ in.} + 0.75\left(\tfrac{7}{16} \text{ in.}\right)\right] - \left(4.00 \text{ in.} - 2.81 \text{ in.}\right)\left(0.268\right)\right\}\left(0.259\right) + \left(\frac{4.00 \text{ in.} - 2.81 \text{ in.}}{0.966}\right) \\ &= 1.45 \text{ in.} \end{aligned}$$

$$\begin{aligned} e_z &= \left(\bar{x} + 0.75t\right)\cos\alpha - \left(0.5b - \bar{y}\right)\sin\alpha \\ &= \left[0.829 \text{ in.} + \left(0.75\right)\left(\tfrac{7}{16} \text{ in.}\right)\right]\left(0.966\right) - \left(4.00 \text{ in.} - 2.81 \text{ in.}\right)\left(0.259\right) \\ &= 0.810 \text{ in.} \end{aligned}$$

Because of these eccentricities, the moment resultant has components about both principal axes; therefore, the combined stress provisions of AISC *Specification* Section H2 must be followed.

$$\left| \frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}} \right| \le 1.0$$

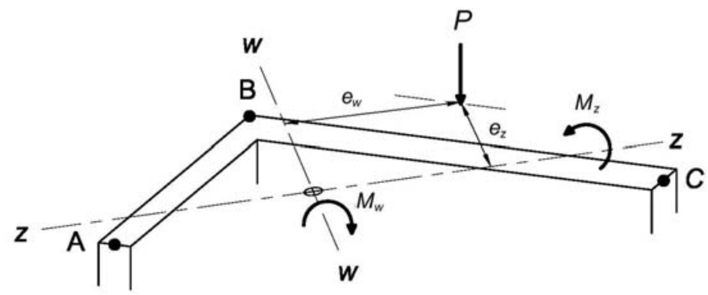


Fig. E.14-2. Applied moments and eccentric axial load.

Due to the load and the given eccentricities, moments about the *w-w* and *z-z* axes will have different effects on points A, B and C. The axial force will produce a compressive stress and the moments, where positive moments are in the direction shown in Figure E.14-2, will produce stresses with a sign indicated by the sense given in the following. In this example, compressive stresses will be taken as positive and tensile stresses will be taken as negative.

Point	Caused by M_w	Caused by M_z
A	tension	tension
B	tension	compression
C	compression	tension

Available Compressive Strength

Check the slenderness of the longest leg for uniform compression.

$$\begin{aligned}\lambda &= \frac{b}{t} \\ &= \frac{8.00\text{ in.}}{7/16\text{ in.}} \\ &= 18.3\end{aligned}$$

From AISC *Specification* Table B4.1a, the limiting width-to-thickness ratio is:

$$\begin{aligned}\lambda_r &= 0.45\sqrt{\frac{E}{F_y}} \\ &= 0.45\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}} \\ &= 12.8\end{aligned}$$

Since b/t = 18.3 > 12.8, this angle must be treated as a slender element compression member according to AISC *Specification* Section E7.1(c). To determine the appropriate equation for determination of Q_s, compare b/t to $0.91\sqrt{\frac{E}{F_y}}$:

$$\frac{b}{t} = 18.3 < 0.91\sqrt{\frac{E}{F_y}}$$
$$< 0.91\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$< 25.8$$

Thus, Q_s is computed for a slender unstiffened element in compression from AISC *Specification* Equation E7-11.

$$Q = Q_s$$
$$= 1.34 - 0.76\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}$$ (*Spec*. Eq. E7-11)
$$= 1.34 - 0.76(18.3)\sqrt{\frac{36 \text{ ksi}}{29{,}000 \text{ ksi}}}$$
$$= 0.850$$

Determine the critical stress, F_{cr}, with KL = 60.0 in. for buckling about the *z-z* axis.

$$\frac{KL}{r_z} = \frac{60.0 \text{ in.}}{0.867 \text{ in.}}$$
$$= 69.2$$
$$< 4.71\sqrt{\frac{E}{QF_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{(0.850)(36 \text{ ksi})}}$$
$$< 145$$

Therefore, use Equation E7-2:

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y$$ (*Spec*. Eq. E7-2)

where

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r_z}\right)^2}$$ (*Spec*. Eq. E3-4)
$$= \frac{\pi^2(29{,}000 \text{ ksi})}{(69.2)^2}$$
$$= 59.8 \text{ ksi}$$

Therefore:

$$F_{cr} = Q\left[0.658^{\frac{QF_y}{F_e}}\right]F_y$$ (*Spec*. Eq. E7-2)
$$= 0.850\left[0.658^{\frac{(0.850)(36 \text{ ksi})}{59.8 \text{ ksi}}}\right](36 \text{ ksi})$$
$$= 24.7 \text{ ksi}$$

The nominal strength, P_n, is:

$$P_n = F_{cr} A_g$$
$$= 24.7 \text{ ksi}(5.11 \text{ in.})$$
$$= 126 \text{ kips}$$
(*Spec*. Eq. E7–1)

From AISC *Specification* Section E1, the available compressive strength is:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c P_n = 0.90(126 \text{ kips})$ $= 113 \text{ kips}$	$\Omega_c = 1.67$ $\dfrac{P_n}{\Omega_c} = \dfrac{126 \text{ kips}}{1.67}$ $= 75.4 \text{ kips}$

Determine the available flexural strengths, M_{cbw} and M_{cbz}, and the available flexural stresses at each point on the cross section.

Limit State of Yielding

Consider the limit state of yielding for bending about the *w-w* and *z-z* axes at points A, B and C, according to AISC *Specification* Section F10.1.

w-w axis:

$$S_{wA} = 11.0 \text{ in.}^3$$
$$M_{ywA} = F_y S_{wA}$$
$$= (36 \text{ ksi})(11.0 \text{ in.}^3)$$
$$= 396 \text{ kip-in.}$$
$$M_{nwA} = 1.5 M_{ywA}$$
$$= 1.5(396 \text{ kip-in.})$$
$$= 594 \text{ kip-in.}$$
(from *Spec*. Eq. F10-1)

$$S_{wB} = 14.6 \text{ in.}^3$$
$$M_{ywB} = F_y S_{wB}$$
$$= (36 \text{ ksi})(14.6 \text{ in.}^3)$$
$$= 526 \text{ kip-in.}$$
$$M_{nwB} = 1.5 M_{ywB}$$
$$= 1.5(526 \text{ kip-in.})$$
$$= 789 \text{ kip-in.}$$
(from *Spec*. Eq. F10-1)

$$S_{wC} = 7.04 \text{ in.}^3$$
$$M_{ywC} = F_y S_{wC}$$
$$= (36 \text{ ksi})(7.04 \text{ in.}^3)$$
$$= 253 \text{ kip-in.}$$
$$M_{nwC} = 1.5 M_{ywC}$$
$$= 1.5(253 \text{ kip-in.})$$
$$= 380 \text{ kip-in.}$$
(from *Spec*. Eq. F10-1)

z-z axis:

$S_{zA} = 1.61 \text{ in.}^3$

$M_{yzA} = F_y S_{zA}$

$= (36 \text{ ksi})(1.61 \text{ in.}^3)$

$= 58.0 \text{ kip-in.}$

$M_{nzA} = 1.5M_{yzA}$ (from *Spec*. Eq. F10-1)

$= 1.5(58.0 \text{ kip-in.})$

$= 87.0 \text{ kip-in.}$

$S_{zB} = 2.51 \text{ in.}^3$

$M_{yzB} = F_y S_{zB}$

$= (36 \text{ ksi})(2.51 \text{ in.}^3)$

$= 90.4 \text{ kip-in.}$

$M_{nzB} = 1.5M_{yzB}$ (from *Spec*. Eq. F10-1)

$= 1.5(90.4 \text{ kip-in.})$

$= 136 \text{ kip-in.}$

$S_{zC} = 5.09 \text{ in.}^3$

$M_{yzC} = F_y S_{zC}$

$= (36 \text{ ksi})(5.09 \text{ in.}^3)$

$= 183 \text{ kip-in.}$

$M_{nzC} = 1.5M_{yzC}$

$= 1.5(183 \text{ kip-in.})$

$= 275 \text{ kip-in.}$

Select the least M_n for each axis:

For the limit state of yielding about the *w-w* axis,

$M_{nw} = 380 \text{ kip-in.}$

For the limit state of yielding about the *z-z* axis,

$M_{nz} = 87.0 \text{ kip-in.}$

Limit State of Lateral-Torsional Buckling

From AISC *Specification* Section F10.2, the limit state of lateral-torsional buckling of a single angle without continuous restraint along its length is a function of the elastic lateral-torsional buckling moment about the major principal axis. For bending about the major principal axis for an unequal leg angle:

$$M_e = \frac{4.9EI_zC_b}{L_b^2}\left[\sqrt{\beta_w^2 + 0.052\left(\frac{L_b t}{r_z}\right)^2} + \beta_w\right] \quad \text{(Spec. Eq. F10-5)}$$

From AISC *Specification* Section F1, for uniform moment along the member length, C_b = 1.0. From AISC *Specification* Commentary Table C-F10.1, an L8×4×7⁄16 has β_w =5.48 in. From AISC *Specification* Commentary

Figure C-F10.4b, with the tip of the long leg (point C) in compression for bending about the *w*-axis, β_w is taken as negative. Thus:

$$M_e = \frac{4.9(29{,}000\text{ ksi})(3.84\text{ in.}^4)(1.0)}{(60.0\text{ in.})^2}\left[\sqrt{(-5.48\text{ in.})^2 + (0.052)\left[\frac{(60.0\text{ in.})(\tfrac{7}{16}\text{ in.})}{0.867\text{ in.}}\right]^2} + (-5.48\text{ in.})\right]$$
$$= 505\text{ kip-in.}$$

Because $M_{ywC} = 253\text{ kip-in.} < M_e = 505\text{ kip-in.}$, determine M_n from AISC *Specification* Equation F10-3:

$$M_{nwC} = \left(1.92 - 1.17\sqrt{\frac{M_{ywC}}{M_e}}\right)M_{ywC} \le 1.5M_{ywC} \qquad (\textit{Spec}.\text{ Eq. F10-3})$$
$$= \left(1.92 - 1.17\sqrt{\frac{253\text{ kip-in.}}{505\text{ kip-in.}}}\right)(253\text{ kip-in.}) \le 1.5(253\text{ kip-in.})$$
$$= 276\text{ kip-in.} \le 380\text{ kip-in.}$$

Limit State of Leg Local Buckling

From AISC *Specification* Section F10.3, the limit state of leg local buckling applies when the toe of the leg is in compression. As discussed previously and indicated in Table E.14-1, the only case in which a toe is in compression is point C for bending about the *w-w* axis. Thus, determine the slenderness of the long leg. From AISC *Specification* Table B4.1b:

$$\lambda_p = 0.54\sqrt{\frac{E}{F_y}}$$
$$= 0.54\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}}$$
$$= 15.3$$

$$\lambda_r = 0.91\sqrt{\frac{E}{F_y}}$$
$$= 0.91\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}}$$
$$= 25.8$$

$$\lambda = \frac{b}{t}$$
$$= \frac{8.0\text{ in.}}{\tfrac{7}{16}\text{ in.}}$$
$$= 18.3$$

Because $\lambda_p < \lambda < \lambda_r$, the angle is noncompact for flexure for this loading. From AISC *Specification* Equation F10-7:

$$M_{nwC} = F_y S_{wC}\left(2.43 - 1.72\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}\right) \qquad (\textit{Spec}.\text{ Eq. F10-7})$$

$$= (36 \text{ ksi})(7.04 \text{ in.}^3)\left[2.43 - 1.72(18.3)\sqrt{\frac{36 \text{ ksi}}{29{,}000 \text{ ksi}}}\right]$$
$$= 335 \text{ kip-in.}$$

Table E.14-1 provides a summary of nominal flexural strength at each point. T indicates the point is in tension and C indicates it is in compression.

Table E.14-1						
	Yielding		Lateral-Torsional Buckling		Leg Local Buckling	
	M_{nw}, kip-in.	M_{nz}, kip-in.	M_{nw}, kip-in.	M_{nz}, kip-in.	M_{nw}, kip-in.	M_{nz}, kip-in.
Point A	594 T	87.0 T	–	–	–	–
Point B	789 T	136 C	–	–	–	–
Point C	380 C	275 T	276 C	–	335 C	–
Note: (–) indicates that the limit state is not applicable to this point.						

Available Flexural Strength

Select the controlling nominal flexural strength for the *w-w* and *z-z* axes.

For the *w-w* axis:

$$M_{nw} = 276 \text{ kip-in.}$$

For the *z-z* axis:

$$M_{nz} = 87.0 \text{ kip-in.}$$

From AISC *Specification* Section F1, determine the available flexural strength for each axis, *w-w* and *z-z* as follows:

LRFD	ASD
$\phi_b = 0.90$ $M_{cbw} = \phi_b M_{nw}$ $= 0.90(276 \text{ kip-in.})$ $= 248 \text{ kip-in.}$ $M_{cbz} = \phi_b M_{nz}$ $= 0.90(87.0 \text{ kip-in.})$ $= 78.3 \text{ kip-in.}$	$\Omega_b = 1.67$ $M_{cbw} = \dfrac{M_{nw}}{\Omega_b}$ $= \dfrac{276 \text{ kip-in.}}{1.67}$ $= 165 \text{ kip-in.}$ $M_{cbz} = \dfrac{M_{nz}}{\Omega_b}$ $= \dfrac{87.0 \text{ kip-in.}}{1.67}$ $= 52.1 \text{ kip-in.}$

Required Flexural Strength

The load on the column is applied at eccentricities about the *w-w* and *z-z* axes resulting in the following moments:

$$M_w = P_r e_w$$
$$= P_r(1.45 \text{ in.})$$

and

$$M_z = P_r e_z$$
$$= P_r\left(0.810 \text{ in.}\right)$$

The combination of axial load and moment will produce second-order effects in the column which must be accounted for.

Using AISC *Specification* Appendix 8.2, an approximate second-order analysis can be performed. The required second-order flexural strengths will be $B_{1w}\,M_w$ and $B_{1z}\,M_z$, respectively, where

$$B_1 = \frac{C_m}{1-\dfrac{\alpha P_r}{P_{e1}}} \qquad \text{(}\textit{Spec}\text{. Eq. A-8-3)}$$

and

$$\alpha = 1.0 \text{ (LRFD)}$$
$$\alpha = 1.6 \text{ (ASD)}$$

C_m = 1.0 for a column with uniform moment along its length

For each axis, parameters P_{e1w} and P_{e1z}, as used in the moment magnification terms, B_{1w} and B_{1z}, are:

$$P_{e1w} = \frac{\pi^2 EI_w}{\left(KL\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. A-8-5)}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})(36.4 \text{ in.}^4)}{\left(60.0 \text{ in.}\right)^2}$$
$$= 2{,}890 \text{ kips}$$

$$P_{e1z} = \frac{\pi^2 EI_z}{\left(KL\right)^2}$$
$$= \frac{\pi^2 (29{,}000 \text{ ksi})(3.84 \text{ in.}^4)}{\left(60.0 \text{ in.}\right)^2}$$
$$= 305 \text{ kips}$$

and

$$B_{1w} = \frac{C_m}{1-\dfrac{\alpha P_r}{P_{e1w}}} \qquad \text{(from }\textit{Spec}\text{. Eq. A-8-3)}$$
$$= \frac{1.0}{1-\dfrac{\alpha P_r}{2{,}890 \text{ kips}}}$$

$$B_{1z} = \frac{C_m}{1-\dfrac{\alpha P_r}{P_{e1z}}} \qquad \text{(from }\textit{Spec}\text{. Eq. A-8-3)}$$

Thus, the required second-order flexural strengths are:

$$M_{rw} = P_r e_w \left(\frac{1.0}{1 - \dfrac{\alpha P_r}{2,890 \text{ kips}}} \right)$$

$$M_{rz} = P_r e_z \left(\frac{1.0}{1 - \dfrac{\alpha P_r}{305 \text{ kips}}} \right)$$

Interaction of Axial and Flexural Strength

Evaluate the interaction of axial and flexural stresses according to the provisions of AISC *Specification* Section H2.

The interaction equation is given as:

$$\left| \frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}} \right| \le 1.0 \qquad \text{(\textit{Spec}. Eq. H2-1)}$$

where the stresses are to be considered at each point on the cross section with the appropriate sign representing the sense of the stress. Because the required stress and available stress at any point are both functions of the same section property, A or S, it is possible to convert Equation H2-1 from a stress based equation to a force based equation where the section properties will cancel.

Substituting the available strengths and the expressions for the required second-order flexural strengths into AISC *Specification* Equation H2-1 yields:

LRFD	ASD
$\left\| \frac{P_u}{113 \text{ kips}} + \frac{P_u (1.45 \text{ in.})}{248 \text{ kip-in.}} \left(\frac{1.0}{1 - \frac{1.00 P_u}{2,890 \text{ kips}}} \right) + \left[\frac{P_u (0.810 \text{ in.})}{78.3 \text{ kip-in.}} \right] \left(\frac{1}{1 - \frac{1.00 P_u}{305 \text{ kips}}} \right) \right\| \le 1.0$	$\left\| \frac{P_a}{75.4 \text{ kips}} + \frac{P_a (1.45 \text{ in.})}{165 \text{ kip-in.}} \left(\frac{1.0}{1 - \frac{1.60 P_a}{2,890 \text{ kips}}} \right) + \left[\frac{P_a (0.810 \text{ in.})}{52.1 \text{ kip-in.}} \right] \left(\frac{1}{1 - \frac{1.60 P_a}{305 \text{ kips}}} \right) \right\| \le 1.0$

These interaction equations must now be applied at each critical point on the section, points A, B and C using the appropriate sign for the sense of the resulting stress, with compression taken as positive.

For point A, the w term is negative and the z term is negative. Thus:

LRFD	ASD
$\left\lvert \dfrac{P_u}{113\text{ kips}} - \dfrac{P_u\left(1.45\text{ in.}\right)}{248\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.00P_u}{2{,}890\text{ kips}}}\right) - \left[\dfrac{P_u\left(0.810\text{ in.}\right)}{78.3\text{ kip-in.}}\right]\left(\dfrac{1.0}{1-\dfrac{P_u}{305\text{ kips}}}\right)\right\rvert \le 1.0$	$\left\lvert \dfrac{P_a}{75.4\text{ kips}} - \dfrac{P_a\left(1.45\text{ in.}\right)}{165\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.60P_a}{2{,}890\text{ kips}}}\right) - \left[\dfrac{P_a\left(0.810\text{ in.}\right)}{52.1\text{ kip-in.}}\right]\left(\dfrac{1.0}{1-\dfrac{1.60P_a}{305\text{ kips}}}\right)\right\rvert \le 1.0$
By iteration, P_u = 86.2 kips.	By iteration, P_a = 56.1 kips.

For point B, the *w* term is negative and the *z* term is positive. Thus:

LRFD	ASD
$\left\lvert \dfrac{P_u}{113\text{ kips}} - \dfrac{P_u\left(1.45\text{ in.}\right)}{248\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.00P_u}{2{,}890\text{ kips}}}\right) + \left[\dfrac{P_u\left(0.810\text{ in.}\right)}{78.3\text{ kip-in.}}\right]\left(\dfrac{1.0}{1-\dfrac{1.00P_u}{305\text{ kips}}}\right)\right\rvert \le 1.0$	$\left\lvert \dfrac{P_a}{75.4\text{ kips}} - \dfrac{P_a\left(1.45\text{ in.}\right)}{165\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.60P_a}{2{,}890\text{ kips}}}\right) + \left[\dfrac{P_a\left(0.810\text{ in.}\right)}{52.1\text{ kip-in.}}\right]\left(\dfrac{1.0}{1-\dfrac{1.60P_a}{305\text{ kips}}}\right)\right\rvert \le 1.0$
By iteration, P_u = 62.9 kips.	By iteration, P_a = 41.1 kips.

For point C, the *w* term is positive and the *z* term is negative. Thus:

LRFD	ASD
$\left\lvert \dfrac{P_u}{113\text{ kips}} + \dfrac{P_u\left(1.45\text{ in.}\right)}{248\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.00P_u}{2{,}890\text{ kips}}}\right) - \left[\dfrac{P_u\left(0.810\text{ in.}\right)}{78.3\text{ kip-in.}}\right]\left(\dfrac{1.0}{1-\dfrac{1.00P_u}{305\text{ kips}}}\right)\right\rvert \le 1.0$	$\left\lvert \dfrac{P_a}{75.4\text{ kips}} + \dfrac{P_a\left(1.45\text{ in.}\right)}{165\text{ kip-in.}}\left(\dfrac{1.0}{1-\dfrac{1.60P_a}{2{,}890\text{ kips}}}\right) - \left[\dfrac{P_a\left(0.810\text{ in.}\right)}{52.1\text{ kip-in.}}\right]\left(\dfrac{1.0}{1-\dfrac{1.60P_a}{305\text{ kips}}}\right)\right\rvert \le 1.0$
By iteration, P_u = 158 kips.	By iteration, P_a = 99.7 kips.

Governing Available Strength

LRFD	ASD
From the above iterations, P_u = 62.9 kips. From AISC *Manual* Table 4-12, ϕP_n = 62.8 kips.	From the above iterations, P_a = 41.1 kips. From AISC *Manual* Table 4-12, P_n/Ω = 41.4 kips.

Thus, the calculations demonstrate how the values for this member in AISC *Manual* Table 4-12 can be confirmed. The slight variations between the calculated solutions and those from AISC *Manual* Table 4-12 are due to rounding.

Chapter F
Design of Members for Flexure

INTRODUCTION

This *Specification* chapter contains provisions for calculating the flexural strength of members subject to simple bending about one principal axis. Included are specific provisions for I-shaped members, channels, HSS, tees, double angles, single angles, rectangular bars, rounds and unsymmetrical shapes. Also included is a section with proportioning requirements for beams and girders.

There are selection tables in the AISC *Manual* for standard beams in the commonly available yield strengths. The section property tables for most cross sections provide information that can be used to conveniently identify noncompact and slender element sections. LRFD and ASD information is presented side-by-side.

Most of the formulas from this chapter are illustrated by the following examples. The design and selection techniques illustrated in the examples for both LRFD and ASD will result in similar designs.

F1. GENERAL PROVISIONS

Selection and evaluation of all members is based on deflection requirements and strength, which is determined as the design flexural strength, $\phi_b M_n$, or the allowable flexural strength, M_n/Ω_b,
where

M_n = the lowest nominal flexural strength based on the limit states of yielding, lateral torsional-buckling, and local buckling, where applicable

$\phi_b = 0.90$ (LRFD) $\Omega_b = 1.67$ (ASD)

This design approach is followed in all examples.

The term L_b is used throughout this chapter to describe the length between points which are either braced against lateral displacement of the compression flange or braced against twist of the cross section. Requirements for bracing systems and the required strength and stiffness at brace points are given in AISC *Specification* Appendix 6.

The use of C_b is illustrated in several of the following examples. AISC *Manual* Table 3-1 provides tabulated C_b values for some common situations.

F2. DOUBLY SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

AISC *Specification* Section F2 applies to the design of compact beams and channels. As indicated in the User Note in Section F2 of the AISC *Specification*, the vast majority of rolled I-shaped beams and channels fall into this category. The curve presented as a solid line in Figure F-1 is a generic plot of the nominal flexural strength, M_n, as a function of the unbraced length, L_b. The horizontal segment of the curve at the far left, between $L_b = 0$ ft and L_p, is the range where the strength is limited by flexural yielding. In this region, the nominal strength is taken as the full plastic moment strength of the section as given by AISC *Specification* Equation F2-1. In the range of the curve at the far right, starting at L_r, the strength is limited by elastic buckling. The strength in this region is given by AISC *Specification* Equation F2-3. Between these regions, within the linear region of the curve between $M_n = M_p$ at L_p on the left, and $M_n = 0.7M_y = 0.7F_yS_x$ at L_r on the right, the strength is limited by inelastic buckling. The strength in this region is provided in AISC *Specification* Equation F2-2.

The curve plotted as a heavy solid line represents the case where C_b = 1.0, while the heavy dashed line represents the case where C_b exceeds 1.0. The nominal strengths calculated in both AISC *Specification* Equations F2-2 and F2-3 are linearly proportional to C_b, but are limited to M_p as shown in the figure.

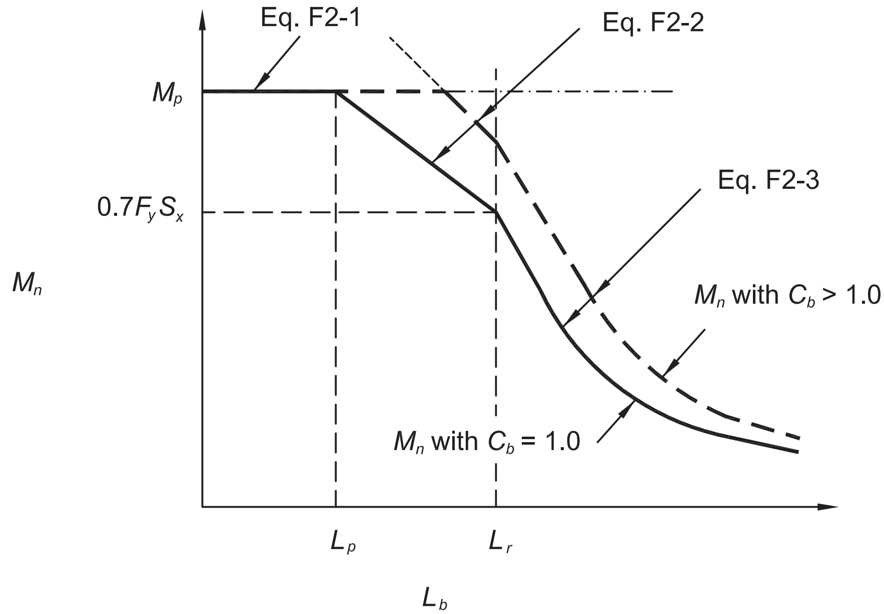


Fig. F-1. Beam strength versus unbraced length.

$$M_n = M_p = F_yZ_x \qquad \text{(}\textit{Spec}\text{. Eq. F2-1)}$$

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(}\textit{Spec}\text{. Eq. F2-2)}$$

$$M_n = F_{cr}S_x \le M_p \qquad \text{(}\textit{Spec}\text{. Eq. F2-3)}$$

where

$$F_{cr} = \frac{C_b\pi^2E}{\left(\dfrac{L_b}{r_{ts}}\right)^2}\sqrt{1 + 0.078\frac{Jc}{S_xh_o}\left(\frac{L_b}{r_{ts}}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. F2-4)}$$

The provisions of this section are illustrated in Example F.1(W-shape beam) and Example F.2 (channel).

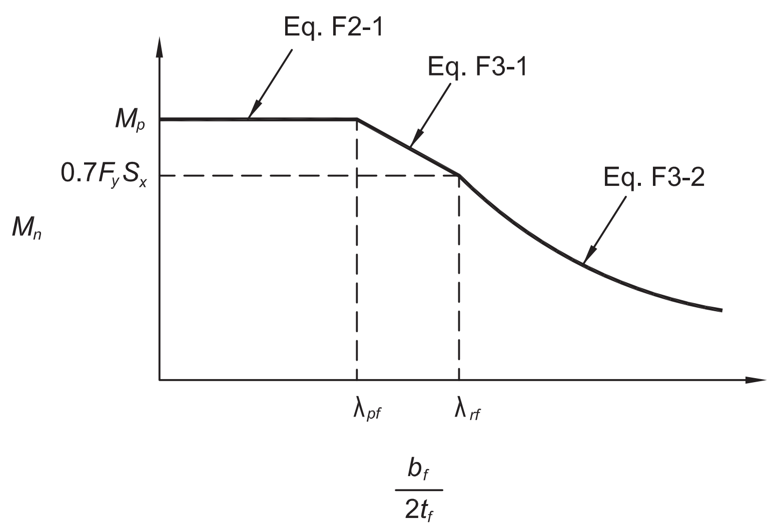
Plastic design provisions are given in AISC *Specification* Appendix 1. L_{pd}, the maximum unbraced length for prismatic member segments containing plastic hinges is less than L_p.

F3. DOUBLY SYMMETRIC I-SHAPED MEMBERS WITH COMPACT WEBS AND NONCOMPACT OR SLENDER FLANGES BENT ABOUT THEIR MAJOR AXIS

The strength of shapes designed according to this section is limited by local buckling of the compression flange. Only a few standard wide flange shapes have noncompact flanges. For these sections, the strength reduction for F_y = 50 ksi steel varies. The approximate percentages of M_p about the strong axis that can be developed by noncompact members when braced such that $L_b \le L_p$ are shown as follows:

W21×48 = 99%	W14×99 = 99%	W14×90 = 97%	W12×65 = 98%
W10×12 = 99%	W8×31 = 99%	W8×10 = 99%	W6×15 = 94%
W6×8.5 = 97%			

The strength curve for the flange local buckling limit state, shown in Figure F-2, is similar in nature to that of the lateral-torsional buckling curve. The horizontal axis parameter is $\lambda = b_f/2t_f$. The flat portion of the curve to the left of λ_{pf} is the plastic yielding strength, M_p. The curved portion to the right of λ_{rf} is the strength limited by elastic buckling of the flange. The linear transition between these two regions is the strength limited by inelastic flange buckling.



Fig, F-2. Flange local buckling strength.

$$M_n = M_p = F_yZ_x \quad \text{(}Spec\text{. Eq. F2-1)}$$

$$M_n = \left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \quad \text{(}Spec\text{. Eq. F3-1)}$$

$$M_n = \frac{0.9Ek_cS_x}{\lambda^2} \quad \text{(}Spec\text{. Eq. F3-2)}$$

where

$k_c = \dfrac{4}{\sqrt{h/t_w}}$ from AISC *Specification* Table B4.1b footnote [a], where $0.35 \le k_c \le 0.76$

The strength reductions due to flange local buckling of the few standard rolled shapes with noncompact flanges are incorporated into the design tables in Chapter 3 of the AISC *Manual*.

There are no standard I-shaped members with slender flanges. The noncompact flange provisions of this section are illustrated in Example F.3.

F4. OTHER I-SHAPED MEMBERS WITH COMPACT OR NONCOMPACT WEBS BENT ABOUT THEIR MAJOR AXIS

This section of the AISC *Specification* applies to doubly symmetric I-shaped members with noncompact webs and singly symmetric I-shaped members (those having different flanges) with compact or noncompact webs.

F5. DOUBLY SYMMETRIC AND SINGLY SYMMETRIC I-SHAPED MEMBERS WITH SLENDER WEBS BENT ABOUT THEIR MAJOR AXIS

This section applies to I-shaped members with slender webs, formerly designated as "plate girders".

F6. I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MINOR AXIS

I-shaped members and channels bent about their minor axis are not subject to lateral-torsional buckling. Rolled or built-up shapes with noncompact or slender flanges, as determined by AISC *Specification* Tables B4.1a and B4.1b, must be checked for strength based on the limit state of flange local buckling using Equations F6-2 or F6-3 as applicable.

The vast majority of W, M, C and MC shapes have compact flanges, and can therefore develop the full plastic moment, M_p, about the minor axis. The provisions of this section are illustrated in Example F.5.

F7. SQUARE AND RECTANGULAR HSS AND BOX-SHAPED MEMBERS

Square and rectangular HSS need only be checked for the limit states of yielding and local buckling. Although lateral-torsional buckling is theoretically possible for very long rectangular HSS bent about the strong axis, deflection will control the design as a practical matter.

The design and section property tables in the AISC *Manual* were calculated using a design wall thickness of 93% of the nominal wall thickness. Strength reductions due to local buckling have been accounted for in the AISC *Manual* design tables. The selection of rectangular or square HSS with compact flanges is illustrated in Example F.6. The provisions for rectangular or square HSS with noncompact flanges are illustrated in Example F.7. The provisions for HSS with slender flanges are illustrated in Example F.8. Available flexural strengths of rectangular and square HSS are listed in Tables 3-12 and 3-13, respectively.

F8. ROUND HSS

The definition of HSS encompasses both tube and pipe products. The lateral-torsional buckling limit state does not apply, but round HSS are subject to strength reductions from local buckling. Available strengths of round HSS and Pipes are listed in AISC *Manual* Tables 3-14 and 3-15, respectively. The tabulated properties and available flexural strengths of these shapes in the AISC *Manual* are calculated using a design wall thickness of 93% of the nominal wall thickness. The design of a Pipe is illustrated in Example F.9.

F9. TEES AND DOUBLE ANGLES LOADED IN THE PLANE OF SYMMETRY

The AISC *Specification* provides a check for flange local buckling, which applies only when the flange is in compression due to flexure. This limit state will seldom govern. A check for local buckling of the web was added in the 2010 edition of the *Specification*. Attention should be given to end conditions of tees to avoid inadvertent fixed end moments which induce compression in the web unless this limit state is checked. The design of a WT-shape in bending is illustrated in Example F.10.

F10. SINGLE ANGLES

Section F10 permits the flexural design of single angles using either the principal axes or geometric axes (*x-x* and *y-y* axes). When designing single angles without continuous bracing using the geometric axis design provisions, M_y must be multiplied by 0.80 for use in Equations F10-1, F10-2 and F10-3. The design of a single angle in bending is illustrated in Example F.11.

F11. RECTANGULAR BARS AND ROUNDS

There are no design tables in the AISC *Manual* for these shapes. The local buckling limit state does not apply to any bars. With the exception of rectangular bars bent about the strong axis, solid square, rectangular and round bars are not subject to lateral-torsional buckling and are governed by the yielding limit state only. Rectangular bars bent about the strong axis are subject to lateral-torsional buckling and are checked for this limit state with Equations F11-2 and F11-3, as applicable.

These provisions can be used to check plates and webs of tees in connections. A design example of a rectangular bar in bending is illustrated in Example F.12. A design example of a round bar in bending is illustrated in Example F.13.

F12. UNSYMMETRICAL SHAPES

Due to the wide range of possible unsymmetrical cross sections, specific lateral-torsional and local buckling provisions are not provided in this *Specification* section. A general template is provided, but appropriate literature investigation and engineering judgment are required for the application of this section. A Z-shaped section is designed in Example F.14.

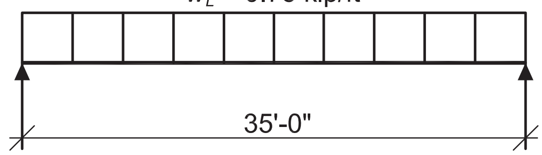
F13. PROPORTIONS OF BEAMS AND GIRDERS

This section of the *Specification* includes a limit state check for tensile rupture due to holes in the tension flange of beams, proportioning limits for I-shaped members, detail requirements for cover plates and connection requirements for built-up beams connected side-to-side. Also included are unbraced length requirements for beams designed using the moment redistribution provisions of AISC *Specification* Section B3.7.

EXAMPLE F.1-1A W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, CONTINUOUSLY BRACED

Given:

Select an ASTM A992 W-shape beam with a simple span of 35 ft. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to L/360. The nominal loads are a uniform dead load of 0.45 kip/ft and a uniform live load of 0.75 kip/ft. Assume the beam is continuously braced.



Beam Loading & Bracing Diagram
(*full lateral support*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
w_u = 1.2(0.45 kip/ft) +1.6 (0.75 kip/ft) = 1.74 kip/ft $M_u = \dfrac{1.74\text{ kip/ft}\left(35.0\text{ ft}\right)^2}{8}$ = 266 kip-ft	w_a = 0.45 kip/ft + 0.75 kip/ft = 1.20 kip/ft $M_a = \dfrac{1.20\text{ kip/ft}\left(35.0\text{ ft}\right)^2}{8}$ = 184 kip-ft

Required Moment of Inertia for Live-Load Deflection Criterion of L/360

$$\Delta_{max} = \frac{L}{360} = \frac{35.0\text{ ft}(12\text{ in./ft})}{360} = 1.17\text{ in.}$$

$$I_{x(reqd)} = \frac{5w_L l^4}{384E\Delta_{max}} \text{ from AISC } \textit{Manual} \text{ Table 3-23 Case 1}$$

$$= \frac{5(0.750\text{ kip/ft})(35.0\text{ ft})^4(12\text{ in./ft})^3}{384\,(29{,}000\text{ ksi})(1.17\text{ in.})} = 746\text{ in.}^4$$

Beam Selection

Select a W18×50 from AISC *Manual* Table 3-2.

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 379 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 252 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

From AISC *Manual* Table 3-2, $I_x = 800 \text{ in.}^4 > 746 \text{ in.}^4$ **o.k.**

EXAMPLE F.1-1B W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, CONTINUOUSLY BRACED

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A by applying the requirements of the AISC *Specification* directly.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

W18×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
Z_x = 101 in.^{3}

The required flexural strength from Example F.1-1A is:

LRFD	ASD
M_u = 266 kip-ft	M_a = 184 kip-ft

Nominal Flexural Strength, M_n

Per the User Note in AISC *Specification* Section F2, the section is compact. Because the beam is continuously braced and compact, only the yielding limit state applies.

$$\begin{aligned} M_n &= M_p \\ &= F_y Z_x \qquad \text{(Spec. Eq. F2-1)} \\ &= 50 \text{ ksi}(101 \text{ in.}^3) \\ &= 5{,}050 \text{ kip-in. or } 421 \text{ kip-ft} \end{aligned}$$

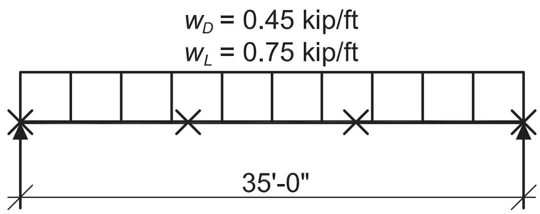
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(421 \text{ kip-ft})$ $= 379 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{421 \text{ kip-ft}}{1.67}$ $= 252 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-2A W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT THIRD POINTS

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and third points. Use the AISC *Manual* tables.



Beam Loading & Bracing Diagram
(*bracing at ends and third points*)

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
M_u = 266 kip-ft	M_a = 184 kip-ft

Unbraced Length

$$L_b = \frac{35.0\text{ ft}}{3} = 11.7\text{ ft}$$

By inspection, the middle segment will govern. From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and third points, C_b = 1.01 in the middle segment. Conservatively neglect this small adjustment in this case.

Available Strength

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft. Obtain the available strength from the appropriate vertical scale to the left.

From AISC *Manual* Table 3-10, the available flexural strength is:

LRFD	ASD
$\phi_b M_n \approx 302\text{ kip-ft} > 266\text{ kip-ft}$ **o.k.**	$\frac{M_n}{\Omega_b} \approx 201\text{kip-ft} > 184\text{ kip-ft}$ **o.k.**

EXAMPLE F.1-2B W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT THIRD POINTS

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and third points. Apply the requirements of the AISC *Specification* directly.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
S_x = 88.9 in.^{3}

The required flexural strength from Example F.1-1A is:

LRFD	ASD
M_u = 266 kip-ft	M_a = 184 kip-ft

Nominal Flexural Strength, M_n

Calculate C_b.

For the lateral-torsional buckling limit state, the nonuniform moment modification factor can be calculated using AISC *Specification* Equation F1-1.

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad \text{(Spec. Eq. F1-1)}$$

For the center segment of the beam, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 1.00, M_A = 0.972, M_B = 1.00, and M_C = 0.972.

$$C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.972) + 4(1.00) + 3(0.972)} = 1.01$$

For the end-span beam segments, the required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max} = 0.889, M_A = 0.306, M_B = 0.556, and M_C = 0.750.

$$C_b = \frac{12.5(0.889)}{2.5(0.889) + 3(0.306) + 4(0.556) + 3(0.750)} = 1.46$$

Thus, the center span, with the higher required strength and lower C_b, will govern.

From AISC *Manual* Table 3-2:

L_p = 5.83 ft
L_r = 16.9 ft

For a compact beam with an unbraced length of $L_p < L_b \le L_r$, the lesser of either the flexural yielding limit state or the inelastic lateral-torsional buckling limit state controls the nominal strength.

M_p = 5,050 kip-in. (from Example F.1-1B)

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p \qquad \text{(}Spec\text{. Eq. F2-2)}$$

$$= 1.01\left\{5{,}050 \text{ kip-in.} - \left[5{,}050 \text{ kip-in.} - 0.7(50 \text{ ksi})(88.9 \text{ in.}^3)\right]\left(\frac{11.7 \text{ ft} - 5.83 \text{ ft}}{16.9 \text{ ft} - 5.83 \text{ ft}}\right)\right\} \le 5{,}050 \text{kip-in.}$$

$$= 4{,}060 \text{ kip-in. or } 339 \text{ kip-ft}$$

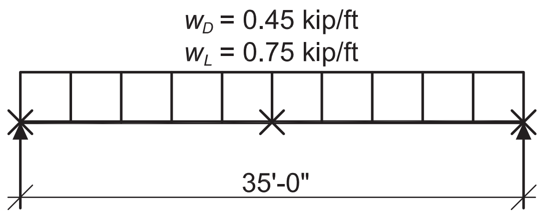
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(339 \text{ kip-ft})$ $= 305 \text{ kip-ft} > 266 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{339 \text{ kip-ft}}{1.67}$ $= 203 \text{ kip-ft} > 184 \text{ kip-ft}$ **o.k.**

EXAMPLE F.1-3A W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT MIDSPAN

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and center point. Use the AISC *Manual* tables.



Beam Loading & Bracing Diagram
(*bracing at ends & midpoint*)

Solution:

The required flexural strength at midspan from Example F.1-1A is:

LRFD	ASD
M_u = 266 kip-ft	M_a = 184 kip-ft

Unbraced Length

$$L_b = \frac{35.0\text{ ft}}{2}$$
$$= 17.5\text{ ft}$$

From AISC *Manual* Table 3-1, for a uniformly loaded beam braced at the ends and at the center point, C_b = 1.30. There are several ways to make adjustments to AISC *Manual* Table 3-10 to account for C_b greater than 1.0.

Procedure A

Available moments from the sloped and curved portions of the plots from AISC *Manual* Table 3-10 may be multiplied by C_b, but may not exceed the value of the horizontal portion (ϕM_p for LRFD, M_p/Ω for ASD).

Obtain the available strength of a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

Enter AISC *Manual* Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 17.5 ft. Obtain the available strength from the appropriate vertical scale to the left.

LRFD	ASD
$\phi_b M_n \approx 222$ kip-ft From *Manual* Table 3-2, $\phi_b M_p = 379$ kip-ft (upper limit on $C_b M_n$)	$\dfrac{M_n}{\Omega_b} \approx 148$ kip-ft From *Manual* Table 3-2, $\dfrac{M_p}{\Omega_b} = 252$ kip-ft (upper limit on $C_b M_n$)

LRFD	ASD
Adjust for C_b. 1.30(222 kip-ft) = 289 kip-ft Check Limit. 289 kip-ft ≤ $\phi_b M_p$ = 379 kip-ft **o.k.** Check available versus required strength. 289 kip-ft > 266 kip-ft **o.k.**	Adjust for C_b. 1.30(147 kip-ft) = 191 kip-ft Check Limit. 191 kip-ft ≤ $\dfrac{M_p}{\Omega_b}$ = 252 kip-ft **o.k.** Check available versus required strength. 191 kip-ft > 184 kip-ft **o.k.**

Procedure B

For preliminary selection, the required strength can be divided by C_b and directly compared to the strengths in AISC *Manual* Table 3-10. Members selected in this way must be checked to ensure that the required strength does not exceed the available plastic moment strength of the section.

Calculate the adjusted required strength.

LRFD	ASD
$M_u{}'$ = 266 kip-ft/1.30 = 205 kip-ft	$M_a{}'$ = 184 kip-ft/1.30 = 142 kip-ft

Obtain the available strength for a W18×50 with an unbraced length of 17.5 ft from AISC *Manual* Table 3-10.

LRFD	ASD
$\phi_b M_n$ ≈ 222 kip-ft > 205 kip-ft **o.k.** $\phi_b M_p$ = 379 kip-ft > 266 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b}$ ≈ 148 kip-ft > 142 kip-ft **o.k.** $\dfrac{M_p}{\Omega_b}$ = 252 kip-ft > 184 kip-ft **o.k.**

EXAMPLE F.1-3B W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT MIDSPAN

Given:

Verify the available flexural strength of the W18×50, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and center point. Apply the requirements of the AISC *Specification* directly.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W18×50
r_{ts} = 1.98 in.
S_x = 88.9 in.^{3}
J = 1.24 in.^{4}
h_o = 17.4 in.

The required flexural strength from Example F.1-1A is:

LRFD	ASD
M_u = 266 kip-ft	M_a = 184 kip-ft

Nominal Flexural Strength, M_n

Calculate C_b.

$$C_b = \frac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C} \quad \text{(Spec. Eq. F1-1)}$$

The required moments for AISC *Specification* Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: M_{max}= 1.00, M_A = 0.438, M_B = 0.751, and M_C = 0.938.

$$C_b = \frac{12.5(1.00)}{2.5(1.00)+3(0.438)+4(0.751)+3(0.938)}$$
$$= 1.30$$

From AISC *Manual* Table 3-2:

L_p = 5.83 ft
L_r = 16.9 ft

For a compact beam with an unbraced length $L_b > L_r$, the limit state of elastic lateral-torsional buckling applies.

Calculate F_{cr} with L_b = 17.5 ft.

$$F_{cr} = \frac{C_b \pi^2 E}{\left(\frac{L_b}{r_{ts}}\right)^2} \sqrt{1 + 0.078 \frac{Jc}{S_x h_o} \left(\frac{L_b}{r_{ts}}\right)^2} \quad \text{where } c = 1.0 \text{ for doubly symmetric I-shapes} \qquad (Spec.\ \text{Eq. F2-4})$$

$$= \frac{1.30\pi^2 (29{,}000 \text{ ksi})}{\left(\frac{17.5 \text{ ft}(12 \text{ in./ft})}{1.98 \text{ in.}}\right)^2} \sqrt{1 + 0.078 \frac{1.24 \text{ in.}^4 (1.0)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})} \left(\frac{17.5 \text{ ft}(12 \text{ in./ft})}{1.98 \text{ in.}}\right)^2}$$

$$= 43.2 \text{ ksi}$$

$M_n = F_{cr} S_x \le M_p$ (*Spec*. Eq. F2-3)
= 43.2 ksi(88.9 in.^{3})
= 3,840 kip-in. ≤ 5,050 kip-in. (from Example F.1-1B)

M_n = 3,840 kip-in or 320 kip-ft

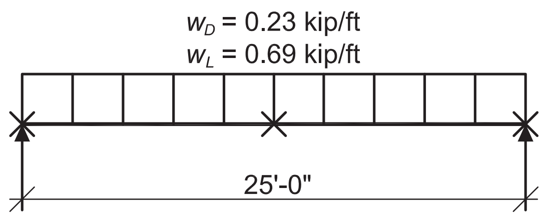
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(320 \text{ kip-ft})$ $= 288$ kip-ft 288 kip-ft > 266 kip-ft **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{320 \text{ kip-ft}}{1.67}$ $= 192$ kip-ft 192 kip-ft > 184 kip-ft **o.k.**

EXAMPLE F.2-1A COMPACT CHANNEL FLEXURAL MEMBER, CONTINUOUSLY BRACED

Given:

Select an ASTM A36 channel to serve as a roof edge beam with a simple span of 25 ft. Limit the live load deflection to *L*/360. The nominal loads are a uniform dead load of 0.23 kip/ft and a uniform live load of 0.69 kip/ft. The beam is continuously braced.



Beam Loading & Bracing Diagram
(*full lateral support*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.23\text{ kip/ft}) + 1.6(0.69\text{ kip/ft})$ $= 1.38\text{ kip/ft}$ $M_u = \dfrac{1.38\text{ kip/ft}\left(25.0\text{ ft}\right)^2}{8}$ $= 108\text{ kip-ft}$	$w_a = 0.23\text{ kip/ft} + 0.69\text{ kip/ft}$ $= 0.920\text{ kip/ft}$ $M_a = \dfrac{0.920\text{ kip/ft}\left(25.0\text{ ft}\right)^2}{8}$ $= 71.9\text{ kip-ft}$

Beam Selection

Per the User Note in AISC *Specification* Section F2, all ASTM A36 channels are compact. Because the beam is compact and continuously braced, the yielding limit state governs and $M_n = M_p$. Try C15×33.9 from AISC *Manual* Table 3-8.

LRFD	ASD
$\phi_b M_n = \phi_b M_p$ $= 137\text{ kip-ft} > 108\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_p}{\Omega_b}$ $= 91.3\text{ kip-ft} > 71.9\text{ kip-ft}$ **o.k.**

Live Load Deflection

Assume the live load deflection at the center of the beam is limited to $L/360$.

$$\Delta_{max} = \frac{L}{360}$$
$$= \frac{25.0 \text{ ft}\left(12 \text{ in./ft}\right)}{360}$$
$$= 0.833 \text{ in.}$$

For C15×33.9, I_x = 315 in.^{4} from AISC *Manual* Table 1-5.

The maximum calculated deflection is:

$$\Delta_{max} = \frac{5 w_L l^4}{384 EI} \text{ from AISC } \textit{Manual} \text{ Table 3-23 Case 1}$$
$$= \frac{5\left(0.69 \text{ kip/ft}\right)\left(25.0 \text{ ft}\right)^4 \left(12 \text{ in./ft}\right)^3}{384\left(29{,}000 \text{ ksi}\right)\left(315 \text{ in.}^4\right)}$$
$$= 0.664 \text{ in.} < 0.833 \text{ in.} \qquad \textbf{o.k.}$$

EXAMPLE F.2-1B COMPACT CHANNEL FLEXURAL MEMBER, CONTINUOUSLY BRACED

Given:

Example F.2-1A can be easily solved by utilizing the tables of the AISC *Manual*. Verify the results by applying the requirements of the AISC *Specification* directly.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-5, the geometric properties are as follows:

C15×33.9
Z_x = 50.8 in.^{3}

The required flexural strength from Example F.2-1A is:

LRFD	ASD
M_u = 108 kip-ft	M_a = 71.9 kip-ft

Nominal Flexural Strength, M_n

Per the User Note in AISC *Specification* Section F2, all ASTM A36 C- and MC-shapes are compact.

A channel that is continuously braced and compact is governed by the yielding limit state.

$$M_n = M_p = F_y Z_x$$ (*Spec*. Eq. F2-1)
$$= 36 \text{ ksi}(50.8 \text{ in.}^3)$$
$$= 1{,}830 \text{ kip-in. or } 152 \text{ kip-ft}$$

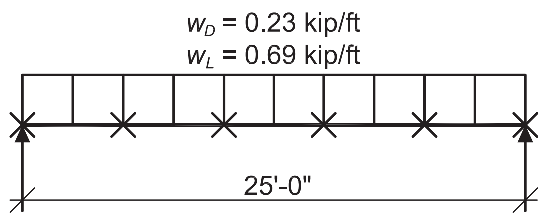
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
ϕ_b = 0.90 $\phi_b M_n$ = 0.90(152 kip-ft) = 137 kip-ft > 108 kip-ft **o.k.**	Ω_b = 1.67 $\dfrac{M_n}{\Omega_b} = \dfrac{152 \text{ kip-ft}}{1.67}$ = 91.0 kip-ft > 71.9 kip-ft **o.k.**

EXAMPLE F.2-2A COMPACT CHANNEL FLEXURAL MEMBER WITH BRACING AT ENDS AND FIFTH POINTS

Given:

Check the C15×33.9 beam selected in Example F.2-1A, assuming it is braced at the ends and the fifth points rather than continuously braced.



Beam Loading & Bracing Diagram
(*bracing at ends & 1/5 points*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The center segment will govern by inspection.

The required flexural strength at midspan from Example F.2-1A is:

LRFD	ASD
M_u = 108 kip-ft	M_a = 71.9 kip-ft

From AISC *Manual* Table 3-1, with an almost uniform moment across the center segment, C_b = 1.00; therefore, no adjustment is required.

Unbraced Length

$$L_b = \frac{25.0\text{ ft}}{5}$$
$$= 5.00\text{ ft}$$

Obtain the strength of the C15×33.9 with an unbraced length of 5.00 ft from AISC *Manual* Table 3-11.

Enter AISC *Manual* Table 3-11 and find the intersection of the curve for the C15×33.9 with an unbraced length of 5.00 ft. Obtain the available strength from the appropriate vertical scale to the left.

LRFD	ASD
$\phi_b M_n \approx 130$ kip-ft > 108 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b} \approx 87.0$ kip-ft > 71.9 kip-ft **o.k.**

EXAMPLE F.2-2B COMPACT CHANNEL FLEXURAL MEMBER WITH BRACING AT ENDS AND FIFTH POINTS

Given:

Verify the results from Example F.2-2A by calculation using the provisions of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-5, the geometric properties are as follows:

C15×33.9
S_x = 42.0 in.^{3}

The required flexural strength from Example F.2-1A is:

LRFD	ASD
M_u = 108 kip-ft	M_a = 71.9 kip-ft

Nominal Flexural Strength, M_n

Per the User Note in AISC *Specification* Section F2, all ASTM A36 C- and MC-shapes are compact.

From AISC *Manual* Table 3-1, for the center segment of a uniformly loaded beam braced at the ends and the fifth points:

C_b = 1.00

From AISC *Manual* Table 3-8, for a C15×33.9:

L_p = 3.75 ft
L_r = 14.5 ft

For a compact channel with $L_p < L_b \le L_r$, the lesser of the flexural yielding limit state or the inelastic lateral-torsional buckling limit-state controls the available flexural strength.

The nominal flexural strength based on the flexural yielding limit state, from Example F.2-1B, is:

$M_n = M_p$ = 1,830 kip-in.

The nominal flexural strength based on the lateral-torsional buckling limit state is:

$$M_n = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \le M_p$$ (*Spec*. Eq. F2-2)

$$= 1.0\left\{1,830 \text{ kip-in.} - \left[1,830 \text{ kip-in.} - 0.7\left(36 \text{ ksi}\right)\left(42.0 \text{ in.}^3\right)\right]\left(\frac{5.00 \text{ ft} - 3.75 \text{ ft}}{14.5 \text{ ft} - 3.75 \text{ ft}}\right)\right\} \le 1,830 \text{ kip-in.}$$

$= 1,740$ kip-in. $\le 1,830$ kip-in. **o.k.**

$M_n = 1,740$ kip-in. or 145 kip-ft

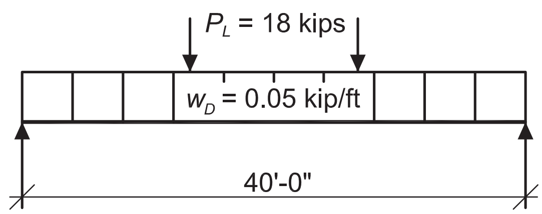
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(145 \text{ kip-ft})$ $= 131$ kip-ft 131 kip-ft > 108 kip-ft **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{145 \text{ kip-ft}}{1.67}$ $= 86.8$ kip-ft 86.8 kip-ft > 71.9 kip-ft **o.k.**

EXAMPLE F.3A W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN STRONG-AXIS BENDING

Given:

Select an ASTM A992 W-shape beam with a simple span of 40 ft. The nominal loads are a uniform dead load of 0.05 kip/ft and two equal 18 kip concentrated live loads acting at the third points of the beam. The beam is continuously braced. Also calculate the deflection.



Beam Loading & Bracing Diagram
(*continuous bracing*)

Note: A beam with noncompact flanges will be selected to demonstrate that the tabulated values of the AISC *Manual* account for flange compactness.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength at midspan is:

LRFD	ASD
$w_u = 1.2(0.05\text{ kip/ft})$ $= 0.0600\text{ kip/ft}$ $P_u = 1.6(18\text{ kips})$ $= 28.8\text{ kips}$ $M_u = \dfrac{(0.0600\text{ kip/ft})(40.0\text{ ft})^2}{8} + (28.8\text{ kips})\dfrac{40.0\text{ ft}}{3}$ $= 396\text{ kip-ft}$	$w_a = 0.05\text{ kip/ft}$ $P_a = 18\text{ kips}$ $M_a = \dfrac{(0.0500\text{ kip/ft})(40.0\text{ ft})^2}{8} + (18.0\text{ kips})\dfrac{40.0\text{ ft}}{3}$ $= 250\text{ kip-ft}$

Beam Selection

For a continuously braced W-shape, the available flexural strength equals the available plastic flexural strength.

Select the lightest section providing the required strength from the bold entries in AISC *Manual* Table 3-2.

Try a W21×48.

This beam has a noncompact compression flange at F_y = 50 ksi as indicated by footnote “f” in AISC *Manual* Table 3-2. This shape is also footnoted in AISC *Manual* Table 1-1.

From AISC *Manual* Table 3-2, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 398$ kip-ft > 396 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 265$ kip-ft > 250 kip-ft **o.k.**

Note: The value M_{px} in AISC *Manual* Table 3-2 includes the strength reductions due to the noncompact nature of the shape.

Deflection

$I_x = 959$ in.^{4} from AISC *Manual* Table 1-1

The maximum deflection occurs at the center of the beam.

$$\Delta_{max} = \frac{5w_D l^4}{384EI} + \frac{P_L l^3}{28EI} \text{ from AISC } \textit{Manual} \text{ Table 3-23 cases 1 and 9}$$

$$= \frac{5\left(0.0500\text{ kip/ft}\right)\left(40.0\text{ ft}\right)^4\left(12\text{ in./ft}\right)^3}{384\left(29{,}000\text{ ksi}\right)\left(959\text{ in.}^4\right)} + \frac{18.0\text{ kips}\left(40.0\text{ ft}\right)^3\left(12\text{ in./ft}\right)^3}{28\left(29{,}000\text{ ksi}\right)\left(959\text{ in}^4\right)}$$

$$= 2.66\text{ in.}$$

This deflection can be compared with the appropriate deflection limit for the application. Deflection will often be more critical than strength in beam design.

Return to Table of Contents

EXAMPLE F.3B W-SHAPE FLEXURAL MEMBER WITH NONCOMPACT FLANGES IN STRONG-AXIS BENDING

Given:

Verify the results from Example F.3A by calculation using the provisions of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×48
S_x = 93.0 in.^{3}
Z_x = 107 in.^{3}
$$\frac{b_f}{2t_f} = 9.47$$

The required flexural strength from Example F.3A is:

LRFD	ASD
M_u = 396 kip-ft	M_a = 250 kip-ft

Flange Slenderness

$$\lambda = \frac{b_f}{2t_f}$$
$$= 9.47$$

The limiting width-to-thickness ratios for the compression flange are:

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}} \quad \text{from AISC } \textit{Specification} \text{ Table B4.1b Case 10}$$
$$= 0.38\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 9.15$$

$$\lambda_{rf} = 1.0\sqrt{\frac{E}{F_y}} \quad \text{from AISC } \textit{Specification} \text{ Table B4.1b Case 10}$$
$$= 1.0\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 24.1$$

$\lambda_{rf} > \lambda > \lambda_{pf}$, therefore, the compression flange is noncompact. This could also be determined from the footnote "f" in AISC *Manual* Table 1-1.

Nominal Flexural Strength, M_n

Because the beam is continuously braced, and therefore not subject to lateral-torsional buckling, the available strength is governed by AISC *Specification* Section F3.2, Compression Flange Local Buckling.

$$M_p = F_y Z_x$$
$$= 50 \text{ ksi}(107 \text{ in.}^3)$$
$$= 5{,}350 \text{ kip-in. or } 446 \text{ kip-ft}$$

$$M_n = \left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \quad \text{(Spec. Eq. F3-1)}$$
$$= \left\{5{,}350 \text{ kip-in.} - \left[5{,}350 \text{ kip-in} - 0.7\left(50 \text{ ksi}\right)\left(93.0 \text{ in.}^3\right)\right]\left(\frac{9.47-9.15}{24.1-9.15}\right)\right\}$$
$$= 5{,}310 \text{ kip-in. or } 442 \text{ kip-ft}$$

From AISC *Specification* Section F1, the available flexural strength is:

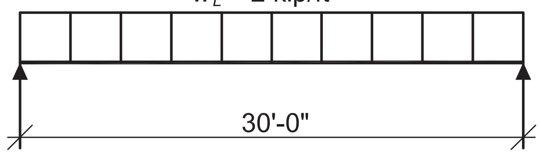
LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90\left(442 \text{ kip-ft}\right)$ $= 398 \text{ kip-ft} > 396 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{442 \text{ kip-ft}}{1.67}$ $= 265 \text{ kip-ft} > 250 \text{ kip-ft}$ **o.k.**

Note that these available strengths are identical to the tabulated values in AISC *Manual* Table 3-2, which account for the noncompact flange.

EXAMPLE F.4 W-SHAPE FLEXURAL MEMBER, SELECTION BY MOMENT OF INERTIA FOR STRONG-AXIS BENDING

Given:

Select an ASTM A992 W-shape flexural member by the moment of inertia, to limit the live load deflection to 1 in. The span length is 30 ft. The loads are a uniform dead load of 0.80 kip/ft and a uniform live load of 2 kip/ft. The beam is continuously braced.



Beam Loading & Bracing Diagram
(*full lateral support*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.800\text{ kip/ft}) + 1.6(2\text{ kip/ft})$ $= 4.16\text{ kip/ft}$ $M_u = \dfrac{4.16\text{ kip/ft}(30.0\text{ ft})^2}{8}$ $= 468\text{ kip-ft}$	$w_a = 0.80\text{ kip/ft} + 2\text{ kip/ft}$ $= 2.80\text{ kip/ft}$ $M_a = \dfrac{2.80\text{ kip/ft}(30.0\text{ ft})^2}{8}$ $= 315\text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection, Δ_{max}, occurs at midspan and is calculated as:

$$\Delta_{max} = \frac{5w_L l^4}{384EI} \text{ from AISC \textit{Manual} Table 3-23 case 1}$$

Rearranging and substituting Δ_{max} = 1.00 in.,

$$I_{min} = \frac{5(2\text{ kips/ft})(30.0\text{ ft})^4(12\text{ in./ft})^3}{384(29{,}000\text{ ksi})(1.00\text{ in.})}$$
$$= 1{,}260\text{ in.}^4$$

Beam Selection

Select the lightest section with the required moment of inertia from the bold entries in AISC *Manual* Table 3-3.

Try a W24×55.

$I_x = 1{,}350 \text{ in.}^4 > 1{,}260 \text{ in.}^4$ **o.k.**

Because the W24×55 is continuously braced and compact, its strength is governed by the yielding limit state and AISC *Specification* Section F2.1

From AISC *Manual* Table 3-2, the available flexural strength is:

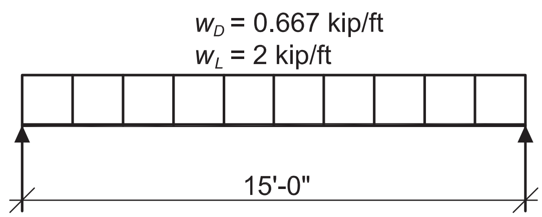
LRFD	ASD
$\phi_b M_n = \phi_b M_{px}$ $= 503$ kip-ft 503 kip-ft > 468 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b} = \dfrac{M_{px}}{\Omega_b}$ $= 334$ kip-ft 334 kip-ft > 315 kip-ft **o.k.**

EXAMPLE F.5 I-SHAPED FLEXURAL MEMBER IN MINOR-AXIS BENDING

Given:

Select an ASTM A992 W-shape beam loaded in its minor axis with a simple span of 15 ft. The loads are a total uniform dead load of 0.667 kip/ft and a uniform live load of 2 kip/ft. Limit the live load deflection to *L*/240. The beam is braced at the ends only.

Note: Although not a common design case, this example is being used to illustrate AISC *Specification* Section F6 (I-shaped members and channels bent about their minor axis).



Beam Loading & Bracing Diagram
(*braced at ends only*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.667\text{ kip/ft}) + 1.6(2\text{ kip/ft})$ $= 4.00\text{ kip/ft}$ $M_u = \dfrac{4.00\text{kip/ft}\left(15.0\text{ ft}\right)^2}{8}$ $= 113\text{ kip-ft}$	$w_a = 0.667\text{ kip/ft} + 2\text{ kip/ft}$ $= 2.67\text{ kip/ft}$ $M_a = \dfrac{2.67\text{kip/ft}\left(15.0\text{ ft}\right)^2}{8}$ $= 75.1\text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240}$$
$$= \frac{15.0\text{ ft(12 in./ft)}}{240}$$
$$= 0.750\text{ in.}$$

$$I_{req} = \frac{5w_L l^4}{384E\Delta_{max}} \quad \text{from AISC } \textit{Manual} \text{ Table 3-23 case 1}$$
$$= \frac{5\left(2.00\text{ kip/ft}\right)\left(15.0\text{ ft}\right)^4\left(12\text{ in./ft}\right)^3}{384\left(29{,}000\text{ ksi}\right)\left(0.750\text{ in.}\right)}$$

$= 105 \text{ in.}^4$

Beam Selection

Select the lightest section from the bold entries in AISC *Manual* Table 3-5, due to the likelihood that deflection will govern this design.

Try a W12×58.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W12×58
$S_y = 21.4 \text{ in.}^3$
$Z_y = 32.5 \text{ in.}^3$
$I_y = 107 \text{ in.}^4 > 105 \text{ in.}^4$ **o.k.**

AISC *Specification* Section F6 applies. Because the W12×58 has compact flanges per the User Note in this Section, the yielding limit state governs the design.

$$M_n = M_p = F_y Z_y \leq 1.6 F_y S_y \quad \text{(Spec. Eq. F6-1)}$$
$$= 50 \text{ ksi}(32.5 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(21.4 \text{ in.}^3)$$
$$= 1{,}630 \text{ kip-in.} \leq 1{,}710 \text{ kip-in.} \quad \textbf{o.k.}$$

$M_n = 1{,}630$ kip-in. or 136 kip-ft

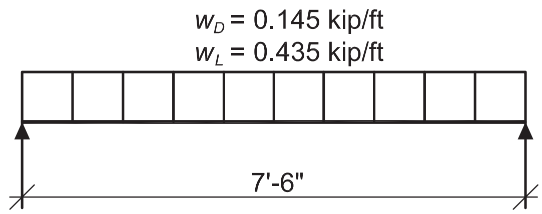
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(136 \text{ kip-ft})$ $= 122 \text{ kip-ft}$ 122 kip-ft > 113 kip-ft **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{136 \text{ kip-ft}}{1.67}$ $= 81.4 \text{ kip-ft}$ 81.4 kip-ft > 75.1 kip-ft **o.k.**

EXAMPLE F.6 HSS FLEXURAL MEMBER WITH COMPACT FLANGES

Given:

Select a square ASTM A500 Grade B HSS beam to span 7.5 ft. The loads are a uniform dead load of 0.145 kip/ft and a uniform live load of 0.435 kip/ft. Limit the live load deflection to *L*/240. The beam is continuously braced.



Beam Loading & Bracing Diagram
(*full lateral support*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.145\text{ kip/ft}) + 1.6(0.435\text{ kip/ft})$ $= 0.870\text{ kip/ft}$ $M_u = \dfrac{(0.870\text{ kip/ft})(7.50\text{ ft})^2}{8}$ $= 6.12\text{ kip-ft}$	$w_a = 0.145\text{ kip/ft} + 0.435\text{ kip/ft}$ $= 0.580\text{ kip/ft}$ $M_a = \dfrac{(0.580\text{ kip/ft})(7.50\text{ ft})^2}{8}$ $= 4.08\text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240}$$
$$= \frac{7.50\text{ ft}(12\text{ in./ft})}{240}$$
$$= 0.375\text{ in.}$$

Determine the minimum required *I* as follows.

$$I_{req} = \frac{5w_L l^4}{384E\Delta_{max}} \quad \text{from AISC } \textit{Manual} \text{ Table 3-23 Case 1}$$
$$= \frac{5(0.435\text{ kip/ft})(7.50\text{ ft})^4(12\text{ in./ft})^3}{384(29{,}000\text{ ksi})(0.375\text{ in.})}$$
$$= 2.85\text{ in.}^4$$

Beam Selection

Select an HSS with a minimum I_x of 2.85 in.^{4}, using AISC *Manual* Table 1-12, and having adequate available strength, using AISC *Manual* Table 3-13.

Try an HSS3½×3½×⅛.

From AISC *Manual* Table 1-12, I_x = 2.90 in.^{4} > 2.85 in.^{4} **o.k.**

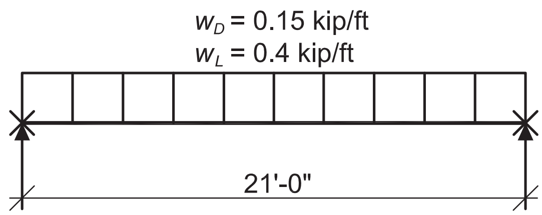
From AISC *Manual* Table 3-13, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = 6.67$ kip-ft > 6.12 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b} = 4.44$ kip-ft > 4.08 kip-ft **o.k.**

EXAMPLE F.7A HSS FLEXURAL MEMBER WITH NONCOMPACT FLANGES

Given:

Select a rectangular ASTM A500 Grade B HSS beam with a span of 21 ft. The loads include a uniform dead load of 0.15 kip/ft and a uniform live load of 0.4 kip/ft. Limit the live load deflection to *L*/240. The beam is braced at the end points only. A noncompact member was selected here to illustrate the relative ease of selecting noncompact shapes from the AISC *Manual*, as compared to designing a similar shape by applying the AISC *Specification* requirements directly, as shown in Example F.7B.



Beam Loading & Bracing Diagram
(*braced at end points only*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.15 \text{ kip/ft}) + 1.6(0.4 \text{ kip/ft})$ $= 0.820 \text{ kip/ft}$ $M_u = \dfrac{0.820 \text{ kip/ft}(21.0 \text{ ft})^2}{8}$ $= 45.2 \text{ kip-ft}$	$w_a = 0.15 \text{ kip/ft} + 0.4 \text{ kip/ft}$ $= 0.550 \text{ kip/ft}$ $M_a = \dfrac{0.550 \text{ kip/ft}(21.0 \text{ ft})^2}{8}$ $= 30.3 \text{ kip-ft}$

Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240}$$
$$= \frac{21.0 \text{ ft}(12 \text{ in./ft})}{240}$$
$$= 1.05 \text{ in.}$$

The maximum calculated deflection is:

$$\Delta_{max} = \frac{5w_L l^4}{384EI} \quad \text{from AISC } \textit{Manual} \text{ Table 3-23 case 1}$$

Rearranging and substituting Δ_{max} = 1.05 in.,

$$I_{min} = \frac{5(0.4\text{ kip/ft})(21.0\text{ ft})^4(12\text{ in./ft})^3}{384(29{,}000\text{ ksi})(1.05\text{ in.})}$$
$$= 57.5\text{ in.}^4$$

Beam Selection

Select a rectangular HSS with a minimum I_x of 57.5 in.^{4}, using AISC *Manual* Table 1-11, and having adequate available strength, using AISC *Manual* Table 3-12.

Try an HSS10×6×3⁄16 oriented in the strong direction. This rectangular HSS section was purposely selected for illustration purposes because it has a noncompact flange. See AISC *Manual* Table 1-12A for compactness criteria.

$I_x = 74.6\text{ in.}^4 > 57.5\text{ in.}^4$ **o.k.**

From AISC *Manual* Table 3-12, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = 57.0\text{ kip-ft} > 45.2\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 37.9\text{ kip-ft} > 30.3\text{ kip-ft}$ **o.k.**

EXAMPLE F.7B HSS FLEXURAL MEMBER WITH NONCOMPACT FLANGES

Given:

Notice that in Example F.7A the required information was easily determined by consulting the tables of the AISC *Manual*. The purpose of the following calculation is to demonstrate the use of the AISC *Specification* equations to calculate the flexural strength of an HSS member with a noncompact compression flange.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS10×6×3⁄16
$Z_x = 18.0 \text{ in.}^3$
$S_x = 14.9 \text{ in.}^3$

Flange Compactness

$$\lambda = \frac{b}{t}$$
$$= 31.5 \text{ from AISC } \textit{Manual} \text{ Table 1-11}$$

Determine the limiting ratio for a compact HSS flange in flexure from AISC *Specification* Table B4.1b Case 17.

$$\lambda_p = 1.12\sqrt{\frac{E}{F_y}}$$
$$= 1.12\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}$$
$$= 28.1$$

Flange Slenderness

Determine the limiting ratio for a slender HSS flange in flexure from AISC *Specification* Table B4.1b Case 17.

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$
$$= 1.40\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}$$
$$= 35.2$$

$\lambda_p < \lambda < \lambda_r$; therefore, the flange is noncompact. For this situation, AISC *Specification* Equation F7-2 applies.

Web Slenderness

$$\lambda = \frac{h}{t}$$
$$= 54.5 \text{ from AISC } \textit{Manual} \text{ Table 1-11}$$

Determine the limiting ratio for a compact HSS web in flexure from AISC *Specification* Table B4.1b Case 19.

$$\lambda_p = 2.42\sqrt{\frac{E}{F_y}}$$
$$= 2.42\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}$$
$$= 60.8$$

$\lambda < \lambda_p$; therefore, the web is compact

For HSS with noncompact flanges and compact webs, AISC *Specification* Section F7.2(b) applies.

$$M_p = F_y Z_x$$
$$= 46 \text{ ksi}(18.0 \text{ in.}^3)$$
$$= 828 \text{ kip-in.}$$

$$M_n = M_p - \left(M_p - F_y S\right)\left(3.57\frac{b}{t_f}\sqrt{\frac{F_y}{E}} - 4.0\right) \leq M_p \qquad \text{(\textit{Spec}. Eq. F7-2)}$$
$$= 828 \text{ kip-in.} - \left[828 \text{ kip-in.} - 46 \text{ ksi}\left(14.9 \text{ in.}^3\right)\right]\left(3.57\left(31.5\right)\sqrt{\frac{46 \text{ ksi}}{29{,}000 \text{ ksi}}} - 4.0\right)$$
$$= 760 \text{ kip-in. or } 63.3 \text{ kip-ft}$$

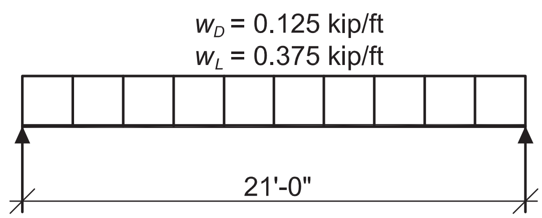
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90\left(63.3 \text{ kip-ft}\right)$ $= 57.0 \text{ kip-ft}$	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{63.3 \text{ kip-ft}}{1.67}$ $= 37.9 \text{ kip-ft}$

EXAMPLE F.8A HSS FLEXURAL MEMBER WITH SLENDER FLANGES

Given:

Verify the strength of an ASTM A500 Grade B HSS8×8×3⁄16 with a span of 21 ft. The loads are a dead load of 0.125 kip/ft and a live load of 0.375 kip/ft. Limit the live load deflection to *L*/240. The beam is continuously braced.



Beam Loading & Bracing Diagram
(*full lateral support*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B (rectangular HSS)
F_y = 46 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
w_u = 1.2(0.125 kip/ft) + 1.6(0.375 kip/ft) = 0.750 kip/ft $M_u = \dfrac{0.750 \text{ kip/ft}(21.0 \text{ ft})^2}{8}$ = 41.3 kip-ft	w_a = 0.125 kip/ft + 0.375 kip/ft = 0.500 kip/ft $M_a = \dfrac{0.500 \text{ kip/ft}(21.0 \text{ ft})^2}{8}$ = 27.6 kip-ft

Obtain the available flexural strength of the HSS8×8×3⁄16 from AISC *Manual* Table 3-13.

LRFD	ASD
$\phi_b M_n$ = 43.3 kip-ft > 41.3 kip-ft **o.k.**	$\dfrac{M_n}{\Omega_b}$ = 28.8 kip-ft > 27.6 kip-ft **o.k.**

Note that the strengths given in AISC *Manual* Table 3-13 incorporate the effects of noncompact and slender elements.

Deflection

The maximum live load deflection permitted is:

$$\begin{aligned}\Delta_{max} &= \frac{L}{240} \\ &= \frac{21.0\text{ ft}\left(12\text{ in./ft}\right)}{240} \\ &= 1.05\text{ in.}\end{aligned}$$

I_x = 54.4 in.^{4} from AISC *Manual* Table 1-12

The maximum calculated deflection is:

$$\begin{aligned}\Delta_{max} &= \frac{5w_L l^4}{384EI}\text{ from AISC } \textit{Manual} \text{ Table 3-23 Case 1} \\ &= \frac{5\left(0.375\text{ kip/ft}\right)\left(21.0\text{ ft}\right)^4\left(12\text{ in./ft}\right)^3}{384\left(29{,}000\text{ ksi}\right)\left(54.4\text{ in.}^4\right)} \\ &= 1.04\text{ in.} < 1.05\text{ in.} \qquad \textbf{o.k.}\end{aligned}$$

EXAMPLE F.8B HSS FLEXURAL MEMBER WITH SLENDER FLANGES

Given:

In Example F.8A the available strengths were easily determined from the tables of the AISC *Manual*. The purpose of the following calculation is to demonstrate the use of the AISC *Specification* equations to calculate the flexural strength of an HSS member with slender flanges.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B (rectangular HSS)
$F_y = 46$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-12, the geometric properties are as follows:

HSS8×8×3⁄16
$I_x = 54.4$ in.^{4}
$Z_x = 15.7$ in.^{3}
$S_x = 13.6$ in.^{3}
$B = 8.00$ in.
$H = 8.00$ in.
$t = 0.174$ in.
$b/t = 43.0$
$h/t = 43.0$

The required flexural strength from Example F.8A is:

LRFD	ASD
$M_u = 41.3$ kip-ft	$M_a = 27.6$ kip-ft

Flange Slenderness

The assumed outside radius of the corners of HSS shapes is 1.5*t*. The design thickness is used to check compactness.

Determine the limiting ratio for a slender HSS flange in flexure from AISC *Specification* Table B4.1b Case 17.

$$\lambda_r = 1.40\sqrt{\frac{E}{F_y}}$$
$$= 1.40\sqrt{\frac{29{,}000\text{ ksi}}{46\text{ ksi}}}$$
$$= 35.2$$

$$\lambda = \frac{b}{t}$$
$= 43.0 > \lambda_r$; therefore, the flange is slender

Web Slenderness

Determine the limiting ratio for a compact web in flexure from AISC *Specification* Table B4.1b Case 19.

$$\lambda_p = 2.42\sqrt{\frac{E}{F_y}}$$
$$= 2.42\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}$$
$$= 60.8$$

$$\lambda = \frac{h}{t}$$
$$= 43.0 < \lambda_p \text{, therefore the web is compact}$$

Nominal Flexural Strength, M_n

For HSS sections with slender flanges and compact webs, AISC *Specification* Section F7.2(c) applies.

$$M_n = F_y S_e \qquad \text{(}Spec\text{. Eq. F7-3)}$$

Where S_e is the effective section modulus determined with the effective width of the compression flange taken as:

$$b_e = 1.92 t_f \sqrt{\frac{E}{F_y}}\left[1 - \frac{0.38}{b/t_f}\sqrt{\frac{E}{F_y}}\right] \le b \qquad \text{(}Spec\text{. Eq. F7-4)}$$
$$= 1.92(0.174 \text{ in.})\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}\left[1 - \frac{0.38}{43.0}\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}\right]$$
$$= 6.53 \text{ in.}$$

$$b = 8.00 \text{ in.} - 3(0.174 \text{ in.}) \text{ from AISC } Specification \text{ Section B4.1b(d)}$$
$$= 7.48 \text{ in.} > 6.53 \text{ in.} \quad \textbf{o.k.}$$

The ineffective width of the compression flange is:

$$b - b_e = 7.48 \text{ in.} - 6.53 \text{ in.}$$
$$= 0.950 \text{ in.}$$

An exact calculation of the effective moment of inertia and section modulus could be performed taking into account the ineffective width of the compression flange and the resulting neutral axis shift. Alternatively, a simpler but slightly conservative calculation can be performed by removing the ineffective width symmetrically from both the top and bottom flanges.

$$I_{eff} \approx 54.4 \text{ in.}^4 - 2\left[\frac{(0.950 \text{ in.})(0.174 \text{ in.})^3}{12} + (0.950 \text{ in.})(0.174 \text{ in.})(3.91)^2\right]$$
$$= 49.3 \text{ in.}^4$$

The effective section modulus can now be calculated as follows:

$$S_e = \frac{I_{eff}}{d/2}$$
$$= \frac{49.3 \text{ in.}^4}{8.00 \text{ in.}/2}$$
$$= 12.3 \text{ in.}^3$$

$M_n = F_y S_e$ (*Spec*. Eq. F7-3)
$= 46 \text{ ksi}(12.3 \text{ in.}^3)$
$= 566$ kip-in. or 47.2 kip-ft

From AISC *Specification* Section F1, the available flexural strength is:

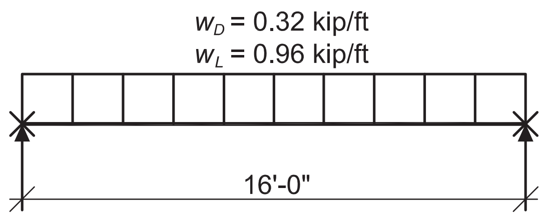
LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(47.2 \text{ kip-ft})$ $= 42.5 \text{ kip-ft} > 41.3 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{47.2 \text{ kip-ft}}{1.67}$ $= 28.3 \text{ kip-ft} > 27.6 \text{ kip-ft}$ **o.k.**

Note that the calculated available strengths are somewhat lower than those in AISC *Manual* Table 3-13 due to the use of the conservative calculation of the effective section modulus.

EXAMPLE F.9A PIPE FLEXURAL MEMBER

Given:

Select an ASTM A53 Grade B Pipe shape with an 8-in. nominal depth and a simple span of 16 ft. The loads are a total uniform dead load of 0.32 kip/ft and a uniform live load of 0.96 kip/ft. There is no deflection limit for this beam. The beam is braced only at the ends.



Beam Loading & Bracing Diagram
(*braced at end points only*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A53 Grade B
F_y = 35 ksi
F_u = 60 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
w_u = 1.2(0.32 kip/ft) + 1.6(0.96 kip/ft) = 1.92 kip/ft $M_u = \dfrac{1.92\text{ kip/ft}\left(16.0\text{ ft}\right)^2}{8}$ = 61.4 kip-ft	w_a = 0.32 kip/ft + 0.96 kip/ft = 1.28 kip/ft $M_a = \dfrac{1.28\text{ kip/ft}\left(16.0\text{ ft}\right)^2}{8}$ = 41.0 kip-ft

Pipe Selection

Select a member from AISC *Manual* Table 3-15 having the required strength.

Select Pipe 8 x-Strong.

From AISC *Manual* Table 3-15, the available flexural strength is:

LRFD	ASD
$\phi_b M_n = 81.4\text{ kip-ft} > 61.4\text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = 54.1\text{ kip-ft} > 41.0\text{ kip-ft}$ **o.k.**

EXAMPLE F.9B PIPE FLEXURAL MEMBER

Given:

The available strength in Example F.9A was easily determined using AISC *Manual* Table 3-15. The following example demonstrates the calculation of the available strength by directly applying the requirements of the AISC *Specification*.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A53 Grade B
$F_y = 35$ ksi
$F_u = 60$ ksi

From AISC *Manual* Table 1-14, the geometric properties are as follows:

Pipe 8 x-Strong
$Z = 31.0$ in.^{3}
$D = 8.63$ in.
$t = 0.465$ in.
$D/t = 18.5$

The required flexural strength from Example F.9A is:

LRFD	ASD
$M_u = 61.4$ kip-ft	$M_a = 41.0$ kip-ft

Slenderness Check

Determine the limiting diameter-to-thickness ratio for a compact section from AISC *Specification* Table B4.1b Case 20.

$$\lambda_p = \frac{0.07E}{F_y}$$
$$= \frac{0.07(29{,}000\text{ ksi})}{35\text{ ksi}}$$
$$= 58.0$$

$$\lambda = \frac{D}{t}$$
$= 18.5 < \lambda_p$; therefore, the section is compact and the limit state of flange local buckling does not apply

$$\frac{D}{t} < \frac{0.45E}{F_y}$$
$= 373$, therefore AISC *Specification* Section F8 applies

Nominal Flexural Strength Based on Flexural Yielding

$$M_n = M_p \qquad \text{(Spec. Eq. F8-1)}$$
$$= F_y Z_x$$
$$= 35\text{ ksi}\left(31.0\text{ in.}^3\right)$$
$$= 1{,}090\text{ kip-in. or }90.4\text{ kip-ft}$$

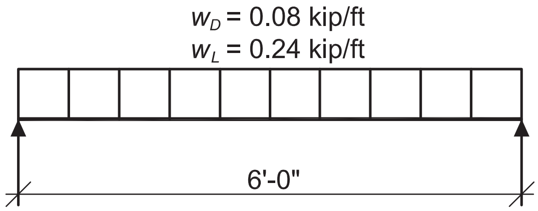
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$\phi_b M_n = 0.90\left(90.4\text{ kip-ft}\right)$	$\dfrac{M_n}{\Omega_b} = \dfrac{90.4\text{ kip-ft}}{1.67}$
$= 81.4\text{ kip-ft} > 61.4\text{ kip-ft}$ **o.k.**	$= 54.1\text{ kip-ft} > 41.0\text{ kip-ft}$ **o.k.**

EXAMPLE F.10 WT-SHAPE FLEXURAL MEMBER

Given:

Select an ASTM A992 WT beam with a 5-in. nominal depth and a simple span of 6 ft. The toe of the stem of the WT is in tension. The loads are a uniform dead load of 0.08 kip/ft and a uniform live load of 0.24 kip/ft. There is no deflection limit for this member. The beam is continuously braced.



Beam Loading & Bracing Diagram
(*full lateral support*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.08\text{ kip/ft}) + 1.6(0.24\text{ kip/ft})$ $= 0.480\text{ kip/ft}$ $M_u = \dfrac{0.480\text{ kip/ft}(6.00\text{ ft})^2}{8}$ $= 2.16\text{ kip-ft}$	$w_a = 0.08\text{ kip/ft} + 0.24\text{ kip/ft}$ $= 0.320\text{ kip/ft}$ $M_a = \dfrac{0.320\text{ kip/ft}(6.00\text{ ft})^2}{8}$ $= 1.44\text{ kip-ft}$

Try a WT5×6.

From AISC *Manual* Table 1-8, the geometric properties are as follows:

WT5×6
d = 4.94 in.
I_x = 4.35 in.^{4}
Z_x = 2.20 in.^{3}
S_x = 1.22 in.^{3}
b_f = 3.96 in.
t_f = 0.210 in.
$\bar{y}$ = 1.36 in.
$b_f/2t_f$ = 9.43

$$S_{xc} = \frac{I_x}{\bar{y}}$$
$$= \frac{4.35 \text{ in.}^4}{1.36 \text{ in.}}$$
$$= 3.20 \text{ in.}^3$$

Flexural Yielding

$$M_n = M_p \qquad \text{(}\textit{Spec}\text{. Eq. F9-1)}$$

$$M_p = F_y Z_x \leq 1.6M_y \text{ for stems in tension} \qquad \text{(}\textit{Spec}\text{. Eq. F9-2)}$$

$$1.6M_y = 1.6F_y S_x$$
$$= 1.6(50 \text{ ksi})(1.22 \text{ in.}^3)$$
$$= 97.6 \text{ kip-in.}$$

$$M_p = F_y Z_x$$
$$= 50 \text{ ksi}(2.20 \text{ in.}^3)$$
$$= 110 \text{ kip-in.} > 97.6 \text{ kip-in., therefore, use}$$

M_p = 97.6 kip-in. or 8.13 kip-ft

Lateral-Torsional Buckling (AISC Specification Section F9.2)

Because the WT is continuously braced, no check of the lateral-torsional buckling limit state is required.

Flange Local Buckling (AISC Specification Section F9.3)

Check flange compactness.

Determine the limiting slenderness ratio for a compact flange from AISC *Specification* Table B4.1b Case 10.

$$\lambda_{pf} = 0.38\sqrt{\frac{E}{F_y}}$$
$$= 0.38\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 9.15$$

$$\lambda = \frac{b_f}{2t_f}$$
$$= 9.43 > \lambda_{pf} \text{ ; therefore, the flange is not compact}$$

Check flange slenderness.

$$\lambda_{rf} = 1.0\sqrt{\frac{E}{F_y}}$$ from AISC *Specification* Table B4.1b Case 10

$$= 1.0\sqrt{\frac{29{,}000 \text{ ksi}}{50 \text{ ksi}}}$$
$$= 24.1$$

$$\lambda = \frac{b_f}{2t_f}$$
$= 9.43 < \lambda_{rf}$; therefore, the flange is not slender

For a WT with a noncompact flange, the nominal flexural strength due to flange local buckling is:

$$M_n = \left[M_p - \left(M_p - 0.7F_yS_{xc}\right)\left(\frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}}\right)\right] \le 1.6M_y \qquad (Spec.\ \text{Eq. F9-6})$$
$$= \left\{110 \text{ kip-in.} - \left[110 \text{ kip-in.} - 0.7\left(50 \text{ ksi}\right)\left(3.20 \text{ in.}^3\right)\right]\left(\frac{9.43 - 9.15}{24.1 - 9.15}\right)\right\} \le 97.6 \text{ kip-in.}$$
$= 110$ kip-in. > 97.6 kip-in.

Therefore use:

$M_n = 97.6$ kip-in. or 8.13 kip-ft

$M_n = M_p$
$= 8.13$ kip-ft **yielding limit state controls** (*Spec*. Eq. F9-1)

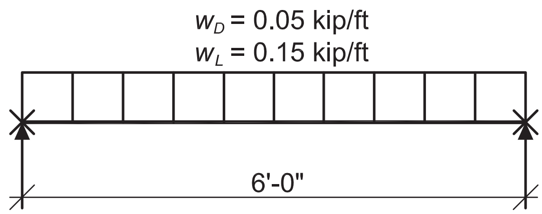
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90\left(8.13 \text{ kip-ft}\right)$ $= 7.32 \text{ kip-ft} > 2.16 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{8.13 \text{ kip-ft}}{1.67}$ $= 4.87 \text{ kip-ft} > 1.44 \text{ kip-ft}$ **o.k.**

EXAMPLE F.11A SINGLE ANGLE FLEXURAL MEMBER

Given:

Select an ASTM A36 single angle with a simple span of 6 ft. The vertical leg of the single angle is up and the toe is in compression. The vertical loads are a uniform dead load of 0.05 kip/ft and a uniform live load of 0.15 kip/ft. There are no horizontal loads. There is no deflection limit for this angle. The angle is braced at the end points only. Assume bending about the geometric *x-x* axis and that there is no lateral-torsional restraint.



Beam Loading & Bracing Diagram
(*braced at end points only*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_{ux} = 1.2(0.05\text{ kip/ft}) + 1.6(0.15\text{ kip/ft})$ $= 0.300\text{ kip/ft}$ $M_{ux} = \dfrac{0.300\text{ kip/ft}(6\text{ ft})^2}{8}$ $= 1.35\text{ kip-ft}$	$w_{ax} = 0.05\text{ kip/ft} + 0.15\text{ kip/ft}$ $= 0.200\text{ kip/ft}$ $M_{ax} = \dfrac{0.200\text{ kip/ft}(6\text{ ft})^2}{8}$ $= 0.900\text{ kip-ft}$

Try a L4×4×¼.

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×¼
S_x = 1.03 in.^{3}

Nominal Flexural Strength, M_n

Flexural Yielding

From AISC *Specification* Section F10.1, the nominal flexural strength due to the limit state of flexural yielding is:

$$\begin{aligned} M_n &= 1.5M_y \qquad \text{(Spec. Eq. F10-1)} \\ &= 1.5F_yS_x \\ &= 1.5(36\text{ ksi})(1.03\text{ in.}^3) \\ &= 55.6\text{ kip-in.} \end{aligned}$$

Lateral-Torsional Buckling

From AISC *Specification* Section F10.2, for single angles bending about a geometric axis with no lateral-torsional restraint, M_y is taken as 0.80 times the yield moment calculated using the geometric section modulus.

$$\begin{aligned} M_y &= 0.80F_yS_x \\ &= 0.80(36\text{ ksi})(1.03\text{ in.}^3) \\ &= 29.7\text{ kip-in.} \end{aligned}$$

Determine M_e.

For bending moment about one of the geometric axes of an equal-leg angle with no axial compression, with no lateral-torsional restraint, and with maximum compression at the toe, use AISC *Specification* Section F10.2(b)(iii)(a)(i), Equation F10-6a.

C_b = 1.14 from AISC *Manual* Table 3-1

$$M_e = \frac{0.66Eb^4tC_b}{L_b^2}\left(\sqrt{1+0.78\left(\frac{L_bt}{b^2}\right)^2}-1\right) \qquad (\textit{Spec}.\text{ Eq. F10-6a})$$

$$= \frac{0.66(29{,}000\text{ ksi})(4.00\text{ in.})^4(\tfrac{1}{4}\text{ in.})(1.14)}{(72.0\text{ in.})^2}\left(\sqrt{1+0.78\left(\frac{(72.0\text{ in.})(\tfrac{1}{4}\text{ in.})}{(4.00\text{ in.})^2}\right)^2}-1\right)$$

= 110 kip-in. > 29.7 kip-in.; therefore, AISC *Specification* Equation F10-3 is applicable

$$M_n = \left(1.92-1.17\sqrt{\frac{M_y}{M_e}}\right)M_y \le 1.5M_y \qquad (\textit{Spec}.\text{ Eq. F10-3})$$

$$= \left(1.92-1.17\sqrt{\frac{29.7\text{ kip-in.}}{110\text{ kip-in.}}}\right)29.7\text{ kip-in.} \le 1.5(29.7\text{ kip-in.})$$

= 39.0 kip-in. ≤ 44.6 kip-in.; therefore, M_n = 39.0 kip-in.

Leg Local Buckling

AISC *Specification* Section F10.3 applies when the toe of the leg is in compression.

Check slenderness of the leg in compression.

$$\begin{aligned} \lambda &= \frac{b}{t} \\ &= \frac{4.00\text{ in.}}{\tfrac{1}{4}\text{ in.}} \\ &= 16.0 \end{aligned}$$

Determine the limiting compact slenderness ratios from AISC *Specification* Table B4.1b Case 12.

$$\lambda_p = 0.54\sqrt{\frac{E}{F_y}}$$

$$= 0.54\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}}$$
$$= 15.3$$

Determine the limiting noncompact slenderness ratios from AISC *Specification* Table B4.1b Case 12.

$$\lambda_r = 0.91\sqrt{\frac{E}{F_y}}$$
$$= 0.91\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}}$$
$$= 25.8$$

$\lambda_p < \lambda < \lambda_r$, therefore, the leg is noncompact in flexure

$$M_n = F_y S_c\left(2.43 - 1.72\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}\right) \quad \text{(}\textit{Spec}\text{. Eq. F10-7)}$$

$$S_c = 0.80S_x$$
$$= 0.80\left(1.03\text{ in.}^3\right)$$
$$= 0.824\text{ in.}^3$$

$$M_n = 36\text{ ksi}\left(0.824\text{ in.}^3\right)\left(2.43 - 1.72(16.0)\sqrt{\frac{36\text{ ksi}}{29{,}000\text{ ksi}}}\right)$$
$$= 43.3\text{ kip-in.}$$

The lateral-torsional buckling limit state controls.

M_n = 39.0 kip-in. or 3.25 kip-ft

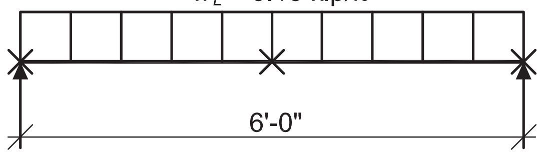
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90\left(3.25\text{ kip-ft}\right)$ $= 2.93\text{ kip-ft} > 1.35\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{3.25\text{ kip-ft}}{1.67}$ $= 1.95\text{ kip-ft} > 0.900\text{ kip-ft}$ **o.k.**

EXAMPLE F.11B SINGLE ANGLE FLEXURAL MEMBER

Given:

Select an ASTM A36 single angle with a simple span of 6 ft. The vertical leg of the single angle is up and the toe is in compression. The vertical loads are a uniform dead load of 0.05 kip/ft and a uniform live load of 0.15 kip/ft. There are no horizontal loads. There is no deflection limit for this angle. The angle is braced at the end points and at the midspan. Assume bending about the geometric *x-x* axis and that there is lateral-torsional restraint at the midspan and ends only.



Beam Loading & Bracing Diagram
(*braced at end points and midspan*)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_{ux} = 1.2(0.05\text{ kip/ft}) + 1.6(0.15\text{ kip/ft})$ $= 0.300\text{ kip/ft}$ $M_{ux} = \dfrac{0.300\text{ kip/ft}(6\text{ ft})^2}{8}$ $= 1.35\text{ kip-ft}$	$w_{ax} = 0.05\text{ kip/ft} + 0.15\text{ kip/ft}$ $= 0.200\text{ kip/ft}$ $M_{ax} = \dfrac{0.200\text{ kip/ft}(6\text{ ft})^2}{8}$ $= 0.900\text{ kip-ft}$

Try a L4×4×¼.

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×¼
$S_x = 1.03\text{ in.}^3$

Nominal Flexural Strength, M_n

Flexural Yielding

From AISC *Specification* Section F10.1, the nominal flexural strength due to the limit state of flexural yielding is:

$$M_n = 1.5M_y \qquad (Spec.\ \text{Eq. F10-1})$$
$$= 1.5F_yS_x$$

$$= 1.5(36 \text{ ksi})(1.03 \text{ in.}^3)$$
$$= 55.6 \text{ kip-in.}$$

Lateral-Torsional Buckling

From AISC *Specification* Section F10.2(b)(iii)(b), for single angles with lateral-torsional restraint at the point of maximum moment, M_y is taken as the yield moment calculated using the geometric section modulus.

$$M_y = F_y S_x$$
$$= 36 \text{ ksi}(1.03 \text{ in.}^3)$$
$$= 37.1 \text{ kip-in.}$$

Determine M_e.

For bending moment about one of the geometric axes of an equal-leg angle with no axial compression, with lateral-torsional restraint at the point of maximum moment only (at midspan in this case), and with maximum compression at the toe, M_e shall be taken as 1.25 times M_e computed using AISC *Specification* Equation F10-6a.

C_b = 1.30 from AISC *Manual* Table 3-1

$$M_e = 1.25\left(\frac{0.66Eb^4tC_b}{L_b^2}\right)\left(\sqrt{1+0.78\left(\frac{L_bt}{b^2}\right)^2}-1\right) \quad \text{(Spec. Eq. F10-6a)}$$

$$= 1.25\left[\frac{0.66(29{,}000 \text{ ksi})(4.00 \text{ in.})^4(\tfrac{1}{4} \text{ in.})(1.30)}{(36.0 \text{ in.})^2}\right]\left(\sqrt{1+0.78\left(\frac{(36.0 \text{ in.})(\tfrac{1}{4} \text{ in.})}{(4.00 \text{ in.})^2}\right)^2}-1\right)$$

= 179 kip-in. > 37.1 kip-in., therefore, AISC *Specification* Equation F10-3 is applicable

$$M_n = \left(1.92 - 1.17\sqrt{\frac{M_y}{M_e}}\right)M_y \le 1.5M_y \quad \text{(Spec. Eq. F10-3)}$$

$$= \left(1.92 - 1.17\sqrt{\frac{37.1 \text{ kip-in.}}{179 \text{ kip-in.}}}\right)37.1 \text{ kip-in.} \le 1.5(37.1 \text{ kip-in.})$$

$= 51.5$ kip-in. ≤ 55.7 kip-in., therefore, M_n = 51.5 kip-in.

Leg Local Buckling

M_n = 43.3 kip-in. from Example F.11A.

The leg local buckling limit state controls.

M_n = 43.3 kip-in. or 3.61 kip-ft

From AISC *Specification* Section F1, the available flexural strength is:

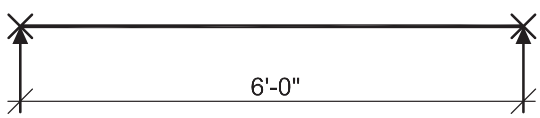
LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(3.61\text{ kip-ft})$ $= 3.25\text{ kip-ft} > 1.35\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{3.61\text{ kip-ft}}{1.67}$ $= 2.16\text{ kip-ft} > 0.900\text{ kip-ft}$ **o.k.**

Return to Table of Contents

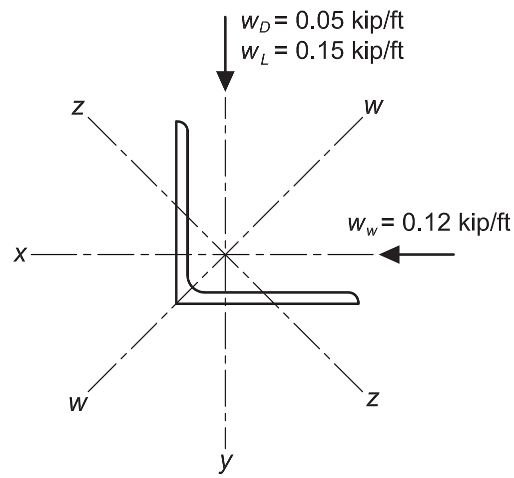
EXAMPLE F.11C SINGLE ANGLE FLEXURAL MEMBER

Given:

Select an ASTM A36 single angle with a simple span of 6 ft. The vertical loads are a uniform dead load of 0.05 kip/ft and a uniform live load of 0.15 kip/ft. The horizontal load is a uniform wind load of 0.12 kip/ft. There is no deflection limit for this angle. The angle is braced at the end points only and there is no lateral-torsional restraint. Use load combination 4 from Section 2.3.2 of ASCE/SEI 7 for LRFD and load combination 6a from Section 2.4.1 of ASCE/SEI 7 for ASD.



Beam Bracing Diagram
(*braced at end points only*)



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_{ux} = 1.2(0.05 \text{ kip/ft}) + 0.15 \text{ kip/ft}$ $= 0.210 \text{ kip/ft}$	$w_{ax} = 0.05 \text{ kip/ft} + 0.75(0.15 \text{ kip/ft})$ $= 0.163 \text{ kip/ft}$
$w_{uy} = 1.0(0.12 \text{ kip/ft})$ $= 0.12 \text{ kip/ft}$	$w_{ay} = 0.75[(0.6)(0.12 \text{ kip/ft})]$ $= 0.054 \text{ kip/ft}$
$M_{ux} = \dfrac{0.210 \text{ kip/ft}(6 \text{ ft})^2}{8}$ $= 0.945 \text{ kip-ft}$	$M_{ax} = \dfrac{0.163 \text{ kip/ft}(6 \text{ ft})^2}{8}$ $= 0.734 \text{ kip-ft}$
$M_{uy} = \dfrac{0.12 \text{ kip/ft}(6 \text{ ft})^2}{8}$ $= 0.540 \text{ kip-ft}$	$M_{ay} = \dfrac{0.054 \text{ kip/ft}(6 \text{ ft})^2}{8}$ $= 0.243 \text{ kip-ft}$

Try a L4×4×¼.

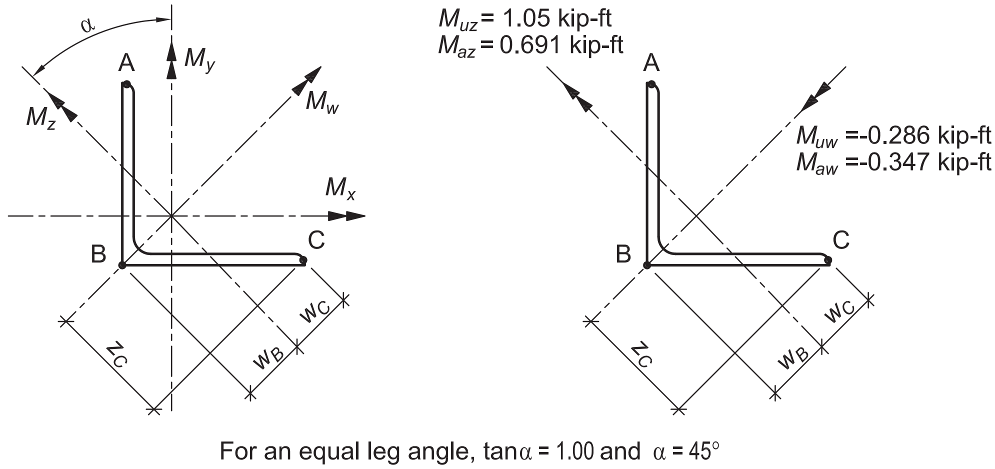


Fig. F.11C-1. Example F.11C single angle geometric and principal axes moments.

Sign convention for geometric axes moments are:

LRFD	ASD
$M_{ux} = -0.945$ kip-ft $M_{uy} = 0.540$ kip-ft	$M_{ax} = -0.734$ kip-ft $M_{ay} = 0.243$ kip-ft

Principal axes moments are:

LRFD	ASD
$M_{uw} = M_{ux} \cos \alpha + M_{uy} \sin \alpha$ $= -0.945$ kip-ft (cos 45°) + 0.540 kip-ft (sin 45°) $= -0.286$ kip-ft $M_{uz} = -M_{ux} \sin \alpha + M_{uy} \cos \alpha$ $= -(-0.945$ kip-ft)(sin 45°) + 0.540kip-ft (cos 45°) = 1.05 kip-ft	$M_{aw} = M_{ax} \cos \alpha + M_{ay} \sin \alpha$ $= -0.734$ kip-ft (cos 45°) + 0.243 kip-ft (sin 45°) $= -0.347$ kip-ft $M_{az} = -M_{ax} \sin \alpha + M_{ay} \cos \alpha$ $= -(-0.734$ kip-ft)(sin 45°) + 0.243 kip-ft (cos 45°) = 0.691 kip-ft

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L4×4×¼
$S_x = S_y = 1.03$ in.^{3}
$I_x = I_y = 3.00$ in.^{4}
$I_z = 1.18$ in.^{4}

Additional properties from the angle geometry are as follows:

$w_B = 1.53$ in.
$w_C = 1.39$ in.
$z_C = 2.74$ in.

Additional principal axes properties from the AISC *Shapes Database* are as follows:

I_w = 4.82 in.^{4}
S_{zB} = 0.779 in.^{3}
S_{zC} = 0.857 in.^{3}
S_{wC} = 1.76 in.^{3}

Z-Axis Nominal Flexural Strength, M_{nz}

Note that M_{uz} and M_{az} are positive; therefore, the toes of the angle are in compression.

Flexural Yielding

From AISC *Specification* Section F10.1, the nominal flexural strength due to the limit state of flexural yielding is:

$$\begin{aligned} M_{nz} &= 1.5M_y \\ &= 1.5F_yS_{zB} \\ &= 1.5(36\text{ ksi})(0.779\text{ in.}^3) \\ &= 42.1\text{ kip-in.} \end{aligned}$$ (*Spec*. Eq. F10-1)

Lateral-Torsional Buckling

From the User Note in AISC *Specification* Section F10, the limit state of lateral-torsional buckling does not apply for bending about the minor axis.

Leg Local Buckling

Check slenderness of outstanding leg in compression.

$$\begin{aligned} \lambda &= \frac{b}{t} \\ &= \frac{4.00\text{ in.}}{\frac{1}{4}\text{ in.}} \\ &= 16.0 \end{aligned}$$

The limiting width-to-thickness ratios are:

$\lambda_p = 0.54\sqrt{\dfrac{E}{F_y}}$ from AISC *Specification* Table B4.1b Case 12

$$\begin{aligned} &= 0.54\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}} \\ &= 15.3 \end{aligned}$$

$\lambda_r = 0.91\sqrt{\dfrac{E}{F_y}}$ from AISC *Specification* Table B4.1b Case 12

$$\begin{aligned} &= 0.91\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}} \\ &= 25.8 \end{aligned}$$

$\lambda_p < \lambda < \lambda_r$, therefore, the leg is noncompact in flexure

$$M_{nz} = F_y S_c \left(2.43 - 1.72\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}\right) \quad \text{(Spec. Eq. F10-7)}$$

$S_c = S_{zC}$ (to toe in compression)
$= 0.857 \text{ in.}^3$

$$M_{nz} = 36 \text{ ksi}\left(0.857 \text{ in.}^3\right)\left[2.43 - 1.72(16.0)\sqrt{\frac{36 \text{ ksi}}{29{,}000 \text{ ksi}}}\right]$$
$$= 45.1 \text{ kip-in.}$$

The flexural yielding limit state controls.

$M_{nz} = 42.1$ kip-in.

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_{nz} = 0.90(42.1 \text{ kip-in.})$ $= 37.9$ kip-in.	$\Omega_b = 1.67$ $\dfrac{M_{nz}}{\Omega_b} = \dfrac{42.1 \text{ kip-in.}}{1.67}$ $= 25.2$ kip-in..

W-Axis Nominal Flexural Strength, M_{nw}

Flexural Yielding

$$M_{nw} = 1.5M_y \quad \text{(Spec. Eq. F10-1)}$$
$$= 1.5F_y S_{wC}$$
$$= 1.5(36 \text{ ksi})\left(1.76 \text{ in.}^3\right)$$
$$= 95.0 \text{ kip-in.}$$

Lateral-Torsional Buckling

Determine M_e.

For bending about the major principal axis of an equal-leg angle without continuous lateral-torsional restraint, use AISC *Specification* Equation F10-4.

$C_b = 1.14$ from AISC *Manual* Table 3-1

$$M_e = \frac{0.46Eb^2t^2C_b}{L_b} \quad \text{(Spec. Eq. F10-4)}$$
$$= \frac{0.46(29{,}000 \text{ ksi})(4.00 \text{ in.})^2 (\tfrac{1}{4} \text{ in.})^2 (1.14)}{72.0 \text{ in.}}$$
$$= 211 \text{ kip-in.}$$

$M_y = F_yS_{wC}$
$= 36 \text{ ksi}(1.76 \text{ in.}^3)$
$= 63.4 \text{ kip-in.}$

$M_e > M_y$, therefore, AISC *Specification* Equation F10-3 is applicable

$$M_{nw} = \left(1.92 - 1.17\sqrt{\frac{M_y}{M_e}}\right)M_y \le 1.5M_y \qquad (\textit{Spec}.\text{ Eq. F10-3})$$

$$= \left(1.92 - 1.17\sqrt{\frac{63.4 \text{ kip-in.}}{211 \text{ kip-in.}}}\right)63.4 \text{ kip-in.} \le 1.5(63.4 \text{ kip-in.})$$

$= 81.1$ kip-in. ≤ 95.1 kip-in., therefore, $M_{nw} = 81.1$ kip-in.

Leg Local Buckling

From the preceding calculations, the leg is noncompact in flexure.

$$M_{nw} = F_yS_c\left(2.43 - 1.72\left(\frac{b}{t}\right)\sqrt{\frac{F_y}{E}}\right) \qquad (\textit{Spec}.\text{ Eq. F10-7})$$

$S_c = S_{wC}$ (to toe in compression)
$= 1.76 \text{ in.}^3$

$$M_{nw} = 36 \text{ ksi}\left(1.76 \text{ in.}^3\right)\left[2.43 - 1.72(16.0)\sqrt{\frac{36 \text{ ksi}}{29{,}000 \text{ ksi}}}\right]$$

$= 92.5$ kip-in.

The lateral-torsional buckling limit state controls.

$M_{nw} = 81.1$ kip-in.

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_bM_{nw} = 0.90(81.1 \text{ kip-in.})$ $= 73.0$ kip-in.	$\Omega_b = 1.67$ $\dfrac{M_{nw}}{\Omega_b} = \dfrac{81.1 \text{ kip-in.}}{1.67}$ $= 48.6$ kip-in.

The moment resultant has components about both principal axes; therefore, the combined stress ratio must be checked using the provisions of AISC *Specification* Section H2.

$$\left|\frac{f_{ra}}{F_{ca}} + \frac{f_{rbw}}{F_{cbw}} + \frac{f_{rbz}}{F_{cbz}}\right| \le 1.0 \qquad (\textit{Spec}.\text{ Eq. H2-1})$$

Note: Rather than convert moments into stresses, it is acceptable to simply use the moments in the interaction equation because the section properties that would be used to convert the moments to stresses are the same in the numerator and denominator of each term. It is also important for the designer to keep track of the signs of the stresses at each point so that the proper sign is applied when the terms are combined. The sign of the moments used to convert geometric axis moments to principal axis moments will indicate which points are in tension and which are in compression but those signs will not be used in the interaction equations directly.

Based on Figure F.11C-1, the required flexural strength and available flexural strength for this beam can be summarized as:

LRFD	ASD
$M_{uw} = 0.286 \text{ kip-ft}$ $\phi_b M_{nw} = \dfrac{73.0 \text{ kip-in.}}{12 \text{ in./ft}} = 6.08 \text{ kip-ft}$ $M_{uz} = 1.05 \text{ kip-ft}$ $\phi_b M_{nz} = \dfrac{37.9 \text{ kip-in.}}{12 \text{ in./ft}} = 3.16 \text{ kip-ft}$	$M_{aw} = 0.347 \text{ kip-ft}$ $\dfrac{M_{nw}}{\Omega_b} = \dfrac{48.6 \text{ kip-in.}}{12 \text{ in./ft}} = 4.05 \text{ kip-ft}$ $M_{az} = 0.691 \text{ kip-ft}$ $\dfrac{M_{nz}}{\Omega_b} = \dfrac{25.2 \text{ kip-in.}}{12 \text{ in./ft}} = 2.10 \text{ kip-ft}$

At point B:

M_w causes no stress at point B; therefore, the stress ratio is set to zero. M_z causes tension at point B; therefore it will be taken as negative.

LRFD	ASD
$\left\lvert \dfrac{0 - 1.05 \text{ kip-ft}}{3.16 \text{ kip-ft}} \right\rvert = 0.332 \le 1.0$ **o.k.**	$\left\lvert \dfrac{0 - 0.691 \text{ kip-ft}}{2.10 \text{ kip-ft}} \right\rvert = 0.329 \le 1.0$ **o.k.**

At point C:

M_w causes tension at point C; therefore, it will be taken as negative. M_z causes compression at point C; therefore, it will be taken as positive.

LRFD	ASD
$\left\lvert -\dfrac{0.286 \text{ kip-ft}}{6.08 \text{ kip-ft}} + \dfrac{1.05 \text{ kip-ft}}{3.16 \text{ kip-ft}} \right\rvert = 0.285 \le 1.0$ **o.k.**	$\left\lvert -\dfrac{0.347 \text{ kip-ft}}{4.05 \text{ kip-ft}} + \dfrac{0.691 \text{ kip-ft}}{2.10 \text{ kip-ft}} \right\rvert = 0.243 \le 1.0$ **o.k.**

At point A:

M_w and M_z cause compression at point A; therefore, both will be taken as positive.

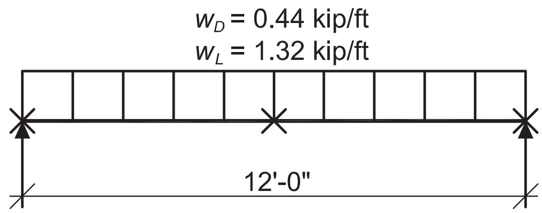
LRFD	ASD
$\left\lvert \dfrac{0.286 \text{ kip-ft}}{6.08 \text{ kip-ft}} + \dfrac{1.05 \text{ kip-ft}}{3.16 \text{ kip-ft}} \right\rvert = 0.379 \le 1.0$ **o.k.**	$\left\lvert \dfrac{0.347 \text{ kip-ft}}{4.05 \text{ kip-ft}} + \dfrac{0.691 \text{ kip-ft}}{2.10 \text{ kip-ft}} \right\rvert = 0.415 \le 1.0$ **o.k.**

Thus, the interaction of stresses at each point is seen to be less than 1.0 and this member is adequate to carry the required load. Although all three points were checked, it was expected that point A would be the controlling point because compressive stresses add at this point.

EXAMPLE F.12 RECTANGULAR BAR IN STRONG-AXIS BENDING

Given:

Select an ASTM A36 rectangular bar with a span of 12 ft. The bar is braced at the ends and at the midpoint. Conservatively use C_b = 1.0. Limit the depth of the member to 5 in. The loads are a total uniform dead load of 0.44 kip/ft and a uniform live load of 1.32 kip/ft.



Beam Loading & Bracing Diagram
(*braced at end points and midspan*)

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.44 \text{ kip/ft}) + 1.6(1.32 \text{ kip/ft})$ $= 2.64 \text{ kip/ft}$ $M_u = \dfrac{2.64 \text{ kip/ft}(12.0 \text{ ft})^2}{8}$ $= 47.5 \text{ kip-ft}$	$w_a = 0.44 \text{ kip/ft} + 1.32 \text{ kip/ft}$ $= 1.76 \text{ kip/ft}$ $M_a = \dfrac{1.76 \text{ kip/ft}(12.0 \text{ ft})^2}{8}$ $= 31.7 \text{ kip-ft}$

Try a BAR 5 in.×3 in.

From AISC *Manual* Table 17-27, the geometric properties are as follows:

$$S_x = \frac{bd^2}{6}$$
$$= \frac{(3.00 \text{ in.})(5.00 \text{ in.})^2}{6}$$
$$= 12.5 \text{ in.}^3$$

$$Z_x = \frac{bd^2}{4}$$
$$= \frac{(3.00\text{ in.})(5.00\text{ in.})^2}{4}$$
$$= 18.8\text{ in.}^3$$

Nominal Flexural Strength, M_n

Flexural Yielding

Check limit from AISC *Specification* Section F11.1.

$$\frac{L_b d}{t^2} \le \frac{0.08E}{F_y}$$
$$\frac{72.0\text{ in.}(5.00\text{ in.})}{(3.00\text{ in.})^2} \le \frac{0.08(29{,}000\text{ ksi})}{36\text{ ksi}}$$

$40.0 \le 64.4$, therefore, the yielding limit state applies

$$M_n = M_p \qquad \text{(}Spec\text{. Eq. F11-1)}$$
$$= F_y Z \le 1.6M_y$$

$$1.6M_y = 1.6F_y S_x$$
$$= 1.6(36\text{ ksi})(12.5\text{ in.}^3)$$
$$= 720\text{ kip-in.}$$

$$M_p = F_y Z_x$$
$$= 36\text{ ksi}(18.8\text{ in.}^3)$$
$$= 677\text{ kip-in.} \le 720\text{ kip-in.}$$

Use $M_n = M_p$
$$= 677\text{ kip-in. or } 56.4\text{ kip-ft}$$

Lateral-Torsional Buckling (AISC *Specification* Section F11.2)

As previously calculated, $L_b d/t^2 \le 0.08E/F_y$, therefore, the lateral-torsional buckling limit state does not apply.

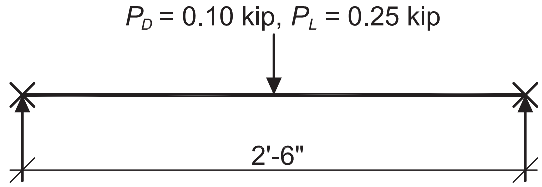
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(56.4\text{ kip-ft})$ $= 50.8\text{ kip-ft} > 47.5\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{56.4\text{ kip-ft}}{1.67}$ $= 33.8\text{ kip-ft} > 31.7\text{ kip-ft}$ **o.k.**

EXAMPLE F.13 ROUND BAR IN BENDING

Given:

Select an ASTM A36 round bar with a span of 2.50 ft. The bar is braced at end points only. Assume C_b = 1.0. Limit the diameter to 2 in. The loads are a concentrated dead load of 0.10 kip and a concentrated live load of 0.25 kip at the center. The weight of the bar is negligible.



Beam Loading & Bracing Diagram
(*braced at end points only*)

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7 and AISC *Manual* Table 3-23 diagram 7, the required flexural strength is:

LRFD	ASD
$P_u = 1.2(0.10 \text{ kip}) + 1.6(0.25 \text{ kip})$ $= 0.520 \text{ kip}$ $M_u = \dfrac{(0.520 \text{ kip})(2.50 \text{ ft})}{4}$ $= 0.325 \text{ kip-ft}$	$P_a = 0.10 \text{ kip} + 0.25 \text{ kip}$ $= 0.350 \text{ kip}$ $M_a = \dfrac{(0.350 \text{ kip})(2.50 \text{ ft})}{4}$ $= 0.219 \text{ kip-ft}$

Try a BAR 1 in. diameter.

From AISC *Manual* Table 17-27, the geometric properties are as follows:

Round bar

$$S_x = \frac{\pi d^3}{32}$$
$$= \frac{\pi (1.00 \text{ in.})^3}{32}$$
$$= 0.0982 \text{ in.}^3$$

$$Z_x = \frac{d^3}{6}$$
$$= \frac{(1.00\text{ in.})^3}{6}$$
$$= 0.167\text{ in.}^3$$

Nominal Flexural Strength, M_n

Flexural Yielding

From AISC *Specification* Section F11.1, the nominal flexural strength based on the limit state of flexural yielding is,

$$M_n = M_p$$
$$= F_y Z \leq 1.6M_y \qquad (Spec.\ \text{Eq. F11-1})$$

$$1.6M_y = 1.6F_y S_x$$
$$= 1.6(36\text{ ksi})(0.0982\text{ in.}^3)$$
$$= 5.66\text{ kip-in.}$$

$$F_y Z_x = 36\text{ ksi}(0.167\text{ in.}^3)$$
$$= 6.01\text{ kip-in.} > 5.66\text{ kip-in.}$$

Therefore, M_n = 5.66 kip-in.

The limit state lateral-torsional buckling (AISC Specification Section F11.2) need not be considered for rounds.

The flexural yielding limit state controls.

$$M_n = 5.66\text{ kip-in. or } 0.472\text{ kip-ft}$$

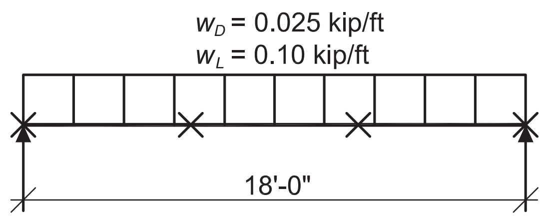
From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(0.472\text{ kip-ft})$ $= 0.425\text{ kip-ft} > 0.325\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{0.472\text{ kip-ft}}{1.67}$ $= 0.283\text{ kip-ft} > 0.219\text{ kip-ft}$ **o.k.**

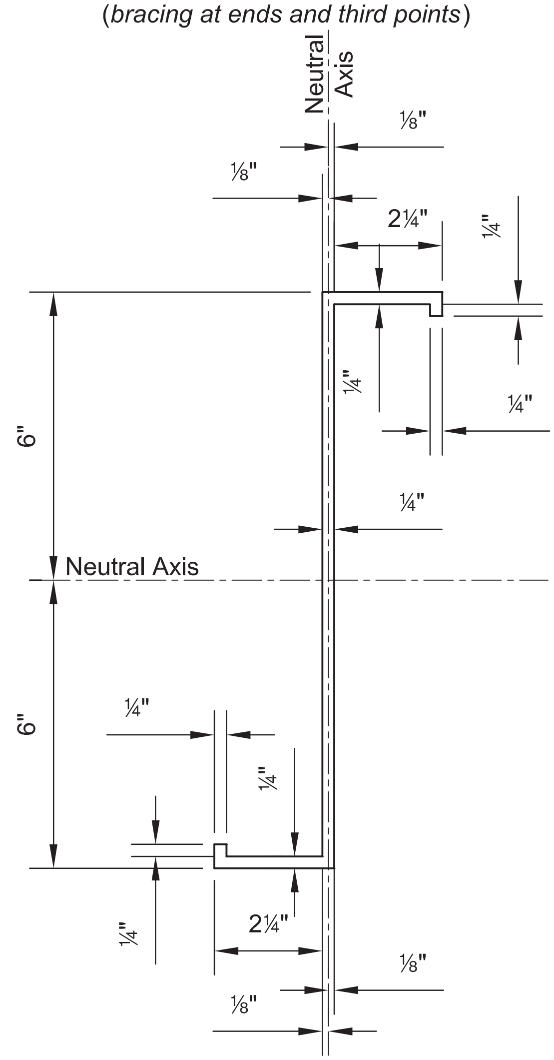
EXAMPLE F.14 POINT-SYMMETRICAL Z-SHAPE IN STRONG-AXIS BENDING

Given:

Determine the available flexural strength of the ASTM A36 Z-shape shown for a simple span of 18 ft. The Z-shape is braced at 6 ft on center. Assume C_b = 1.0. The loads are a uniform dead load of 0.025 kip/ft and a uniform live load of 0.10 kip/ft. Assume the beam is loaded through the shear center. The profile of the purlin is shown below.



Beam Loading & Bracing Diagram
(*bracing at ends and third points*)



Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The geometric properties are as follows:

$$t_w = t_f$$
$$= \text{¼ in.}$$

$$A = (2.50\text{ in.})(\text{¼ in.})(2) + (\text{¼ in.})(\text{¼ in.})(2) + (11.5\text{ in.})(\text{¼ in.})$$
$$= 4.25\text{ in.}^2$$

$$I_x = \left[\frac{(\text{¼ in.})(\text{¼ in.})^3}{12} + (0.25\text{ in.})^2(5.63\text{ in.})^2\right](2)$$
$$+\left[\frac{(2.50\text{ in.})(\text{¼ in.})^3}{12} + (2.50\text{ in.})(\text{¼ in.})(5.88\text{ in.})^2\right](2)$$
$$+\frac{(\text{¼ in.})(11.5\text{ in.})^3}{12}$$
$$= 78.9\text{ in.}^4$$

$$\bar{y} = 6.00\text{ in.}$$

$$S_x = \frac{I_x}{\bar{y}}$$
$$= \frac{78.9\text{ in.}^4}{6.00\text{ in.}}$$
$$= 13.2\text{ in.}^3$$

$$I_y = \left[\frac{(\text{¼ in.})(\text{¼ in.})^3}{12} + (\text{¼ in.})^2(2.25\text{ in.})^2\right](2)$$
$$+\left[\frac{(\text{¼ in.})(2.50\text{ in.})^3}{12} + (2.50\text{ in.})(\text{¼ in.})(1.13\text{ in.})^2\right](2)$$
$$+\frac{(11.5\text{ in.})(\text{¼ in.})^3}{12}$$
$$= 2.90\text{ in.}^4$$

$$r_y = \sqrt{\frac{I_y}{A}}$$
$$= \sqrt{\frac{2.90\text{ in.}^4}{4.25\text{ in.}^2}}$$
$$= 0.826\text{ in.}$$

$$r_{ts} \approx \frac{b_f}{\sqrt{12\left(1+\frac{1}{6}\frac{ht_w}{b_f t_f}\right)}}$$ from AISC *Specification* Section F2.2 User Note

$$= \frac{2.50 \text{ in.}}{\sqrt{12\left\{1+\frac{1}{6}\left[\frac{(11.5 \text{ in.})(\frac{1}{4} \text{ in.})}{(2.50 \text{ in.})(\frac{1}{4} \text{ in.})}\right]\right\}}}$$
$$= 0.543 \text{ in.}$$

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.025 \text{ kip/ft}) + 1.6(0.10 \text{ kip/ft})$ $= 0.190 \text{ kip/ft}$ $M_u = \frac{(0.190 \text{ kip/ft})(18.0 \text{ ft})^2}{8}$ $= 7.70 \text{ kip-ft}$	$w_a = 0.025 \text{ kip/ft} + 0.10 \text{ kip/ft}$ $= 0.125 \text{ kip/ft}$ $M_a = \frac{(0.125 \text{ kip/ft})(18.0 \text{ ft})^2}{8}$ $= 5.06 \text{ kip-ft}$

Nominal Flexural Strength, M_n

Flexural Yielding

From AISC *Specification* Section F12.1, the nominal flexural strength based on the limit state of flexural yielding is,

$$F_n = F_y \qquad (Spec.\ \text{Eq. F12-2})$$
$$= 36 \text{ ksi}$$

$$M_n = F_n S_{min} \qquad (Spec.\ \text{Eq. F12-1})$$
$$= 36 \text{ ksi}(13.2 \text{ in.}^3)$$
$$= 475 \text{ kip-in.}$$

Local Buckling

There are no specific local buckling provisions for Z-shapes in the AISC *Specification*. Use provisions for rolled channels from AISC *Specification* Table B4.1b, Cases 10 and 15.

Flange Slenderness

Conservatively neglecting the end return,

$$\lambda = \frac{b}{t_f}$$
$$= \frac{2.50 \text{ in.}}{\frac{1}{4} \text{ in.}}$$
$$= 10.0$$

$$\lambda_p = 0.38\sqrt{\frac{E}{F_y}}$$ from AISC *Specification* Table B4.1b case 10

$$= 0.38\sqrt{\frac{29{,}000\,\text{ksi}}{36\,\text{ksi}}}$$
$$= 10.8$$

$\lambda < \lambda_p$; therefore, the flange is compact

Web Slenderness

$$\lambda = \frac{h}{t_w}$$
$$= \frac{11.5 \text{ in.}}{\frac{1}{4} \text{ in.}}$$
$$= 46.0$$

$\lambda_p = 3.76\sqrt{\frac{E}{F_y}}$ from AISC *Specification* Table B4.1b case 15

$$= 3.76\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 107$$

$\lambda < \lambda_p$; therefore, the web is compact

Therefore, the local buckling limit state does not apply.

Lateral-Torsional Buckling

Per the User Note in AISC *Specification* Section F12, take the critical lateral-torsional buckling stress as half that of the equivalent channel. This is a conservative approximation of the lateral-torsional buckling strength which accounts for the rotation between the geometric and principal axes of a Z-shaped cross-section, and is adopted from the *North American Specification for the Design of Cold-Formed Steel Structural Members* (AISC, 2007).

Calculate limiting unbraced lengths.

For bracing at 6 ft on center,

$$L_b = 6.00 \text{ ft}\left(12 \text{ in./ft}\right)$$
$$= 72.0 \text{ in.}$$

$$L_p = 1.76 r_y \sqrt{\frac{E}{F_y}} \qquad \text{(}\textit{Spec}\text{. Eq. F2-5)}$$

$$= 1.76\left(0.826 \text{ in.}\right)\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 41.3 \text{ in.} < 72.0 \text{ in.}$$

$$L_r = 1.95 r_{ts}\left(\frac{E}{0.7F_y}\right)\sqrt{\frac{Jc}{S_x h_0} + \sqrt{\left(\frac{Jc}{S_x h_0}\right)^2 + 6.76\left(\frac{0.7F_y}{E}\right)^2}} \qquad \text{(}\textit{Spec}\text{. Eq. F2-6)}$$

Per the User Note in AISC *Specification* Section F2, the square root term in AISC *Specification* Equation F2-4 can conservatively be taken equal to one. Therefore, Equation F2-6 can also be simplified. Substituting $0.7F_y$ for F_{cr} in Equation F2-4 and solving for $L_b = L_r$, AISC *Specification* Equation F2-6 becomes:

$$L_r = \pi r_{ts}\sqrt{\frac{E}{0.7F_y}}$$
$$= \pi(0.543\text{ in.})\sqrt{\frac{29{,}000\text{ ksi}}{0.7(36\text{ ksi})}}$$
$$= 57.9\text{ in.} < 72.0\text{ in.}$$

Calculate one half of the critical lateral-torsional buckling stress of the equivalent channel.

$L_b > L_r$, therefore,

$$F_{cr} = (0.5)\frac{C_b\pi^2E}{\left(\frac{L_b}{r_{ts}}\right)^2}\sqrt{1+0.078\left(\frac{Jc}{S_xh_0}\right)\left(\frac{L_b}{r_{ts}}\right)^2} \quad \text{(}Spec\text{. Eq. F2-4)}$$

Conservatively taking the square root term as 1.0,

$$F_{cr} = (0.5)\frac{C_b\pi^2E}{\left(\frac{L_b}{r_{ts}}\right)^2}$$
$$= (0.5)\frac{1.0(\pi)^2(29{,}000\text{ ksi})}{\left(\frac{72.0\text{ in.}}{0.543\text{ in.}}\right)^2}$$
$$= 8.14\text{ ksi}$$

$$F_n = F_{cr} \le F_y \quad \text{(}Spec\text{. Eq. F12-3)}$$
$$= 8.14\text{ ksi} \le 36\text{ ksi} \quad \textbf{o.k.}$$

$$M_n = F_nS_{min} \quad \text{(}Spec\text{. Eq. F12-1)}$$
$$= 8.14\text{ ksi}(13.2\text{ in.}^3)$$
$$= 107\text{ kip-in.}$$

The lateral-torsional buckling limit state controls.

M_n = 107 kip-in. or 8.95 kip-ft

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_bM_n = 0.90(8.95\text{ kip-ft})$ $= 8.06\text{ kip-ft} > 7.70\text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{8.95\text{ kip-ft}}{1.67}$ $= 5.36\text{ kip-ft} > 5.06\text{ kip-ft}$ **o.k.**

Because the beam is loaded through the shear center, consideration of a torsional moment is unnecessary. If the loading produced torsion, the torsional effects should be evaluated using AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997).

CHAPTER F DESIGN EXAMPLE REFERENCES

AISI (2007), *North American Specification for the Design of Cold-Formed Steel Structural Members*, ANSI/AISI Standard S100, Washington D.C.

Seaburg, P.A. and Carter, C.J. (1997), *Torsional Analysis of Structural Steel Members*, Design Guide 9, AISC, Chicago, IL.

Chapter G
Design of Members for Shear

INTRODUCTION

This chapter covers webs of singly or doubly symmetric members subject to shear in the plane of the web, single angles, HSS sections, and shear in the weak direction of singly or doubly symmetric shapes.

Most of the equations from this chapter are illustrated by example. Tables for all standard ASTM A992 W-shapes and ASTM A36 channels are included in the AISC *Manual*. In the tables, where applicable, LRFD and ASD shear information is presented side-by-side for quick selection, design and verification.

LRFD and ASD will produce identical designs for the case where the live load effect is approximately three times the dead load effect.

G1. GENERAL PROVISIONS

The design shear strength, $\phi_v V_n$, and the allowable shear strength, V_n/Ω_v, are determined as follows:

V_n = nominal shear strength based on shear yielding or shear buckling

$$V_n = 0.6F_y A_w C_v \quad \text{(Spec. Eq. G2-1)}$$

$\phi_v = 0.90$ (LRFD) $\qquad \Omega_v = 1.67$ (ASD)

Exception: For all current ASTM A6, W, S and HP shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26 and W12×14 for F_y = 50 ksi:

$\phi_v = 1.00$ (LRFD) $\qquad \Omega_v = 1.50$ (ASD)

AISC *Specification* Section G2 does not utilize tension field action. AISC *Specification* Section G3 specifically addresses the use of tension field action.

Strong axis shear values are tabulated for W-shapes in AISC *Manual* Tables 3-2 and 3-6, for S-shapes in AISC *Manual* Table 3-7, for C-shapes in AISC *Manual* Table 3-8, and for MC-shapes in AISC *Manual* Table 3-9. Weak axis shear values for W-shapes, S-shapes, C-shapes and MC-shapes, and shear values for angles, rectangular HSS and box members, and round HSS are not tabulated.

G2. MEMBERS WITH UNSTIFFENED OR STIFFENED WEBS

As indicated in the User Note of this section, virtually all W, S and HP shapes are not subject to shear buckling and are also eligible for the more liberal safety and resistance factors, ϕ_v = 1.00 (LRFD) and Ω_v = 1.50 (ASD). This is presented in Example G.1 for a W-shape. A channel shear strength design is presented in Example G.2.

G3. TENSION FIELD ACTION

A built-up girder with a thin web and transverse stiffeners is presented in Example G.8.

G4. SINGLE ANGLES

Rolled angles are typically made from ASTM A36 steel. A single angle example is illustrated in Example G.3.

G5. RECTANGULAR HSS AND BOX-SHAPED MEMBERS

The shear height, h, is taken as the clear distance between the flanges less the inside corner radius on each side. If the corner radii are unknown, h shall be taken as the corresponding outside dimension minus 3 times the thickness. A rectangular HSS example is provided in Example G.4.

G6. ROUND HSS

For all round HSS and pipes of ordinary length listed in the AISC *Manual*, F_{cr} can be taken as $0.6F_y$ in AISC *Specification* Equation G6-1. A round HSS example is illustrated in Example G.5.

G7. WEAK AXIS SHEAR IN DOUBLY SYMMETRIC AND SINGLY SYMMETRIC SHAPES

For examples of weak axis shear, see Example G.6 and Example G.7.

G8. BEAMS AND GIRDERS WITH WEB OPENINGS

For a beam and girder with web openings example, see AISC Design Guide 2, *Steel and Composite Beams with Web Openings* (Darwin, 1990).

EXAMPLE G.1A W-SHAPE IN STRONG AXIS SHEAR

Given:

Determine the available shear strength and adequacy of a W24×62 ASTM A992 beam using the AISC *Manual* with end shears of 48 kips from dead load and 145 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
V_u = 1.2(48.0 kips) + 1.6(145 kips) = 290 kips	V_a = 48.0 kips + 145 kips = 193 kips

From AISC *Manual* Table 3-2, the available shear strength is:

LRFD	ASD
$\phi_v V_n$ = 306 kips 306 kips > 290 kips **o.k.**	$\dfrac{V_n}{\Omega_v}$ = 204 kips 204 kips > 193 kips **o.k.**

EXAMPLE G.1B W-SHAPE IN STRONG AXIS SHEAR

Given:

The available shear strength, which can be easily determined by the tabulated values of the AISC *Manual*, can be verified by directly applying the provisions of the AISC *Specification*. Determine the available shear strength for the W-shape in Example G.1A by applying the provisions of the AISC *Specification*.

Solution:

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W24×62
$d = 23.7$ in.
$t_w = 0.430$ in.

Except for very few sections, which are listed in the User Note, AISC *Specification* Section G2.1(a) is applicable to the I-shaped beams published in the AISC *Manual* for $F_y = 50$ ksi.

$C_v = 1.0$ (*Spec*. Eq. G2-2)

Calculate A_w.

$A_w = dt_w$ from AISC *Specification* Section G2.1b
$= 23.7 \text{ in.}(0.430 \text{ in.})$
$= 10.2 \text{ in.}^2$

Calculate V_n.

$V_n = 0.6F_yA_wC_v$ (*Spec*. Eq. G2-1)
$= 0.6(50 \text{ ksi})(10.2 \text{ in.}^2)(1.0)$
$= 306$ kips

From AISC *Specification* Section G2.1a, the available shear strength is:

LRFD	ASD
$\phi_v = 1.00$ $\phi_vV_n = 1.00(306 \text{ kips})$ $= 306$ kips	$\Omega_v = 1.50$ $\dfrac{V_n}{\Omega_v} = \dfrac{306 \text{ kips}}{1.50}$ $= 204$ kips

EXAMPLE G.2A C-SHAPE IN STRONG AXIS SHEAR

Given:

Verify the available shear strength and adequacy of a C15×33.9 ASTM A36 channel with end shears of 17.5 kips from dead load and 52.5 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
V_u = 1.2(17.5 kips) + 1.6(52.5 kips) = 105 kips	V_a = 17.5 kips + 52.5 kips = 70.0 kips

From AISC *Manual* Table 3-8, the available shear strength is:

LRFD	ASD
$\phi_v V_n$ = 117 kips 117 kips > 105 kips **o.k.**	$\dfrac{V_n}{\Omega_v}$ = 77.6 kips 77.6 kips > 70.0 kips **o.k.**

EXAMPLE G.2B C-SHAPE IN STRONG AXIS SHEAR

Given:

The available shear strength, which can be easily determined by the tabulated values of the AISC *Manual*, can be verified by directly applying the provisions of the AISC *Specification*. Determine the available shear strength for the channel in Example G.2A.

Solution:

From AISC *Manual* Table 1-5, the geometric properties are as follows:

C15×33.9
d = 15.0 in.
t_w = 0.400 in.

AISC *Specification* Equation G2-1 is applicable. All ASTM A36 channels listed in the AISC *Manual* have $h/t_w \leq 1.10\sqrt{k_v E / F_y}$; therefore,

$C_v = 1.0$ (*Spec*. Eq. G2-3)

Calculate A_w.

$A_w = dt_w$ from AISC *Specification* Section G2.1b
$= 15.0 \text{ in.}(0.400 \text{ in.})$
$= 6.00 \text{ in.}^2$

Calculate V_n.

$V_n = 0.6F_yA_wC_v$ (*Spec*. Eq. G2-1)
$= 0.6(36 \text{ ksi})(6.00 \text{ in.}^2)(1.0)$
$= 130 \text{ kips}$

Available Shear Strength

The values of ϕ_v = 1.00 (LRFD) and Ω_v = 1.50 (ASD) do not apply to channels. The general values ϕ_v = 0.90 (LRFD) and Ω_v = 1.67 (ASD) must be used.

LRFD	ASD
$\phi_v V_n = 0.90(130 \text{ kips})$ $= 117 \text{ kips}$	$\dfrac{V_n}{\Omega_v} = \dfrac{130 \text{ kips}}{1.67}$ $= 77.8 \text{ kips}$

EXAMPLE G.3 ANGLE IN SHEAR

Given:

Determine the available shear strength and adequacy of a L5×3×¼ (LLV) ASTM A36 with end shears of 3.50 kips from dead load and 10.5 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-7, the geometric properties are as follows:

L5×3×¼
b = 5.00 in.
t = ¼ in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
V_u = 1.2(3.50 kips) + 1.6(10.5 kips) = 21.0 kips	V_a = 3.50 kips + 10.5 kips = 14.0 kips

Note: There are no tables for angles in shear, but the available shear strength can be calculated according to AISC *Specification* Section G4, as follows.

AISC *Specification* Section G4 stipulates k_v = 1.2.

Calculate A_w.

$$\begin{aligned} A_w &= bt \\ &= 5.00\text{ in.}(\tfrac{1}{4}\text{ in.}) \\ &= 1.25\text{ in.}^2 \end{aligned}$$

Determine C_v from AISC *Specification* Section G2.1(b).

$$\begin{aligned} h/t_w &= b/t \\ &= 5.0\text{ in.}/\tfrac{1}{4}\text{ in.} \\ &= 20 \end{aligned}$$

$$\begin{aligned} 1.10\sqrt{k_v E / F_y} &= 1.10\sqrt{1.2(29{,}000\text{ ksi}/36\text{ ksi})} \\ &= 34.2 \end{aligned}$$

$20 < 34.2$; therefore, C_v = 1.0 (*Spec*. Eq. G2-3)

Calculate V_n.

$$\begin{aligned} V_n &= 0.6F_yA_wC_v \\ &= 0.6(36\text{ ksi})(1.25\text{ in.}^2)(1.0) \\ &= 27.0\text{ kips} \end{aligned}$$ (*Spec*. Eq. G2-1)

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(27.0 \text{ kips})$ $= 24.3 \text{ kips}$ 24.3 kips > 21.0 kips **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{27.0 \text{ kips}}{1.67}$ $= 16.2 \text{ kips}$ 16.2 kips > 14.0 kips **o.k.**

EXAMPLE G.4 RECTANGULAR HSS IN SHEAR

Given:

Determine the available shear strength and adequacy of an HSS6×4×⅜ ASTM A500 Grade B member with end shears of 11.0 kips from dead load and 33.0 kips from live load. The beam is oriented with the shear parallel to the 6 in. dimension.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS6×4×⅜
$H = 6.00$ in.
$B = 4.00$ in.
$t = 0.349$ in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
$V_u = 1.2(11.0\text{ kips}) + 1.6(33.0\text{ kips})$ $= 66.0$ kips	$V_a = 11.0\text{ kips} + 33.0\text{ kips}$ $= 44.0$ kips

Note: There are no AISC *Manual* Tables for shear in HSS shapes, but the available shear strength can be determined from AISC *Specification* Section G5, as follows.

Nominal Shear Strength

For rectangular HSS in shear, use AISC *Specification* Section G2.1 with $A_w = 2ht$ (per AISC *Specification* Section G5) and $k_v = 5$.

From AISC *Specification* Section G5, if the exact radius is unknown, h shall be taken as the corresponding outside dimension minus three times the design thickness.

$$h = H - 3t$$
$$= 6.00\text{ in.} - 3(0.349\text{ in.})$$
$$= 4.95\text{ in.}$$

$$\frac{h}{t_w} = \frac{4.95\text{ in.}}{0.349\text{ in.}}$$
$$= 14.2$$

$$1.10\sqrt{k_v E/F_y} = 1.10\sqrt{5(29{,}000\text{ ksi}/46\text{ ksi})}$$
$$= 61.8$$

$14.2 \le 61.8$, therefore, $C_v = 1.0$ (*Spec*. Eq. G2-3)

Note: Most standard HSS sections listed in the AISC *Manual* have $C_v = 1.0$ at $F_y \le 46$ ksi.

Calculate A_w.

$$A_w = 2ht$$
$$= 2(4.95 \text{ in.})(0.349 \text{ in.})$$
$$= 3.46 \text{ in.}^2$$

Calculate V_n.

$$V_n = 0.6F_yA_wC_v \quad \text{(Spec. Eq. G2-1)}$$
$$= 0.6(46 \text{ ksi})(3.46 \text{ in.}^2)(1.0)$$
$$= 95.5 \text{ kips}$$

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(95.5 \text{ kips})$ $= 86.0 \text{ kips}$ 86.0 kips > 66.0 kips **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{95.5 \text{ kips}}{1.67}$ $= 57.2 \text{ kips}$ 57.2 kips > 44.0 kips **o.k.**

EXAMPLE G.5 ROUND HSS IN SHEAR

Given:

Verify the available shear strength and adequacy of a round HSS16.000×0.375 ASTM A500 Grade B member spanning 32 ft with end shears of 30.0 kips from dead load and 90.0 kips from live load.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
F_y = 42 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-13, the geometric properties are as follows:

HSS16.000×0.375
D = 16.0 in.
t = 0.349 in.
A_g = 17.2 in.^{2}

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
V_u = 1.2(30.0 kips) + 1.6(90.0 kips) = 180 kips	V_a = 30.0 kips + 90.0 kips = 120 kips

There are no AISC *Manual* tables for round HSS in shear, but the available strength can be determined from AISC *Specification* Section G6, as follows:

Using AISC *Specification* Section G6, calculate F_{cr} as the larger of:

$$F_{cr} = \frac{1.60E}{\sqrt{\frac{L_v}{D}}\left(\frac{D}{t}\right)^{\frac{5}{4}}} \quad \text{where } L_v = \text{half the span} = 192 \text{ in.} \qquad (Spec.\ \text{Eq. G6-2a})$$

$$= \frac{1.60(29{,}000 \text{ ksi})}{\sqrt{\frac{192 \text{ in.}}{16.0 \text{ in.}}}\left(\frac{16.0 \text{ in.}}{0.349 \text{ in.}}\right)^{\frac{5}{4}}}$$

$$= 112 \text{ ksi}$$

or

$$F_{cr} = \frac{0.78E}{(D/t)^{\frac{3}{2}}} \qquad (Spec.\ \text{Eq. G6-2b})$$

$$= \frac{0.78(29{,}000 \text{ ksi})}{\left(\frac{16.0 \text{ in.}}{0.349 \text{ in.}}\right)^{\frac{3}{2}}}$$

$$= 72.9 \text{ ksi}$$

The maximum value of F_{cr} permitted is,

$$\begin{aligned} F_{cr} &= 0.6F_y \\ &= 0.6(42 \text{ ksi}) \\ &= 25.2 \text{ ksi} \quad \textbf{controls} \end{aligned}$$

Note: AISC *Specification* Equations G6-2a and G6-2b will not normally control for the sections published in the AISC *Manual* except when high strength steel is used or the span is unusually long.

Calculate V_n using AISC *Specification* Section G6.

$$\begin{aligned} V_n &= \frac{F_{cr} A_g}{2} \quad \text{(Spec. Eq. G6-1)} \\ &= \frac{(25.2 \text{ ksi})(17.2 \text{ in.}^2)}{2} \\ &= 217 \text{ kips} \end{aligned}$$

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(217 \text{ kips})$ $= 195 \text{ kips}$ 195 kips > 180 kips **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{217 \text{ kips}}{1.67}$ $= 130 \text{ kips}$ 130 kips > 120 kips **o.k.**

EXAMPLE G.6 DOUBLY SYMMETRIC SHAPE IN WEAK AXIS SHEAR

Given:

Verify the available shear strength and adequacy of a W21×48 ASTM A992 beam with end shears of 20.0 kips from dead load and 60.0 kips from live load in the weak direction.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×48
b_f = 8.14 in.
t_f = 0.430 in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
V_u = 1.2(20.0 kips) + 1.6(60.0 kips) = 120 kips	V_a = 20.0 kips + 60.0 kips = 80.0 kips

From AISC *Specification* Section G7, for weak axis shear, use AISC *Specification* Equation G2-1 and AISC *Specification* Section G2.1(b) with $A_w = b_f t_f$ for each flange, $h/t_w = b/t_f$, $b = b_f/2$ and $k_v = 1.2$.

Calculate A_w. (Multiply by 2 for both shear resisting elements.)

$$\begin{aligned} A_w &= 2b_f t_f \\ &= 2(8.14 \text{ in.})(0.430 \text{ in.}) \\ &= 7.00 \text{ in.}^2 \end{aligned}$$

Calculate C_v.

$$\begin{aligned} h/t_w &= b/t_f \\ &= \frac{(8.14 \text{ in.})/2}{0.430 \text{ in.}} \\ &= 9.47 \end{aligned}$$

$$\begin{aligned} 1.10\sqrt{k_v E/F_y} &= 1.10\sqrt{1.2(29{,}000 \text{ ksi}/50 \text{ ksi})} \\ &= 29.0 \geq 9.47\text{, therefore, } C_v = 1.0 \end{aligned}$$ (*Spec*. Eq. G2-3)

Note: For all ASTM A6 W-, S-, M- and HP-shapes when $F_y \leq$ 50 ksi, C_v = 1.0, except some M-shapes noted in the User Note at the end of AISC *Specification* Section G2.1.

Calculate V_n.

$$\begin{aligned} V_n &= 0.6F_y A_w C_v \\ &= 0.6(50 \text{ ksi})(7.00 \text{ in.}^2)(1.0) \\ &= 210 \text{ kips} \end{aligned}$$ (*Spec*. Eq. G2-1)

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(210 \text{ kips})$ $= 189 \text{ kips}$ 189 kips > 120 kips **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{210 \text{ kips}}{1.67}$ $= 126 \text{ kips}$ 126 kips > 80.0 kips **o.k.**

EXAMPLE G.7 SINGLY SYMMETRIC SHAPE IN WEAK AXIS SHEAR

Given:

Verify the available shear strength and adequacy of a C9×20 ASTM A36 channel with end shears of 5.00 kips from dead load and 15.0 kips from live load in the weak direction.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-5, the geometric properties are as follows:

C9×20
b_f = 2.65 in.
t_f = 0.413 in.

From Chapter 2 of ASCE/SEI 7, the required shear strength is:

LRFD	ASD
V_u = 1.2(5.00 kips) + 1.6(15.0 kips) = 30.0 kips	V_a = 5.00 kips + 15.0 kips = 20.0 kips

Note: There are no AISC *Manual* tables for weak-axis shear in channel sections, but the available strength can be determined from AISC *Specification* Section G7.

From AISC *Specification* Section G7, for weak axis shear, use AISC *Specification* Equation G2-1 and AISC *Specification* Section G2.1(b) with $A_w = b_f t_f$ for each flange, $h/t_w = b/t_f$, $b = b_f$ and $k_v = 1.2$.

Calculate A_w. (Multiply by 2 for both shear resisting elements.)

$$A_w = 2b_f t_f$$
$$= 2(2.65 \text{ in.})(0.413 \text{ in.})$$
$$= 2.19 \text{ in.}^2$$

Calculate C_v.

$$\frac{b_f}{t_f} = \frac{2.65 \text{ in.}}{0.413 \text{ in.}}$$
$$= 6.42$$

$$1.10\sqrt{k_v E/F_y} = 1.10\sqrt{1.2(29{,}000 \text{ ksi}/36 \text{ ksi})}$$
$$= 34.2 \geq 6.42, \text{ therefore, } C_v = 1.0$$ (*Spec*. Eq. G2-3)

Calculate V_n.

$$V_n = 0.6F_y A_w C_v$$ (*Spec*. Eq. G2-1)
$$= 0.6(36 \text{ ksi})(2.19 \text{ in.}^2)(1.0)$$
$$= 47.3 \text{ kips}$$

From AISC *Specification* Section G1, the available shear strength is:

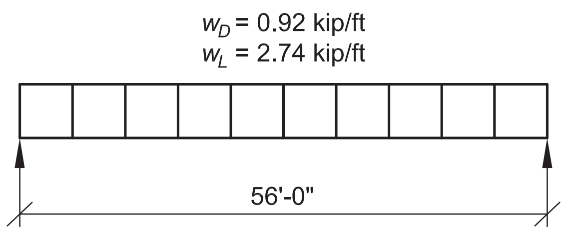
LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(47.3\text{ kips})$ $= 42.6\text{ kips}$ $42.6\text{ kips} > 30.0\text{ kips}$ **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{47.3\text{ kips}}{1.67}$ $= 28.3\text{ kips}$ $28.3\text{ kips} > 20.0\text{ kips}$ **o.k.**

EXAMPLE G.8A BUILT-UP GIRDER WITH TRANSVERSE STIFFENERS

Given:

A built-up ASTM A36 I-shaped girder spanning 56 ft has a uniformly distributed dead load of 0.920 klf and a live load of 2.74 klf in the strong direction. The girder is 36 in. deep with 12-in. × 1½-in. flanges and a 5⁄16-in. web. Determine if the member has sufficient available shear strength to support the end shear, without and with tension field action. Use transverse stiffeners, as required.

Note: This built-up girder was purposely selected with a thin web in order to illustrate the design of transverse stiffeners. A more conventionally proportioned plate girder would have at least a ½-in. web and slightly smaller flanges.



Beam loading and bracing diagram
(continuously braced)

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

The geometric properties are as follows:

Built-up girder
$t_w = 5/16$ in.
$d = 36.0$ in.
$b_{ft} = b_{fc} = 12.0$ in.
$t_f = 1\frac{1}{2}$ in.
$h = 33.0$ in.

From Chapter 2 of ASCE/SEI 7, the required shear strength at the support is:

LRFD	ASD
$R_u = wl/2$ $= [1.2(0.920 \text{ klf}) + 1.6(2.74 \text{ klf})](56.0 \text{ ft}/2)$ $= 154$ kips	$R_a = wl/2$ $= (0.920 \text{ klf} + 2.74 \text{ klf})(56.0 \text{ ft}/2)$ $= 102$ kips

Stiffener Requirement Check

$A_w = dt_w$ from AISC *Specification* Section G2.1(b)
$= 36.0 \text{ in.}(5/16 \text{ in.})$
$= 11.3 \text{ in.}^2$

$$\frac{h}{t_w} = \frac{33.0 \text{ in.}}{5/16 \text{ in.}}$$
$$= 106$$

106 < 260; therefore k_v = 5 for webs without transverse stiffeners from AISC *Specification* Section G2.1(b)

$$1.37\sqrt{k_v E / F_y} = 1.37\sqrt{5(29{,}000 \text{ ksi} / 36 \text{ ksi})}$$
$$= 86.9$$

106 > 86.9; therefore, use AISC *Specification* Equation G2-5 to calculate C_v

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \quad \text{(Spec. Eq. G2-5)}$$
$$= \frac{1.51(5)(29{,}000 \text{ ksi})}{(106)^2 (36 \text{ ksi})}$$
$$= 0.541$$

Calculate V_n.

$$V_n = 0.6 F_y A_w C_v \quad \text{(Spec. Eq. G2-1)}$$
$$= 0.6(36 \text{ ksi})(11.3 \text{ in.}^2)(0.541)$$
$$= 132 \text{ kips}$$

From AISC *Specification* Section G1, the available shear strength without stiffeners is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(132 \text{ kips})$ $= 119 \text{ kips}$ 119 kips < 154 kips **n.g.** **Therefore, stiffeners are required.**	$\Omega_v = 1.67$ $\frac{V_n}{\Omega_v} = \frac{132 \text{ kips}}{1.67}$ $= 79.0 \text{ kips}$ 79.0 kips < 102 kips **n.g.** **Therefore, stiffeners are required.**

Limits on the Use of Tension Field

AISC *Manual* Tables 3-16a and 3-16b can be used to select stiffener spacings needed to develop the required stress in the web.

From AISC *Specification* Section G3.1, consideration of tension field action is not permitted for any of the following conditions:

(a) end panels in all members with transverse stiffeners
(b) members when a/h exceeds 3.0 or $[260/(h/t_w)]^2$
(c) $2A_w/(A_{fc} + A_{ft}) > 2.5$; 2(11.3)/[2(12 in.)(1½ in.)] = 0.628 < 2.5
(d) h/b_{fc} or $h/b_{ft} > 6.0$; 33 in./12 in. = 2.75 < 6.0

Items (c) and (d) are satisfied by the configuration provided. Item (b) is accounted for in AISC *Manual* Tables 3-16a and 3-16b.

Stiffener Spacing for End Panel

Tension field action is not permitted for end panels, therefore use AISC *Manual* Table 3-16a.

LRFD	ASD
Use $V_u = \phi_v V_n$ to determine the required stress in the web by dividing by the web area. $\frac{\phi_v V_n}{A_w} = \frac{V_u}{A_w}$ $= \frac{154 \text{ kips}}{11.3 \text{ in.}^2}$ $= 13.6$ ksi	Use $V_a = V_n / \Omega_v$ to determine the required stress in the web by dividing by the web area. $\frac{V_n}{\Omega_v A_w} = \frac{V_a}{A_w}$ $= \frac{102 \text{ kips}}{11.3 \text{ in.}^2}$ $= 9.03$ ksi

Use Table 3-16a from the AISC *Manual* to select the required stiffener ratio a/h based on the h/t_w ratio of the girder and the required stress. Interpolate and follow an available stress curve, $\phi_v V_n/A_w$= 13.6 ksi for LRFD, $V_n/\Omega_v A_w$ = 9.03 ksi for ASD, until it intersects the horizontal line for a h/t_w value of 106. Project down from this intersection and take the maximum a/h value of 2.00 from the axis across the bottom. Because h = 33.0 in., stiffeners are required at (2.00)(33.0 in.) = 66.0 in. maximum. Conservatively, use 60.0 in. spacing.

Stiffener Spacing for the Second Panel

From AISC *Specification* Section G3.1, tension field action is allowed because the second panel is not an end panel.

The required shear strength at the start of the second panel, 60 in. from the end is:

LRFD	ASD
$V_u = 154 \text{ kips} - \left[1.2(0.920 \text{ klf}) + 1.6(2.74 \text{ klf})\right]\left(\frac{60.0 \text{ in.}}{12 \text{ in./ft}}\right)$ $= 127$ kips	$V_a = 102 \text{ kips} - (0.920 \text{ klf} + 2.74 \text{ klf})\left(\frac{60.0 \text{ in.}}{12 \text{ in./ft}}\right)$ $= 83.7$ kips

From AISC *Specification* Section G1, the available shear strength without stiffeners is:

LRFD	ASD
$\phi_v = 0.90$ From previous calculations, $\phi_v V_n = 119$ kips 119 kips < 127 kips **n.g.** **Therefore additional stiffeners are required.** Use $V_u = \phi_v V_n$ to determine the required stress in the web by dividing by the web area. $\frac{\phi_v V_n}{A_w} = \frac{V_u}{A_w}$ $= \frac{127 \text{ kips}}{11.3 \text{ in.}^2}$ $= 11.2$ ksi	$\Omega_v = 1.67$ From previous calculations, $\frac{V_n}{\Omega_v} = 79.0$ kips 79.0 kips < 83.7 kips **n.g.** **Therefore additional stiffeners are required.** Use $V_a = V_n / \Omega_v$ to determine the required stress in the web by dividing by the web area. $\frac{V_n}{\Omega_v A_w} = \frac{V_a}{A_w}$ $= \frac{83.7 \text{ kips}}{11.3 \text{ in.}^2}$ $= 7.41$ ksi

Use Table 3-16b from the AISC *Manual*, including tension field action, to select the required stiffener ratio a/h based on the h/t_w ratio of the girder and the required stress. Interpolate and follow an available stress curve, $\phi_v V_n/A_w$ = 11.2 ksi for LRFD, $V_n/\Omega_v A_w$ = 7.41 ksi for ASD, until it intersects the horizontal line for a h/t_w value of 106. Because the available stress does not intersect the h/t_w value of 106, the maximum value of 3.0 for a/h may be used. Because h = 33.0 in., an additional stiffener is required at (3.0)(33.0 in.) = 99.0 in. maximum from the previous one.

Stiffener Spacing for the Third Panel

From AISC *Specification* Section G3.1, tension field action is allowed because the next panel is not an end panel.

The required shear strength at the start of the third panel, 159 in. from the end is:

LRFD	ASD
$V_u = 154 \text{ kips} - [1.2(0.920 \text{ klf}) + 1.6(2.74 \text{ klf})] \times \left(\dfrac{159 \text{ in.}}{12 \text{ in./ft}}\right)$ $= 81.3 \text{ kips}$	$V_a = 102 \text{ kips} - (0.920 \text{ klf} + 2.74 \text{ klf})\left(\dfrac{159 \text{ in.}}{12 \text{ in./ft}}\right)$ $= 53.5 \text{ kips}$

From AISC *Specification* Section G1, the available shear strength without stiffeners is:

LRFD	ASD
$\phi_v = 0.90$ From previous calculations, $\phi_v V_n = 119 \text{ kips}$ 119 kips > 81.3 kips **o.k.** **Therefore additional stiffeners are not required.**	$\Omega_v = 1.67$ From previous calculations, $\dfrac{V_n}{\Omega_v} = 79.0 \text{ kips}$ 79.0 kips > 53.5 kips **o.k.** **Therefore additional stiffeners are not required.**

The four Available Shear Stress tables, AISC *Manual* Tables 3-16a, 3-16b, 3-17a and 3-17b, are useful because they permit a direct solution for the required stiffener spacing. Alternatively, you can select a stiffener spacing and check the resulting strength, although this process is likely to be iterative. In Example G.8B, the stiffener spacings used are taken from this example.

EXAMPLE G.8B BUILT-UP GIRDER WITH TRANSVERSE STIFFENERS

Given:

Verify the stiffener spacings from Example G.8A, which were easily determined from the tabulated values of the AISC *Manual*, by directly applying the provisions of the AISC *Specification*.

Solution:

Shear Strength of End Panel

Determine k_v based on AISC *Specification* Section G2.1(b) and check a/h limits.

$$a/h = \frac{60.0 \text{ in.}}{33.0 \text{ in.}} = 1.82$$

$$k_v = 5 + \frac{5}{(a/h)^2} \qquad \text{(Spec. Eq. G2-6)}$$

$$= 5 + \frac{5}{(1.82)^2} = 6.51$$

Based on AISC *Specification* Section G2.1, $k_v = 5$ when $a/h > 3.0$ or $a/h > \left[\frac{260}{(h/t_w)}\right]^2$

$$\frac{h}{t_w} = \frac{33.0 \text{ in.}}{5/16 \text{ in.}} = 106$$

$$a/h = 1.82 \leq 3.0$$

$$a/h = 1.82 \leq \left[\frac{260}{(h/t_w)}\right]^2$$

$$\left[\frac{260}{(h/t_w)}\right]^2 = \left[\frac{260}{106}\right]^2 = 6.02$$

$$1.82 \leq 6.02$$

Therefore, use $k_v = 6.51$.

Tension field action is not allowed because the panel is an end panel.

Because $h/t_w > 1.37\sqrt{k_v E/F_y}$

$$= 1.37\sqrt{6.51(29{,}000 \text{ ksi}/36 \text{ ksi})} = 99.2$$

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \qquad \text{(Spec. Eq. G2-5)}$$

$$= \frac{1.51(6.51)(29{,}000 \text{ ksi})}{(106)^2(36 \text{ ksi})} = 0.705$$

$V_n = 0.6F_yA_wC_v$ (*Spec*. Eq. G2-1)
$= 0.6(36 \text{ ksi})(11.3 \text{ in.}^2)(0.705)$
$= 172 \text{ kips}$

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(172 \text{ kips})$ $= 155 \text{ kips}$ 155 kips > 154 kips **o.k.**	$\Omega_v = 1.67$ $\dfrac{V_n}{\Omega_v} = \dfrac{172 \text{ kips}}{1.67}$ $= 103 \text{ kips}$ 103 kips > 102 kips **o.k.**

Shear Strength of the Second Panel (AISC Specification Section G2.1b)

Determine k_v and check a/h limits based on AISC *Specification* Section G2.1(b).

a/h for the second panel is 3.0

$$k_v = 5 + \frac{5}{(a/h)^2} \quad \text{(Spec. Eq. G2-6)}$$
$$= 5 + \frac{5}{(3.0)^2}$$
$$= 5.56$$

Check a/h limits.

$a/h = 3.00 \leq 3.0$

$$a/h = 3.00 \leq \left[\frac{260}{(h/t_w)}\right]^2$$
≤ 6.02 as previously calculated

Therefore, use $k_v = 5.56$.

Because $h/t_w > 1.37\sqrt{k_v E / F_y}$
$$= 1.37\sqrt{5.56(29{,}000 \text{ ksi}/36 \text{ ksi})}$$
$$= 91.7$$

$$C_v = \frac{1.51 k_v E}{(h/t_w)^2 F_y} \quad \text{(Spec. Eq. G2-5)}$$
$$= \frac{1.51(5.56)(29{,}000 \text{ ksi})}{(106)^2(36 \text{ ksi})}$$
$$= 0.602$$

Check the additional limits from AISC *Specification* Section G3.1 for the use of tension field action:

Note the limits of $a/h \le 3.0$ and $a/h \le [260/(h/t_w)]^2$ have already been calculated.

$$\frac{2A_w}{\left(A_{fc}+A_{ft}\right)} = \frac{2\left(11.3\text{ in.}^2\right)}{2\left(12.0\text{ in.}\right)\left(1\tfrac{1}{2}\text{ in.}\right)}$$
$$= 0.628 \le 2.5$$

$$\frac{h}{b_{fc}} = \frac{h}{b_{ft}}$$
$$= \frac{33.0\text{ in.}}{12.0\text{ in.}}$$
$$= 2.75 \le 6.0$$

Tension field action is permitted because the panel under consideration is not an end panel and the other limits indicated in AISC *Specification* Section G3.1 have been met.

From AISC *Specification* Section G3.2,

$$1.10\sqrt{k_v E / F_y} = 1.10\sqrt{5.56(29{,}000\text{ ksi} / 36\text{ ksi})}$$
$$= 73.6$$

because $h/t_w > 73.6$, use AISC *Specification* Equation G3-2

$$V_n = 0.6F_yA_w\left[C_v + \frac{1-C_v}{1.15\sqrt{1+(a/h)^2}}\right] \qquad (\textit{Spec}.\text{ Eq. G3-2})$$
$$= 0.6\left(36\text{ ksi}\right)\left(11.3\text{ in.}^2\right)\left[0.602 + \frac{1-0.602}{1.15\sqrt{1+\left(3.00\right)^2}}\right]$$
$$= 174\text{ kips}$$

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $\phi_v V_n = 0.90(174\text{ kips})$ $= 157\text{ kips}$ 157 kips > 127 kips **o.k.**	$\Omega_v = 1.67$ $\frac{V_n}{\Omega_v} = \frac{174\text{ kips}}{1.67}$ $= 104\text{ kips}$ 104 kips > 83.7 kips **o.k.**

CHAPTER G DESIGN EXAMPLE REFERENCES

Darwin, D. (1990), *Steel and Composite Beams with Web Openings,* Design Guide 2, AISC, Chicago, IL.

Chapter H
Design of Members for Combined Forces and Torsion

For all interaction equations in AISC *Specification* Chapter H, the required forces and moments must include second-order effects, as required by Chapter C of the AISC *Specification*. ASD users of the 1989 AISC *Specification* are accustomed to using an interaction equation that includes a partial second-order amplification. Second order effects are now calculated in the analysis and are not included in these interaction equations.

EXAMPLE H.1A W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING ABOUT BOTH AXES (BRACED FRAME)

Given:

Using AISC *Manual* Table 6-1, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b = 14.0$ ft.

LRFD	ASD
P_u = 400 kips	P_a = 267 kips
M_{ux} = 250 kip-ft	M_{ax} = 167 kip-ft
M_{uy} = 80.0 kip-ft	M_{ay} = 53.3 kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

The combined strength parameters from AISC *Manual* Table 6-1 are:

LRFD	ASD
$p = \dfrac{0.887}{10^3 \text{ kips}}$ at 14.0 ft	$p = \dfrac{1.33}{10^3 \text{ kips}}$ at 14.0 ft
$b_x = \dfrac{1.38}{10^3 \text{ kip-ft}}$ at 14.0 ft	$b_x = \dfrac{2.08}{10^3 \text{ kip-ft}}$ at 14.0 ft
$b_y = \dfrac{2.85}{10^3 \text{ kip-ft}}$	$b_y = \dfrac{4.29}{10^3 \text{ kip-ft}}$
Check limit for AISC *Specification* Equation H1-1a.	Check limit for AISC *Specification* Equation H1-1a.
From AISC *Manual* Part 6,	From AISC *Manual* Part 6,
$\dfrac{P_u}{\phi_c P_n} = pP_u = \left(\dfrac{0.887}{10^3 \text{ kips}}\right)(400 \text{ kips}) = 0.355$	$\dfrac{P_a}{P_n/\Omega_c} = pP_a = \left(\dfrac{1.33}{10^3 \text{ kips}}\right)(267 \text{ kips}) = 0.355$
Because $pP_u \geq 0.2$,	Because $pP_a \geq 0.2$,
$pP_u + b_xM_{ux} + b_yM_{uy} \leq 1.0$ (*Manual* Eq. 6-1)	$pP_a + b_xM_{ax} + b_yM_{ay} \leq 1.0$ (*Manual* Eq. 6-1)

LRFD	ASD
$0.355+\left(\dfrac{1.38}{10^3\text{ kip-ft}}\right)(250\text{ kip-ft})$ $+\left(\dfrac{2.85}{10^3\text{ kip-ft}}\right)(80.0\text{ kip-ft})\leq 1.0$ = 0.355 + 0.345 + 0.228 = 0.928 ≤ 1.0 **o.k.**	$0.355+\left(\dfrac{2.08}{10^3\text{ kip-ft}}\right)(167\text{ kip-ft})$ $+\left(\dfrac{4.29}{10^3\text{ kip-ft}}\right)(53.3\text{ kip-ft})\leq 1.0$ = 0.355 + 0.347 + 0.229 = 0.931 ≤ 1.0 **o.k.**

AISC *Manual* Table 6-1 simplifies the calculation of AISC *Specification* Equations H1-1a and H1-1b. A direct application of these equations is shown in Example H.1B.

EXAMPLE H.1B W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BRACED FRAME)

Given:

Using AISC *Manual* tables to determine the available compressive and flexural strengths, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b = 14.0$ ft.

LRFD	ASD
$P_u = 400$ kips $M_{ux} = 250$ kip-ft $M_{uy} = 80.0$ kip-ft	$P_a = 267$ kips $M_{ax} = 167$ kip-ft $M_{ay} = 53.3$ kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

The available axial and flexural strengths from AISC *Manual* Tables 4-1, 3-10 and 3-4 are:

LRFD	ASD
at $KL_y = 14.0$ ft, $P_c = \phi_c P_n = 1{,}130$ kips at $L_b = 14.0$ ft, $M_{cx} = \phi M_{nx} = 642$ kip-ft $M_{cy} = \phi M_{ny} = 311$ kip-ft $\dfrac{P_u}{\phi_c P_n} = \dfrac{400 \text{ kips}}{1{,}130 \text{ kips}} = 0.354$ Because $\dfrac{P_u}{\phi_c P_n} \geq 0.2$, use AISC *Specification* Equation H1-1a. $\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ $\dfrac{400 \text{ kips}}{1{,}130 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{250 \text{ kip-ft}}{642 \text{ kip-ft}} + \dfrac{80.0 \text{ kip-ft}}{311 \text{ kip-ft}}\right)$ $= 0.354 + \dfrac{8}{9}(0.389 + 0.257)$ $= 0.928 < 1.0$ **o.k.**	at $KL_y = 14.0$ ft, $P_c = \dfrac{P_n}{\Omega_c} = 750$ kips at $L_b = 14.0$ ft, $M_{cx} = M_{nx}/\Omega = 428$ kip-ft $M_{cy} = \dfrac{M_{ny}}{\Omega} = 207$ kip-ft $\dfrac{P_a}{P_n/\Omega_c} = \dfrac{267 \text{ kips}}{750 \text{ kips}} = 0.356$ Because $\dfrac{P_a}{P_n/\Omega_c} \geq 0.2$, use AISC *Specification* Equation H1-1a. $\dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ $\dfrac{267 \text{ kips}}{750 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{167 \text{ kip-ft}}{428 \text{ kip-ft}} + \dfrac{53.3 \text{ kip-ft}}{207 \text{ kip-ft}}\right)$ $= 0.356 + \dfrac{8}{9}(0.390 + 0.257)$ $= 0.931 < 1.0$ **o.k.**

EXAMPLE H.2 W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES (BY AISC *SPECIFICATION* SECTION H2)

Given:

Using AISC *Specification* Section H2, determine if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b$ = 14.0 ft. This example is included primarily to illustrate the use of AISC *Specification* Section H2.

LRFD	ASD
P_u = 360 kips M_{ux} = 250 kip-ft M_{uy} = 80.0 kip-ft	P_a = 240 kips M_{ax} = 167 kip-ft M_{ay} = 53.3 kip-ft

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×99
A = 29.1 in.^{2}
S_x = 157 in.^{3}
S_y = 55.2 in.^{3}

The required flexural and axial stresses are:

LRFD	ASD
$f_{ra} = \dfrac{P_u}{A}$ $= \dfrac{360\text{ kips}}{29.1\text{ in.}^2}$ $= 12.4\text{ ksi}$ $f_{rbx} = \dfrac{M_{ux}}{S_x}$ $= \dfrac{250\text{ kip-ft}}{157\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 19.1\text{ ksi}$	$f_{ra} = \dfrac{P_a}{A}$ $= \dfrac{240\text{ kips}}{29.1\text{ in.}^2}$ $= 8.25\text{ ksi}$ $f_{rbx} = \dfrac{M_{ax}}{S_x}$ $= \dfrac{167\text{ kip-ft}}{157\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 12.8\text{ ksi}$

LRFD	ASD
$f_{rby} = \dfrac{M_{uy}}{S_y}$ $= \dfrac{80.0\text{ kip-ft}}{55.2\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 17.4\text{ ksi}$	$f_{rby} = \dfrac{M_{ay}}{S_y}$ $= \dfrac{53.3\text{ kip-ft}}{55.2\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 11.6\text{ ksi}$

Calculate the available flexural and axial stresses from the available strengths in Example H.1B.

LRFD	ASD
$F_{ca} = \phi_c F_{cr}$ $= \dfrac{\phi_c P_n}{A}$ $= \dfrac{1{,}130\text{ kips}}{29.1\text{ in.}^2}$ $= 38.8\text{ ksi}$ $F_{cbx} = \dfrac{\phi_b M_{nx}}{S_x}$ $= \dfrac{642\text{ kip-ft}}{157\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 49.1\text{ ksi}$ $F_{cby} = \dfrac{\phi_b M_{ny}}{S_y}$ $= \dfrac{311\text{ kip-ft}}{55.2\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 67.6\text{ ksi}$	$F_{ca} = \dfrac{F_{cr}}{\Omega_c}$ $= \dfrac{P_n}{\Omega_c A}$ $= \dfrac{750\text{ kips}}{29.1\text{ in.}^2}$ $= 25.8\text{ ksi}$ $F_{cbx} = \dfrac{M_{nx}}{\Omega_b S_x}$ $= \dfrac{428\text{ kip-ft}}{157\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 32.7\text{ ksi}$ $F_{cby} = \dfrac{M_{ny}}{\Omega_b S_y}$ $= \dfrac{207\text{ kip-ft}}{55.2\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 45.0\text{ ksi}$

As shown in the LRFD calculation of F_{cby} in the preceding text, the available flexural stresses can exceed the yield stress in cases where the available strength is governed by yielding and the yielding strength is calculated using the plastic section modulus.

Combined Stress Ratio

From AISC *Specification* Section H2, check the combined stress ratios as follows:

LRFD	ASD
$\left\lvert \dfrac{f_{ra}}{F_{ca}} + \dfrac{f_{rbx}}{F_{cbx}} + \dfrac{f_{rby}}{F_{cby}} \right\rvert \le 1.0$ (from *Spec.* Eq. H2-1) $\left\lvert \dfrac{12.4\text{ ksi}}{38.8\text{ ksi}} + \dfrac{19.1\text{ ksi}}{49.1\text{ ksi}} + \dfrac{17.4\text{ ksi}}{67.6\text{ ksi}} \right\rvert = 0.966 \le 1.0$ **o.k.**	$\left\lvert \dfrac{f_{ra}}{F_{ca}} + \dfrac{f_{rbx}}{F_{cbx}} + \dfrac{f_{rby}}{F_{cby}} \right\rvert \le 1.0$ (from *Spec.* Eq. H2-1) $\left\lvert \dfrac{8.25\text{ ksi}}{25.8\text{ ksi}} + \dfrac{12.8\text{ ksi}}{32.7\text{ ksi}} + \dfrac{11.6\text{ ksi}}{45.0\text{ ksi}} \right\rvert = 0.969 \le 1.0$ **o.k.**

A comparison of these results with those from Example H.1B shows that AISC *Specification* Equation H1-1a will produce less conservative results than AISC *Specification* Equation H2-1 when its use is permitted.

Note: This check is made at a point on the cross-section (extreme fiber, in this example). The designer must therefore determine which point on the cross-section is critical, or check multiple points if the critical point cannot be readily determined.

EXAMPLE H.3 W-SHAPE SUBJECT TO COMBINED AXIAL TENSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 14-in. nominal depth to carry forces of 29.0 kips from dead load and 87.0 kips from live load in axial tension, as well as the following moments due to uniformly distributed loads:

M_{xD} = 32.0 kip-ft
M_{xL} = 96.0 kip-ft

M_{yD} = 11.3 kip-ft
M_{yL} = 33.8 kip-ft

The unbraced length is 30.0 ft and the ends are pinned. Assume the connections are made with no holes.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
P_u = 1.2(29.0 kips) + 1.6(87.0 kips) = 174 kips M_{ux} = 1.2(32.0 kip-ft) + 1.6(96.0 kip-ft) = 192 kip-ft M_{uy} = 1.2(11.3 kip-ft) + 1.6(33.8 kip-ft) = 67.6 kip-ft	P_a = 29.0 kips + 87.0 kips = 116 kips M_{ax} = 32.0 kip-ft + 96 kip-ft = 128 kip-ft M_{ay} = 11.3 kip-ft + 33.8 kip-ft = 45.1 kip-ft

Try a W14×82.

From AISC *Manual* Tables 1-1 and 3-2, the geometric properties are as follows:

W14×82
A = 24.0 in.^{2}
S_x = 123 in.^{3}
Z_x = 139 in.^{3}
S_y = 29.3 in.^{3}
Z_y = 44.8 in.^{3}
I_y = 148 in.^{4}
L_p = 8.76 ft
L_r = 33.2 ft

Nominal Tensile Strength

From AISC *Specification* Section D2(a), the nominal tensile strength due to tensile yielding on the gross section is:

$$P_n = F_yA_g \qquad \text{(Spec. Eq. D2-1)}$$
$$= 50 \text{ ksi}(24.0 \text{ in.}^2)$$
$$= 1{,}200 \text{ kips}$$

Note that for a member with holes, the rupture strength of the member would also have to be computed using AISC *Specification* Equation D2-2.

Nominal Flexural Strength for Bending About the X-X Axis

Yielding

From AISC *Specification* Section F2.1, the nominal flexural strength due to yielding (plastic moment) is:

$$\begin{aligned}M_{nx} &= M_p \\ &= F_y Z_x \qquad \text{(Spec. Eq. F2-1)} \\ &= 50 \text{ ksi}(139 \text{ in.}^3) \\ &= 6{,}950 \text{ kip-in.}\end{aligned}$$

Lateral-Torsional Buckling

From AISC *Specification* Section F2.2, the nominal flexural strength due to lateral-torsional buckling is determined as follows:

Because $L_p < L_b \leq L_r$, i.e., 8.76 ft < 30.0 ft < 33.2 ft, AISC *Specification* Equation F2-2 applies.

Lateral-Torsional Buckling Modification Factor, C_b

From AISC *Manual* Table 3-1, C_b = 1.14, without considering the beneficial effects of the tension force. However, per AISC *Specification* Section H1.2, C_b may be increased because the column is in axial tension.

$$\begin{aligned}P_{ey} &= \frac{\pi^2 EI_y}{L_b{}^2} \\ &= \frac{\pi^2(29{,}000 \text{ ksi})(148 \text{ in.}^4)}{\left[30.0 \text{ ft}(12.0 \text{ in./ft})\right]^2} \\ &= 327 \text{ kips}\end{aligned}$$

LRFD	ASD
$\sqrt{1+\frac{\alpha P_u}{P_{ey}}} = \sqrt{1+\frac{1.0(174 \text{ kips})}{327 \text{ kips}}} = 1.24$	$\sqrt{1+\frac{\alpha P_a}{P_{ey}}} = \sqrt{1+\frac{1.6(116 \text{ kips})}{327 \text{ kips}}} = 1.25$

$$\begin{aligned}C_b &= 1.24(1.14) \\ &= 1.41\end{aligned}$$

$$M_n = C_b\left[M_p - (M_p - 0.7F_yS_x)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \qquad \text{(Spec. Eq. F2-2)}$$

$$\begin{aligned}&= 1.41\left\{6{,}950 \text{ kip-in.} - \left[6{,}950 \text{ kip-in.} - 0.7(50 \text{ ksi})(123 \text{ in.}^3)\right]\left(\frac{30.0 \text{ ft} - 8.76 \text{ ft}}{33.2 \text{ ft} - 8.76 \text{ ft}}\right)\right\} \\ &= 6{,}560 \text{ kip-in.} \leq M_p\end{aligned}$$

Therefore, use M_n = 6,560 kip-in. or 547 kip-ft **controls**

Local Buckling

Per AISC *Manual* Table 1-1, the cross section is compact at F_y = 50 ksi; therefore, the local buckling limit state does not apply.

Nominal Flexural Strength for Bending About the Y-Y Axis and the Interaction of Flexure and Tension

Because a W14×82 has compact flanges, only the limit state of yielding applies for bending about the *y-y* axis.

$$M_{ny} = M_p = F_y Z_y \le 1.6 F_y S_y \quad \text{(Spec. Eq. F6-1)}$$
$$= 50 \text{ ksi}(44.8 \text{ in.}^3) \le 1.6(50 \text{ ksi})(29.3 \text{ in.}^3)$$
$$= 2{,}240 \text{ kip-in.} \le 2{,}340 \text{ kip-in.}$$

Therefore, use M_{ny} = 2,240 kip-in. or 187 kip-ft

Available Strength

From AISC *Specification* Sections D2 and F1, the available strength is:

LRFD	ASD
$\phi_b = \phi_t = 0.90$	$\Omega_b = \Omega_t = 1.67$
$P_c = \phi_t P_n$ $= 0.90(1{,}200 \text{ kips})$ $= 1{,}080 \text{ kips}$	$P_c = \dfrac{P_n}{\Omega_t}$ $= \dfrac{1{,}200 \text{ kips}}{1.67}$ $= 719 \text{ kips}$
$M_{cx} = \phi_b M_{nx}$ $= 0.90(547 \text{ kip-ft})$ $= 492 \text{ kip-ft}$	$M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= \dfrac{547 \text{ kip-ft}}{1.67}$ $= 328 \text{ kip-ft}$
$M_{cy} = \phi_b M_{ny}$ $= 0.90(187 \text{ kip-ft})$ $= 168 \text{ kip-ft}$	$M_{cy} = \dfrac{M_{ny}}{\Omega_b}$ $= \dfrac{187 \text{ kip-ft}}{1.67}$ $= 112 \text{ kip-ft}$

Interaction of Tension and Flexure

Check limit for AISC *Specification* Equation H1-1a.

LRFD	ASD
$\dfrac{P_r}{\phi_t P_n} = \dfrac{P_u}{\phi_t P_n}$ $= \dfrac{174 \text{ kips}}{1{,}080 \text{ kips}}$ $= 0.161 < 0.2$	$\dfrac{P_r}{P_n / \Omega_t} = \dfrac{P_a}{P_n / \Omega_t}$ $= \dfrac{116 \text{ kips}}{719 \text{ kips}}$ $= 0.161 < 0.2$

Therefore, AISC *Specification* Equation H1-1b applies.

$$\frac{P_r}{2P_c}+\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\leq 1.0$$ (*Spec*. Eq. H1-1b)

LRFD	ASD
$\frac{174\text{ kips}}{2(1{,}080\text{ kips})}+\frac{192\text{ kip-ft}}{492\text{ kip-ft}}+\frac{67.6\text{ kip-ft}}{168\text{ kip-ft}}\leq 1.0$	$\frac{116\text{ kips}}{2(719\text{ kips})}+\frac{128\text{ kip-ft}}{328\text{ kip-ft}}+\frac{45.1\text{ kip-ft}}{112\text{ kip-ft}}\leq 1.0$
$0.873\leq 1.0$ **o.k.**	$0.874\leq 1.0$ **o.k.**

EXAMPLE H.4 W-SHAPE SUBJECT TO COMBINED AXIAL COMPRESSION AND FLEXURE

Given:

Select an ASTM A992 W-shape with a 10-in. nominal depth to carry axial compression forces of 5.00 kips from dead load and 15.0 kips from live load. The unbraced length is 14.0 ft and the ends are pinned. The member also has the following required moment strengths due to uniformly distributed loads, not including second-order effects:

M_{xD} = 15 kip-ft
M_{xL} = 45 kip-ft

M_{yD} = 2 kip-ft
M_{yL} = 6 kip-ft

The member is not subject to sidesway (no lateral translation).

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From Chapter 2 of ASCE/SEI 7, the required strength (not considering second-order effects) is:

LRFD	ASD
P_u = 1.2(5.00 kips) + 1.6(15.0 kips) = 30.0 kips	P_a = 5.00 kips + 15.0 kips = 20.0 kips
M_{ux} = 1.2(15.0 kip-ft) + 1.6(45.0 kip-ft) = 90.0 kip-ft	M_{ax} = 15.0 kip-ft + 45.0 kip-ft = 60.0 kip-ft
M_{uy} = 1.2(2.00 kip-ft) + 1.6(6.00 kip-ft) = 12.0 kip-ft	M_{ay} = 2.00 kip-ft + 6.00 kip-ft = 8.00 kip-ft

Try a W10×33.

From AISC *Manual* Tables 1-1 and 3-2, the geometric properties are as follows:

W10×33
A = 9.71 in.^{2}
S_x = 35.0 in.^{3}
Z_x = 38.8 in.^{3}
I_x = 171 in.^{4}
r_x = 4.19 in.
S_y = 9.20 in.^{3}
Z_y = 14.0 in.^{3}
I_y = 36.6 in.^{4}
r_y = 1.94 in.
L_p = 6.85 ft
L_r = 21.8 ft

Available Axial Strength

From AISC *Specification* Commentary Table C-A-7.1, for a pinned-pinned condition, K = 1.0.

Because $KL_x = KL_y$ = 14.0 ft and $r_x > r_y$, the y-y axis will govern.

From AISC *Manual* Table 4-1, the available axial strength is:

LRFD	ASD
$P_c = \phi_c P_n$ $= 253$ kips	$P_c = \dfrac{P_n}{\Omega_c}$ $= 168$ kips

Required Flexural Strength (including second-order amplification)

Use the approximate method of second-order analysis procedure from AISC *Specification* Appendix 8. Because the member is not subject to sidesway, only P-δ amplifiers need to be added.

$$B_1 = \frac{C_m}{1 - \alpha P_r / P_{e1}} \geq 1$$ (*Spec*. Eq. A-8-3)

$C_m = 1.0$

The x-x axis flexural magnifier is,

$$P_{e1} = \frac{\pi^2 EI_x}{(K_1 L_x)^2}$$ (from *Spec.* Eq. A-8-5)

$$= \frac{\pi^2 (29{,}000\text{ ksi})(171\text{ in.}^4)}{\left[(1.0)(14.0\text{ ft})(12\text{ in./ft})\right]^2}$$

$$= 1{,}730\text{ kips}$$

LRFD	ASD
$\alpha = 1.0$ $B_1 = \dfrac{1.0}{1 - 1.0(30.0\text{ kips} / 1{,}730\text{ kips})}$ $= 1.02$ $M_{ux} = 1.02(90.0\text{ kip-ft})$ $= 91.8$ kip-ft	$\alpha = 1.6$ $B_1 = \dfrac{1.0}{1 - 1.6(20.0\text{ kips} / 1{,}730\text{ kips})}$ $= 1.02$ $M_{ax} = 1.02(60.0\text{ kip-ft})$ $= 61.2$ kip-ft

The y-y axis flexural magnifier is,

$$P_{e1} = \frac{\pi^2 EI_y}{(K_1 L_y)^2}$$ (from *Spec.* Eq. A-8-5)

$$= \frac{\pi^2 (29{,}000\text{ ksi})(36.6\text{ in.}^4)}{\left[(1.0)(14.0\text{ ft})(12\text{ in./ft})\right]^2}$$

$$= 371\text{ kips}$$

LRFD	ASD
$\alpha = 1.0$ $B_1 = \dfrac{1.0}{1-1.0(30.0\text{ kips}/371\text{ kips})}$ $= 1.09$ $M_{uy} = 1.09(12.0\text{ kip-ft})$ $= 13.1\text{ kip-ft}$	$\alpha = 1.6$ $B_1 = \dfrac{1.0}{1-1.6(20.0\text{ kips}/371\text{ kips})}$ $= 1.09$ $M_{ay} = 1.09(8.00\text{ kip-ft})$ $= 8.72\text{ kip-ft}$

Nominal Flexural Strength about the X-X Axis

Yielding

$$M_{nx} = M_p = F_yZ_x \qquad (\textit{Spec}.\text{ Eq. F2-1})$$
$$= 50\text{ ksi}(38.8\text{ in.}^3)$$
$$= 1{,}940\text{ kip-in}$$

Lateral-Torsional Buckling

Because $L_p < L_b \leq L_r$, i.e., 6.85 ft < 14.0 ft < 21.8 ft, AISC *Specification* Equation F2-2 applies.

From AISC *Manual* Table 3-1, $C_b = 1.14$

$$M_{nx} = C_b\left[M_p - \left(M_p - 0.7F_yS_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)\right] \leq M_p \qquad (\textit{Spec}.\text{ Eq. F2-2})$$

$$= 1.14\left\{1{,}940\text{ kip-in.} - \left[1{,}940\text{ kip-in.} - 0.7(50\text{ ksi})(35.0\text{ in.}^3)\right]\left(\frac{14.0\text{ ft} - 6.85\text{ ft}}{21.8\text{ ft} - 6.85\text{ ft}}\right)\right\}$$
$$= 1{,}820\text{ kip-in.} \leq 1{,}940\text{ kip-in.}$$

Therefore, use M_{nx} = 1,820 kip-in. or 152 kip-ft **controls**

Local Buckling

Per AISC *Manual* Table 1-1, the member is compact for F_y = 50 ksi, so the local buckling limit state does not apply.

Nominal Flexural Strength about the Y-Y Axis

Determine the nominal flexural strength for bending about the *y-y* axis from AISC *Specification* Section F6. Because a W10×33 has compact flanges, only the yielding limit state applies.

From AISC *Specification* Section F6.2,

$$M_{ny} = M_p = F_yZ_y \leq 1.6F_yS_y \qquad (\textit{Spec}.\text{ Eq. F6-1})$$
$$= 50\text{ ksi}(14.0\text{ in.}^3) \leq 1.6(50\text{ ksi})(9.20\text{ in.}^3)$$
$$= 700\text{ kip-in} \leq 736\text{ kip-in.}$$

Therefore, use M_{ny} = 700 kip-in. or 58.3 kip-ft

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $M_{cx} = \phi_b M_{nx}$ $= 0.90(152\text{ kip-ft})$ $= 137\text{ kip-ft}$ $M_{cy} = \phi_b M_{ny}$ $= 0.90(58.3\text{ kip-ft})$ $= 52.5\text{ kip-ft}$	$\Omega_b = 1.67$ $M_{cx} = \dfrac{M_{nx}}{\Omega_b}$ $= \dfrac{152\text{ kip-ft}}{1.67}$ $= 91.0\text{ kip-ft}$ $M_{cy} = \dfrac{M_{ny}}{\Omega_b}$ $= \dfrac{58.3\text{ kip-ft}}{1.67}$ $= 34.9\text{ kip-ft}$

Check limit for AISC *Specification* Equations H1-1a and H1-1b.

LRFD	ASD
$\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{30.0\text{ kips}}{253\text{ kips}}$ $= 0.119 < 0.2$, therefore, use AISC *Specification* Equation H1-1b $\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1b) $\dfrac{30.0\text{ kips}}{2(253\text{ kips})} + \left(\dfrac{91.8\text{ kip-ft}}{137\text{ kip-ft}} + \dfrac{13.1\text{ kip-ft}}{52.5\text{ kip-ft}}\right)$ $0.0593 + 0.920 = 0.979 \leq 1.0$ **o.k.**	$\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{20.0\text{ kips}}{168\text{ kips}}$ $= 0.119 < 0.2$, therefore, use AISC *Specification* Equation H1-1b $\dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \leq 1.0$ (*Spec*. Eq. H1-1b) $\dfrac{20.0\text{ kips}}{2(168\text{ kips})} + \left(\dfrac{61.2\text{ kip-ft}}{91.0\text{ kip-ft}} + \dfrac{8.72\text{ kip-ft}}{34.9\text{ kip-ft}}\right)$ $0.0595 + 0.922 = 0.982 \leq 1.0$ **o.k.**

EXAMPLE H.5A RECTANGULAR HSS TORSIONAL STRENGTH

Given:

Determine the available torsional strength of an ASTM A500 Grade B HSS6×4×¼.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS6×4×¼
$h/t = 22.8$
$b/t = 14.2$
$t = 0.233$ in.
$C = 10.1$ in.^{3}

The available torsional strength for rectangular HSS is stipulated in AISC *Specification* Section H3.1(b).

$h/t > b/t$, therefore, h/t governs

$$h/t \leq 2.45\sqrt{\frac{E}{F_y}}$$

$$22.8 \leq 2.45\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}$$
$$= 61.5\text{, therefore, use AISC } \textit{Specification} \text{ Equation H3-3}$$

$$F_{cr} = 0.6F_y \qquad (\textit{Spec}.\ \text{Eq. H3-3})$$
$$= 0.6(46 \text{ ksi})$$
$$= 27.6 \text{ ksi}$$

The nominal torsional strength is,

$$T_n = F_{cr}C \qquad (\textit{Spec}.\ \text{Eq. H3-1})$$
$$= 27.6 \text{ ksi } (10.1 \text{ in.}^3)$$
$$= 279 \text{ kip-in.}$$

From AISC *Specification* Section H3.1, the available torsional strength is:

LRFD	ASD
$\phi_T = 0.90$ $\phi_T T_n = 0.90(279 \text{ kip-in.})$ $= 251$ kip-in.	$\Omega_T = 1.67$ $\frac{T_n}{\Omega_T} = \frac{279 \text{ kip-in.}}{1.67}$ $= 167$ kip-in.

Note: For more complete guidance on designing for torsion, see AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997).

EXAMPLE H.5B ROUND HSS TORSIONAL STRENGTH

Given:

Determine the available torsional strength of an ASTM A500 Grade B HSS5.000×0.250 that is 14 ft long.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
F_y = 42 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-13, the geometric properties are as follows:

HSS5.000×0.250
D/t = 21.5
t = 0.233 in.
D = 5.00 in.
C = 7.95 in.^{3}

The available torsional strength for round HSS is stipulated in AISC *Specification* Section H3.1(a).

Calculate the critical stress as the larger of:

$$F_{cr} = \frac{1.23E}{\sqrt{\frac{L}{D}}\left(\frac{D}{t}\right)^{\frac{5}{4}}} \qquad (Spec.\ \text{Eq. H3-2a})$$

$$= \frac{1.23(29{,}000\text{ ksi})}{\sqrt{\frac{14.0\text{ ft }(12\text{ in./ft})}{5.00\text{ in.}}}(21.5)^{\frac{5}{4}}}$$

$$= 133\text{ ksi}$$

and

$$F_{cr} = \frac{0.60E}{\left(\frac{D}{t}\right)^{\frac{3}{2}}} \qquad (Spec.\ \text{Eq. H3-2b})$$

$$= \frac{0.60(29{,}000\text{ ksi})}{(21.5)^{\frac{3}{2}}}$$

$$= 175\text{ ksi}$$

However, F_{cr} shall not exceed the following:

$$0.6F_y = 0.6(42\text{ ksi}) = 25.2\text{ ksi}$$

Therefore, F_{cr} = 25.2 ksi.

The nominal torsional strength is,

$$T_n = F_{cr}C \qquad (Spec.\ \text{Eq. H3-1})$$
$$= 25.2 \text{ ksi } (7.95 \text{ in.}^3)$$
$$= 200 \text{ kip-in.}$$

From AISC *Specification* Section H3.1, the available torsional strength is:

LRFD	ASD
$\phi_T = 0.90$ $\phi_T T_n = 0.90(200 \text{ kip-in.})$ $= 180 \text{ kip-in.}$	$\Omega_T = 1.67$ $\dfrac{T_n}{\Omega_T} = \dfrac{200 \text{ kip-in.}}{1.67}$ $= 120 \text{ kip-in.}$

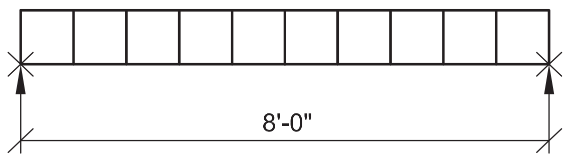
Note: For more complete guidance on designing for torsion, see AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997).

EXAMPLE H.5C RECTANGULAR HSS COMBINED TORSIONAL AND FLEXURAL STRENGTH

Given:

Verify the strength of an ASTM A500 Grade B HSS6×4×¼ loaded as shown. The beam is simply supported and is torsionally fixed at the ends. Bending is about the strong axis.

w_D = 0.460 kip/ft applied 6 in. off centerline
w_L = 1.38 kip/ft applied 6 in. off centerline



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS6×4×¼
$h/t = 22.8$
$b/t = 14.2$
$t = 0.233$ in.
$Z_x = 8.53$ in.^{3}

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$w_u = 1.2(0.460\text{ kip/ft}) + 1.6(1.38\text{ kip/ft})$ $= 2.76$ kip/ft	$w_a = 0.460\text{ kip/ft} + 1.38\text{ kip/ft}$ $= 1.84$ kip/ft

Calculate the maximum shear (at the supports) using AISC *Manual* Table 3-23, Case 1.

LRFD	ASD
$V_r = V_u$ $= \dfrac{w_u l}{2}$ $= \dfrac{2.76\text{ kip/ft}\left(8.00\text{ ft}\right)}{2}$ $= 11.0$ kips	$V_r = V_a$ $= \dfrac{w_a l}{2}$ $= \dfrac{1.84\text{ kip/ft}\left(8.00\text{ ft}\right)}{2}$ $= 7.36$ kips

Calculate the maximum torsion (at the supports).

LRFD	ASD
$T_r = T_u$ $= \dfrac{w_u le}{2}$	$T_r = T_a$ $= \dfrac{w_a le}{2}$

$= \dfrac{2.76\text{ kip/ft}\left(8.00\text{ ft}\right)\left(6.00\text{ in.}\right)}{2}$ $= 66.2\text{ kip-in.}$	$= \dfrac{1.84\text{ kip/ft}\left(8.00\text{ ft}\right)\left(6.00\text{ in.}\right)}{2}$ $= 44.2\text{ kip-in.}$

Available Shear Strength

Determine the available shear strength from AISC *Specification* Section G5.

$$h = 6.00\text{ in.} - 3(0.233\text{ in.}) = 5.30\text{ in.}$$

$A_w = 2ht$ from AISC *Specification* Section G5

$$\begin{aligned} A_w &= 2(5.30\text{ in.})(0.233\text{ in.}) \\ &= 2.47\text{ in.}^2 \end{aligned}$$

$$k_v = 5$$

The web shear coefficient is determined from AISC *Specification* Section G2.1(b).

$$\begin{aligned} \frac{h}{t_w} &= 22.8 \;\le\; 1.10\sqrt{\frac{k_v E}{F_y}} \\ &= 1.10\sqrt{\frac{5(29{,}000\text{ ksi})}{46\text{ ksi}}} \\ &= 61.8\text{, therefore, } C_v = 1.0 \end{aligned}$$

(*Spec.* Eq. G2-3)

The nominal shear strength from AISC *Specification* Section G2.1 is,

$$\begin{aligned} V_n &= 0.6F_yA_wC_v \\ &= 0.6(46\text{ ksi})(2.47\text{ in.}^2)(1.0) \\ &= 68.2\text{ kips} \end{aligned}$$

(*Spec.* Eq. G2-1)

From AISC *Specification* Section G1, the available shear strength is:

LRFD	ASD
$\phi_v = 0.90$ $V_c = \phi_v V_n$ $= 0.90(68.2\text{ kips})$ $= 61.4\text{ kips}$	$\Omega_v = 1.67$ $V_c = \dfrac{V_n}{\Omega_v}$ $= \dfrac{68.2\text{ kips}}{1.67}$ $= 40.8\text{ kips}$

Available Flexural Strength

The available flexural strength is determined from AISC *Specification* Section F7 for rectangular HSS. For the limit state of flexural yielding, the nominal flexural strength is,

$$\begin{aligned} M_n = M_p &= F_yZ_x \\ &= 46\text{ ksi}(8.53\text{ in.}^3) \\ &= 392\text{ kip-in.} \end{aligned}$$

(*Spec.* Eq. F7-1)

Determine if the limit state of flange local buckling applies as follows:

$$\lambda = \frac{b}{t}$$
$$= 14.2$$

Determine the flange compact slenderness limit from AISC *Specification* Table B4.1b Case 17.

$$\lambda_p = 1.12\sqrt{\frac{E}{F_y}}$$
$$= 1.12\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}$$
$$= 28.1$$

$\lambda < \lambda_p$; therefore, the flange is compact and the flange local buckling limit state does not apply.

Determine if the limit state of web local buckling applies as follows:

$$\lambda = \frac{h}{t}$$
$$= 22.8$$

Determine the web compact slenderness limit from AISC *Specification* Table B4.1b Case 19.

$$\lambda_p = 2.42\sqrt{\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}}$$
$$= 60.8$$

$\lambda < \lambda_p$; therefore, the web is compact and the web local buckling limit state does not apply.

Therefore, M_n = 392 kip-in., controlled by the flexural yielding limit state.

From AISC *Specification* Section F1, the available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $M_c = \phi_b M_n$ $= 0.90(392 \text{ kip-in.})$ $= 353 \text{ kip-in.}$	$\Omega_b = 1.67$ $M_c = \dfrac{M_n}{\Omega_b}$ $= \dfrac{392 \text{ kip-in.}}{1.67}$ $= 235 \text{ kip-in.}$

From Example H.5A, the available torsional strength is:

LRFD	ASD
$T_c = \phi_T T_n$ $= 0.90(279 \text{ kip-in.})$ $= 251 \text{ kip-in.}$	$T_c = \dfrac{T_n}{\Omega_T}$ $= \dfrac{279 \text{ kip-in.}}{1.67}$

Return to Table of Contents

	$= 167$ kip-in.

Using AISC *Specification* Section H3.2, check combined strength at several locations where $T_r > 0.2T_c$.

Check at the supports, the point of maximum shear and torsion.

LRFD	ASD
$\frac{T_r}{T_c} = \frac{66.2 \text{ kip-in.}}{251 \text{ kip-in.}}$ $= 0.264 > 0.20$ Therefore, use AISC *Specification* Equation H3-6 $\left(\frac{P_r}{P_c} + \frac{M_r}{M_c}\right) + \left(\frac{V_r}{V_c} + \frac{T_r}{T_c}\right)^2 \leq 1.0$ $(0+0) + \left(\frac{11.0 \text{ kips}}{61.4 \text{ kips}} + \frac{66.2 \text{ kip-in.}}{251 \text{ kip-in.}}\right)^2$ $= 0.196 \leq 1.0$ **o.k.**	$\frac{T_r}{T_c} = \frac{44.2 \text{ kip-in.}}{167 \text{ kip-in.}}$ $= 0.265 > 0.20$ Therefore, use AISC *Specification* Equation H3-6 $\left(\frac{P_r}{P_c} + \frac{M_r}{M_c}\right) + \left(\frac{V_r}{V_c} + \frac{T_r}{T_c}\right)^2 \leq 1.0$ $(0+0) + \left(\frac{7.36 \text{ kips}}{40.8 \text{ kips}} + \frac{44.2 \text{ kip-in.}}{167 \text{ kip-in.}}\right)^2$ $= 0.198 \leq 1.0$ **o.k.**

Check near the location where $T_r = 0.2T_c$. This is the location with the largest bending moment required to be considered in the interaction.

Calculate the shear and moment at this location, x.

LRFD	ASD
$x = \frac{66.2 \text{ kip-in.} - (0.20)(251 \text{ kip-in.})}{2.76 \text{ kip/ft}(6.00 \text{ in.})}$ $= 0.966$ ft $\frac{T_r}{T_c} = 0.20$ $V_r = 11.0 \text{ kips} - 0.966 \text{ ft}(2.76 \text{ kips/ft})$ $= 8.33 \text{ kips}$ $M_r = \frac{2.76 \text{ kip/ft}(0.966 \text{ ft})^2}{2} + 8.33 \text{ kips}(0.966 \text{ ft})$ $= 9.33 \text{ kip-ft} = 112$ kip-in. $\left(0 + \frac{112 \text{ kip-in.}}{353 \text{ kip-in.}}\right) + \left(\frac{8.33 \text{ kips}}{61.4 \text{ kips}} + 0.20\right)^2$ $= 0.430 \leq 1.0$ **o.k.**	$x = \frac{44.2 \text{ kip-in.} - (0.20)(167 \text{ kip-in.})}{1.84 \text{ kip/ft}(6.00 \text{ in.})}$ $= 0.978$ ft $\frac{T_r}{T_c} = 0.20$ $V_r = 7.36 \text{ kips} - 0.978 \text{ ft}(1.84 \text{ kips/ft})$ $= 5.56 \text{ kips}$ $M_r = \frac{1.84 \text{ kip/ft}(0.978 \text{ ft})^2}{2} + 5.56 \text{ kips}(0.978 \text{ ft})$ $= 6.32 \text{ kip-ft} = 75.8$ kip-in. $\left(0 + \frac{75.8 \text{ kip-in.}}{235 \text{ kip-in.}}\right) + \left(\frac{5.56 \text{ kips}}{40.8 \text{ kips}} + 0.20\right)^2$ $= 0.436 \leq 1.0$ **o.k.**

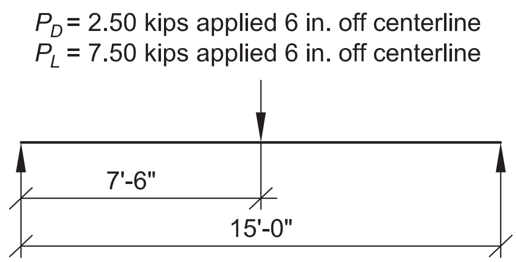
Note: The remainder of the beam, where $T_r \leq T_c$, must also be checked to determine if the strength without torsion controls over the interaction with torsion.

Return to Table of Contents

EXAMPLE H.6 W-SHAPE TORSIONAL STRENGTH

Given:

This design example is taken from AISC Design Guide 9, *Torsional Analysis of Structural Steel Members*. As shown in the following diagram, an ASTM A992 W10×49 spans 15 ft and supports concentrated loads at midspan that act at a 6-in. eccentricity with respect to the shear center. Determine the stresses on the cross section, the adequacy of the section to support the loads, and the maximum rotation.



Beam loading diagram

The end conditions are assumed to be flexurally pinned and unrestrained for warping torsion. The eccentric load can be resolved into a torsional moment and a load applied through the shear center.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W10×49
I_x = 272 in.^{4}
S_x = 54.6 in.^{3}
t_f = 0.560 in.
t_w = 0.340 in.
J = 1.39 in.^{4}
C_w = 2,070 in.^{6}
Z_x = 60.4 in.^{3}

From the AISC Shapes Database, the additional torsional properties are as follows:

W10×49
S_{w1} = 33.0 in.^{4}
W_{no} = 23.6 in.^{2}
Q_f = 12.8 in.^{3}
Q_w = 29.8 in.^{3}

From AISC Design Guide 9 (Seaburg and Carter, 1997), the torsional property, a, is calculated as follows:

$$a = \sqrt{\frac{EC_w}{GJ}}$$
$$= \sqrt{\frac{(29{,}000\text{ ksi})(2{,}070\text{ in.}^6)}{(11{,}200\text{ ksi})(1.39\text{ in.}^4)}}$$
$$= 62.1\text{ in.}$$

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$P_u = 1.2(2.50\text{ kips}) + 1.6(7.50\text{ kips})$ $= 15.0\text{ kips}$ $V_u = \dfrac{P_u}{2}$ $= \dfrac{15.0\text{ kips}}{2}$ $= 7.50\text{ kips}$ $M_u = \dfrac{P_u l}{4}$ $= \dfrac{15.0\text{ kips}(15.0\text{ ft})(12\text{ in./ft})}{4}$ $= 675\text{ kip-in.}$ $T_u = P_u e$ $= 15.0\text{ kips}(6.00\text{ in.})$ $= 90.0\text{ kip-in.}$	$P_a = 2.50\text{ kips} + 7.50\text{ kips}$ $= 10.0\text{ kips}$ $V_a = \dfrac{P_a}{2}$ $= \dfrac{10.0\text{ kips}}{2}$ $= 5.00\text{ kips}$ $M_a = \dfrac{P_a l}{4}$ $= \dfrac{10.0\text{ kips}(15.0\text{ ft})(12\text{ in./ft})}{4}$ $= 450\text{ kip-in.}$ $T_a = P_a e$ $= 10.0\text{ kips}(6.00\text{ in.})$ $= 60.0\text{ kip-in.}$

Normal and Shear Stresses from Flexure

The normal and shear stresses from flexure are determined from AISC Design Guide 9, as follows:

LRFD	ASD
$\sigma_{ub} = \dfrac{M_u}{S_x}$ (from Design Guide 9 Eq. 4.5) $= \dfrac{675\text{ kip-in.}}{54.6\text{ in.}^3}$ $= 12.4\text{ ksi}$ (compression at top, tension at bottom) $\tau_{ub\ web} = \dfrac{V_u Q_w}{I_x t_w}$ (from Design Guide 9 Eq. 4.6) $= \dfrac{7.50\text{ kips}\left(29.8\text{ in.}^3\right)}{272\text{ in.}^4\left(0.340\text{ in.}\right)}$ $= 2.42\text{ ksi}$ $\tau_{ub\ flange} = \dfrac{V_u Q_f}{I_x t_f}$ (from Design Guide 9 Eq. 4.6)	$\sigma_{ab} = \dfrac{M_a}{S_x}$ (from Design Guide 9 Eq. 4.5) $= \dfrac{450\text{ kip-in.}}{54.6\text{ in.}^3}$ $= 8.24\text{ ksi}$ (compression at top, tension at bottom) $\tau_{ab\ web} = \dfrac{V_a Q_w}{I_x t_w}$ (from Design Guide 9 Eq. 4.6) $= \dfrac{5.00\text{ kips}\left(29.8\text{ in.}^3\right)}{272\text{ in.}^4\left(0.340\text{ in.}\right)}$ $= 1.61\text{ ksi}$ $\tau_{ab\ flange} = \dfrac{V_a Q_f}{I_x t_f}$ (from Design Guide 9 Eq. 4.6)

$= \dfrac{7.50\text{ kips}\left(12.8\text{ in.}^3\right)}{272\text{ in.}^4\left(0.560\text{ in.}\right)}$ $= 0.630\text{ ksi}$	$= \dfrac{5.00\text{ kips}\left(12.8\text{ in.}^3\right)}{272\text{ in.}^4\left(0.560\text{ in.}\right)}$ $= 0.420\text{ ksi}$

Torsional Stresses

The following functions are taken from AISC Design Guide 9, *Torsional Analysis of Structural Steel Members*, Appendix B, Case 3, with $\alpha = 0.5$.

$$\frac{l}{a} = \frac{180\text{ in.}}{62.1\text{ in.}} = 2.90$$

At midspan ($z/l = 0.5$):

Using the graphs for θ, θ″, θ′ and θ‴, select values

For θ: $\theta \times \left(\dfrac{GJ}{T_r}\right)\left(\dfrac{1}{l}\right) = +0.09$ Solve for $\theta = +0.09\dfrac{T_r l}{GJ}$

For θ″: $\theta'' \times \left(\dfrac{GJ}{T_r}\right)a = -0.44$ Solve for $\theta'' = -0.44\dfrac{T_r}{GJa}$

For θ′: $\theta' \times \left(\dfrac{GJ}{T_r}\right) = 0$ Therefore $\theta' = 0$

For θ‴: $\theta''' \times \left(\dfrac{GJ}{T_r}\right)a^2 = -0.50$ Solve for $\theta''' = -0.50\dfrac{T_r}{GJa^2}$

At the support ($z/l = 0$):

For θ: $\theta \times \left(\dfrac{GJ}{T_r}\right)\left(\dfrac{1}{l}\right) = 0$ Therefore $\theta = 0$

For θ″: $\theta'' \times \left(\dfrac{GJ}{T_r}\right)a = 0$ Therefore $\theta'' = 0$

For θ′: $\theta' \times \left(\dfrac{GJ}{T_r}\right) = +0.28$ Solve for $\theta' = +0.28\dfrac{T_r}{GJ}$

For θ‴: $\theta''' \times \left(\dfrac{GJ}{T_r}\right)a^2 = -0.22$ Solve for $\theta''' = -0.22\dfrac{T_r}{GJa^2}$

In the preceding calculations, note that the applied torque is negative with the sign convention used.

Calculate T_r/GJ for use as follows:

LRFD	ASD
$\dfrac{T_u}{GJ} = \dfrac{-90.0\text{ kip-in.}}{11{,}200\text{ ksi}\left(1.39\text{ in.}^4\right)}$ $= -5.78 \times 10^{-3}\text{ rad/in.}$	$\dfrac{T_a}{GJ} = \dfrac{-60.0\text{ kip-in.}}{11{,}200\text{ ksi}\left(1.39\text{ in.}^4\right)}$ $= -3.85 \times 10^{-3}\text{ rad/in.}$

Shear Stresses Due to Pure Torsion

The shear stresses due to pure torsion are determined from AISC Design Guide 9 as follows:

$\tau_t = Gt\theta'$ (Design Guide 9 Eq. 4.1)

LRFD	ASD
At midspan: $\theta' = 0;\ \tau_{ut} = 0$ At the support, for the web: $\tau_{ut} = 11{,}200\text{ ksi}(0.340\text{ in.})(0.28)\left(\dfrac{-5.78\text{ rad}}{10^3\text{ in.}}\right)$ $= -6.16\text{ ksi}$ At the support, for the flange: $\tau_{ut} = 11{,}200\text{ ksi}(0.560\text{ in.})(0.28)\left(\dfrac{-5.78\text{ rad}}{10^3\text{ in.}}\right)$ $= -10.2\text{ ksi}$	At midspan: $\theta' = 0;\ \tau_{at} = 0$ At the support, for the web: $\tau_{at} = 11{,}200\text{ ksi}(0.340\text{ in.})(0.28)\left(\dfrac{-3.85\text{ rad}}{10^3\text{ in.}}\right)$ $= -4.11\text{ ksi}$ At the support, for the flange: $\tau_{at} = 11{,}200\text{ ksi}(0.560\text{ in.})(0.28)\left(\dfrac{-3.85\text{rad}}{10^3\text{ in.}}\right)$ $= -6.76\text{ ksi}$

Shear Stresses Due to Warping

The shear stresses due to warping are determined from AISC Design Guide 9 as follows:

$$\tau_w = \frac{-ES_{w1}\theta'''}{t_f}$$ (from Design Guide 9 Eq. 4.2a)

LRFD	ASD
At midspan: $\tau_{uw} = \dfrac{-29{,}000\text{ ksi}(33.0\text{ in.}^4)}{0.560\text{ in.}}\left[\dfrac{-0.50(-5.78\text{ rad})}{(62.1\text{ in.})^2(10^3\text{ in.})}\right]$ $= -1.28\text{ ksi}$ At the support: $\tau_{uw} = \dfrac{-29{,}000\text{ ksi}(33.0\text{ in.}^4)}{0.560\text{ in.}}\left[\dfrac{-0.22(-5.78\text{ rad})}{(62.1\text{ in.})^2(10^3\text{ in.})}\right]$ $= -0.563\text{ ksi}$	At midspan: $\tau_{aw} = \dfrac{-29{,}000\text{ ksi}(33.0\text{ in.}^4)}{0.560\text{ in.}}\left[\dfrac{-0.50(-3.85\text{ rad})}{(62.1\text{ in.})^2(10^3\text{ in.})}\right]$ $= -0.853\text{ ksi}$ At the support: $\tau_{aw} = \dfrac{-29{,}000\text{ ksi}(33.0\text{ in.}^4)}{0.560\text{ in.}}\left[\dfrac{-0.22(-3.85\text{ rad})}{(62.1\text{ in.})^2(10^3\text{in.})}\right]$ $= -0.375\text{ ksi}$

Normal Stresses Due to Warping

The normal stresses due to warping are determined from AISC Design Guide 9 as follows:

$\sigma_w = EW_{no}\theta''$ (from Design Guide 9 Eq. 4.3a)

LRFD	ASD
At midspan: $\sigma_{uw} = 29{,}000\text{ ksi}(23.6\text{ in.}^2)\left[\dfrac{-0.44(-5.78\text{ rad})}{62.1\text{ in.}(10^3\text{ in.})}\right]$ $= 28.0\text{ ksi}$ At the support: Because $\theta'' = 0,\ \sigma_{uw} = 0$	At midspan: $\sigma_{aw} = 29{,}000\text{ ksi}(23.6\text{in.}^2)\left[\dfrac{-0.44(-3.85\text{ rad})}{62.1\text{ in.}(10^3\text{ in.})}\right]$ $= 18.7\text{ ksi}$ At the support: Because $\theta'' = 0,\ \sigma_{aw} = 0$

Combined Stresses

The stresses are summarized in the following table and shown in Figure H.6-1.

Summary of Stresses Due to Flexure and Torsion, ksi														
	LFRD							ASD						
Location	Normal Stresses			Shear Stresses				Normal Stresses			Shear Stresses			
	σ_{uw}	σ_{ub}	f_{un}	τ_{ut}	τ_{uw}	τ_{ub}	f_{uv}	σ_{aw}	σ_{ab}	f_{an}	τ_{at}	τ_{aw}	τ_{ab}	f_{av}
Midspan														
Flange	±28.0	±12.4	±40.4	0	-1.28	±0.630	-1.91	±18.7	±8.24	±26.9	0	-0.853	±0.420	-1.27
Web	----	----	----	0	----	±2.42	±2.42	----	----	----	0	----	±1.61	±1.61
Support														
Flange	0	0	0	-10.2	-0.563	±0.630	-11.4	0	0	0	-6.76	-0.375	±0.420	-7.56
Web	----	----	----	-6.16	----	±2.42	-8.58	----	----	----	-4.11	----	±1.61	-5.72
Maximum			±40.4				-11.4			±26.9				-7.56

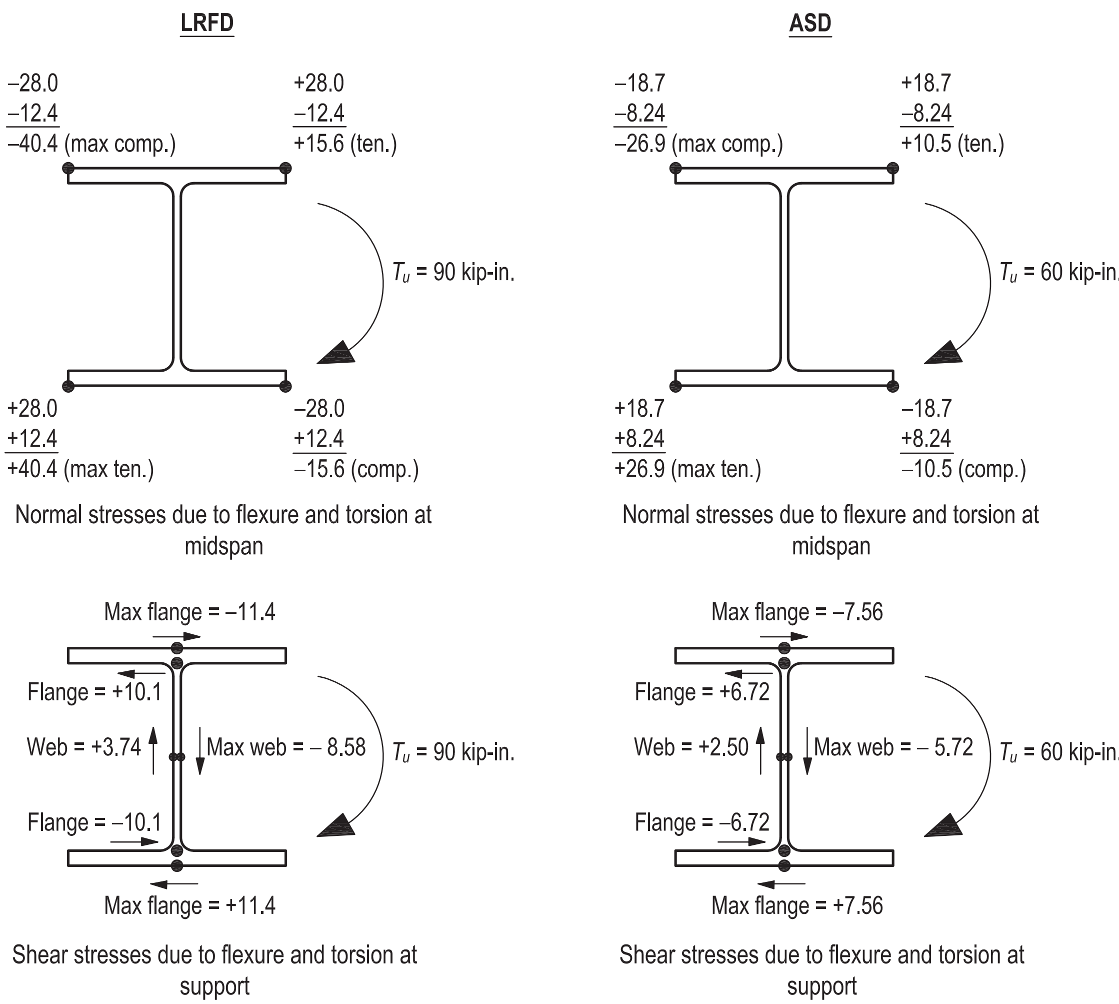


Fig. H.6-1. Stresses due to flexure and torsion.

LRFD	ASD
The maximum normal stress due to flexure and torsion occurs at the edge of the flange at midspan and is equal to 40.4 ksi. The maximum shear stress due to flexure and torsion occurs in the middle of the flange at the support and is equal to 11.4 ksi.	The maximum normal stress due to flexure and torsion occurs at the edge of the flange at midspan and is equal to 26.9 ksi. The maximum shear stress due to flexure and torsion occurs in the middle of the flange at the support and is equal to 7.56 ksi.

Available Torsional Strength

The available torsional strength is the lowest value determined for the limit states of yielding under normal stress, shear yielding under shear stress, or buckling in accordance with AISC *Specification* Section H3.3. The nominal torsional strength due to the limit states of yielding under normal stress and shear yielding under shear stress are compared to the applicable buckling limit states.

Buckling

For the buckling limit state, lateral-torsional buckling and local buckling must be evaluated. The nominal torsional strength due to the limit state of lateral-torsional buckling is determined as follows:

LRFD	ASD
C_b = 1.32 from AISC *Manual* Table 3-1. Compute F_n for a W10×49 using values from AISC *Manual* Table 3-10 with L_b = 15.0 ft and C_b = 1.0. $\phi_b M_n = 204$ kip-ft $F_n = F_{cr}$ (*Spec*. Eq. H3-9) $= C_b \dfrac{\phi_b M_n}{\phi_b S_x}$ $= 1.32 \dfrac{204\text{ kip-ft}}{0.90\left(54.6\text{ in.}^3\right)}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 65.8$ ksi	C_b = 1.32 from AISC *Manual* Table 3-1. Compute F_n for a W10×49 using values from AISC *Manual* Table 3-10 with L_b = 15.0 ft and C_b = 1.0. $\dfrac{M_n}{\Omega_b} = 136$ kip-ft $F_n = F_{cr}$ (*Spec*. Eq. H3-9) $= C_b \Omega_b \dfrac{M_n / \Omega_b}{S_x}$ $= 1.32\left(1.67\right)\dfrac{136\text{ kip-ft}}{54.6\text{ in.}^3}\left(\dfrac{12\text{ in.}}{\text{ft}}\right)$ $= 65.9$ ksi

The limit state of local buckling does not apply because a W10×49 is compact in flexure per the user note in AISC *Specification* Section F2.

Yielding Under Normal Stress

The nominal torsional strength due to the limit state of yielding under normal stress is determined as follows:

$$F_n = F_y = 50 \text{ ksi} \quad (\textit{Spec}.\ \text{Eq. H3-7})$$

Therefore, the limit state of yielding under normal stress controls over buckling. The available torsional strength for yielding under normal stress is determined as follows, from AISC *Specification* Section H3:

LRFD	ASD
$\phi_T = 0.90$ $\phi_T F_n = 0.90(50\text{ ksi})$ $= 45.0\text{ ksi} > 40.4\text{ ksi}$ **o.k.**	$\Omega_T = 1.67$ $\dfrac{F_n}{\Omega_T} = \dfrac{50\text{ ksi}}{1.67}$ $= 29.9\text{ ksi} > 26.9\text{ ksi}$ **o.k.**

Shear Yielding Under Shear Stress

The nominal torsional strength due to the limit state of shear yielding under shear stress is:

$$\begin{aligned} F_n &= 0.6F_y \\ &= 0.6(50\text{ ksi}) \\ &= 30\text{ ksi} \end{aligned}$$ (*Spec*. Eq. H3-8)

The limit state of shear yielding under shear stress controls over buckling. The available torsional strength for shear yielding under shear stress determined as follows, from AISC *Specification* Section H3:

LRFD	ASD
$\phi_T = 0.90$ $\phi_T F_n = 0.90(0.6)(50\text{ ksi})$ $= 27.0\text{ ksi} > 11.4\text{ ksi}$ **o.k.**	$\Omega_T = 1.67$ $\dfrac{F_n}{\Omega_T} = \dfrac{0.6(50\text{ ksi})}{1.67}$ $= 18.0\text{ ksi} > 7.56\text{ ksi}$ **o.k.**

Maximum Rotation at Service Load

The maximum rotation occurs at midspan. The service load torque is:

$$\begin{aligned} T &= Pe \\ &= -(2.50\text{ kips} + 7.50\text{ kips})(6.00\text{ in.}) \\ &= -60.0\text{ kip-in.} \end{aligned}$$

From AISC Design Guide 9, Appendix B, Case 3 with $\alpha = 0.5$, the maximum rotation is:

$$\begin{aligned} \theta &= +0.09\frac{Tl}{GJ} \\ &= \frac{0.09(-60.0\text{ kip-in.})(180\text{ in.})}{11{,}200\text{ ksi}(1.39\text{ in.}^4)} \\ &= -0.0624\text{ rads or } -3.58^\circ \end{aligned}$$

See AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* for additional guidance.

CHAPTER H DESIGN EXAMPLE REFERENCES

Seaburg, P.A. and Carter, C.J. (1997), *Torsional Analysis of Structural Steel Members,* Design Guide 9, AISC, Chicago, IL.

Chapter I
Design of Composite Members

I1. GENERAL PROVISIONS

Design, detailing, and material properties related to the concrete and steel reinforcing portions of composite members are governed by ACI 318 as modified with composite-specific provisions by the AISC *Specification*.

The available strength of composite sections may be calculated by one of two methods; the plastic stress distribution method, or the strain-compatibility method. The composite design tables in the *Steel Construction Manual* and the Examples are based on the plastic stress distribution method.

Filled composite sections are classified for local buckling according to the slenderness of the compression steel elements as illustrated in AISC *Specification* Table I1.1 and **Examples I.4, I.6 and I.7**. Local buckling effects do not need to be considered for encased composite members.

Terminology used within the Examples for filled composite section geometry is illustrated in Figure I-2.

I2. AXIAL FORCE

The available compressive strength of a composite member is based on a summation of the strengths of all of the components of the column with reductions applied for member slenderness and local buckling effects where applicable.

For tension members, the concrete tensile strength is ignored and only the strength of the steel member and properly connected reinforcing is permitted to be used in the calculation of available tensile strength.

The available compressive strengths given in AISC *Manual* Tables 4-13 through 4-20 reflect the requirements given in AISC *Specification* Sections I1.4 and I2.2. The design of filled composite compression and tension members is presented in **Examples I.4 and I.5.**

The design of encased composite compression and tension members is presented in **Examples I.9 and I.10.** There are no tables in the Manual for the design of these members.

Note that the AISC *Specification* stipulates that the available compressive strength need not be less than that specified for the bare steel member.

I3. FLEXURE

The design of typical composite beams with steel anchors is illustrated in **Examples I.1 and I.2**. AISC *Manual* Table 3-19 provides available flexural strengths for composite beams, Table 3-20 provides lower-bound moments of inertia for plastic composite sections, and Table 3-21 provides shear strengths of steel stud anchors utilized for composite action in composite beams.

The design of filled composite members for flexure is illustrated within **Examples I.6 and I.7**, and the design of encased composite members for flexure is illustrated within **Example I.11**.

I4. SHEAR

For composite beams with formed steel deck, the available shear strength is based upon the properties of the steel section alone in accordance with AISC *Specification* Chapter G as illustrated in **Examples I.1 and I.2**.

For filled and encased composite members, either the shear strength of the steel section alone, the steel section plus the reinforcing steel, or the reinforced concrete alone are permitted to be used in the calculation of available shear strength. The calculation of shear strength for filled composite members is illustrated within **Examples I.6 and I.7** and for encased composite members within **Example I.11**.

I5. COMBINED FLEXURE AND AXIAL FORCE

Design for combined axial force and flexure may be accomplished using either the strain compatibility method or the plastic-distribution method. Several different procedures for employing the plastic-distribution method are outlined in the *Commentary*, and each of these procedures is demonstrated for concrete filled members in **Example I.6** and for concrete encased members in **Example I.11**. Interaction calculations for non-compact and slender concrete filled members are illustrated in **Example I.7**.

To assist in developing the interaction curves illustrated within the design examples, a series of equations is provided in Figure I-1 (Geschwindner, 2010). These equations define selected points on the interaction curve, without consideration of slenderness effects. Figures I-1a through I-1d outline specific cases, and the applicability of the equations to a cross-section that differs should be carefully considered. As an example, the equations in Figure I-1a are appropriate for the case of side bars located at the centerline, but not for other side bar locations. In contrast, these equations are appropriate for any amount of reinforcing at the extreme reinforcing bar location. In Figure I-1b, the equations are appropriate only for the case of 4 reinforcing bars at the corners of the encased section. When design cases deviate from those presented the appropriate interaction equations can be derived from first principles.

I6. LOAD TRANSFER

The AISC *Specification* provides several requirements to ensure that the concrete and steel portions of the section act together. These requirements address both force allocation - how much of the applied loads are resisted by the steel versus the reinforced concrete, and force transfer mechanisms - how the force is transferred between the two materials. These requirements are illustrated in **Example I.3** for concrete filled members and **Example I.8** for encased composite members.

I7. COMPOSITE DIAPHRAGMS AND COLLECTOR BEAMS

The *Commentary* provides guidance on design methodologies for both composite diaphragms and composite collector beams.

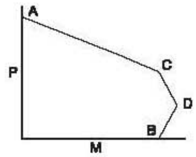
I8. STEEL ANCHORS

AISC *Specification* Section I8 addresses the strength of steel anchors in composite beams and in composite components. **Examples I.1 and I.2** illustrates the design of composite beams with steel headed stud anchors.

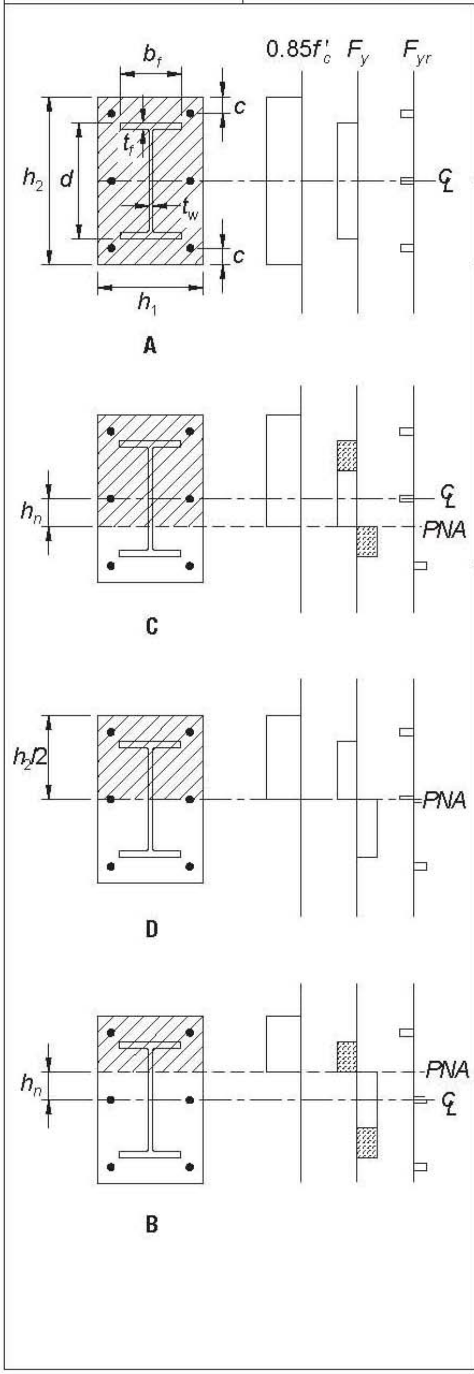
The application of steel anchors in composite component provisions have strict limitations as summarized in the User Note provided at the beginning of AISC *Specification* Section I8.3. These provisions do not apply to typical composite beam designs nor do they apply to hybrid construction where the steel and concrete do not resist loads together via composite action such as in embed plates. The most common application for these provisions is for the transfer of longitudinal shear within the load introduction length of composite columns as demonstrated in **Example I.8**. The application of these provisions to an isolated anchor within an applicable composite system is illustrated in **Example I.12**.

Return to Table of Contents

Plastic Capacities for Rectangular, Encased W-Shapes Bent About the *X-X* Axis



Section	Stress Distribution	Pt.	Defining Equations



A

$P_A = A_sF_y + A_{sr}F_{yr} + 0.85f'_cA_c$

$M_A = 0$

A_s = area of steel shape

A_{sr} = area of all continuous reinforcing bars

$A_c = h_1h_2 - A_s - A_{sr}$

C

$P_C = 0.85f'_cA_c$

$M_C = M_B$

D

$P_D = \dfrac{0.85f'_cA_c}{2}$

$M_D = Z_sF_y + Z_rF_{yr} + \dfrac{Z_c}{2}(0.85f'_c)$

Z_s = full *x*-axis plastic section modulus of steel shape

A_{srs} = area of continuous reinforcing bars at the centerline

$Z_r = (A_{sr} - A_{srs})\left(\dfrac{h_2}{2} - c\right)$

$Z_c = \dfrac{h_1h_2^2}{4} - Z_s - Z_r$

B

$P_B = 0$

$M_B = M_D - Z_{sn}F_y - \dfrac{Z_{cn}}{2}(0.85f'_c)$

$Z_{cn} = h_1h_n^2 - Z_{sn}$

For h_n below the flange $\left(h_n \le \dfrac{d}{2} - t_f\right)$

$$h_n = \frac{0.85f'_c(A_c + A_{srs}) - 2F_{yr}A_{srs}}{2\left[0.85f'_c(h_1 - t_w) + 2F_yt_w\right]}$$

$Z_{sn} = t_wh_n^2$

For h_n within the flange $\left(\dfrac{d}{2} - t_f < h_n \le \dfrac{d}{2}\right)$

$$h_n = \frac{0.85f'_c(A_c + A_s - db_f + A_{srs}) - 2F_y(A_s - db_f) - 2F_{yr}A_{srs}}{2\left[0.85f'_c(h_1 - b_f) + 2F_yb_f\right]}$$

$Z_{sn} = Z_s - b_f\left(\dfrac{d}{2} - h_n\right)\left(\dfrac{d}{2} + h_n\right)$

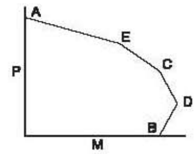
For h_n above the flange $\left(h_n > \dfrac{d}{2}\right)$

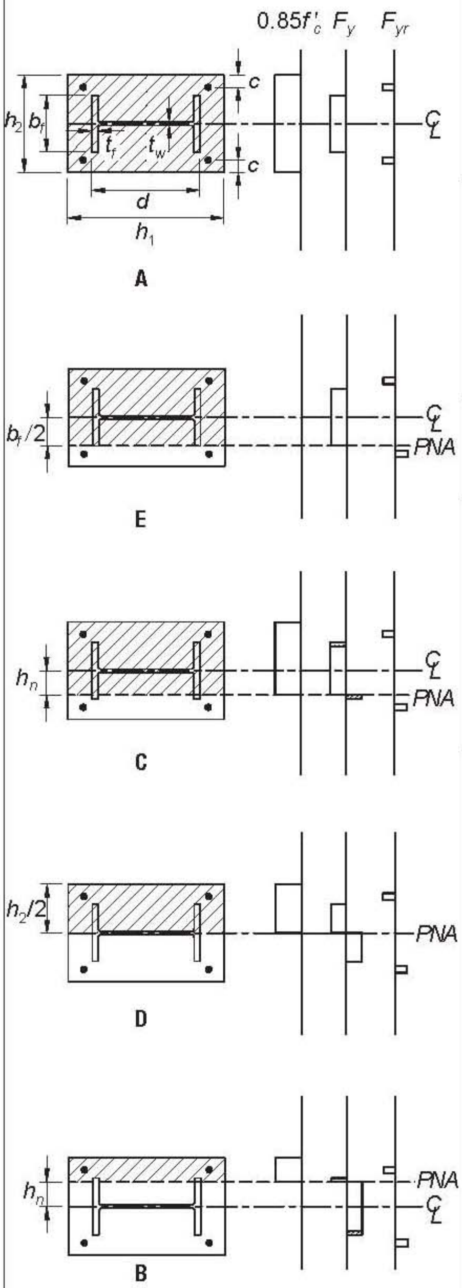
$$h_n = \frac{0.85f'_c(A_c + A_s + A_{srs}) - 2F_yA_s - 2F_{yr}A_{srs}}{2(0.85f'_ch_1)}$$

$Z_{sn} = Z_s$

Fig. I-1a. W-shapes, strong-axis anchor points.

Plastic Capacities for Rectangular, Encased W-Shapes Bent About the *Y-Y* Axis



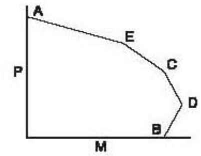


Pt.	Defining Equations
A	$P_A = A_sF_y + A_{sr}F_{yr} + 0.85f'_cA_c$ $M_A = 0$ A_s = area of steel shape A_{sr} = area of continuous reinforcing bars $A_c = h_1h_2 - A_s - A_{sr}$
E	$P_E = A_sF_y + (0.85f'_c)\left[A_c - \frac{h_1}{2}(h_2 - b_f) + \frac{A_{sr}}{2}\right]$ $M_E = M_D - Z_{sE}F_y - \frac{Z_{cE}}{2}(0.85f'_c)$ $Z_{sE} = Z_s$ = full *y*-axis plastic section modulus of steel shape $Z_{cE} = \frac{h_1b_f^2}{4} - Z_{sE}$
C	$P_C = 0.85f'_cA_c$ $M_C = M_B$
D	$P_D = \frac{0.85f'_cA_c}{2}$ $M_D = Z_sF_y + Z_rF_{sr} + \frac{Z_c}{2}(0.85f'_c)$ $Z_r = A_{sr}\left(\frac{h_2}{2} - c\right)$ $Z_c = \frac{h_1h_2^2}{4} - Z_s - Z_r$
B	$P_B = 0$ $M_B = M_D - Z_{sn}F_y - \frac{Z_{cn}}{2}(0.85f'_c)$ $Z_{cn} = h_1h_n^2 - Z_{sn}$ For h_n below the flange $\left(\frac{t_w}{2} < h_n \le \frac{b_f}{2}\right)$ $h_n = \frac{0.85f'_c(A_c + A_s - 2t_fb_f) - 2F_y(A_s - 2t_fb_f)}{2\left[4t_fF_y + (h_1 - 2t_f)0.85f'_c\right]}$ $Z_{sn} = Z_s - 2t_f\left(\frac{b_f}{2} + h_n\right)\left(\frac{b_f}{2} - h_n\right)$ For h_n above the flange $\left(h_n > \frac{b_f}{2}\right)$ $h_n = \frac{0.85f'_c(A_c + A_s) - 2F_yA_s}{2\left[0.85f'_ch_1\right]}$ $Z_{sn} = Z_s$

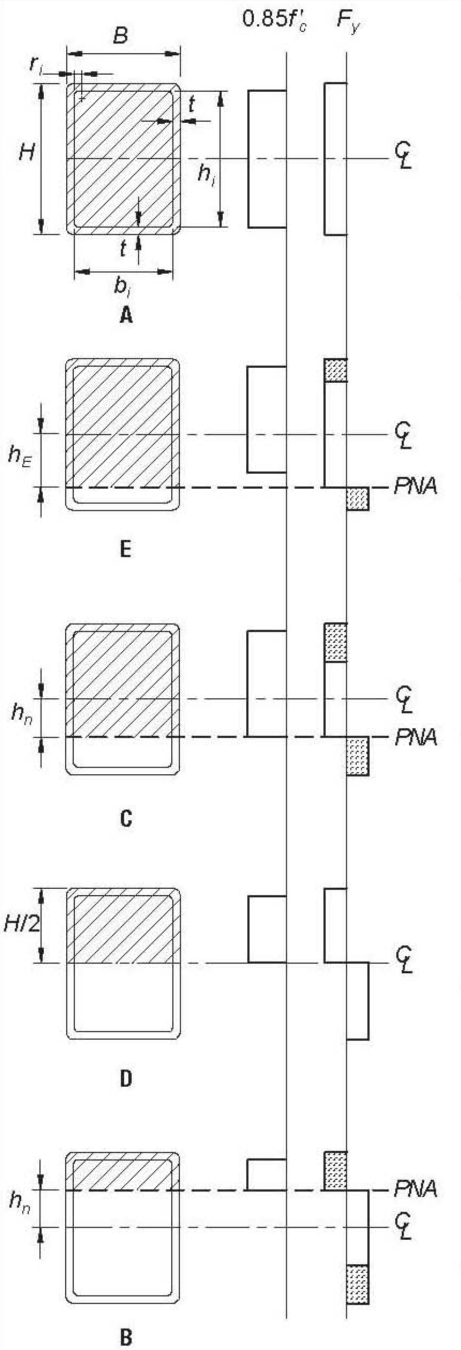
Fig. I-1b. W-shapes, weak-axis anchor points.

Return to Table of Contents

Plastic Capacities for Composite Filled HSS Bent About Either Principal Axis



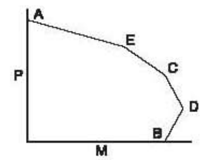
Section | **Stress Distribution**



Pt.	Defining Equations
A	$P_A = F_y A_s + 0.85 f'_c A_c$ $M_A = 0$ A_s = area of steel shape $A_c = b_i h_i - 0.858 r_i^2$ $b_i = B - 2t$ $h_i = H - 2t$ $r_i = t$
E	$P_E = \frac{0.85 f'_c A_c}{2} + 0.85 f'_c b_i h_E + 4 F_y t h_E$ $M_E = M_D - F_y Z_{sE} - \frac{0.85 f'_c Z_{cE}}{2}$ $Z_{cE} = b_i h_E^2$ $Z_{sE} = 2 t h_E^2$ $h_E = \frac{h_n}{2} + \frac{H}{4}$
C	$P_C = 0.85 f'_c A_c$ $M_C = M_B$
D	$P_D = \frac{0.85 f'_c A_c}{2}$ $M_D = F_y Z_s + \frac{0.85 f'_c Z_c}{2}$ Z_s = full *x*-axis plastic section modulus of HSS $Z_c = \frac{b_i h_i^2}{4} - 0.192 r_i^3$
B	$P_B = 0$ $M_B = M_D - F_y Z_{sn} - \frac{0.85 f'_c Z_{cn}}{2}$ $Z_{sn} = 2 t h_n^2$ $Z_{cn} = b_i h_n^2$ $h_n = \frac{0.85 f'_c A_c}{2\left[0.85 f'_c b_i + 4 t F_y\right]} \le \frac{h_i}{2}$
	Note: Equations in this table are equally applicable to bending about the shape's *X-X* axis (when $H \ge B$) and to bending about the shape's *Y-Y* axis (when $B > H$).

Fig. I-1c. Filled rectangular or square HSS, strong-axis anchor points.

Plastic Capacities for Composite Filled Round HSS Bent About Any Axis



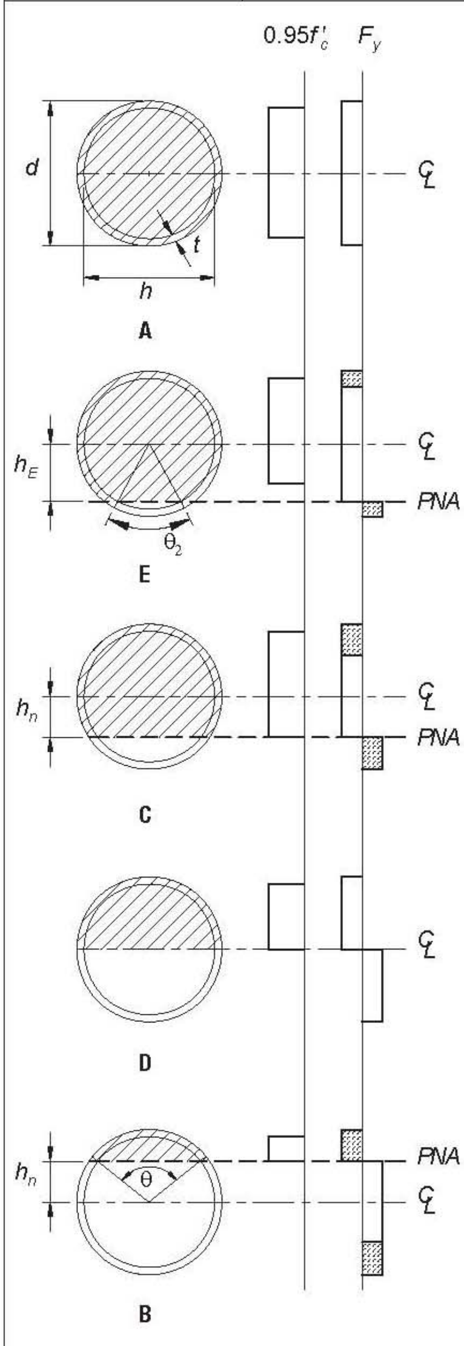
Section	Stress Distribution	Pt.	Defining Equations
		A	$P_A = F_y A_s + 0.95 f'_c A_c$ $M_A = 0$ $A_s = \pi\left(dt - t^2\right)$ $A_c = \dfrac{\pi h^2}{4}$
		E	$P_E = P_A - \frac{1}{4}\left[F_y\left(d^2 - h^2\right) + \dfrac{0.95 f'_c}{2} h^2\right]\left(\theta_2 - \sin\theta_2\right)$ $M_E = F_y Z_{sE} + \dfrac{0.95 f'_c Z_{cE}}{2}$ $Z_{cE} = \dfrac{h^3}{6}\sin^3\left(\dfrac{\theta_2}{2}\right)$ $Z_{sE} = \dfrac{\left(d^3 - h^3\right)}{6}\sin\left(\dfrac{\theta_2}{2}\right)$ $h_E = \dfrac{h_n}{2} + \dfrac{h}{4}$ $\theta_2 = \pi - 2\arcsin\left(\dfrac{2h_E}{h}\right)$
		C	$P_C = 0.95 f'_c A_c$ $M_C = M_B$
		D	$P_D = \dfrac{0.95 f'_c A_c}{2}$ $M_D = F_y Z_s + \dfrac{0.95 f'_c Z_c}{2}$ Z_s = plastic section modulus of steel shape $= \dfrac{d^3}{6} - Z_c$ $Z_c = \dfrac{h^3}{6}$
		B	$P_B = 0$ $M_B = F_y Z_{sB} + \dfrac{0.95 f'_c Z_{cB}}{2}$ $Z_{sB} = \dfrac{\left(d^3 - h^3\right)}{6}\sin\left(\dfrac{\theta}{2}\right)$ $Z_{cB} = \dfrac{h^3 \sin^3\left(\dfrac{\theta}{2}\right)}{6}$ $\theta = \dfrac{0.0260K_c - 2K_s}{0.0848K_c} + \dfrac{\sqrt{\left(0.0260K_c + 2K_s\right)^2 + 0.857K_c K_s}}{0.0848K_c}$ (rad) $K_c = f'_c h^2$ $K_s = F_y\left(\dfrac{d-t}{2}\right)t$ ("thin" HSS wall assumed) $h_n = \dfrac{h}{2}\sin\left(\dfrac{\pi - \theta}{2}\right) \le \dfrac{h}{2}$

Fig. I-1d. Filled round HSS anchor points.

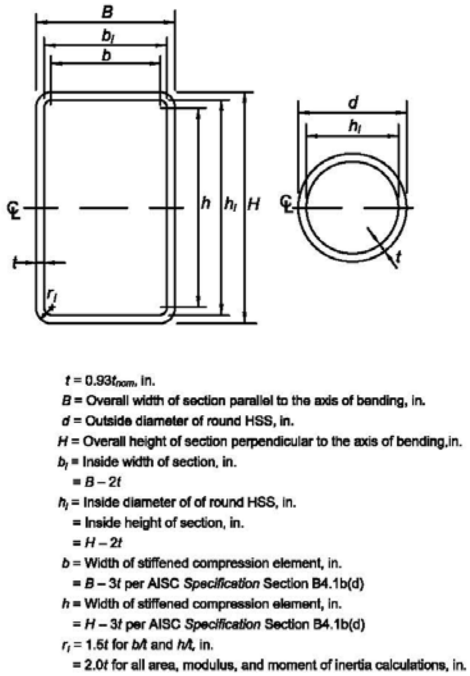


Fig. I-2. Terminology used for filled members.

REFERENCES

Geschwindner, L.F. (2010), "Discussion of Limit State Response of Composite Columns and Beam-Columns Part II: Application of Design Provisions for the 2005 AISC Specification," *Engineering Journal*, AISC, Vol. 47, No. 2, 2nd Quarter, pp. 131–140.

EXAMPLE I.1 COMPOSITE BEAM DESIGN

Given:

A typical bay of a composite floor system is illustrated in Figure I.1-1. Select an appropriate ASTM A992 W-shaped beam and determine the required number of ¾-in.-diameter steel headed stud anchors. The beam will not be shored during construction.

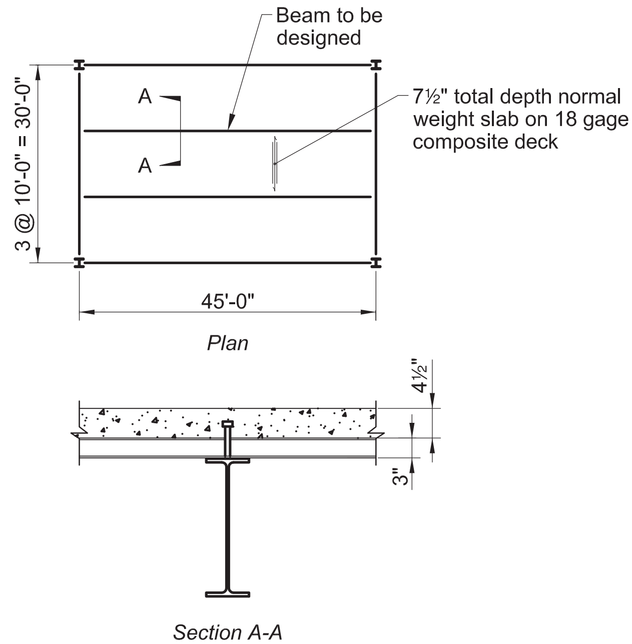


Fig. I.1-1. Composite bay and beam section.

To achieve a two-hour fire rating without the application of spray applied fire protection material to the composite deck, 4½ in. of normal weight (145 lb/ft^3) concrete will be placed above the top of the deck. The concrete has a specified compressive strength, $f_c' = 4$ ksi.

Applied loads are as follows:

Dead Loads:

Pre-composite:

Slab = 75 lb/ft^2 (in accordance with metal deck manufacturer's data)

Self weight = 5 lb/ft^2 (assumed uniform load to account for beam weight)

Composite (applied after composite action has been achieved):

Miscellaneous = 10 lb/ft^2 (HVAC, ceiling, floor covering, etc.)

Live Loads:

Pre-composite:

Construction = 25 lb/ft^2 (temporary loads during concrete placement)

Composite (applied after composite action has been achieved):

Non-reducible = 100 lb/ft^2 (assembly occupancy)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Applied Loads

For slabs that are to be placed at a constant elevation, AISC Design Guide 3 (West and Fisher, 2003) recommends an additional 10% of the nominal slab weight be applied to account for concrete ponding due to deflections resulting from the wet weight of the concrete during placement. For the slab under consideration, this would result in an additional load of 8 lb/ft^2; however, for this design the slab will be placed at a constant thickness thus no additional weight for concrete ponding is required.

For pre-composite construction live loading, 25 lb/ft^2 will be applied in accordance with recommendations from ASCE/SEI 37-02 *Design Loads on Structures During Construction* (ASCE, 2002) for a light duty operational class which includes concrete transport and placement by hose.

Composite Deck and Anchor Requirements

Check composite deck and anchor requirements stipulated in AISC *Specification* Sections I1.3, I3.2c and I8.

(1) Concrete Strength: $3 \text{ ksi} \le f'_c \le 10 \text{ ksi}$

$f'_c = 4 \text{ ksi}$ **o.k.**

(2) Rib height: $h_r \le 3 \text{ in.}$

$h_r = 3 \text{ in.}$ **o.k.**

(3) Average rib width: $w_r \ge 2 \text{ in.}$

$w_r = 6 \text{ in.}$ (from deck manufacturer's literature) **o.k.**

(4) Use steel headed stud anchors ¾ in. or less in diameter.

Use ¾-in.-diameter steel anchors per problem statement **o.k.**

(5) Steel headed stud anchor diameter: $d_{sa} \le 2.5\left(t_f\right)$

In accordance with AISC *Specification* Section I8.1, this limit only applies if steel headed stud anchors are not welded to the flange directly over the web. The ¾-in.-diameter anchors will be placed in pairs transverse to the web in some locations, thus this limit must be satisfied. Select a beam size with a minimum flange thickness of 0.30 in., as determined below:

$$t_f \ge \frac{d_{sa}}{2.5}$$

$$\frac{d_{sa}}{2.5} = \frac{\text{¾ in.}}{2.5}$$
$$= 0.30 \text{ in.}$$

(6) Steel headed stud anchors, after installation, shall extend not less than 1½ in. above the top of the steel deck.

A minimum anchor length of 4½ in. is required to meet this requirement for 3 in. deep deck. From steel headed stud anchor manufacturer's data, a standard stock length of 4⅞ in. is selected. Using a ⅜-in. length reduction to account for burn off during anchor installation through the deck yields a final installed length of 4½ in.

$$4\tfrac{1}{2}\text{ in.} = 4\tfrac{1}{2}\text{ in.} \quad \textbf{o.k.}$$

(7) Minimum length of stud anchors $= 4d_{sa}$

$$4\tfrac{1}{2}\text{ in.} > 4(\tfrac{3}{4}\text{ in.}) = 3.00\text{ in.} \quad \textbf{o.k.}$$

(8) There shall be at least ½ in. of specified concrete cover above the top of the headed stud anchors.

As discussed in AISC *Specification* Commentary to Section I3.2c, it is advisable to provide greater than ½ in. minimum cover to assure anchors are not exposed in the final condition, particularly for intentionally cambered beams.

$$7\tfrac{1}{2}\text{ in.} - 4\tfrac{1}{2}\text{ in.} = 3.00\text{ in.} > \tfrac{1}{2}\text{ in.} \quad \textbf{o.k.}$$

(9) Slab thickness above steel deck ≥ 2 in.

$$4\tfrac{1}{2}\text{ in.} > 2\text{ in.} \quad \textbf{o.k.}$$

Design for Pre-Composite Condition

Construction (Pre-Composite) Loads

The beam is uniformly loaded by its tributary width as follows:

$$\begin{aligned} w_D &= \left[(10\text{ ft})\left(75\text{ lb/ft}^2 + 5\text{ lb/ft}^2\right)\right](0.001\text{ kip/lb}) \\ &= 0.800\text{ kip/ft} \\ w_L &= \left[(10\text{ ft})\left(25\text{ lb/ft}^2\right)\right](0.001\text{ kip/lb}) \\ &= 0.250\text{ kip/ft} \end{aligned}$$

Construction (Pre-Composite) Flexural Strength

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.800\text{ kip/ft}) + 1.6(0.250\text{ kip/ft})$ $= 1.36\text{ kip/ft}$ $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(1.36\text{ kip/ft})(45\text{ ft})^2}{8}$ $= 344\text{ kip-ft}$	$w_a = 0.800\text{ kip/ft} + 0.250\text{ kip/ft}$ $= 1.05\text{ kip/ft}$ $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.05\text{ kip/ft})(45\text{ ft})^2}{8}$ $= 266\text{ kip-ft}$

Beam Selection

Assume that attachment of the deck perpendicular to the beam provides adequate bracing to the compression flange during construction, thus the beam can develop its full plastic moment capacity. The required plastic section modulus, Z_x, is determined as follows, from AISC *Specification* Equation F2-1:

LRFD	ASD
$\phi_b = 0.90$ $Z_{x,min} = \dfrac{M_u}{\phi_b F_y}$ $= \dfrac{(344 \text{ kip-ft})(12 \text{ in./ft})}{0.90(50 \text{ ksi})}$ $= 91.7 \text{ in.}^3$	$\Omega_b = 1.67$ $Z_{x,min} = \dfrac{\Omega_b M_a}{F_y}$ $= \dfrac{1.67(266 \text{ kip-ft})(12 \text{ in./ft})}{50 \text{ ksi}}$ $= 107 \text{ in.}^3$

From AISC *Manual* Table 3-2 select a W21×50 with a Z_x value of 110 in.^{3}

Note that for the member size chosen, the self weight on a pounds per square foot basis is $50 \text{ plf}/10 \text{ ft} = 5.00 \text{ psf}$; thus the initial self weight assumption is adequate.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W21×50
$A = 14.7 \text{ in.}^2$
$I_x = 984 \text{ in.}^4$

Pre-Composite Deflections

AISC Design Guide 3 recommends deflections due to concrete plus self weight not exceed the minimum of L/360 or 1.0 in.

From AISC *Manual* Table 3-23, Case 1:

$$\Delta_{nc} = \frac{5 w_D L^4}{384 EI}$$

Substituting for the moment of inertia of the non-composite section, $I = 984 \text{ in.}^4$, yields a dead load deflection of:

$$\Delta_{nc} = \frac{5\left[\dfrac{(0.800 \text{ kip/ft})}{12 \text{ in./ft}}\right]\left[(45.0 \text{ ft})(12 \text{ in./ft})\right]^4}{384(29{,}000 \text{ ksi})(984 \text{ in.}^4)}$$
$$= 2.59 \text{ in.}$$
$$= L/208 > L/360 \quad \textbf{n.g.}$$

Pre-composite deflections exceed the recommended limit. One possible solution is to increase the member size. A second solution is to induce camber into the member. For this example, the second solution is selected, and the beam will be cambered to reduce the net pre-composite deflections.

Reducing the estimated simple span deflections to 80% of the calculated value to reflect the partial restraint of the end connections as recommended in AISC Design Guide 3 yields a camber of:

$$\begin{aligned}\text{Camber} &= 0.8(2.59\text{ in.})\\ &= 2.07\text{ in.}\end{aligned}$$

Rounding down to the nearest ¼-in. increment yields a specified camber of 2 in.

Select a W21×50 with 2 in. of camber.

Design for Composite Condition

Required Flexural Strength

Using tributary area calculations, the total uniform loads (including pre-composite dead loads in addition to dead and live loads applied after composite action has been achieved) are determined as:

$$\begin{aligned}w_D &= \left[(10.0\text{ ft})\left(75\text{ lb/ft}^2 + 5\text{ lb/ft}^2 + 10\text{ lb/ft}^2\right)\right](0.001\text{ kip/lb})\\ &= 0.900\text{ kip/ft}\\ w_L &= \left[(10.0\text{ ft})\left(100\text{ lb/ft}^2\right)\right](0.001\text{ kip/lb})\\ &= 1.00\text{ kip/ft}\end{aligned}$$

From ASCE/SEI 7 Chapter 2, the required flexural strength is:

LRFD	ASD
$w_u = 1.2(0.900\text{ kip/ft}) + 1.6(1.00\text{ kip/ft})$ $= 2.68\text{ kip/ft}$ $M_u = \dfrac{w_u L^2}{8}$ $= \dfrac{(2.68\text{ kip/ft})(45.0\text{ ft})^2}{8}$ $= 678\text{ kip-ft}$	$w_a = 0.900\text{ kip/ft} + 1.00\text{ kip/ft}$ $= 1.90\text{ kip/ft}$ $M_a = \dfrac{w_a L^2}{8}$ $= \dfrac{(1.90\text{ kip/ft})(45.0\text{ ft})^2}{8}$ $= 481\text{ kip-ft}$

Determine b

The effective width of the concrete slab is the sum of the effective widths to each side of the beam centerline as determined by the minimum value of the three widths set forth in AISC *Specification* Section I3.1a:

(1) one-eighth of the beam span center-to-center of supports

$$\frac{45.0\text{ ft}}{8}(2\text{ sides}) = 11.3\text{ ft}$$

(2) one-half the distance to the centerline of the adjacent beam

$$\frac{10.0\text{ ft}}{2}(2\text{ sides}) = 10.0\text{ ft}\quad \textbf{controls}$$

(3) distance to the edge of the slab

not applicable for an interior member

Available Flexural Strength

According to AISC *Specification* Section I3.2a, the nominal flexural strength shall be determined from the plastic stress distribution on the composite section when $h / t_w \le 3.76\sqrt{E / F_y}$.

From AISC *Manual* Table 1-1, h/t_w for a W21×50 = 49.4.

$$
\begin{aligned}
49.4 &\le 3.76\sqrt{29{,}000 \text{ ksi} / 50 \text{ ksi}} \\
&\le 90.6
\end{aligned}
$$

Therefore, use the plastic stress distribution to determine the nominal flexural strength.

According to the User Note in AISC *Specification* Section I3.2a, this check is generally unnecessary as all current W-shapes satisfy this limit for $F_y \le 50$ ksi.

Flexural strength can be determined using AISC *Manual* Table 3-19 or calculated directly using the provisions of AISC *Specification* Chapter I. This design example illustrates the use of the *Manual* table only. For an illustration of the direct calculation procedure, refer to Design Example I.2.

To utilize AISC *Manual* Table 3-19, the distance from the compressive concrete flange force to beam top flange, $Y2$, must first be determined as illustrated by *Manual* Figure 3-3. Fifty percent composite action [$\Sigma Q_n \approx 0.50(A_s F_y)$] is used to calculate a trial value of the compression block depth, a_{trial}, for determining $Y2$ as follows:

$$
\begin{aligned}
a_{trial} &= \frac{\sum Q_n}{0.85 f_c' b} \qquad \text{(from } \textit{Manual}\text{. Eq. 3-7)} \\
&= \frac{0.50\left(A_s F_y\right)}{0.85 f_c' b} \\
&= \frac{0.50\left(14.7 \text{ in.}^2\right)\left(50 \text{ ksi}\right)}{0.85\left(4 \text{ ksi}\right)\left(10.0 \text{ ft}\right)\left(12 \text{ in./ft}\right)} \\
&= 0.90 \text{ in.} \rightarrow \text{say } 1.0 \text{ in.}
\end{aligned}
$$

Note that a trial value of $a = 1.0$ in. is a common starting point in many design problems.

$$
Y2 = Y_{con} - \frac{a_{trial}}{2} \qquad \text{(from } \textit{Manual}\text{. Eq. 3-6)}
$$

where

Y_{con} = distance from top of steel beam to top of slab, in.
= 7.50 in.

$$
\begin{aligned}
Y2 &= 7.50 \text{ in.} - \frac{1.0 \text{ in.}}{2} \\
&= 7.00 \text{ in.}
\end{aligned}
$$

Enter AISC *Manual* Table 3-19 with the required strength and $Y2$ = 7.00 in. to select a plastic neutral axis location for the W21×50 that provides sufficient available strength.

Selecting PNA location 5 (BFL) with $\sum Q_n = 386$ kips provides a flexural strength of:

LRFD	ASD
$\phi_b M_n \geq M_u$ $\phi_b M_n = 769\text{ kip-ft} \geq 678\text{ kip-ft}$ **o.k.**	$M_n / \Omega_b \geq M_a$ $M_n / \Omega_b = 512\text{ kip-ft} \geq 481\text{ kip-ft}$ **o.k.**

Based on the available flexural strength provided in Table 3-19, the required PNA location for ASD and LRFD design methodologies differ. This discrepancy is due to the live to dead load ratio in this example, which is not equal to the ratio of 3 at which ASD and LRFD design methodologies produce equivalent results as discussed in AISC *Specification* Commentary Section B3.4. The selected PNA location 5 is acceptable for ASD design, and more conservative for LRFD design.

The actual value for the compression block depth, a, is determined as follows:

$$a = \frac{\sum Q_n}{0.85 f_c' b} \qquad \text{(\textit{Manual} Eq. 3-7)}$$

$$= \frac{386\text{ kips}}{0.85(4\text{ ksi})(10\text{ ft})(12\text{ in./ft})}$$

$$= 0.946\text{ in.}$$

$$a = 0.946\text{ in.} < a_{trial} = 1.0\text{ in.} \quad \textbf{o.k.}$$

Live Load Deflection

Deflections due to live load applied after composite action has been achieved will be limited to $L/360$ under the design live load as required by Table 1604.3 of the 2009 *International Building Code* (IBC) (ICC, 2009), or 1 in. using a 50% reduction in design live load as recommended by AISC Design Guide 3.

Deflections for composite members may be determined using the lower bound moment of inertia provided by *Specification* Commentary Equation C-I3-1 and tabulated in AISC *Manual* Table 3-20. The *Specification* Commentary also provides an alternate method for determining deflections of a composite member through the calculation of an effective moment of inertia. This design example illustrates the use of the *Manual* table. For an illustration of the direct calculation procedure for each method, refer to Design Example I.2.

Entering Table 3-20, for a W21×50 with PNA location 5 and $Y2$ = 7.00 in., provides a lower bound moment of inertia of $I_{LB} = 2{,}520\text{ in.}^4$

Inserting I_{LB} into AISC *Manual* Table 3-23, Case 1, to determine the live load deflection under the full design live load for comparison to the IBC limit yields:

$$\Delta_c = \frac{5 w_L L^4}{384 E I_{LB}}$$

$$= \frac{5\left[\frac{(1.00\text{ kip/ft})}{12\text{ in./ft}}\right]\left[(45.0\text{ ft})(12\text{ in./ft})\right]^4}{384(29{,}000\text{ ksi})(2{,}520\text{ in.}^4)}$$

$$= 1.26\text{ in.}$$

$$= L/429 < L/360 \quad \textbf{o.k.}$$

Performing the same check with 50% of the design live load for comparison to the AISC Design Guide 3 limit yields:

$$\Delta_c = 0.50(1.26 \text{ in.})$$
$$= 0.630 \text{ in.} < 1.0 \text{ in.} \quad \textbf{o.k.}$$

Steel Anchor Strength

Steel headed stud anchor strengths are tabulated in AISC *Manual* Table 3-21 for typical conditions. Conservatively assuming that all anchors are placed in the weak position, the strength for ¾-in.-diameter anchors in normal weight concrete with $f'_c = 4$ ksi and deck oriented perpendicular to the beam is:

1 anchor per rib: $Q_n = 17.2$ kips/anchor
2 anchors per rib: $Q_n = 14.6$ kips/anchor

Number and Spacing of Anchors

Deck flutes are spaced at 12 in. on center according to the deck manufacturer's literature. The minimum number of deck flutes along each half of the 45-ft-long beam, assuming the first flute begins a maximum of 12 in. from the support line at each end, is:

$$n_{flutes} = n_{spaces} + 1$$
$$= \frac{45.0 \text{ ft} - 2(12 \text{ in.})(1 \text{ ft/12 in.})}{2(1 \text{ ft per space})} + 1$$
$$= 22.5 \rightarrow \text{say 22 flutes}$$

According to AISC *Specification* Section I8.2c, the number of steel headed stud anchors required between the section of maximum bending moment and the nearest point of zero moment is determined by dividing the required horizontal shear, $\sum Q_n$, by the nominal shear strength per anchor, Q_n. Assuming one anchor per flute:

$$n_{anchors} = \frac{\sum Q_n}{Q_n}$$
$$= \frac{386 \text{ kips}}{17.2 \text{ kips/anchor}}$$
$$= 22.4 \rightarrow \text{place 23 anchors on each side of the beam centerline}$$

As the number of anchors exceeds the number of available flutes by one, place two anchors in the first flute. The revised horizontal shear capacity of the anchors taking into account the reduced strength for two anchors in one flute is:

$$\sum Q_n = 2(14.6 \text{ kips}) + 21(17.2 \text{ kips})$$
$$= 390 \text{ kips} \geq 386 \text{ kips} \quad \textbf{o.k.}$$

The final anchor pattern chosen is illustrated in Figure I.1-2.

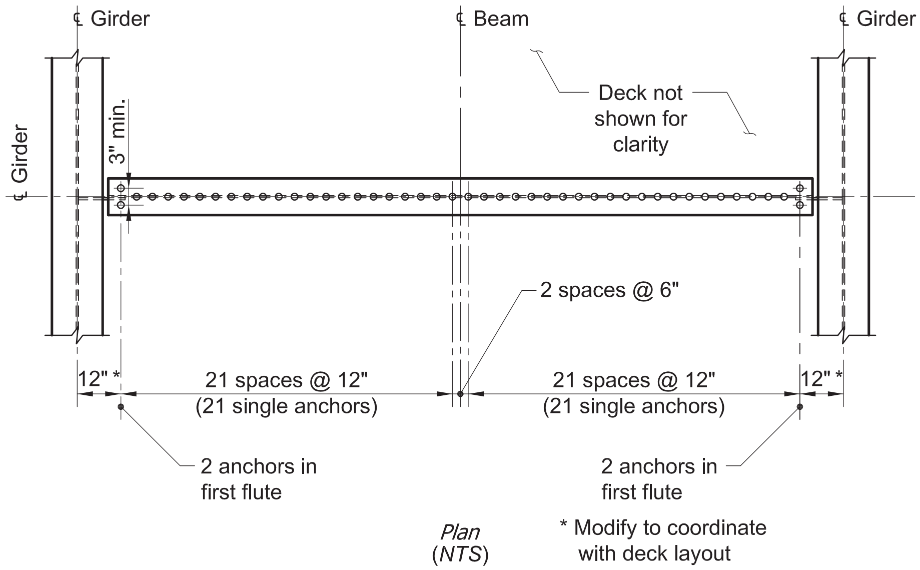


Fig. I.1-2. Steel headed stud anchor layout.

Review steel headed stud anchor spacing requirements of AISC Specification Sections I8.2d and I3.2c.

(1) Maximum anchor spacing along beam: $8t_{slab} = 8(7.5\text{ in.}) = 60.0\text{ in. or }36\text{ in.}$

$12\text{ in.} < 36\text{ in.}$ **o.k.**

(2) Minimum anchor spacing along beam: $6d_{sa} = 6(\frac{3}{4}\text{ in.}) = 4.50\text{ in.}$

$12\text{ in.} > 4.50\text{ in.}$ **o.k.**

(3) Minimum transverse spacing between anchor pairs: $4d_{sa} = 4(\frac{3}{4}\text{ in.}) = 3.00\text{ in.}$

$3.00\text{ in.} = 3.00\text{ in.}$ **o.k.**

(4) Minimum distance to free edge in the direction of the horizontal shear force:

AISC *Specification* Section I8.2d requires that the distance from the center of an anchor to a free edge in the direction of the shear force be a minimum of 8 in. for normal weight concrete slabs.

(5) Maximum spacing of deck attachment:

AISC *Specification* Section I3.2c(4) requires that steel deck be anchored to all supporting members at a maximum spacing of 18 in. The stud anchors are welded through the metal deck at a maximum spacing of 12 inches in this example, thus this limit is met without the need for additional puddle welds or mechanical fasteners.

Available Shear Strength

According to AISC *Specification* Section I4.2, the beam should be assessed for available shear strength as a bare steel beam using the provisions of Chapter G.

Applying the loads previously determined for the governing ASCE/SEI 7-10 load combinations and using available shear strengths from AISC *Manual* Table 3-2 for a W21×50 yields the following:

LRFD	ASD
$V_u = \dfrac{w_u L}{2}$ $= \dfrac{(2.68\text{ kips/ft})(45.0\text{ ft})}{2}$ $= 60.3\text{ kips}$ $\phi_v V_n > V_u$ $\phi_v V_n = 237\text{ kips} \geq 60.3\text{ kips}$ **o.k.**	$V_a = \dfrac{w_a L}{2}$ $= \dfrac{(1.90\text{ kips/ft})(45.0\text{ ft})}{2}$ $= 42.8\text{ kips}$ $V_n / \Omega_v \geq V_a$ $V_n / \Omega_v = 158\text{ kips} > 42.8\text{ kips}$ **o.k.**

Serviceability

Depending on the intended use of this bay, vibrations might need to be considered. See AISC Design Guide 11 (Murray et al., 1997) for additional information.

Summary

From Figure I.1-2, the total number of stud anchors used is equal to $(2)(2+21)=46$. A plan layout illustrating the final beam design is provided in Figure I.1-3:

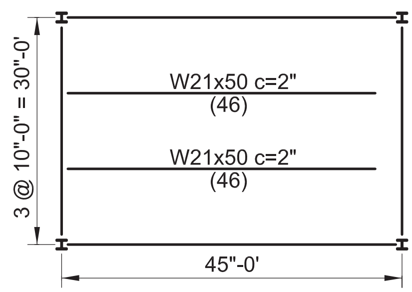


Fig. I.1-3. Revised plan.

A W21×50 with 2 in. of camber and 46, ¾-in.-diameter by 4⅞-in.-long steel headed stud anchors is adequate to resist the imposed loads.

EXAMPLE I.2 COMPOSITE GIRDER DESIGN

Given:

Two typical bays of a composite floor system are illustrated in Figure I.2-1. Select an appropriate ASTM A992 W-shaped girder and determine the required number of steel headed stud anchors. The girder will not be shored during construction.

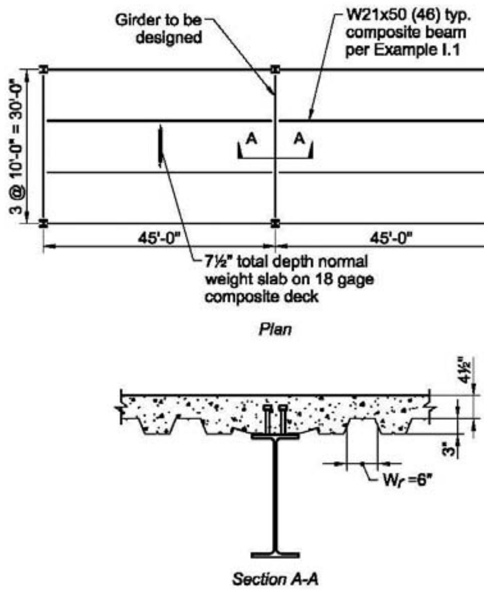


Fig. I.2-1. Composite bay and girder section.

To achieve a two-hour fire rating without the application of spray applied fire protection material to the composite deck, 4½ in. of normal weight (145 lb/ft^3) concrete will be placed above the top of the deck. The concrete has a specified compressive strength, f_c' = 4 ksi.

Applied loads are as follows:

Dead Loads:

Pre-composite:

Slab	= 75 lb/ft^2 (in accordance with metal deck manufacturer's data)
Self weight	= 80 lb/ft (trial girder weight)
	= 50 lb/ft (beam weight from Design Example I.1)

Composite (applied after composite action has been achieved):

Miscellaneous = 10 lb/ft^2 (HVAC, ceiling, floor covering, etc.)

Live Loads:

Pre-composite:

Construction = 25 lb/ft^2 (temporary loads during concrete placement)

Composite (applied after composite action has been achieved):

Non-reducible = 100 lb/ft^2 (assembly occupancy)

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Applied Loads

For slabs that are to be placed at a constant elevation, AISC Design Guide 3 (West and Fisher, 2003) recommends an additional 10% of the nominal slab weight be applied to account for concrete ponding due to deflections resulting from the wet weight of the concrete during placement. For the slab under consideration, this would result in an additional load of 8 lb/ft^2; however, for this design the slab will be placed at a constant thickness thus no additional weight for concrete ponding is required.

For pre-composite construction live loading, 25 lb/ft^2 will be applied in accordance with recommendations from ASCE/SEI 37-02 *Design Loads on Structures During Construction* (ASCE, 2002) for a light duty operational class which includes concrete transport and placement by hose.

Composite Deck and Anchor Requirements

Check composite deck and anchor requirements stipulated in AISC *Specification* Sections I1.3, I3.2c and I8.

(1) Concrete Strength: $3 \text{ ksi} \leq f_c' \leq 10 \text{ ksi}$

$f_c' = 4 \text{ ksi}$ **o.k.**

(2) Rib height: $h_r \leq 3 \text{ in.}$

$h_r = 3 \text{ in.}$ **o.k.**

(3) Average rib width: $w_r \geq 2 \text{ in.}$

$w_r = 6 \text{ in.}$ (See Figure I.2-1) **o.k.**

(4) Use steel headed stud anchors ¾ in. or less in diameter.

Select ¾-in.-diameter steel anchors **o.k.**

(5) Steel headed stud anchor diameter: $d_{sa} \leq 2.5\left(t_f\right)$

In accordance with AISC *Specification* Section I8.1, this limit only applies if steel headed stud anchors are not welded to the flange directly over the web. The ¾-in.-diameter anchors will be attached in a staggered pattern, thus this limit must be satisfied. Select a girder size with a minimum flange thickness of 0.30 in., as determined below:

$$t_f \geq \frac{d_{sa}}{2.5}$$

$$\frac{d_{sa}}{2.5} = \frac{\frac{3}{4} \text{ in.}}{2.5} = 0.30 \text{ in.}$$

(6) Steel headed stud anchors, after installation, shall extend not less than 1½ in. above the top of the steel deck.

A minimum anchor length of 4½ in. is required to meet this requirement for 3 in. deep deck. From steel headed stud anchor manufacturer's data, a standard stock length of 4⅞ in. is selected. Using a 3⁄16-in. length reduction to account for burn off during anchor installation directly to the girder flange yields a final installed length of 411⁄16 in.

$$4^{11}\!/_{16} \text{ in.} > 4\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

(7) Minimum length of stud anchors = $4d_{sa}$

$$4^{11}\!/_{16} \text{ in.} > 4(\tfrac{3}{4} \text{ in.}) = 3.00 \text{ in.} \quad \textbf{o.k.}$$

(8) There shall be at least ½ in. of specified concrete cover above the top of the headed stud anchors.

As discussed in the *Specification* Commentary to Section I3.2c, it is advisable to provide greater than ½ in. minimum cover to assure anchors are not exposed in the final condition.

$$7\tfrac{1}{2} \text{ in.} - 4^{11}\!/_{16} \text{ in.} = 2^{13}\!/_{16} \text{ in.} > \tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}$$

(9) Slab thickness above steel deck ≥ 2 in.

$$4\tfrac{1}{2} \text{ in.} > 2 \text{ in.} \quad \textbf{o.k.}$$

Design for Pre-Composite Condition

Construction (Pre-Composite) Loads

The girder will be loaded at third points by the supported beams. Determine point loads using tributary areas.

$$\begin{aligned} P_D &= \left[(45.0 \text{ ft})(10.0 \text{ ft})(75 \text{ lb/ft}^2) + (45.0 \text{ ft})(50 \text{ lb/ft})\right](0.001 \text{ kip/lb}) \\ &= 36.0 \text{ kips} \\ P_L &= \left[(45.0 \text{ ft})(10.0 \text{ ft})(25 \text{ lb/ft}^2)\right](0.001 \text{ kip/lb}) \\ &= 11.3 \text{ kips} \end{aligned}$$

Construction (Pre-Composite) Flexural Strength

From Chapter 2 of ASCE/SEI 7, the required flexural strength is:

LRFD	ASD
$P_u = 1.2(36.0 \text{ kips}) + 1.6(11.3 \text{ kips})$ $= 61.3 \text{ kips}$ $w_u = 1.2(80 \text{ lb/ft})(0.001 \text{ kip/lb})$ $= 0.0960 \text{ kip/ft}$ $M_u = P_u a + \dfrac{w_u L^2}{8}$ $= (61.3 \text{ kips})(10.0 \text{ ft}) + \dfrac{(0.0960 \text{ kip/ft})(30.0 \text{ ft})^2}{8}$ $= 624 \text{ kip-ft}$	$P_a = 36.0 \text{ kips} + 11.3 \text{ kips}$ $= 47.3 \text{ kips}$ $w_a = (80 \text{ lb/ft})(0.001 \text{ kip/lb})$ $= 0.0800 \text{ kip/ft}$ $M_a = P_a a + \dfrac{w_a L^2}{8}$ $= (47.3 \text{ kips})(10.0 \text{ ft}) + \dfrac{(0.0800 \text{ kip/ft})(30.0 \text{ ft})^2}{8}$ $= 482 \text{ kip-ft}$

Girder Selection

Based on the required flexural strength under construction loading, a trial member can be selected utilizing AISC *Manual* Table 3-2. For the purposes of this example, the unbraced length of the girder prior to hardening of the concrete is taken as the distance between supported beams (one third of the girder length).

Try a W24×76

$$L_b = 10.0 \text{ ft}$$
$$L_p = 6.78 \text{ ft}$$
$$L_r = 19.5 \text{ ft}$$

LRFD	ASD
$\phi_b BF = 22.6$ kips $\phi_b M_{px} = 750$ kip-ft $\phi_b M_{rx} = 462$ kip-ft	$BF/\Omega_b = 15.1$ kips $M_{px}/\Omega_b = 499$ kip-ft $M_{rx}/\Omega_b = 307$ kip-ft

Because L_p<L_b<L_r, use AISC *Manual* Equations 3-4a and 3-4b with $C_b = 1.0$ within the center girder segment in accordance with *Manual* Table 3-1:

LRFD	ASD
$\phi_b M_n = C_b\left[\phi_b M_{px} - \phi_b BF(L_b - L_p)\right] \le \phi_b M_{px}$ $= 1.0[750 \text{ kip-ft} - 22.6 \text{ kips}(10.0 \text{ ft} - 6.78 \text{ ft})]$ $= 677 \text{ kip-ft} \le 750 \text{ kip-ft}$ $\phi_b M_n \ge M_u$ $677 \text{ kip-ft} > 624 \text{ kip-ft}$ **o.k.**	$\dfrac{M_n}{\Omega_b} = C_b\left[\dfrac{M_{px}}{\Omega_b} - \dfrac{BF}{\Omega_b}(L_b - L_p)\right] \le \dfrac{M_{px}}{\Omega_b}$ $= 1.0[499 \text{ kip-ft} - 15.1 \text{ kips}(10.0 \text{ ft} - 6.78 \text{ ft})]$ $= 450 \text{ kip-ft} \le 499 \text{ kip-ft}$ $\dfrac{M_n}{\Omega_b} \ge M_a$ $450 \text{ kip-ft} < 482 \text{ kip-ft}$ **n.g.**

For this example, the relatively low live load to dead load ratio results in a lighter member when LRFD methodology is employed. When ASD methodology is employed, a heavier member is required, and it can be shown that a W24×84 is adequate for pre-composite flexural strength. This example uses a W24×76 member to illustrate the determination of flexural strength of the composite section using both LRFD and ASD methodologies; however, this is done for comparison purposes only, and calculations for a W24×84 would be required to provide a satisfactory ASD design. Calculations for the heavier section are not shown as they would essentially be a duplication of the calculations provided for the W24×76 member.

Note that for the member size chosen, $76 \text{ lb/ft} \le 80 \text{ lb/ft}$, thus the initial weight assumption is adequate.

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W24×76
A = 22.4 in.^{2}
I_x = 2,100 in.^{4}
b_f = 8.99 in.
t_f = 0.680 in.
d = 23.9 in.

Pre-Composite Deflections

AISC Design Guide 3 recommends deflections due to concrete plus self weight not exceed the minimum of L/360 or 1.0 in.

From the superposition of AISC *Manual* Table 3-23, Cases 1 and 9:

$$\Delta_{nc} = \frac{P_D L^3}{28EI} + \frac{5w_D L^4}{384EI}$$

Substituting for the moment of inertia of the non-composite section, I = 2,100 in.^{4}, yields a dead load deflection of:

$$\Delta_{nc} = \frac{36.0\text{ kips}\left[(30.0\text{ ft})(12\text{ in./ft})\right]^3}{28(29{,}000\text{ ksi})(2{,}100\text{ in.}^4)} + \frac{5\left[\dfrac{(0.0760\text{ kip/ft})}{12\text{ in./ft}}\right]\left[(30.0\text{ ft})(12\text{ in./ft})\right]^4}{384(29{,}000\text{ ksi})(2{,}100\text{ in.}^4)}$$
$$= 1.01\text{ in.}$$
$$= L/356 > L/360 \quad \textbf{n.g.}$$

Pre-composite deflections slightly exceed the recommended value. One possible solution is to increase the member size. A second solution is to induce camber into the member. For this example, the second solution is selected, and the girder will be cambered to reduce pre-composite deflections.

Reducing the estimated simple span deflections to 80% of the calculated value to reflect the partial restraint of the end connections as recommended in AISC Design Guide 3 yields a camber of:

$$\text{Camber} = 0.8(1.01\text{ in.})$$
$$= 0.808\text{ in.}$$

Rounding down to the nearest ¼-in. increment yields a specified camber of ¾ in.

Select a W24×76 with ¾ in. of camber.

Design for Composite Flexural Strength

Required Flexural Strength

Using tributary area calculations, the total applied point loads (including pre-composite dead loads in addition to dead and live loads applied after composite action has been achieved) are determined as:

$$P_D = \left[(45.0\text{ ft})(10.0\text{ ft})(75\text{ lb/ft}^2 + 10\text{ lb/ft}^2) + (45.0\text{ ft})(50\text{ lb/ft})\right](0.001\text{ kip/lb})$$
$$= 40.5\text{ kips}$$
$$P_L = \left[(45.0\text{ ft})(10.0\text{ ft})(100\text{ lb/ft}^2)\right](0.001\text{ kip/lb})$$
$$= 45.0\text{ kips}$$

The required flexural strength diagram is illustrated by Figure I.2-2:

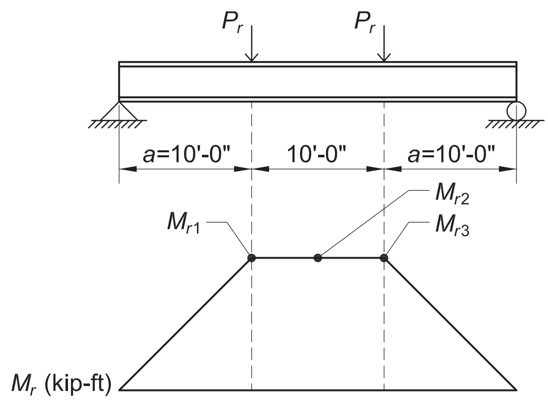


Fig. I.2-2. Required flexural strength.

From ASCE/SEI 7-10 Chapter 2, the required flexural strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(40.5 \text{ kips}) + 1.6(45.0 \text{ kips})$ $= 121 \text{ kips}$ $w_u = 1.2(0.0760 \text{ kip/ft})$ $= 0.0912 \text{ kip/ft}$ from self weight of W24×76 From AISC *Manual* Table 3-23, Case 1 and 9. $M_{u1} = M_{u3}$ $= P_u a + \frac{w_u a}{2}(L-a)$ $= (121 \text{ kips})(10.0 \text{ ft}) + \frac{(0.0912 \text{ kip/ft})(10.0 \text{ ft})}{2}(30.0 \text{ ft} - 10.0 \text{ ft})$ $= 1{,}220 \text{ kip-ft}$ $M_{u2} = P_u a + \frac{w_u L^2}{8}$ $= (121 \text{ kips})(10.0 \text{ ft}) + \frac{(0.0912 \text{ kip/ft})(30.0 \text{ ft})^2}{8}$ $= 1{,}220 \text{ kip-ft}$	$P_r = P_a$ $= 40.5 \text{ kips} + 45.0 \text{ kips}$ $= 85.5 \text{ kips}$ $w_a = 0.0760 \text{ kip/ft}$ from self weight of W24×76 From AISC *Manual* Table 3-23, Case 1 and 9. $M_{a1} = M_{a3}$ $= P_a a + \frac{w_a a}{2}(L-a)$ $= (85.5 \text{ kips})(10.0 \text{ ft}) + \frac{(0.0760 \text{ kip/ft})(10.0 \text{ ft})}{2}(30.0 \text{ ft} - 10.0 \text{ ft})$ $= 863 \text{ kip-ft}$ $M_{a2} = P_a a + \frac{w_a L^2}{8}$ $= (85.5 \text{ kips})(10.0 \text{ ft}) + \frac{(0.0760 \text{ kip/ft})(30.0 \text{ ft})^2}{8}$ $= 864 \text{ kip-ft}$

Determine b

The effective width of the concrete slab is the sum of the effective widths to each side of the beam centerline as determined by the minimum value of the three conditions set forth in AISC *Specification* Section I3.1a:

(1) one-eighth of the girder span center-to-center of supports

$$\frac{30.0 \text{ ft}}{8}(2 \text{ sides}) = 7.50 \text{ ft} \quad \textbf{controls}$$

(2) one-half the distance to the centerline of the adjacent girder

$$\frac{45\text{ ft}}{2}(2\text{ sides}) = 45.0\text{ ft}$$

(3) distance to the edge of the slab

not applicable for an interior member

Available Flexural Strength

According to AISC *Specification* Section I3.2a, the nominal flexural strength shall be determined from the plastic stress distribution on the composite section when $h/t_w \leq 3.76\sqrt{E/F_y}$.

From AISC *Manual* Table 1-1, h/t_w for a W24×76 = 49.0.

$$\begin{aligned} 49.0 &\leq 3.76\sqrt{29{,}000\text{ ksi}/50\text{ ksi}} \\ &\leq 90.6 \end{aligned}$$

Therefore, use the plastic stress distribution to determine the nominal flexural strength.

According to the User Note in AISC *Specification* Section I3.2a, this check is generally unnecessary as all current W-shapes satisfy this limit for $F_y \leq 50$ ksi.

AISC *Manual* Table 3-19 can be used to facilitate the calculation of flexural strength for composite beams. Alternately, the available flexural strength can be determined directly using the provisions of AISC *Specification* Chapter I. Both methods will be illustrated for comparison in the following calculations.

Method 1: AISC *Manual*

To utilize AISC *Manual* Table 3-19, the distance from the compressive concrete flange force to beam top flange, $Y2$, must first be determined as illustrated by *Manual* Figure 3-3. Fifty percent composite action [$\Sigma Q_n \approx 0.50(A_sF_y)$] is used to calculate a trial value of the compression block depth, a_{trial}, for determining $Y2$ as follows:

$$a_{trial} = \frac{\sum Q_n}{0.85 f_c' b} \qquad \text{(from } \textit{Manual}\text{. Eq. 3-7)}$$

$$\begin{aligned} &= \frac{0.50(A_s F_y)}{0.85 f_c' b} \\ &= \frac{0.50(22.4\text{ in.}^2)(50\text{ ksi})}{0.85(4\text{ ksi})(7.50\text{ ft})(12\text{ in./ft})} \\ &= 1.83\text{ in.} \end{aligned}$$

$$Y2 = Y_{con} - \frac{a_{trial}}{2} \qquad \text{(from } \textit{Manual}\text{. Eq. 3-6)}$$

where

Y_{con} = distance from top of steel beam to top of slab
= 7.50 in.

$$Y2 = 7.50 \text{ in.} - \frac{1.83 \text{ in.}}{2}$$
$$= 6.59 \text{ in.}$$

Enter AISC *Manual* Table 3-19 with the required strength and $Y2 = 6.59$ in. to select a plastic neutral axis location for the W24×76 that provides sufficient available strength. Based on the available flexural strength provided in Table 3-19, the required PNA location for ASD and LRFD design methodologies differ. This discrepancy is due to the live to dead load ratio in this example, which is not equal to the ratio of 3 at which ASD and LRFD design methodologies produce equivalent results as discussed in AISC *Specification* Commentary Section B3.4.

Selecting PNA location 5 (BFL) with $\sum Q_n = 509$ kips provides a flexural strength of:

LRFD	ASD
$\phi_b M_n \geq M_u$ $\phi_b M_n = 1{,}240 \text{ kip-ft} > 1{,}220 \text{ kip-ft}$ **o.k.**	$M_n / \Omega_b \geq M_a$ $M_n / \Omega_b = 823 \text{ kip-ft} < 864 \text{ kip-ft}$ **n.g.**

The selected PNA location 5 is acceptable for LRFD design, but inadequate for ASD design. For ASD design, it can be shown that a W24×76 is adequate if a higher composite percentage of approximately 60% is employed. However, as discussed previously, this beam size is not adequate for construction loading and a larger section is necessary when designing utilizing ASD.

The actual value for the compression block depth, a, for the chosen PNA location is determined as follows:

$$a = \frac{\sum Q_n}{0.85 f_c' b} \qquad (\textit{Manual} \text{ Eq. 3-7})$$
$$= \frac{509 \text{ kips}}{0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})}$$
$$= 1.66 \text{ in.}$$
$$a = 1.66 \text{ in.} < a_{trial} = 1.83 \text{ in.} \quad \textbf{o.k. for LRFD design}$$

Method 2: Direct Calculation

According to AISC *Specification* Commentary Section I3.2a, the number and strength of steel headed stud anchors will govern the compressive force, C, for a partially composite beam. The composite percentage is based on the minimum of the limit states of concrete crushing and steel yielding as follows:

(1) Concrete crushing

A_c = Area of concrete slab within effective width. Assume that the deck profile is 50% void and 50% concrete fill.

$$= b_{eff}(4\tfrac{1}{2} \text{ in.}) + (b_{eff}/2)(3.00 \text{ in.})$$
$$= (7.50 \text{ ft})(12 \text{ in./ft})(4\tfrac{1}{2} \text{ in.}) + \left[\frac{(7.50 \text{ ft})(12 \text{ in./ft})}{2}\right](3.00 \text{ in.})$$
$$= 540 \text{ in.}^2$$

$$C = 0.85 f_c' A_c \qquad (\textit{Comm.} \text{ Eq. C-I3-7})$$
$$= 0.85(4 \text{ ksi})(540 \text{ in.}^2)$$
$$= 1{,}840 \text{ kips}$$

(2) Steel yielding

$$C = A_s F_y \qquad \text{(from } \textit{Comm}\text{. Eq. C-I3-6)}$$
$$= (22.4 \text{ in.}^2)(50 \text{ ksi})$$
$$= 1{,}120 \text{ kips}$$

(3) Shear transfer

Fifty percent is used as a trial percentage of composite action as follows:

$$C = \Sigma Q_n \qquad (\textit{Comm}\text{. Eq. C-I3-8})$$
$$= 50\% \left(\text{Min} \begin{Bmatrix} 1{,}840 \text{ kips} \\ 1{,}120 \text{ kips} \end{Bmatrix} \right)$$
$$= 560 \text{ kips to achieve 50\% composite action}$$

Location of the Plastic Neutral Axis

The plastic neutral axis (PNA) is located by determining the axis above and below which the sum of horizontal forces is equal. This concept is illustrated in Figure I.2-3, assuming the trial PNA location is within the top flange of the girder.

$$\Sigma F_{\,above\ PNA} = \Sigma F_{\,below\ PNA}$$
$$C + x b_f F_y = (A_s - b_f x) F_y$$

Solving for x:

$$x = \frac{A_s F_y - C}{2 b_f F_y}$$
$$= \frac{(22.4 \text{ in.}^2)(50 \text{ ksi}) - 560 \text{ kips}}{2(8.99 \text{ in.})(50 \text{ ksi})}$$
$$= 0.623 \text{ in.}$$
$$x = 0.623 \text{ in.} \le t_f = 0.680 \text{ in.} \quad \text{PNA in flange}$$

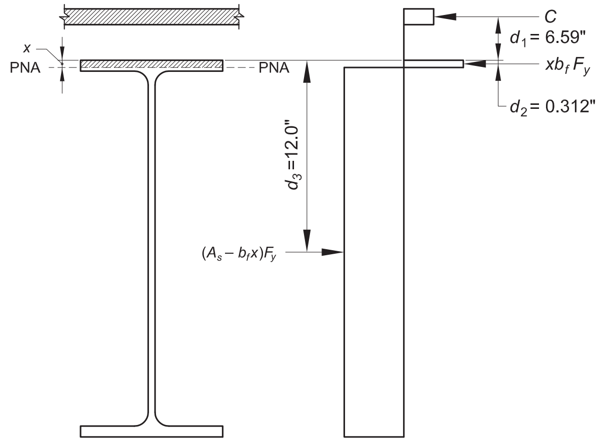


Fig. I.2-3. Plastic neutral axis location.

Return to Table of Contents

Determine the nominal moment resistance of the composite section following the procedure in *Specification* Commentary Section I3.2a as illustrated in Figure C-I3.3.

$$M_n = C\left(d_1 + d_2\right) + P_y\left(d_3 - d_2\right) \qquad \text{(Comm. Eq. C-I3-10)}$$

$$a = \frac{C}{0.85 f'_c b} \qquad \text{(Comm. Eq. C-I3-9)}$$

$$= \frac{560 \text{ kips}}{0.85\left(4 \text{ ksi}\right)\left(7.50 \text{ ft}\right)\left(12 \text{ in./ft}\right)}$$

$$= 1.83 \text{ in.} < 4.5 \text{ in.} \quad \textbf{Above top of deck}$$

$$d_1 = t_{slab} - a/2$$
$$= 7.50 \text{ in.} - 1.83 \text{ in./2}$$
$$= 6.59 \text{ in.}$$

$$d_2 = x/2$$
$$= 0.623 \text{ in./2}$$
$$= 0.312 \text{ in.}$$

$$d_3 = d/2$$
$$= 23.9 \text{ in./2}$$
$$= 12.0 \text{ in.}$$

$$P_y = A_s F_y$$
$$= 22.4 \text{ in.}^2\left(50 \text{ ksi}\right)$$
$$= 1{,}120 \text{ kips}$$

$$M_n = \left[\left(560 \text{ kips}\right)\left(6.59 \text{ in.} + 0.312 \text{ in.}\right) + \left(1{,}120 \text{ kips}\right)\left(12.0 \text{ in.} - 0.312 \text{ in.}\right)\right] \big/ 12 \text{ in./ft}$$

$$= \frac{17{,}000 \text{ kip-in.}}{12 \text{ in./ft}}$$

$$= 1{,}420 \text{ kip-ft}$$

Note that Equation C-I3-10 is based on summation of moments about the centroid of the compression force in the steel; however, the same answer may be obtained by summing moments about any arbitrary point.

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n \geq M_u$ $\phi_b M_n = 0.90\left(1{,}420 \text{ kip-ft}\right)$ $= 1{,}280 \text{ kip-ft} > 1{,}220 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $M_n / \Omega_b \geq M_a$ $M_n / \Omega_b = \dfrac{1{,}420 \text{ kip-ft}}{1.67}$ $= 850 \text{ kip-ft} < 864 \text{ kip-ft}$ **n.g.**

As was determined previously using the Manual Tables, a W24×76 with 50% composite action is acceptable when LRFD methodology is employed, while for ASD design the beam is inadequate at this level of composite action.

Continue with the design using a W24×76 with 50% composite action.

Steel Anchor Strength

Steel headed stud anchor strengths are tabulated in AISC *Manual* Table 3-21 for typical conditions and may be calculated according to AISC *Specification* Section I8.2a as follows:

$$Q_n = 0.5A_{sa}\sqrt{f_c'E_c} \le R_g R_p A_{sa} F_u \qquad (Spec.\ \text{Eq. I8-1})$$

$$A_{sa} = \pi d_{sa}^2 / 4$$
$$= \pi(\text{¾ in.})^2 / 4$$
$$= 0.442 \text{ in.}^2$$

$$f_c' = 4 \text{ ksi}$$

$$E_c = w_c^{1.5}\sqrt{f_c'}$$
$$= \left(145 \text{ lb/ft}^3\right)^{1.5}\sqrt{4 \text{ ksi}}$$
$$= 3{,}490 \text{ ksi}$$

$R_g = 1.0$ Stud anchors welded directly to the steel shape within the slab haunch

$R_p = 0.75$ Stud anchors welded directly to the steel shape

$F_u = 65$ ksi From AISC *Manual* Table 2-6 for ASTM A108 steel anchors

$$Q_n = (0.5)\left(0.442 \text{ in.}^2\right)\sqrt{(4 \text{ ksi})(3{,}490 \text{ ksi})} \le (1.0)(0.75)\left(0.442 \text{ in.}^2\right)(65 \text{ ksi})$$
$$= 26.1 \text{ kips} > 21.5 \text{ kips}$$

use $Q_n = 21.5$ kips

Number and Spacing of Anchors

According to AISC *Specification* Section I8.2c, the number of steel headed stud anchors required between any concentrated load and the nearest point of zero moment shall be sufficient to develop the maximum moment required at the concentrated load point.

From Figure I.2-2 the moment at the concentrated load points, M_{r1} and M_{r3}, is approximately equal to the maximum beam moment, M_{r2} . The number of anchors between the beam ends and the point loads should therefore be adequate to develop the required compressive force associated with the maximum moment, C, previously determined to be 560 kips.

$$N_{anchors} = \frac{\Sigma Q_n}{Q_n}$$
$$= \frac{C}{Q_n}$$
$$= \frac{560 \text{ kips}}{21.5 \text{ kips/anchor}}$$
$$= 26 \text{ anchors from each end to concentrated load points}$$

In accordance with AISC *Specification* Section I8.2d, anchors between point loads should be spaced at a maximum of:

$8t_{slab} = 60.0$ in.

or 36 in. **controls**

For beams with deck running parallel to the span such as the one under consideration, spacing of the stud anchors is independent of the flute spacing of the deck. Single anchors can therefore be spaced as needed along the beam length provided a minimum longitudinal spacing of six anchor diameters in accordance with AISC *Specification* Section I8.2d is maintained. Anchors can also be placed in aligned or staggered pairs provided a minimum transverse spacing of four stud diameters = 3 in. is maintained. For this design, it was chosen to use pairs of anchors along each end of the girder to meet strength requirements and single anchors along the center section of the girder to meet maximum spacing requirements as illustrated in Figure I.2-4.

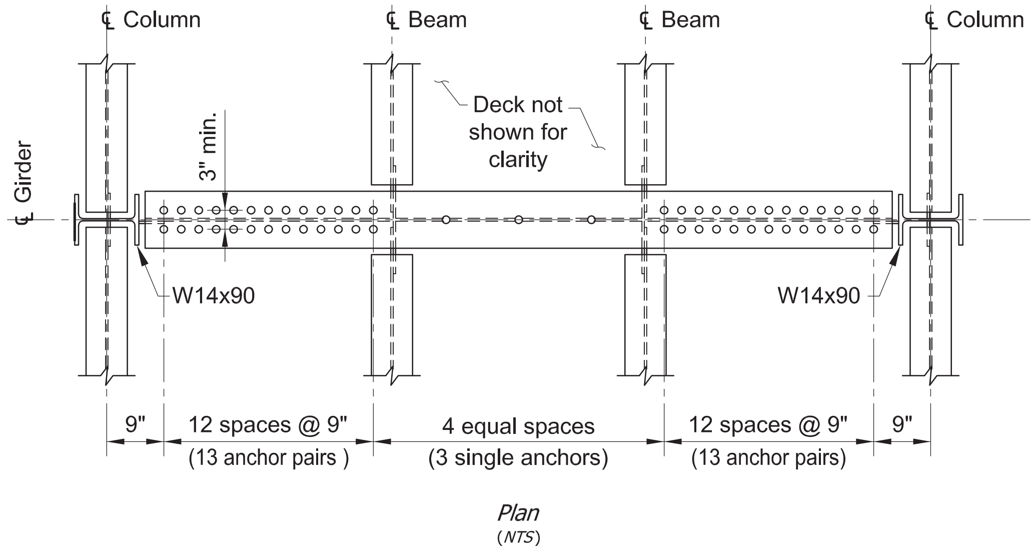


Fig. I.2-4. Steel headed stud anchor layout.

AISC *Specification* Section I8.2d requires that the distance from the center of an anchor to a free edge in the direction of the shear force be a minimum of 8 in. for normal weight concrete slabs. For simply-supported composite beams this provision could apply to the distance between the slab edge and the first anchor at each end of the beam. Assuming the slab edge is coincident to the centerline of support, Figure I.2-4 illustrates an acceptable edge distance of 9 in., though in this case the column flange would prevent breakout and negate the need for this check. The slab edge is often uniformly supported by a column flange or pour stop in typical composite construction thus preventing the possibility of a concrete breakout failure and nullifying the edge distance requirement as discussed in AISC *Specification* Commentary Section I8.3.

For this example, the minimum number of headed stud anchors required to meet the maximum spacing limit previously calculated is used within the middle third of the girder span. Note also that AISC *Specification* Section I3.2c(1)(4) requires that steel deck be anchored to all supporting members at a maximum spacing of 18 in. Additionally, ANSI/SDI C1.0-2006, *Standard for Composite Steel Floor Deck* (SDI, 2006), requires deck attachment at an average of 12 in. but no more than 18 in.

From the previous discussion and Figure I.2-4, the total number of stud anchors used is equal to $(13)(2)+3+(13)(2)=55$. A plan layout illustrating the final girder design is provided in Figure I.2-5.

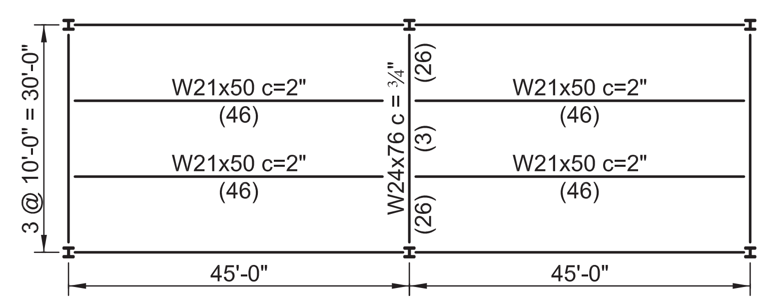


Fig. I.2-5. Revised plan.

Live Load Deflection Criteria

Deflections due to live load applied after composite action has been achieved will be limited to $L/360$ under the design live load as required by Table 1604.3 of the 2009 *International Building Code* (IBC) (ICC, 2009), or 1 in. using a 50% reduction in design live load as recommended by AISC Design Guide 3.

Deflections for composite members may be determined using the lower bound moment of inertia provided in AISC *Specification* Commentary Equation C-I3-1 and tabulated in AISC *Manual* Table 3-20. The *Specification* Commentary also provides an alternate method for determining deflections through the calculation of an effective moment of inertia. Both methods are acceptable and are illustrated in the following calculations for comparison purposes:

Method 1: Calculation of the lower bound moment of inertia, I_{LB}

$$I_{LB} = I_s + A_s\left(Y_{ENA} - d_3\right)^2 + \left(\frac{\Sigma Q_n}{F_y}\right)\left(2d_3 + d_1 - Y_{ENA}\right)^2 \qquad \text{(}\textit{Comm}\text{. Eq. C-I3-1)}$$

Variables d_1, d_2 and d_3 in AISC *Specification* Commentary Equation C-I3-1 are determined using the same procedure previously illustrated for calculating nominal moment resistance. However, for the determination of I_{LB} the nominal strength of steel anchors is calculated between the point of maximum positive moment and the point of zero moment as opposed to between the concentrated load and point of zero moment used previously. The maximum moment is located at the center of the span and it can be seen from Figure I.2-4 that 27 anchors are located between the midpoint of the beam and each end.

$$\begin{aligned}\Sigma Q_n &= \left(27 \text{ anchors}\right)\left(21.5 \text{ kips/anchor}\right)\\ &= 581 \text{ kips}\end{aligned}$$

$$\begin{aligned}a &= \frac{C}{0.85 f_c' b} \qquad \text{(}\textit{Comm}\text{. Eq. C-I3-9)}\\ &= \frac{\Sigma Q_n}{0.85 f_c' b}\\ &= \frac{581 \text{ kips}}{0.85\left(4 \text{ ksi}\right)\left(7.50 \text{ ft}\right)\left(12 \text{ in./ft}\right)}\\ &= 1.90 \text{ in.}\end{aligned}$$

$$\begin{aligned}d_1 &= t_{slab} - a/2\\ &= 7.50 \text{ in.} - 1.90 \text{ in.}/2\\ &= 6.55 \text{ in.}\end{aligned}$$

$$\begin{aligned}x &= \frac{A_s F_y - \Sigma Q_n}{2 b_f F_y}\\ &= \frac{\left(22.4 \text{ in.}^2\right)\left(50 \text{ ksi}\right) - 581 \text{ kips}}{2\left(8.99 \text{ in.}\right)\left(50 \text{ ksi}\right)}\\ &= 0.600 \text{ in.} < t_f = 0.680 \text{ in.} \quad \text{(PNA within flange)}\end{aligned}$$

$$\begin{aligned}d_2 &= x/2\\ &= 0.600 \text{ in.}/2\\ &= 0.300 \text{ in.}\end{aligned}$$

$$\begin{aligned}d_3 &= d/2\\ &= 23.9 \text{ in.}/2\\ &= 12.0 \text{ in.}\end{aligned}$$

The distance from the top of the steel section to the elastic neutral axis, Y_{ENA}, for use in Equation C-I3-1 is calculated using the procedure provided in AISC *Specification* Commentary Section I3.2 as follows:

$$Y_{ENA} = \frac{A_s d_3 + \left(\frac{\Sigma Q_n}{F_y}\right)(2d_3 + d_1)}{A_s + \left(\frac{\Sigma Q_n}{F_y}\right)} \qquad \text{(}Comm.\text{ Eq. C-I3-2)}$$

$$= \frac{(22.4\text{ in.}^2)(12.0\text{ in.}) + \left(\frac{581\text{ kips}}{50\text{ ksi}}\right)\left[2(12.0\text{ in.}) + 6.55\text{ in.}\right]}{22.4\text{ in.}^2 + \left(\frac{581\text{ kips}}{50\text{ ksi}}\right)}$$

$$= 18.3\text{ in.}$$

Substituting these values into AISC *Specification* Commentary Equation C-I3-1 yields the following lower bound moment of inertia:

$$I_{LB} = 2{,}100\text{ in.}^4 + (22.4\text{ in.})(18.3\text{ in.} - 12.0\text{ in.})^2 + \left(\frac{581\text{ kips}}{50\text{ ksi}}\right)\left[2(12.0\text{ in.}) + 6.55\text{ in.} - 18.3\text{ in.}\right]^2$$

$$= 4{,}730\text{ in.}^4$$

Alternately, this value can be determined directly from AISC *Manual* Table 3-20 as illustrated in Design Example I.1.

Method 2: Calculation of the effective moment of inertia, I_{eff}

An alternate procedure for determining a moment of inertia for deflections of the composite section is presented in AISC *Specification* Commentary Section I3.2 as follows:

Transformed Moment of Inertia, I_{tr}

The effective width of the concrete below the top of the deck may be approximated with the deck profile resulting in a 50% effective width as depicted in Figure I.2-6. The effective width, b_{eff} = 7.50 ft(12 in./ft) = 90.0 in.

Transformed slab widths are calculated as follows:

$$n = E_s / E_c$$
$$= 29{,}000\text{ ksi} / 3{,}490\text{ ksi}$$
$$= 8.31$$

$$b_{tr1} = b_{eff} / n$$
$$= 90.0\text{ in.} / 8.31$$
$$= 10.8\text{ in.}$$

$$b_{tr2} = 0.5b_{eff} / n$$
$$= 0.5(90.0\text{ in.}) / 8.31$$
$$= 5.42\text{ in.}$$

The transformed model is illustrated in Figure I.2-7.

Determine the elastic neutral axis of the transformed section (assuming fully composite action) and calculate the transformed moment of inertia using the information provided in Table I.2-1 and Figure I.2-7. For this problem, a trial location for the elastic neutral axis (ENA) is assumed to be within the depth of the composite deck.

Table I.2-1. Properties for Elastic Neutral Axis Determination of Transformed Section

Part	A (in.^{2})	y (in.)	I (in.^{4})
A_1	48.6	$2.25+x$	82.0
A_2	$5.42x$	$x/2$	$0.452x^3$
W24×76	22.4	$x - 15.0$	2,100

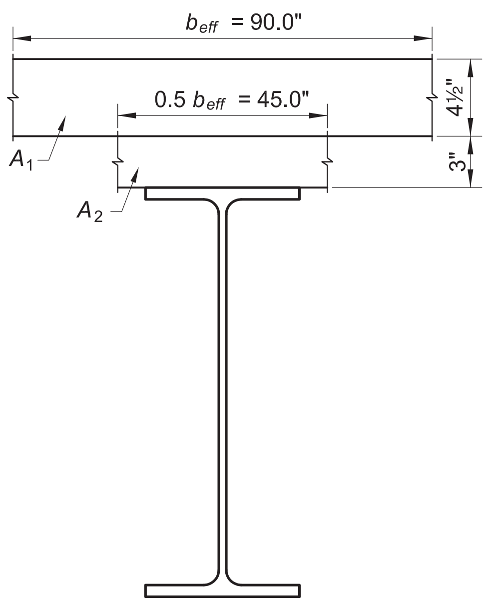


Fig. I.2-6. Effective concrete width.

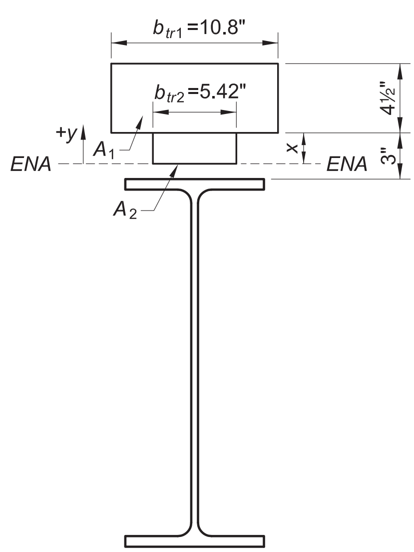


Fig. I.2-7. Transformed area model.

$$\sum Ay \text{ about Elastic Neutral Axis} = 0$$

$$\left(48.6 \text{ in.}^2\right)\left(2.25 \text{ in.} + x\right) + \left(5.42 \text{ in.}\right)\left(\frac{x^2}{2}\right) + \left(22.4 \text{ in.}^2\right)\left(x - 15 \text{ in.}\right) = 0$$

solve for $x \quad \rightarrow \quad x = 2.88$ in.

Verify trial location:

$2.88 \text{ in.} < h_r = 3 \text{ in.}$ **Elastic Neutral Axis within composite deck**

Utilizing the parallel axis theorem and substituting for x yields:

$$\begin{aligned}
I_{tr} &= \Sigma I + \Sigma Ay^2 \\
&= 82.0 \text{ in.}^4 + \left(0.452 \text{ in.}\right)\left(2.88 \text{ in.}\right)^3 + 2{,}100 \text{ in.}^4 + \left(48.6 \text{ in.}^2\right)\left(2.25 \text{ in.} + 2.88 \text{ in.}\right)^2 \\
&\quad + \frac{\left(5.42 \text{ in.}\right)\left(2.88 \text{ in.}\right)^3}{4} + \left(22.4 \text{ in.}^2\right)\left(2.88 \text{ in.} - 15.0 \text{ in.}\right)^2 \\
&= 6{,}800 \text{ in.}^4
\end{aligned}$$

Determine the equivalent moment of inertia, I_{equiv}

$$I_{equiv} = I_s + \sqrt{\left(\Sigma Q_n / C_f\right)}\left(I_{tr} - I_s\right) \qquad \text{(\textit{Comm}. Eq. C-I3-4)}$$

ΣQ_n = 581 kips (previously determined in Method 1)

C_f = Compression force for fully composite beam previously determined to be controlled by $A_s F_y = 1{,}120$ kips

$$\begin{aligned}
I_{equiv} &= 2{,}100 \text{ in.}^4 + \sqrt{\left(581 \text{ kips} / 1{,}120 \text{ kips}\right)}\left(6{,}800 \text{ in.}^4 - 2{,}100 \text{ in.}^4\right) \\
&= 5{,}490 \text{ in.}^4
\end{aligned}$$

According to *Specification* Commentary Section I3.2:

$$\begin{aligned}
I_{eff} &= 0.75 I_{equiv} \\
&= 0.75\left(5{,}490 \text{ in.}^4\right) \\
&= 4{,}120 \text{ in.}^4
\end{aligned}$$

Comparison of Methods and Final Deflection Calculation

I_{LB} was determined to be 4,730 in.^{4} and I_{eff} was determined to be 4,120 in.^{4} I_{LB} will be used for the remainder of this example.

From AISC *Manual* Table 3-23, Case 9:

$$\begin{aligned}
\Delta_{LL} &= \frac{P_L L^3}{28 E I_{LB}} \\
&= \frac{\left(45.0 \text{ kips}\right)\left[\left(30.0 \text{ ft}\right)\left(12 \text{ in./ft}\right)\right]^3}{28\left(29{,}000 \text{ ksi}\right)\left(4{,}730 \text{ in.}^4\right)}
\end{aligned}$$

$= 0.547 \text{ in.} < 1.00 \text{ in.}$ **o.k. for AISC Design Guide 3 limit**

(50% reduction in design live load as allowed by Design Guide 3 was not necessary to meet this limit)

$= L/658 < L/360$ **o.k. for IBC 2009 Table 1604.3 limit**

Available Shear Strength

According to AISC *Specification* Section I4.2, the girder should be assessed for available shear strength as a bare steel beam using the provisions of Chapter G.

Applying the loads previously determined for the governing load combination of ASCE/SEI 7-10 and obtaining available shear strengths from AISC *Manual* Table 3-2 for a W24×76 yields the following:

LRFD	ASD
$V_u = 121 \text{ kips} + (0.0912 \text{ kip/ft})(30.0 \text{ ft/2})$	$V_a = 85.5 \text{ kips} + (0.0760 \text{ kip/ft})(30.0 \text{ ft/2})$
$= 122 \text{ kips}$	$= 86.6 \text{ kips}$
$\phi_v V_n \geq V_u$	$V_n / \Omega_v \geq V_a$
$\phi_v V_n = 315 \text{ kips} > 122 \text{ kips}$ **o.k.**	$V_n / \Omega_v = 210 \text{ kips} > 86.6 \text{ kips}$ **o.k.**

Serviceability

Depending on the intended use of this bay, vibrations might need to be considered. See AISC Design Guide 11 (Murray et al., 1997) for additional information.

It has been observed that cracking of composite slabs can occur over girder lines. The addition of top reinforcing steel transverse to the girder span will aid in mitigating this effect.

Summary

Using LRFD design methodology, it has been determined that a W24×76 with ¾ in. of camber and 55, ¾-in.-diameter by 4⅞-in.-long steel headed stud anchors as depicted in Figure I.2-4, is adequate for the imposed loads and deflection criteria. Using ASD design methodology, a W24×84 with a steel headed stud anchor layout determined using a procedure analogous to the one demonstrated in this example would be required.

EXAMPLE I.3 FILLED COMPOSITE MEMBER FORCE ALLOCATION AND LOAD TRANSFER

Given:

Refer to Figure I.3-1.

Part I: For each loading condition (a) through (c) determine the required longitudinal shear force, V_r', to be transferred between the steel section and concrete fill.

Part II: For loading condition (a), investigate the force transfer mechanisms of direct bearing, shear connection, and direct bond interaction.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, f_c' = 5 ksi. Use ASTM A36 material for the bearing plate.

Applied loading, P_r, for each condition illustrated in Figure I.3-1 is composed of the following nominal loads:

P_D = 32.0 kips
P_L = 84.0 kips

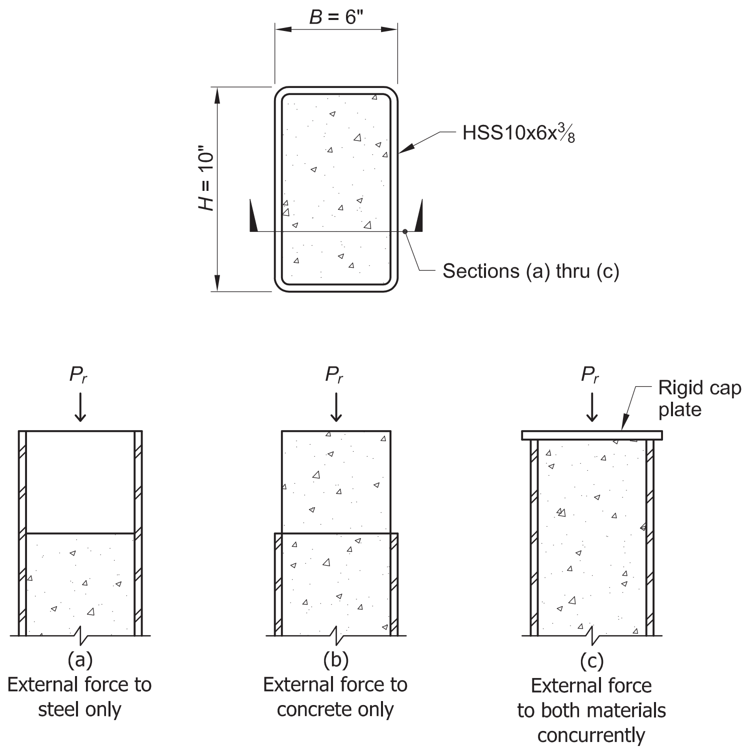


Fig. I.3-1. Concrete filled member in compression.

Solution:

Part I—Force Allocation

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-11 and Figure I.3-1, the geometric properties are as follows:

HSS10×6×⅜
A_s = 10.4 in.^{2}
H = 10.0 in.
B = 6.00 in.
t_{nom} = ⅜ in. (nominal wall thickness)
t = 0.349 in. (design wall thickness in accordance with AISC *Specification* Section B4.2)
h/t = 25.7
b/t = 14.2

Calculate the concrete area using geometry compatible with that used in the calculation of the steel area in AISC *Manual* Table 1-11 (taking into account the design wall thickness and a corner radii of two times the design wall thickness in accordance with AISC *Manual* Part 1), as follows:

$$
\begin{aligned}
h_i &= H - 2t \\
&= 10.0 \text{ in.} - 2(0.349 \text{ in.}) \\
&= 9.30 \text{ in.} \\
b_i &= B - 2t \\
&= 6.00 \text{ in.} - 2(0.349 \text{ in.}) \\
&= 5.30 \text{ in.} \\
A_c &= b_i h_i - t^2(4-\pi) \\
&= (5.30 \text{ in.})(9.30 \text{ in.}) - (0.349)^2(4-\pi) \\
&= 49.2 \text{ in.}^2
\end{aligned}
$$

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(32.0 \text{ kips}) + 1.6(84.0 \text{ kips})$ $= 173 \text{ kips}$	$P_r = P_a$ $= 32.0 \text{ kips} + 84.0 \text{ kips}$ $= 116 \text{ kips}$

Composite Section Strength for Force Allocation

In order to determine the composite section strength, the member is first classified as compact, noncompact or slender in accordance with AISC *Specification* Table I1.1a. However, the results of this check do not affect force allocation calculations as *Specification* Section I6.2 requires the use of Equation I2-9a regardless of the local buckling classification, thus this calculation is omitted for this example. The nominal axial compressive strength without consideration of length effects, P_{no}, used for force allocation calculations is therefore determined as:

$$P_{no} = P_p \qquad (\textit{Spec}.\text{ Eq. I2-9a})$$

$$= F_y A_s + C_2 f_c' \left(A_c + A_{sr} \frac{E_s}{E_c} \right) \qquad (\textit{Spec}.\text{ Eq. I2-9b})$$

where

C_2 = 0.85 for rectangular sections

A_{sr} = 0 when no reinforcing steel is present within the HSS

$$P_{no} = (46\text{ ksi})(10.4\text{ in.}^2) + 0.85(5\text{ ksi})(49.2\text{ in.}^2 + 0.0\text{ in.}^2)$$
$$= 688\text{ kips}$$

Transfer Force for Condition (a)

Refer to Figure I.3-1(a). For this condition, the entire external force is applied to the steel section only, and the provisions of AISC *Specification* Section I6.2a apply.

$$V_r' = P_r \left(1 - \frac{F_y A_s}{P_{no}} \right) \qquad (\textit{Spec}.\text{ Eq. I6-1})$$

$$= P_r \left[1 - \frac{(46\text{ ksi})(10.4\text{ in.}^2)}{688\text{ kips}} \right]$$

$$= 0.305 P_r$$

LRFD	ASD
$V_r' = 0.305(173\text{ kips})$ $= 52.8\text{ kips}$	$V_r' = 0.305(116\text{ kips})$ $= 35.4\text{ kips}$

Transfer Force for Condition (b)

Refer to Figure I.3-1(b). For this condition, the entire external force is applied to the concrete fill only, and the provisions of AISC *Specification* Section I6.2b apply.

$$V_r' = P_r \left(\frac{F_y A_s}{P_{no}} \right) \qquad (\textit{Spec}.\text{ Eq. I6-2})$$

$$= P_r \left[\frac{(46\text{ ksi})(10.4\text{ in.}^2)}{688\text{ kips}} \right]$$

$$= 0.695 P_r$$

LRFD	ASD
$V_r' = 0.695(173\text{ kips})$ $= 120\text{ kips}$	$V_r' = 0.695(116\text{ kips})$ $= 80.6\text{ kips}$

Transfer Force for Condition (c)

Refer to Figure I.3-1(c). For this condition, external force is applied to the steel section and concrete fill concurrently, and the provisions of AISC *Specification* Section I6.2c apply.

AISC *Specification* Commentary Section I6.2 states that when loads are applied to both the steel section and concrete fill concurrently, V_r' can be taken as the difference in magnitudes between the portion of the external force applied directly to the steel section and that required by Equation I6-2. Using the plastic distribution approach employed in *Specification* Equations I6-1 and I6-2, this concept can be written in equation form as follows:

$$V_r' = \left| P_{rs} - P_r\left(\frac{A_s F_y}{P_{no}}\right) \right| \quad \text{(Eq. 1)}$$

where

P_{rs} = portion of external force applied directly to the steel section, kips

Currently the *Specification* provides no specific requirements for determining the distribution of the applied force for the determination of P_{rs}, so it is left to engineering judgment. For a bearing plate condition such as the one represented in Figure I.3-1(c), one possible method for determining the distribution of applied forces is to use an elastic distribution based on the material axial stiffness ratios as follows:

$$\begin{aligned}E_c &= w_c^{1.5}\sqrt{f_c'} \\ &= \left(145\text{ lb/ft}^3\right)^{1.5}\sqrt{5\text{ ksi}} \\ &= 3{,}900\text{ ksi}\end{aligned}$$

$$\begin{aligned}P_{rs} &= \left(\frac{E_s A_s}{E_s A_s + E_c A_c}\right)(P_r) \\ &= \left[\frac{(29{,}000\text{ ksi})(10.4\text{ in.}^2)}{(29{,}000\text{ ksi})(10.4\text{ in.}^2)+(3{,}900\text{ ksi})(49.2\text{ in.}^2)}\right](P_r) \\ &= 0.611P_r\end{aligned}$$

Substituting the results into Equation 1 yields:

$$\begin{aligned}V_r' &= \left|0.611P_r - P_r\left(\frac{A_s F_y}{P_{no}}\right)\right| \\ &= \left|0.611P_r - P_r\left[\frac{(10.4\text{ in.}^2)(46\text{ ksi})}{688\text{ kips}}\right]\right| \\ &= 0.0843P_r\end{aligned}$$

LRFD	ASD
$V_r' = 0.0843(173\text{ kips})$ $= 14.6\text{ kips}$	$V_r' = 0.0843(116\text{ kips})$ $= 9.78\text{ kips}$

An alternate approach would be the use of a plastic distribution method whereby the load is partitioned to each material in accordance with their contribution to the composite section strength given in Equation I2-9b. This method eliminates the need for longitudinal shear transfer provided the local bearing strength of the concrete and steel are adequate to resist the forces resulting from this distribution.

Additional Discussion

- The design and detailing of the connections required to deliver external forces to the composite member should be performed according to the applicable sections of AISC *Specification* Chapters J and K. Note that for checking bearing strength on concrete confined by a steel HSS or box member, the $\sqrt{A_2 / A_1}$ term in Equation J8-2 may be taken as 2.0 according to the User Note in *Specification* Section I6.2.

- The connection cases illustrated by Figure I.3-1 are idealized conditions representative of the mechanics of actual connections. For instance, a standard shear connection welded to the face of an HSS column is an example of a condition where all external force is applied directly to the steel section only. Note that the connection configuration can also impact the strength of the force transfer mechanism as illustrated in Part II of this example.

Solution:

Part II—Load Transfer

The required longitudinal force to be transferred, V_r' , determined in Part I condition (a) will be used to investigate the three applicable force transfer mechanisms of AISC *Specification* Section I6.3: direct bearing, shear connection, and direct bond interaction. As indicated in the *Specification*, these force transfer mechanisms may not be superimposed; however, the mechanism providing the greatest nominal strength may be used.

Direct Bearing

Trial Layout of Bearing Plate

For investigating the direct bearing load transfer mechanism, the external force is delivered directly to the HSS section by standard shear connections on each side of the member as illustrated in Figure I.3-2. One method for utilizing direct bearing in this instance is through the use of an internal bearing plate. Given the small clearance within the HSS section under consideration, internal access for welding is limited to the open ends of the HSS; therefore, the HSS section will be spliced at the bearing plate location. Additionally, it is a practical consideration that no more than 50% of the internal width of the HSS section be obstructed by the bearing plate in order to facilitate concrete placement. It is essential that concrete mix proportions and installation of concrete fill produce full bearing above and below the projecting plate. Based on these considerations, the trial bearing plate layout depicted in Figure I.3-2 was selected using an internal plate protrusion, L_p, of 1.0 in.

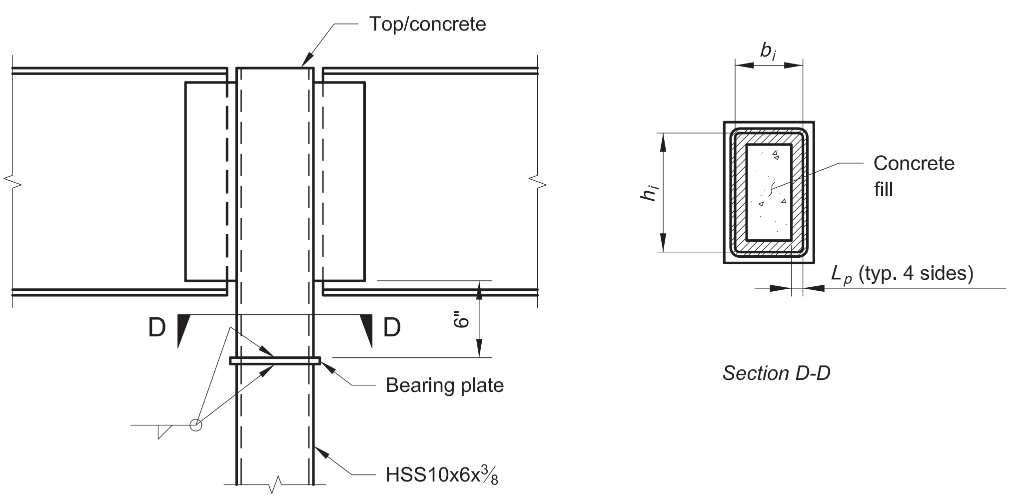


Fig. I.3-2. Internal bearing plate configuration.

Location of Bearing Plate

The bearing plate is placed within the load introduction length discussed in AISC *Specification* Section I6.4b. The load introduction length is defined as two times the minimum transverse dimension of the HSS both above and below the load transfer region. The load transfer region is defined in *Specification* Commentary Section I6.4 as the depth of the connection. For the configuration under consideration, the bearing plate should be located within 2(B = 6 in.) = 12 in. of the bottom of the shear connection. From Figure I.3-2, the location of the bearing plate is 6 in. from the bottom of the shear connection and is therefore adequate.

Return to Table of Contents

Available Strength for the Limit State of Direct Bearing

The contact area between the bearing plate and concrete, A_1, may be determined as follows:

$$A_1 = A_c - (b_i - 2L_p)(h_i - 2L_p) \quad \text{(Eq. 2)}$$

where

L_p = typical protrusion of bearing plate inside HSS
$= 1.0$ in.

Substituting for the appropriate geometric properties previously determined in Part I into Equation 2 yields:

$$\begin{aligned} A_1 &= 49.2 \text{ in.}^2 - \left[5.30 \text{ in.} - 2(1.0 \text{ in.})\right]\left[9.30 \text{ in.} - 2(1.0 \text{ in.})\right] \\ &= 25.1 \text{ in.}^2 \end{aligned}$$

The available strength for the direct bearing force transfer mechanism is:

$$R_n = 1.7 f_c' A_1 \quad \text{(}\textit{Spec}\text{. Eq. I6-3)}$$

LRFD	ASD
$\phi_B = 0.65$ $\phi_B R_n \geq V_r'$ $\phi_B R_n = 0.65(1.7)(5 \text{ ksi})(25.1 \text{ in.}^2)$ $= 139 \text{ kips} > 52.8 \text{ kips}$ **o.k.**	$\Omega_B = 2.31$ $R_n / \Omega_B \geq V_r'$ $R_n / \Omega_B = \dfrac{1.7(5 \text{ ksi})(25.1 \text{ in.}^2)}{2.31}$ $= 92.4 \text{ kips} > 35.4 \text{ kips}$ **o.k.**

Required Thickness of Internal Bearing Plate

There are several methods available for determining the bearing plate thickness. For round HSS sections with circular bearing plate openings, a closed-form elastic solution such as those found in *Roark's Formulas for Stress and Strain* (Young and Budynas, 2002) may be used. Alternately, the use of computational methods such as finite element analysis may be employed.

For this example, yield line theory can be employed to determine a plastic collapse mechanism of the plate. In this case, the walls of the HSS lack sufficient stiffness and strength to develop plastic hinges at the perimeter of the bearing plate. Utilizing only the plate material located within the HSS walls, and ignoring the HSS corner radii, the yield line pattern is as depicted in Figure I.3-3.

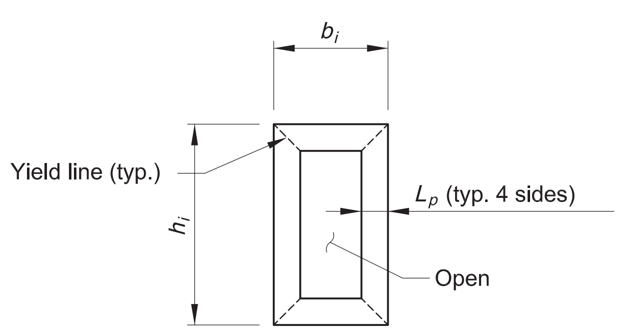


Fig. I.3-3. Yield line pattern.

Utilizing the results of the yield line analysis with $F_y = 36$ ksi plate material, the plate thickness may be determined as follows:

LRFD	ASD
$\phi = 0.90$ $t_p = \sqrt{\dfrac{w_u}{2\phi F_y}\left[L_p\left(b_i + h_i\right) - \dfrac{8L_p^{\ 2}}{3}\right]}$ where w_u = bearing pressure on plate determined using LRFD load combinations $= \dfrac{V_r'}{A_1}$ $= \dfrac{52.8 \text{ kips}}{25.1 \text{ in.}^2}$ $= 2.10$ ksi $t_p = \sqrt{\dfrac{(2.10 \text{ ksi})}{2(0.9)(36 \text{ ksi})}\left[(1.0 \text{ in.})(5.30 \text{ in.} + 9.30 \text{ in.}) - \dfrac{8(1.0 \text{ in.})^2}{3}\right]}$ $= 0.622$ in.	$\Omega = 1.67$ $t_p = \sqrt{\dfrac{\Omega w_a}{2F_y}\left[L_p\left(b_i + h_i\right) - \dfrac{8L_p^{\ 2}}{3}\right]}$ where w_a = bearing pressure on plate determined using ASD load combinations $= \dfrac{V_r'}{A_1}$ $= \dfrac{35.4 \text{ kips}}{25.1 \text{ in.}^2}$ $= 1.41$ ksi $t_p = \sqrt{\dfrac{(1.67)(1.41 \text{ ksi})}{2(36 \text{ ksi})}\left[(1.0 \text{ in.})(5.30 \text{ in.} + 9.30 \text{ in.}) - \dfrac{8(1.0 \text{ in.})^2}{3}\right]}$ $= 0.625$ in.

Thus, select a ¾-in.-thick bearing plate.

Splice Weld

The HSS is in compression due to the imposed loads, therefore the splice weld indicated in Figure I.3-2 is sized according to the minimum weld size requirements of Chapter J. Should uplift or flexure be applied in other loading conditions, the splice should be designed to resist these forces using the applicable provisions of AISC *Specification* Chapters J and K.

Shear Connection

Shear connection involves the use of steel headed stud or channel anchors placed within the HSS section to transfer the required longitudinal shear force. The use of the shear connection mechanism for force transfer in filled HSS is usually limited to large HSS sections and built-up box shapes, and is not practical for the composite member in question. Consultation with the fabricator regarding their specific capabilities is recommended to determine the feasibility of shear connection for HSS and box members. Should shear connection be a feasible load transfer mechanism, AISC *Specification* Section I6.3b in conjunction with the steel anchors in composite component provisions of Section I8.3 apply.

Direct Bond Interaction

The use of direct bond interaction for load transfer is limited to filled HSS and depends upon the location of the load transfer point within the length of the member being considered (end or interior) as well as the number of faces to which load is being transferred.

From AISC *Specification* Section I6.3c, the nominal bond strength for a rectangular section is:

$$R_n = B^2 C_{in} F_{in} \qquad \text{(\textit{Spec}. Eq. I6-5)}$$

where

B = overall width of rectangular steel section along face transferring load, in.

C_{in} = 2 if the filled composite member extends to one side of the point of force transfer

= 4 if the filled composite member extends to both sides of the point of force transfer

F_{in} = 0.06 ksi

For the design of this load transfer mechanism, two possible cases will be considered:

Case 1: End Condition – Load Transferred to Member from Four Sides Simultaneously

For this case the member is loaded at an end condition (the composite member only extends to one side of the point of force transfer). Force is applied to all four sides of the section simultaneously thus allowing the full perimeter of the section to be mobilized for bond strength.

From AISC *Specification* Equation I6-5:

LRFD	ASD
$\phi = 0.45$ $\phi R_n \geq V_r'$ $\phi R_n = 0.45\left[2(6.00 \text{ in.})^2 + 2(10.0 \text{ in.})^2\right](2)(0.06 \text{ ksi})$ $= 14.7 \text{ kips} < 52.8 \text{ kips}$ **n.g.**	$\Omega = 3.33$ $R_n / \Omega \geq V_r'$ $R_n / \Omega = \dfrac{\left[2(6.00 \text{ in.})^2 + 2(10.0 \text{ in.})^2\right](2)(0.06 \text{ ksi})}{3.33}$ $= 9.80 \text{ kips} < 35.4 \text{ kips}$ **n.g.**

Bond strength is inadequate and another force transfer mechanism such as direct bearing must be used to meet the load transfer provisions of AISC *Specification* Section I6.

Alternately, the detail could be revised so that the external force is applied to both the steel section and concrete fill concurrently as schematically illustrated in Figure I.3-1(c). Comparing bond strength to the load transfer requirements for concurrent loading determined in Part I of this example yields:

LRFD	ASD
$\phi = 0.45$ $\phi R_n \geq V_r'$ $\phi R_n = 14.7 \text{ kips} > 14.6 \text{ kips}$ **o.k.**	$\Omega = 3.33$ $R_n / \Omega \geq V_r'$ $R_n / \Omega = 9.80 \text{ kips} > 9.78 \text{ kips}$ **o.k.**

Case 2: Interior Condition – Load Transferred to Three Faces

For this case the composite member is loaded from three sides away from the end of the member (the composite member extends to both sides of the point of load transfer) as indicated in Figure I.3-4.

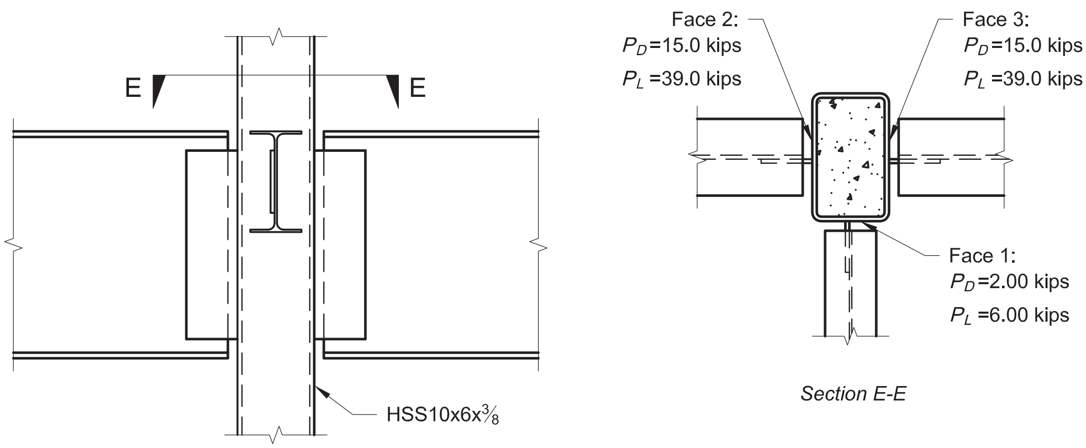


Fig. I.3-4. Case 2 load transfer.

Longitudinal shear forces to be transferred at each face of the HSS are calculated using the relationship to external forces determined in Part I of the example for condition (a) shown in Figure I.3-1, and the applicable ASCE/SEI 7-10 load combinations as follows:

LRFD	ASD
Face 1: $P_{r1} = P_u$ $= 1.2(2.00 \text{ kips}) + 1.6(6.00 \text{ kips})$ $= 12.0 \text{ kips}$ $V'_{r1} = 0.305P_{r1}$ $= 0.305(12.0 \text{ kips})$ $= 3.66 \text{ kips}$ Faces 2 and 3: $P_{r2-3} = P_u$ $= 1.2(15.0 \text{ kips}) + 1.6(39.0 \text{ kips})$ $= 80.4 \text{ kips}$ $V'_{r2-3} = 0.305P_{r2-3}$ $= 0.305(80.4 \text{ kips})$ $= 24.5 \text{ kips}$	Face 1: $P_{r1} = P_a$ $= 2.00 \text{ kips} + 6.00 \text{ kips}$ $= 8.00 \text{ kips}$ $V'_{r1} = 0.305P_{r1}$ $= 0.305(8.00 \text{ kips})$ $= 2.44 \text{ kips}$ Faces 2 and 3: $P_{r2-3} = P_u$ $= 15.0 \text{ kips} + 39.0 \text{ kips}$ $= 54.0 \text{ kips}$ $V'_{r2-3} = 0.305P_{r2-3}$ $= 0.305(54.0 \text{ kips})$ $= 16.5 \text{ kips}$

Load transfer at each face of the section is checked separately for the longitudinal shear at that face using Equation I6-5 as follows:

LRFD	ASD
$\phi = 0.45$ Face 1: $\phi R_{n1} \geq V'_{r1}$ $\phi R_{n1} = 0.45(6.00 \text{ in.})^2(4)(0.06 \text{ ksi})$ $= 3.89 \text{ kips} > 3.66 \text{ kips}$ **o.k.**	$\Omega = 3.33$ Face 1: $R_{n1}/\Omega \geq V'_{r1}$ $R_{n1}/\Omega = \dfrac{(6.00 \text{ in.})^2(4)(0.06 \text{ ksi})}{3.33}$ $= 2.59 \text{ kips} > 2.44 \text{ kips}$ **o.k.**

LRFD	ASD
Faces 2 and 3: $\phi R_{n2-3} \geq V'_{r2-3}$ $\phi R_{n2-3} = 0.45(10.0\text{ in.})^2(4)(0.06\text{ ksi})$ $= 10.8\text{ kips} < 24.5\text{ kips}$ **n.g.**	Faces 2 and 3: $R_{n2-3} / \Omega \geq V'_{r2-3}$ $R_{n2-3} / \Omega = \dfrac{(10.0\text{ in.})^2(4)(0.06\text{ ksi})}{3.33}$ $= 7.21\text{ kips} < 16.5\text{ kips}$ **n.g.**

The calculations indicate that the bond strength is inadequate for two of the three loaded faces, thus an alternate means of load transfer such as the use of internal bearing plates as demonstrated previously in this example is necessary.

As demonstrated by this example, direct bond interaction provides limited available strength for transfer of longitudinal shears and is generally only acceptable for lightly loaded columns or columns with low shear transfer requirements such as those with loads applied to both concrete fill and steel encasement simultaneously.

EXAMPLE I.4 FILLED COMPOSITE MEMBER IN AXIAL COMPRESSION

Given:

Determine if the 14 ft long, filled composite member illustrated in Figure I.4-1 is adequate for the indicated dead and live loads.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, $f_c' = 5$ ksi.

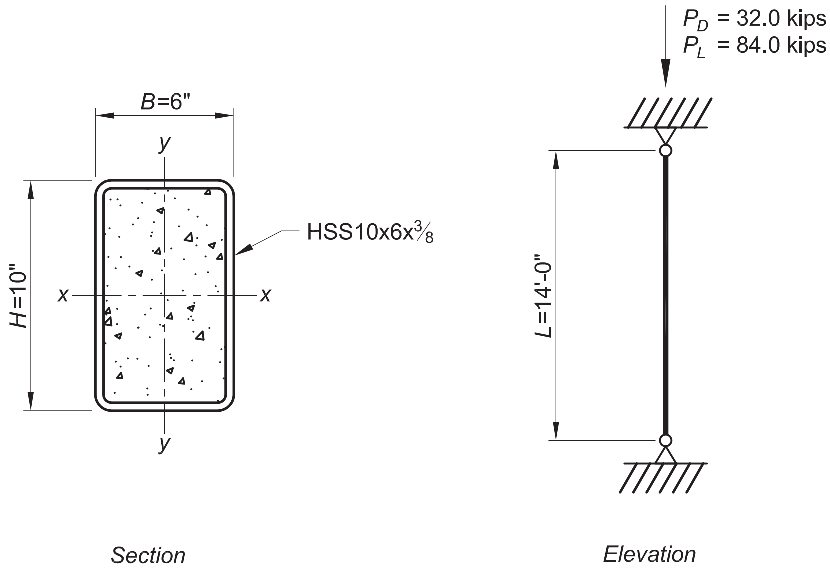


Fig. I.4-1. Concrete filled member section and applied loading.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required compressive strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(32.0\text{ kips}) + 1.6(84.0\text{ kips})$ $= 173\text{ kips}$	$P_r = P_a$ $= 32.0\text{ kips} + 84.0\text{ kips}$ $= 116\text{ kips}$

Method 1: AISC Manual Tables

The most direct method of calculating the available compressive strength is through the use of AISC *Manual* Table 4-14. A *K* factor of 1.0 is used for a pin-ended member. Because the unbraced length is the same in both the *x-x* and *y-y* directions, and I_x exceeds I_y, *y-y* axis buckling will govern.

Entering Table 4-14 with KL_y = 14 ft yields:

LRFD	ASD
$\phi_c P_n = 354$ kips $\phi_c P_n \geq P_u$ $354\text{ kips} > 173\text{ kips}$ **o.k.**	$P_n / \Omega_c = 236$ kips $P_n / \Omega_c \geq P_a$ $236\text{ kips} > 116\text{ kips}$ **o.k.**

Method 2: AISC Specification Calculations

As an alternate to the AISC *Manual* tables, the available compressive strength can be calculated directly using the provisions of AISC *Specification* Chapter I.

From AISC *Manual* Table 1-11 and Figure I.4-1, the geometric properties of an HSS10×6×⅜ are as follows:

A_s = 10.4 in.^{2}
H = 10.0 in.
B = 6.00 in.
t_{nom} = ⅜ in. (nominal wall thickness)
t = 0.349 in. (design wall thickness in accordance with AISC *Specification* Section B4.2)
h/t = 25.7
b/t = 14.2
I_{sx} = 137 in.^{4}
I_{sy} = 61.8 in.^{4}

Internal clear distances are determined as:

$$
\begin{aligned}
h_i &= H - 2t \\
&= 10.0\text{ in.} - 2(0.349\text{ in.}) \\
&= 9.30\text{ in.} \\
b_i &= B - 2t \\
&= 6.0\text{ in.} - 2(0.349\text{ in.}) \\
&= 5.30\text{ in.}
\end{aligned}
$$

From Design Example I.3, the area of concrete, A_c, equals 49.2 in.^{2} The steel and concrete areas can be used to calculate the gross cross-sectional area as follows:

$$
\begin{aligned}
A_g &= A_s + A_c \\
&= 10.4\text{ in.}^2 + 49.2\text{ in.}^2 \\
&= 59.6\text{ in.}^2
\end{aligned}
$$

Calculate the concrete moment of inertia using geometry compatible with that used in the calculation of the steel area in AISC *Manual* Table 1-11 (taking into account the design wall thickness and corner radii of two times the design wall thickness in accordance with AISC *Manual* Part 1), the following equations may be used, based on the terminology given in Figure I-2 of the introduction to these examples:

For bending about the *x-x* axis:

$$
I_{cx} = \frac{(B-4t)h_i^3}{12} + \frac{t(H-4t)^3}{6} + \frac{(9\pi^2 - 64)t^4}{36\pi} + \pi t^2\left(\frac{H-4t}{2} + \frac{4t}{3\pi}\right)^2
$$

$$= \frac{\left[6.00\text{ in.} - 4(0.349\text{ in.})\right](9.30\text{ in.})^3}{12} + \frac{(0.349\text{ in.})\left[10.0\text{ in.} - 4(0.349\text{ in.})\right]^3}{6} + \frac{(9\pi^2 - 64)(0.349\text{ in.})^4}{36\pi}$$

$$+ \pi(0.349\text{ in.})^2 \left(\frac{10.0\text{ in.} - 4(0.349\text{ in.})}{2} + \frac{4(0.349\text{ in.})}{3\pi}\right)^2$$

$$= 353\text{ in.}^4$$

For bending about the *y-y* axis:

$$I_{cy} = \frac{(H - 4t)b_i^3}{12} + \frac{t(B - 4t)^3}{6} + \frac{(9\pi^2 - 64)t^4}{36\pi} + \pi t^2 \left(\frac{B - 4t}{2} + \frac{4t}{3\pi}\right)^2$$

$$= \frac{\left[10.0\text{ in.} - 4(0.349\text{ in.})\right](5.30\text{ in.})^3}{12} + \frac{(0.349\text{ in.})\left[6.00\text{ in.} - 4(0.349\text{ in.})\right]^3}{6} + \frac{(9\pi^2 - 64)(0.349\text{ in.})^4}{36\pi}$$

$$+ \pi(0.349\text{ in.})^2 \left(\frac{6.00\text{ in.} - 4(0.349\text{ in.})}{2} + \frac{4(0.349\text{ in.})}{3\pi}\right)^2$$

$$= 115\text{ in.}^4$$

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3\text{ ksi} \leq f_c' \leq 10\text{ ksi}$

$f_c' = 5\text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \leq 75\text{ ksi}$

$F_y = 46\text{ ksi}$ **o.k.**

(3) Cross-sectional area of steel section: $A_s \geq 0.01A_g$

$$10.4\text{ in.}^2 \geq (0.01)(59.6\text{ in.}^2)$$

$$> 0.596\text{ in.}^2 \quad \textbf{o.k.}$$

There are no minimum longitudinal reinforcement requirements in the AISC *Specification* within filled composite members; therefore, the area of reinforcing bars, A_{sr}, for this example is zero.

Classify Section for Local Buckling

In order to determine the strength of the composite section subject to axial compression, the member is first classified as compact, noncompact or slender in accordance with AISC *Specification* Table I1.1A.

$$\lambda_p = 2.26\sqrt{\frac{E}{F_y}}$$

$$= 2.26\sqrt{\frac{29{,}000\text{ ksi}}{46\text{ ksi}}}$$

$$= 56.7$$

$$\lambda_{controlling} = \max\begin{pmatrix} h/t = 25.7 \\ b/t = 14.2 \end{pmatrix}$$

$$= 25.7$$

$\lambda_{controlling} \leq \lambda_p$ **section is compact**

Available Compressive Strength

The nominal axial compressive strength for compact sections without consideration of length effects, P_{no}, is determined from AISC *Specification* Section I2.2b as:

$$P_{no} = P_p \qquad \text{(}Spec\text{. Eq. I2-9a)}$$

$$= F_y A_s + C_2 f_c' \left(A_c + A_{sr} \frac{E_s}{E_c} \right) \qquad \text{(}Spec\text{. Eq. I2-9b)}$$

where C_2 = 0.85 for rectangular sections

$$P_{no} = (46 \text{ ksi})(10.4 \text{ in.}^2) + 0.85(5 \text{ ksi})(49.2 \text{ in.}^2 + 0.0 \text{ in.}^2)$$
$$= 688 \text{ kips}$$

Because the unbraced length is the same in both the *x-x* and *y-y* directions, the column will buckle about the weaker *y-y* axis (the axis having the lower moment of inertia). I_{cy} and I_{sy} will therefore be used for calculation of length effects in accordance with AISC *Specification* Sections I2.2b and I2.1b as follows:

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) \le 0.9 \qquad \text{(}Spec\text{. Eq. I2-13)}$$

$$= 0.6 + 2\left(\frac{10.4 \text{ in.}^2}{49.2 \text{ in.}^2 + 10.4 \text{ in.}^2}\right) \le 0.9$$

$$= 0.949 > 0.9 \qquad \textbf{0.9 controls}$$

$$E_c = w_c^{1.5}\sqrt{f_c'}$$
$$= (145 \text{ lb/ft}^3)^{1.5}\sqrt{5 \text{ ksi}}$$
$$= 3{,}900 \text{ ksi}$$

$$EI_{eff} = E_s I_{sy} + E_s I_{sr} + C_3 E_c I_{cy} \qquad \text{(}Spec\text{. Eq. I2-12)}$$
$$= (29{,}000 \text{ ksi})(61.8 \text{ in.}^4) + 0 + 0.9(3{,}900 \text{ ksi})(115 \text{ in.}^4)$$
$$= 2{,}200{,}000 \text{ kip-in.}^2$$

$$P_e = \pi^2 (EI_{eff})/(KL)^2 \qquad \text{(from }Spec\text{. Eq. I2-5)}$$

where K=1.0 for a pin-ended member

$$P_e = \frac{\pi^2 (2{,}200{,}000 \text{ kip-in.}^2)}{\left[(1.0)(14.0 \text{ ft})(12 \text{ in./ft})\right]^2}$$
$$= 769 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{688 \text{ kips}}{769 \text{ kips}}$$
$$= 0.895 < 2.25$$

Therefore, use AISC Specification Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] \qquad \text{(}Spec\text{. Eq. I2}-2\text{)}$$
$$= (688 \text{ kips})(0.658)^{0.895}$$
$$= 473 \text{ kips}$$

Check adequacy of the composite column for the required axial compressive strength:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n \geq P_u$ $\phi_c P_n = 0.75(473 \text{ kips})$ $= 355 \text{ kips} > 173 \text{ kips}$ **o.k.**	$\Omega_c = 2.00$ $P_n / \Omega_c \geq P_a$ $P_n / \Omega_c = \dfrac{473 \text{ kips}}{2.00}$ $= 237 \text{ kips} > 116 \text{ kips}$ **o.k.**

The slight differences between these values and those tabulated in the AISC *Manual* are due to the number of significant digits carried through the calculations.

Available Compressive Strength of Bare Steel Section

Due to the differences in resistance and safety factors between composite and noncomposite column provisions, it is possible to calculate a lower available compressive strength for a composite column than one would calculate for the corresponding bare steel section. However, in accordance with AISC *Specification* Section I2.1b, the available compressive strength need not be less than that calculated for the bare steel member in accordance with Chapter E.

From AISC *Manual* Table 4-3, for an HSS10×6×⅜, $KL_y = 14.0$ ft:

LRFD	ASD
$\phi_c P_n = 313 \text{ kips}$ $313 \text{ kips} < 355 \text{ kips}$	$P_n / \Omega_c = 208 \text{ kips}$ $208 \text{ kips} < 237 \text{ kips}$

Thus, the composite section strength controls and is adequate for the required axial compressive strength as previously demonstrated.

Force Allocation and Load Transfer

Load transfer calculations for external axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.

EXAMPLE I.5 FILLED COMPOSITE MEMBER IN AXIAL TENSION

Given:

Determine if the 14 ft long, filled composite member illustrated in Figure I.5-1 is adequate for the indicated dead load compression and wind load tension. The entire load is applied to the steel section.

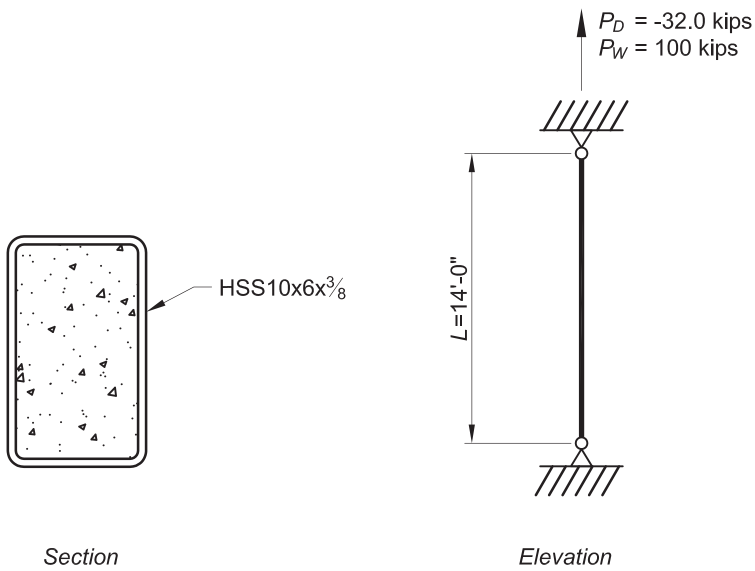


Fig. I.5-1. Concrete filled member section and applied loading.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, f_c' = 5 ksi.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS10×6×3⁄8
A_s = 10.4 in.^{2}

There are no minimum requirements for longitudinal reinforcement in the AISC *Specification*; therefore it is common industry practice to use filled shapes without longitudinal reinforcement, thus A_{sr} =0.

From Chapter 2 of ASCE/SEI 7, the required compressive strength is (taking compression as negative and tension as positive):

LRFD	ASD
Governing Uplift Load Combination $= 0.9D + 1.0W$ $P_r = P_u$ $= 0.9(-32.0 \text{ kips}) + 1.0(100 \text{ kips})$ $= 71.2 \text{ kips}$	Governing Uplift Load Combination $= 0.6D + 0.6W$ $P_r = P_a$ $= 0.6(-32.0 \text{ kips}) + 0.6(100 \text{ kips})$ $= 40.8 \text{ kips}$

Available Tensile Strength

Available tensile strength for a filled composite member is determined in accordance with AISC *Specification* Section I2.2c.

$$P_n = A_s F_y + A_{sr} F_{ysr} \quad \text{(}\textit{Spec}\text{. Eq. I2-14)}$$
$$= (10.4 \text{ in.}^2)(46 \text{ ksi}) + (0.0 \text{ in.}^2)(60 \text{ ksi})$$
$$= 478 \text{ kips}$$

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n \geq P_u$ $\phi_t P_n = 0.90(478 \text{ kips})$ $= 430 \text{ kips} > 71.2 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $P_n / \Omega_t \geq P_a$ $P_n / \Omega_t = \dfrac{478 \text{ kips}}{1.67}$ $= 286 \text{ kips} > 40.8 \text{ kips}$ **o.k.**

For concrete filled HSS members with no internal longitudinal reinforcing, the values for available tensile strength may also be taken directly from AISC *Manual* Table 5-4.

Force Allocation and Load Transfer

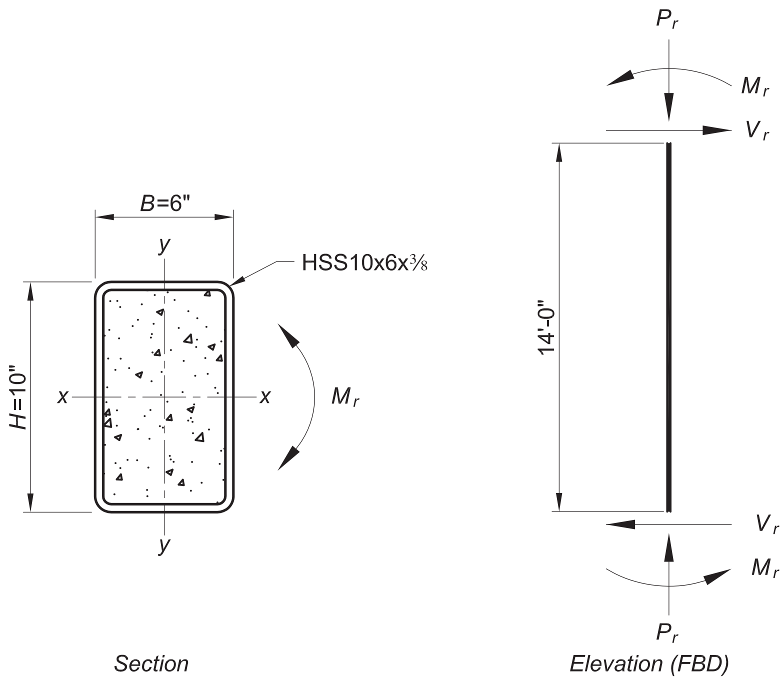
Load transfer calculations are not required for concrete filled members in axial tension that do not contain longitudinal reinforcement, such as the one under investigation, as only the steel section resists tension.

Return to Table of Contents

EXAMPLE I.6 FILLED COMPOSITE MEMBER IN COMBINED AXIAL COMPRESSION, FLEXURE AND SHEAR

Given:

Determine if the 14 ft long, filled composite member illustrated in Figure I.6-1 is adequate for the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC *Specification* Chapter C for the controlling ASCE/SEI 7-10 load combinations.



	LRFD	ASD
P_r (kips)	129	98.2
M_r (kip-ft)	120	54.0
V_r (kips)	17.1	10.3

Fig. I.6-1. Concrete filled member section and member forces.

The composite member consists of an ASTM A500 Grade B HSS with normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, $f_c' = 5$ ksi.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A500 Grade B
$F_y = 46$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-11 and Figure I.6-1, the geometric properties are as follows:

HSS10×6×⅜
H = 10.0 in.
B = 6.00 in.

t_{nom} = ⅜ in. (nominal wall thickness)
t = 0.349 in. (design wall thickness)
h/t = 25.7
b/t = 14.2
A_s = 10.4 in.^{2}
I_{sx} = 137 in.^{4}
I_{sy} = 61.8 in.^{4}
Z_{sx} = 33.8 in.^{3}

Additional geometric properties used for composite design are determined in Design Examples I.3 and I.4 as follows:

h_i	= 9.30 in.	clear distance between HSS walls (longer side)
b_i	= 5.30 in.	clear distance between HSS walls (shorter side)
A_c	= 49.2 in.^{2}	cross-sectional area of concrete fill
A_g	= 59.6 in.^{2}	gross cross-sectional area of composite member
A_{sr}	= 0 in.^{2}	area of longitudinal reinforcement
E_c	= 3,900 ksi	modulus of elasticity of concrete
I_{cx}	= 353 in.^{4}	moment of inertia of concrete fill about the *x-x* axis
I_{cy}	= 115 in.^{4}	moment of inertia of concrete fill about the *y-y* axis

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$$f_c' = 5 \text{ ksi} \quad \textbf{o.k.}$$

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$$F_y = 46 \text{ ksi} \quad \textbf{o.k.}$$

(3) Cross-sectional area of steel section: $A_s \ge 0.01A_g$

$$10.4 \text{ in.}^2 \ge (0.01)\left(59.6 \text{ in.}^2\right)$$
$$> 0.596 \text{ in.}^2 \quad \textbf{o.k.}$$

Classify Section for Local Buckling

The composite member in question was shown to be compact for pure compression in Design Example I.4 in accordance with AISC *Specification* Table I1.1a. The section must also be classified for local buckling due to flexure in accordance with *Specification* Table I1.1b; however, since the limits for members subject to flexure are equal to or less stringent than those for members subject to compression, the member is compact for flexure.

Interaction of Axial Force and Flexure

The interaction between axial forces and flexure in composite members is governed by AISC *Specification* Section I5 which, for compact members, permits the use of a strain compatibility method or plastic stress distribution method, with the option to use the interaction equations of Section H1.1.

The strain compatibility method is a generalized approach that allows for the construction of an interaction diagram based upon the same concepts used for reinforced concrete design. Application of the strain compatibility method is required for irregular/nonsymmetrical sections, and its general application may be found in reinforced concrete design texts and will not be discussed further here.

Plastic stress distribution methods are discussed in AISC *Specification* Commentary Section I5 which provides three acceptable procedures for filled members. The first procedure, Method 1, invokes the interaction equations of

Section H1. This is the only method applicable to sections with noncompact or slender elements. The second procedure, Method 2, involves the construction of a piecewise-linear interaction curve using the plastic strength equations provided in Figure I.1c located within the front matter of the Chapter I Design Examples. The third procedure, Method 2 – Simplified, is a reduction of the piecewise-linear interaction curve that allows for the use of less conservative interaction equations than those presented in Chapter H.

For this design example, each of the three applicable plastic stress distribution procedures are reviewed and compared.

Method 1: Interaction Equations of Section H1

The most direct and conservative method of assessing interaction effects is through the use of the interaction equations of AISC *Specification* Section H1. For HSS shapes, both the available compressive and flexural strengths can be determined from *Manual* Table 4-14. In accordance with the direct analysis method, a K factor of 1 is used. Because the unbraced length is the same in both the x-x and y-y directions, and I_x exceeds I_y, y-y axis buckling will govern for the compressive strength. Flexural strength is determined for the x-x axis to resist the applied moment about this axis indicated in Figure I.6-1.

Entering Table 4-14 with KL_y = 14 ft yields:

LRFD	ASD
$\phi_c P_n = 354\text{ kips}$ $\phi_b M_{nx} = 130\text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n} = \dfrac{129\text{ kips}}{354\text{ kips}} = 0.364 \geq 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \leq 1.0$ $\dfrac{129\text{ kips}}{354\text{ kips}} + \dfrac{8}{9}\left(\dfrac{120\text{ kip-ft}}{130\text{ kip-ft}}\right) \leq 1.0$ $1.18 > 1.0$ **n.g.**	$P_n/\Omega_c = 236\text{ kips}$ $M_{nx}/\Omega_c = 86.6\text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n/\Omega_c} = \dfrac{98.2\text{ kips}}{236\text{ kips}} = 0.416 \geq 0.2$ Therefore, use AISC *Specification* Equation H1-1a. $\dfrac{P_a}{P_n/\Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n/\Omega_b}\right) \leq 1.0$ $\dfrac{98.2\text{ kips}}{236\text{ kips}} + \dfrac{8}{9}\left(\dfrac{54.0\text{ kip-ft}}{86.6\text{ kip-ft}}\right) \leq 1.0$ $0.97 < 1.0$ **o.k.**

Using LRFD methodology, Method 1 indicates that the section is inadequate for the applied loads. The designer can elect to choose a new section that passes the interaction check or re-analyze the current section using a less conservative design method such as Method 2. The use of Method 2 is illustrated in the following section.

Method 2: Interaction Curves from the Plastic Stress Distribution Model

The procedure for creating an interaction curve using the plastic stress distribution model is illustrated graphically in Figure I.6-2.

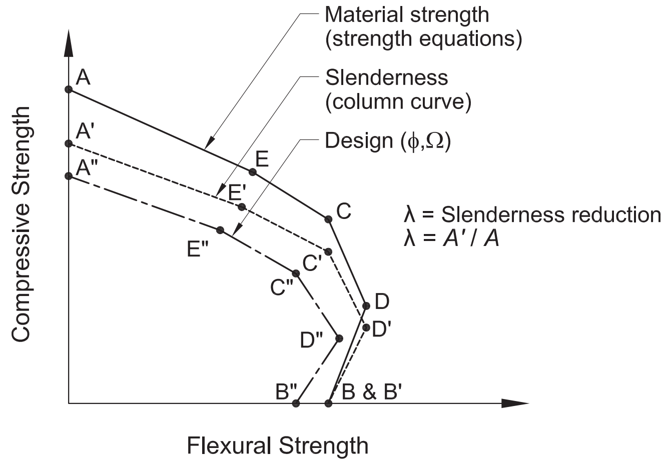


Fig. I.6-2. Interaction diagram for composite beam-column —Method 2.

Referencing Figure I.6-2, the nominal strength interaction surface A,B,C,D,E is first determined using the equations of Figure I-1c found in the introduction of the Chapter I Design Examples. This curve is representative of the short column member strength without consideration of length effects. A slenderness reduction factor, λ, is then calculated and applied to each point to create surface A′, B′, C′, D′, E′. The appropriate resistance or safety factors are then applied to create the design surface A″, B″, C″, D″, E″. Finally, the required axial and flexural strengths from the applicable load combinations of ASCE/SEI 7-10 are plotted on the design surface, and the member is acceptable for the applied loading if all points fall within the design surface. These steps are illustrated in detail by the following calculations.

Step 1: Construct nominal strength interaction surface A, B, C, D, E without length effects

Using the equations provided in Figure I-1c for bending about the *x-x* axis yields:

Point A (pure axial compression):

$$
\begin{aligned}
P_A &= F_y A_s + 0.85 f_c' A_c \\
&= (46\text{ ksi})(10.4\text{ in.}^2) + 0.85(5\text{ ksi})(49.2\text{ in.}^2) \\
&= 688\text{ kips} \\
M_A &= 0\text{ kip-ft}
\end{aligned}
$$

Point D (maximum nominal moment strength):

$$
\begin{aligned}
P_D &= \frac{0.85 f_c' A_c}{2} \\
&= \frac{0.85(5\text{ ksi})(49.2\text{ in.}^2)}{2} \\
&= 105\text{ kips} \\
Z_{sx} &= 33.8\text{ in.}^3
\end{aligned}
$$

$$Z_c = \frac{b_i h_i^2}{4} - 0.192 r_i^3 \quad \text{where } r_i = t$$
$$= \frac{(5.30 \text{ in.})(9.30 \text{ in.})^2}{4} - 0.192(0.349 \text{ in.})^3$$
$$= 115 \text{ in.}^3$$

$$M_D = F_y Z_{sx} + \frac{0.85 f_c' Z_c}{2}$$
$$= (46 \text{ ksi})(33.8 \text{ in.}^3) + \frac{0.85(5 \text{ ksi})(115 \text{ in.}^3)}{2}$$
$$= \frac{1{,}800 \text{ kip-in.}}{12 \text{ in./ft}}$$
$$= 150 \text{ kip-ft}$$

Point B (pure flexure):

$$P_B = 0 \text{ kips}$$
$$h_n = \frac{0.85 f_c' A_c}{2(0.85 f_c' b_i + 4tF_y)} \le \frac{h_i}{2}$$
$$= \frac{0.85(5 \text{ ksi})(49.2 \text{ in.}^2)}{2[0.85(5 \text{ ksi})(5.30 \text{ in.}) + 4(0.349 \text{ in.})(46 \text{ ksi})]} \le \frac{9.30 \text{ in.}}{2}$$
$$= 1.21 \text{ in.} \le 4.65 \text{ in.}$$
$$= 1.21 \text{ in.}$$

$$Z_{sn} = 2th_n^2$$
$$= 2(0.349 \text{ in.})(1.21 \text{ in.})^2$$
$$= 1.02 \text{ in.}^3$$

$$Z_{cn} = b_i h_n^2$$
$$= (5.30 \text{ in.})(1.21 \text{ in.})^2$$
$$= 7.76 \text{ in.}^3$$

$$M_B = M_D - F_y Z_{sn} - \frac{0.85 f_c' Z_{cn}}{2}$$
$$= 1{,}800 \text{ kip-in.} - (46 \text{ ksi})(1.02 \text{ in.}^3) - \frac{0.85(5 \text{ ksi})(7.76 \text{ in.}^3)}{2}$$
$$= \frac{1{,}740 \text{ kip-in.}}{12 \text{ in./ft}}$$
$$= 145 \text{ kip-ft}$$

Point C (intermediate point):

$$P_C = 0.85 f_c' A_c$$
$$= 0.85(5 \text{ ksi})(49.2 \text{ in.}^2)$$
$$= 209 \text{ kips}$$
$$M_C = M_B$$
$$= 145 \text{ kip-ft}$$

Point E (optional):

Point E is an optional point that helps better define the interaction curve.

$$h_E = \frac{h_n}{2} + \frac{H}{4} \quad \text{where } h_n = 1.21 \text{ in. from Point B}$$
$$= \frac{1.21 \text{ in.}}{2} + \frac{10.0 \text{ in.}}{4}$$
$$= 3.11 \text{ in.}$$

$$P_E = \frac{0.85 f'_c A_c}{2} + 0.85 f'_c b_i h_E + 4F_y t h_E$$
$$= \frac{0.85(5 \text{ ksi})(49.2 \text{ in.}^2)}{2} + 0.85(5 \text{ ksi})(5.30 \text{ in.})(3.11 \text{ in.}) + 4(46 \text{ ksi})(0.349 \text{ in.})(3.11 \text{ in.})$$
$$= 374 \text{ kips}$$

$$Z_{cE} = b_i h_E^2$$
$$= (5.30 \text{ in.})(3.11 \text{ in.})^2$$
$$= 51.3 \text{ in.}^3$$

$$Z_{sE} = 2 t h_E^2$$
$$= 2(0.349 \text{ in.})(3.11 \text{ in.})^2$$
$$= 6.75 \text{ in.}^3$$

$$M_E = M_D - F_y Z_{sE} - \frac{0.85 f'_c Z_{cE}}{2}$$
$$= 1{,}800 \text{ kip-in.} - (46 \text{ ksi})(6.75 \text{ in.}^3) - \frac{0.85(5 \text{ ksi})(51.3 \text{ in.}^3)}{2}$$
$$= \frac{1{,}380 \text{ kip-in.}}{12 \text{ in./ft}}$$
$$= 115 \text{ kip-ft}$$

The calculated points are plotted to construct the nominal strength interaction surface without length effects as depicted in Figure I.6-3.

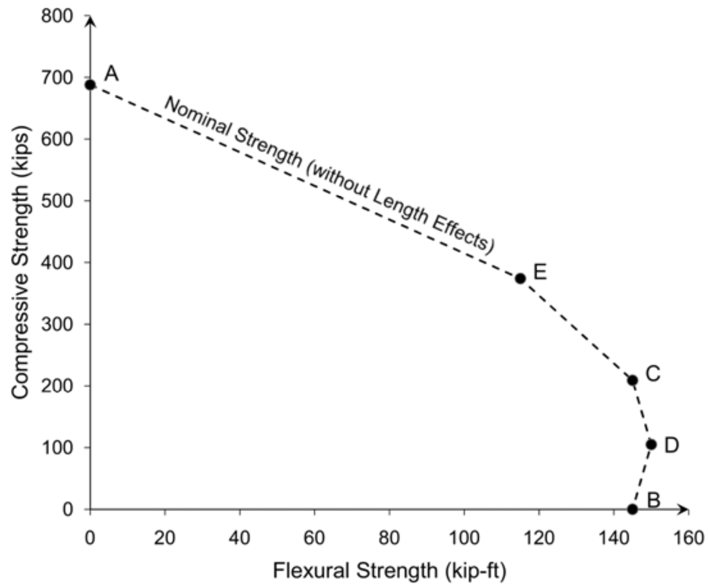


Fig. I.6-3. Nominal strength interaction surface without length effects.

Step 2: Construct nominal strength interaction surface A′, B′, C′, D′, E′ with length effects

The slenderness reduction factor, λ, is calculated for Point A using AISC *Specification* Section I2.2 in accordance with *Specification* Commentary Section I5.

$$P_{no} = P_A$$
$$= 688 \text{ kips}$$

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.9 \qquad \text{(Spec. Eq. I2-13)}$$
$$= 0.6 + 2\left(\frac{10.4 \text{ in.}^2}{49.2 \text{ in.}^2 + 10.4 \text{ in.}^2}\right) \leq 0.9$$
$$= 0.949 > 0.9 \qquad \textbf{0.9 controls}$$

$$EI_{eff} = E_s I_{sy} + E_s I_{sr} + C_3 E_c I_{cy} \qquad \text{(from Spec. Eq. I2-12)}$$
$$= (29{,}000 \text{ ksi})(61.8 \text{ in.}^4) + 0 + 0.9(3{,}900 \text{ ksi})(115 \text{ in.}^4)$$
$$= 2{,}200{,}000 \text{ ksi}$$

$$P_e = \pi^2 (EI_{eff}) / (KL)^2 \text{ where } K = 1.0 \text{ in accordance with the direct analysis method} \qquad \text{(Spec. Eq. I2-5)}$$
$$= \frac{\pi^2 (2{,}200{,}000 \text{ ksi})}{\left[(14.0 \text{ ft})(12 \text{ in./ft})\right]^2}$$
$$= 769 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{688 \text{ kips}}{769 \text{ kips}}$$
$$= 0.895 < 2.25$$

Use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] \qquad \text{(}Spec\text{. Eq. I2-2)}$$

$$= 688 \text{ kips}(0.658)^{0.895}$$

$$= 473 \text{ kips}$$

$$\lambda = \frac{P_n}{P_{no}}$$

$$= \frac{473 \text{ kips}}{688 \text{ kips}}$$

$$= 0.688$$

In accordance with AISC *Specification* Commentary Section I5, the same slenderness reduction is applied to each of the remaining points on the interaction surface as follows:

$$P_{A'} = \lambda P_A$$

$$= 0.688(688 \text{ kips})$$

$$= 473 \text{ kips}$$

$$P_{B'} = \lambda P_B$$

$$= 0.688(0 \text{ kips})$$

$$= 0 \text{ kips}$$

$$P_{C'} = \lambda P_C$$

$$= 0.688(209 \text{ kips})$$

$$= 144 \text{ kips}$$

$$P_{D'} = \lambda P_D$$

$$= 0.688(105 \text{ kips})$$

$$= 72.2 \text{ kips}$$

$$P_{E'} = \lambda P_E$$

$$= 0.688(374 \text{ kips})$$

$$= 257 \text{ kips}$$

The modified axial strength values are plotted with the flexural strength values previously calculated to construct the nominal strength interaction surface including length effects. These values are superimposed on the nominal strength surface not including length effects for comparison purposes in Figure I.6-4.

Return to Table of Contents

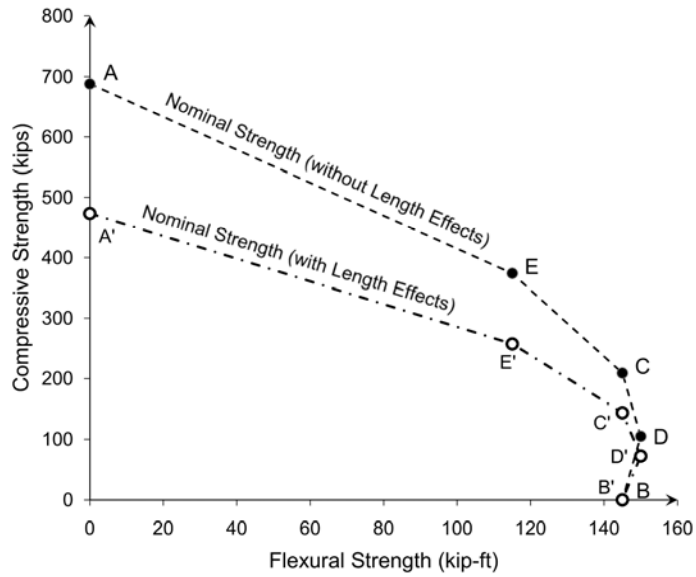


Fig. I.6-4. Nominal strength interaction surfaces (with and without length effects).

Step 3: Construct design interaction surface A″, B″, C″, D″, E″ and verify member adequacy

The final step in the Method 2 procedure is to reduce the interaction surface for design using the appropriate resistance or safety factors.

LRFD	ASD
Design compressive strength: $\phi_c = 0.75$ $P_{X''} = \phi_c P_{X'}$ where X = A, B, C, D or E $P_{A''} = 0.75(473 \text{ kips})$ $= 355 \text{ kips}$ $P_{B''} = 0.75(0 \text{ kips})$ $= 0 \text{ kips}$ $P_{C''} = 0.75(144 \text{ kips})$ $= 108 \text{ kips}$ $P_{D''} = 0.75(72.2 \text{ kips})$ $= 54.2 \text{ kips}$ $P_{E''} = 0.75(257 \text{ kips})$ $= 193 \text{ kips}$ Design flexural strength: $\phi_b = 0.90$ $M_{X''} = \phi_b M_{X'}$ where X = A, B, C, D or E	Allowable compressive strength: $\Omega_c = 2.00$ $P_{X''} = P_{X'} / \Omega_c$ where X = A, B, C, D or E $P_{A''} = 473 \text{ kips} / 2.00$ $= 237 \text{ kips}$ $P_{B''} = 0 \text{ kips} / 2.00$ $= 0 \text{ kips}$ $P_{C''} = 144 \text{ kips} / 2.00$ $= 72 \text{ kips}$ $P_{D''} = 72.2 \text{ kips} / 2.00$ $= 36.1 \text{ kips}$ $P_{E''} = 257 \text{ kips} / 2.00$ $= 129 \text{ kips}$ Allowable flexural strength: $\Omega_b = 1.67$ $M_{X''} = M_{X'} / \Omega_b$ where X = A, B, C, D or E

LRFD	ASD
$M_{A''} = 0.90(0 \text{ kip-ft})$ $= 0 \text{ kip-ft}$ $M_{B''} = 0.90(145 \text{ kip-ft})$ $= 131 \text{ kip-ft}$ $M_{C''} = 0.90(145 \text{ kip-ft})$ $= 131 \text{ kip-ft}$ $M_{D''} = 0.90(150 \text{ kip-ft})$ $= 135 \text{ kip-ft}$ $M_{E''} = 0.90(115 \text{ kip-ft})$ $= 104 \text{ kip-ft}$	$M_{A''} = 0 \text{ kip-ft} / 1.67$ $= 0 \text{ kip-ft}$ $M_{B''} = 145 \text{ kip-ft} / 1.67$ $= 86.8 \text{ kip-ft}$ $M_{C''} = 145 \text{ kip-ft} / 1.67$ $= 86.8 \text{ kip-ft}$ $M_{D''} = 150 \text{ kip-ft} / 1.67$ $= 89.8 \text{ kip-ft}$ $M_{E''} = 115 \text{ kip-ft} / 1.67$ $= 68.9 \text{ kip-ft}$

The available strength values for each design method can now be plotted. These values are superimposed on the nominal strength surfaces (with and without length effects) previously calculated for comparison purposes in Figure I.6-5.

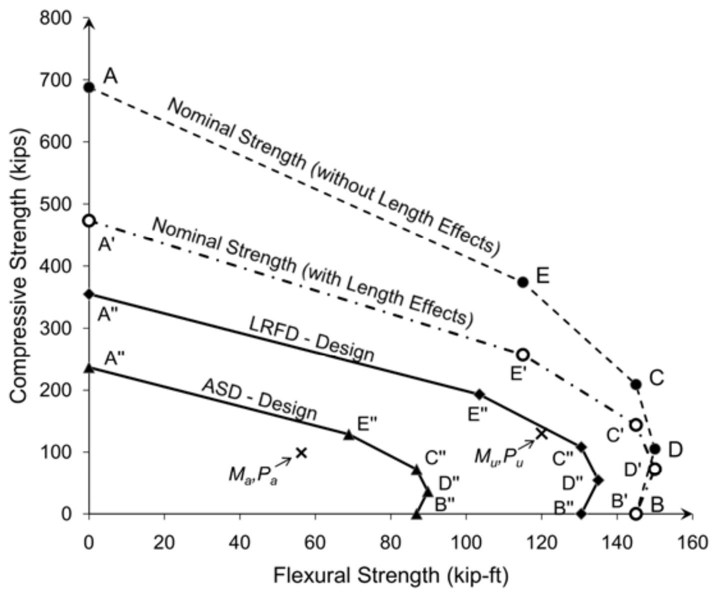


Fig. I.6-5. Available and nominal interaction surfaces.

By plotting the required axial and flexural strength values determined for the governing load combinations on the available strength surfaces indicated in Figure I.6-5, it can be seen that both ASD (M_a, P_a) and LRFD (M_u, P_u) points lie within their respective design surfaces. The member in question is therefore adequate for the applied loads.

Designers should carefully review the proximity of the available strength values in relation to point D'' on Figure I.6-5 as it is possible for point D'' to fall outside of the nominal strength curve, thus resulting in an unsafe design. This possibility is discussed further in AISC Commentary Section I5 and is avoided through the use of Method 2 – Simplified as illustrated in the following section.

Method 2: Simplified

The simplified version of Method 2 involves the removal of points D″ and E″ from the Method 2 interaction surface leaving only points A″,B″ and C″ as illustrated in the comparison of the two methods in Figure I.6-6.

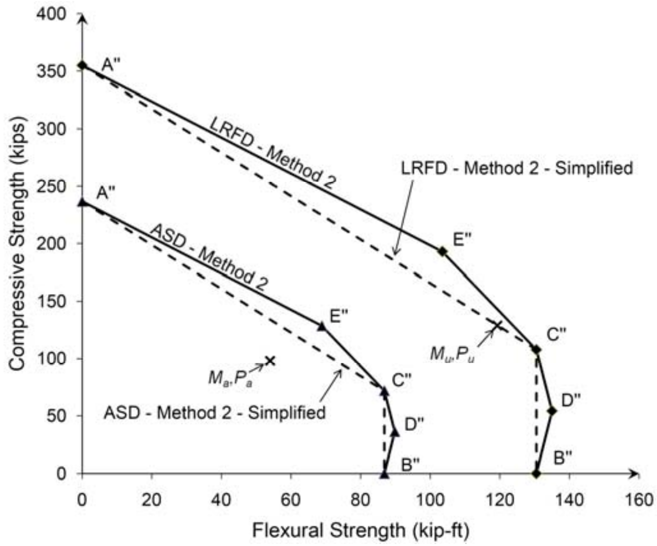


Fig. I.6-6. Comparison of Method 2 and Method 2 – Simplified.

Reducing the number of interaction points allows for a bilinear interaction check defined by AISC *Specification* Commentary Equations C-I5-1a and C-I5-1b to be performed. Using the available strength values previously calculated in conjunction with the Commentary equations, interaction ratios are determined as follows:

LRFD	ASD
$P_r = P_u$ $= 129 \text{ kips}$ $P_r \geq P_{C''}$ $\geq 108 \text{ kips}$ Use AISC *Specification* Commentary Equation C-I5-1b. $\dfrac{P_r - P_C}{P_A - P_C} + \dfrac{M_r}{M_C} \leq 1.0$ which for LRFD equals: $\dfrac{P_u - P_{C''}}{P_{A''} - P_{C''}} + \dfrac{M_u}{M_{C''}} \leq 1.0$ $\dfrac{129 \text{ kips} - 108 \text{ kips}}{355 \text{ kips} - 108 \text{ kips}} + \dfrac{120 \text{ kip-ft}}{131 \text{ kip-ft}} \leq 1.0$ $1.00 = 1.0$ **o.k.**	$P_r = P_a$ $= 98.2 \text{ kips}$ $P_r \geq P_{C''}$ $\geq 72 \text{ kips}$ Use AISC *Specification* Commentary Equation C-I5-1b. $\dfrac{P_r - P_C}{P_A - P_C} + \dfrac{M_r}{M_C} \leq 1.0$ which for ASD equals: $\dfrac{P_a - P_{C''}}{P_{A''} - P_{C''}} + \dfrac{M_a}{M_{C''}} \leq 1.0$ $\dfrac{98.2 \text{ kips} - 72.0 \text{ kips}}{237 \text{ kips} - 72.0 \text{ kips}} + \dfrac{54.0 \text{ kip-ft}}{86.8 \text{ kip-ft}} \leq 1.0$ $0.781 < 1.0$ **o.k.**

Thus, the member is adequate for the applied loads.

Comparison of Methods

The composite member was found to be inadequate using Method 1—Chapter H interaction equations, but was found to be adequate using both Method 2 and Method 2—Simplified procedures. A comparison between the methods is most easily made by overlaying the design curves from each method as illustrated in Figure I.6-7 for LRFD design.

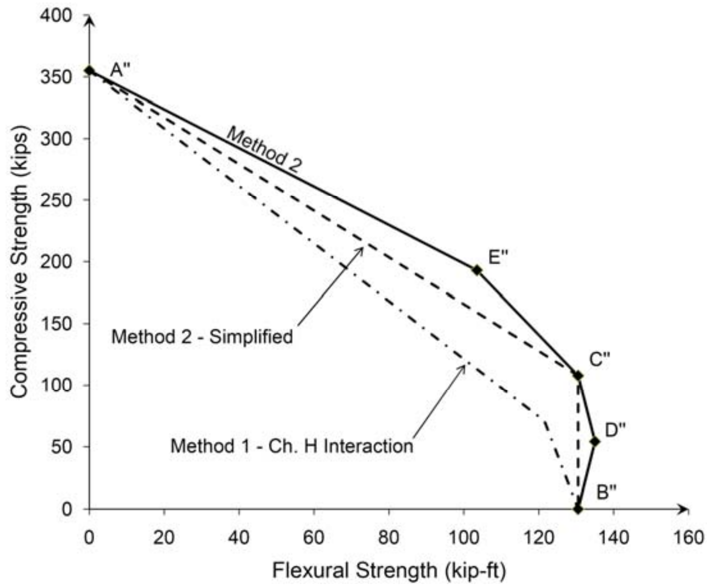


Fig. I.6-7. Comparison of interaction methods (LRFD).

From Figure I.6-7, the conservative nature of the Chapter H interaction equations can be seen. Method 2 provides the highest available strength; however, the Method 2—Simplified procedure also provides a good representation of the complete design curve. By using Part 4 of the AISC *Manual* to determine the available strength of the composite member in compression and flexure (Points A'' and B'' respectively), the modest additional effort required to calculate the available compressive strength at Point C'' can result in appreciable gains in member strength when using Method 2—Simplified as opposed to Method 1.

Available Shear Strength

AISC *Specification* Section I4.1 provides three methods for determining the available shear strength of a filled member: available shear strength of the steel section alone in accordance with Chapter G, available shear strength of the reinforced concrete portion alone per ACI 318, or available shear strength of the steel section plus the reinforcing steel ignoring the contribution of the concrete.

Available Shear Strength of Steel Section

From AISC *Specification* Section G5, the nominal shear strength, V_n, of HSS members is determined using the provisions of Section G2.1(b) with $k_v = 5$. The provisions define the width of web resisting the shear force, h, as the outside dimension minus three times the design wall thickness.

$$
\begin{aligned}
h &= H - 3t \\
&= 10.0 \text{ in.} - 3(0.349 \text{ in.}) \\
&= 8.95 \text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
A_w &= 2ht \\
&= 2(8.95 \text{ in.})(0.349 \text{ in.}) \\
&= 6.25 \text{ in.}^2
\end{aligned}
$$

The slenderness value, h/t_w, used to determine the web shear coefficient, C_v, is provided in AISC *Manual* Table 1-11 as 25.7.

$$\frac{h}{t_w} \leq 1.10\sqrt{k_v E/F_y}$$

$$\leq 1.10\sqrt{5\left(\frac{29{,}000 \text{ ksi}}{46 \text{ ksi}}\right)}$$

$$25.7 < 61.8$$

Use AISC *Specification* Equation G2-3.

$$C_v = 1.0$$ (*Spec*. Eq. G2-3)

The nominal shear strength is calculated as:

$$V_n = 0.6F_y A_w C_v$$ (*Spec*. Eq. G2-1)

$$= 0.6(46 \text{ ksi})(6.25 \text{ in.}^2)(1.0)$$

$$= 173 \text{ kips}$$

The available shear strength of the steel section is:

LRFD	ASD
$V_u = 17.1$ kips $\phi_v = 0.90$ $\phi_v V_n \geq V_u$ $\phi_v V_n = 0.90(173 \text{ kips})$ $= 156 \text{ kips} > 17.1 \text{ kips}$ **o.k.**	$V_a = 10.3$ kips $\Omega_v = 1.67$ $V_n / \Omega_v \geq V_a$ $V_n / \Omega_v = \dfrac{173 \text{ kips}}{1.67}$ $= 104 \text{ kips} > 10.3 \text{ kips}$ **o.k.**

Available Shear Strength of the Reinforced Concrete

The available shear strength of the steel section alone has been shown to be sufficient, but the available shear strength of the concrete will be calculated for demonstration purposes. Considering that the member does not have longitudinal reinforcing, the method of shear strength calculation involving reinforced concrete is not valid; however, the design shear strength of the plain concrete using Chapter 22 of ACI 318 can be determined as follows:

$\phi = 0.60$ for plain concrete design from ACI 318 Section 9.3.5
$\lambda = 1.0$ for normal weight concrete from ACI 318 Section 8.6.1

$$V_n = \left(\frac{4}{3}\right)\lambda\sqrt{f_c'}b_w h$$ (ACI 318 Eq. 22-9)

$$b_w = b_i$$

$$h = h_i$$

$$V_n = \left(\frac{4}{3}\right)(1.0)\sqrt{5{,}000 \text{ psi}}(5.30 \text{ in.})(9.30 \text{ in.})\left(\frac{1 \text{ kip}}{1{,}000 \text{ lb}}\right)$$

$$= 4.65 \text{ kips}$$

$$\phi V_n = 0.60(4.65 \text{ kips})$$

$$= 2.79 \text{ kips}$$

$$\phi V_n \geq V_u$$ (ACI 318 Eq. 22-8)

$2.79 \text{ kips} < 17.1 \text{ kips}$ **n.g.**

As can be seen from this calculation, the shear resistance provided by plain concrete is small and the strength of the steel section alone is generally sufficient.

Force Allocation and Load Transfer

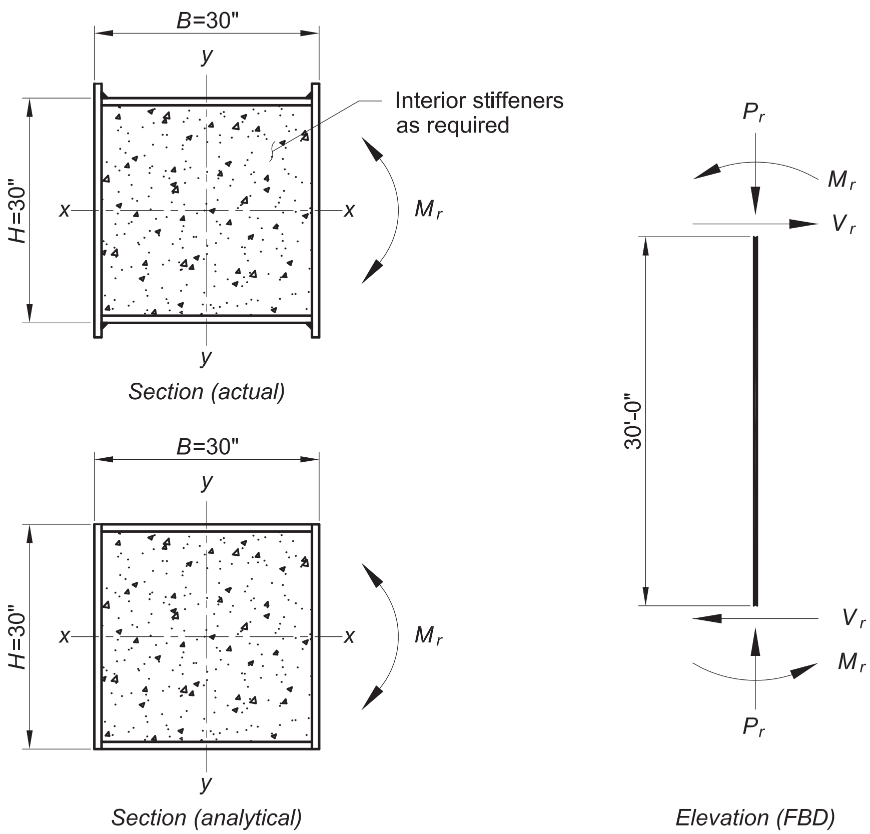
Load transfer calculations for applied axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.

Return to Table of Contents

EXAMPLE I.7 FILLED BOX COLUMN WITH NONCOMPACT/SLENDER ELEMENTS

Given:

Determine the required ASTM A36 plate thickness of the 30 ft long, composite box column illustrated in Figure I.7-1 to resist the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC *Specification* Chapter C for the controlling ASCE/SEI 7-10 load combinations. The core is composed of normal weight (145 lb/ft^3) concrete fill having a specified concrete compressive strength, $f_c' = 7$ ksi.



	LRFD	ASD
P_r (kips)	1,310	1,370
M_r (kip-ft)	552	248
V_r (kips)	36.8	22.1

Fig. I.7-1. Composite box column section and member forces.

Solution:

From AISC *Manual* Table 2-4, the material properties are:

ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

Trial Size 1 (Noncompact)

For ease of calculation the contribution of the plate extensions to the member strength will be ignored as illustrated by the analytical model in Figure I.7-1.

Trial Plate Thickness and Geometric Section Properties of the Composite Member

Select a trial plate thickness, t, of ⅜ in. Note that the design wall thickness reduction of AISC *Specification* Section B4.2 applies only to electric-resistance-welded HSS members and does not apply to built-up sections such as the one under consideration.

The calculated geometric properties of the 30 in. by 30 in. steel box column are:

$$B = 30.0 \text{ in.}$$
$$H = 30.0 \text{ in.}$$
$$b_i = B - 2t = 29.25 \text{ in.}$$
$$h_i = H - 2t = 29.25 \text{ in.}$$

$$A_g = 900 \text{ in.}^2$$
$$A_c = 856 \text{ in.}^2$$
$$A_s = 44.4 \text{ in.}^2$$

$$\begin{aligned} E_c &= w_c^{1.5}\sqrt{f_c'} \\ &= \left(145 \text{ lb/ft}^3\right)^{1.5}\sqrt{7 \text{ ksi}} \\ &= 4{,}620 \text{ ksi} \end{aligned}$$

$$\begin{aligned} I_{gx} &= BH^3/12 \\ &= 67{,}500 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{cx} &= b_i h_i^3/12 \\ &= 61{,}000 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{sx} &= I_{gx} - I_{cx} \\ &= 6{,}500 \text{ in.}^4 \end{aligned}$$

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 7 \text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$F_y = 36 \text{ ksi}$ **o.k.**

(3) Cross-sectional area of steel section: $A_s \ge 0.01A_g$

$$\begin{aligned} 44.4 \text{ in.}^2 &\ge (0.01)\left(900 \text{ in.}^2\right) \\ &> 9.00 \text{ in.}^2 \quad \textbf{o.k.} \end{aligned}$$

Classify Section for Local Buckling

Classification of the section for local buckling is performed in accordance with AISC *Specification* Table I1.1A for compression and Table I1.1B for flexure. As noted in *Specification* Section I1.4, the definitions of width, depth and thickness used in the evaluation of slenderness are provided in Tables B4.1a and B4.1b.

For box columns, the widths of the stiffened compression elements used for slenderness checks, b and h, are equal to the clear distances between the column walls, b_i and h_i. The slenderness ratios are determined as follows:

$$\begin{aligned} \lambda &= \frac{b_i}{t} \\ &= \frac{h_i}{t} \\ &= \frac{29.25 \text{ in.}}{3/8 \text{ in.}} \\ &= 78.0 \end{aligned}$$

Classify section for local buckling in steel elements subject to axial compression from AISC *Specification* Table I1.1A:

$$\lambda_p = 2.26\sqrt{\frac{E}{F_y}}$$

$$= 2.26\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$

$$= 64.1$$

$$\lambda_r = 3.00\sqrt{\frac{E}{F_y}}$$

$$= 3.00\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$

$$= 85.1$$

$\lambda_p \le \lambda \le \lambda_r$

$64.1 \le 78.0 \le 85.1$; therefore, the section is noncompact for compression.

According to AISC *Specification* Section I1.4, if any side of the section in question is noncompact or slender, then the entire section is treated as noncompact or slender. For the square section under investigation; however, this distinction is unnecessary as all sides are equal in length.

Classification of the section for local buckling in elements subject to flexure is performed in accordance with AISC *Specification* Table I1.1B. Note that flanges and webs are treated separately; however, for the case of a square section only the most stringent limitations, those of the flange, need be applied. Noting that the flange limitations for bending are the same as those for compression,

$\lambda_p \le \lambda \le \lambda_r$

$64.1 \le 78.0 \le 85.1$; therefore, the section is noncompact for flexure

Available Compressive Strength

Compressive strength for noncompact filled members is determined in accordance with AISC *Specification* Section I2.2b(b).

$$P_p = F_y A_s + C_2 f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right) \text{ where } C_2 = 0.85 \text{ for rectangular sections} \qquad (\textit{Spec}.\text{ Eq. I2-9b})$$

$$= (36 \text{ ksi})(44.4 \text{ in.}^2) + 0.85(7 \text{ ksi})(856 \text{ in.}^2 + 0)$$

$$= 6{,}690 \text{ kips}$$

$$P_y = F_y A_s + 0.7 f_c'\left(A_c + A_{sr}\frac{E_s}{E_c}\right) \qquad (\textit{Spec}.\text{ Eq. I2-9d})$$

$$= (36 \text{ ksi})(44.4 \text{ in.}^2) + 0.7(7 \text{ ksi})(856 \text{ in.}^2 + 0)$$

$$= 5{,}790 \text{ kips}$$

$$P_{no} = P_p - \frac{P_p - P_y}{(\lambda_r - \lambda_p)^2}(\lambda - \lambda_p)^2 \qquad (\textit{Spec}.\text{ Eq. I2-9c})$$

$$= 6{,}690 \text{ kips} - \frac{6{,}690 \text{ kips} - 5{,}790 \text{ kips}}{(85.1 - 64.1)^2}(78.0 - 64.1)^2$$

$$= 6{,}300 \text{ kips}$$

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.9$$ (*Spec*. Eq. I2-13)

$$= 0.6 + 2\left(\frac{44.4 \text{ in.}^2}{856 \text{ in.}^2 + 44.4 \text{ in.}^2}\right) \leq 0.9$$

$$= 0.699 \leq 0.9$$

$$EI_{eff} = E_s I_s + E_s I_{sr} + C_3 E_c I_c$$ (*Spec*. Eq. I2-12)

$$= (29{,}000 \text{ ksi})(6{,}500 \text{ in.}^4) + 0.0 + 0.699(4{,}620 \text{ ksi})(61{,}000 \text{ in.}^4)$$

$$= 385{,}000{,}000 \text{ ksi}$$

$$P_e = \pi^2 (EI_{eff})/(KL)^2$$ where K=1.0 in accordance with the direct analysis method (*Spec*. Eq. I2-5)

$$= \frac{\pi^2 (385{,}000{,}000 \text{ ksi})}{\left[(30.0 \text{ ft})(12 \text{ in./ft})\right]^2}$$

$$= 29{,}300 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{6{,}300 \text{ kips}}{29{,}300 \text{ kips}}$$

$$= 0.215 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right]$$ (*Spec*. Eq. I2-2)

$$= 6{,}300 \text{ kips}(0.658)^{0.215}$$

$$= 5{,}760 \text{ kips}$$

According to AISC *Specification* Section I2.2b, the compression strength need not be less than that specified for the bare steel member as determined by *Specification* Chapter E. It can be shown that the compression strength of the bare steel for this section is equal to 955 kips, thus the strength of the composite section controls.

The available compressive strength is:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n = 0.75(5{,}760 \text{ kips})$ $= 4{,}320 \text{ kips}$	$\Omega_c = 2.00$ $P_n / \Omega_c = 5{,}760 \text{ kips}/2.00$ $= 2{,}880 \text{ kips}$

Available Flexural Strength

Flexural strength of noncompact filled composite members is determined in accordance with AISC *Specification* Section I3.4b(b):

$$M_n = M_p - (M_p - M_y)\frac{(\lambda - \lambda_p)}{(\lambda_r - \lambda_p)}$$ (*Spec*. Eq. I3-3b)

In order to utilize Equation I3-3b, both the plastic moment strength of the section, M_p, and the yield moment strength of the section, M_y, must be calculated.

Return to Table of Contents

Plastic Moment Strength

The first step in determining the available flexural strength of a noncompact section is to calculate the moment corresponding to the plastic stress distribution over the composite cross section. This concept is illustrated graphically in AISC *Specification* Commentary Figure C-I3.7(a) and follows the force distribution depicted in Figure I.7-2 and detailed in Table I.7-1.

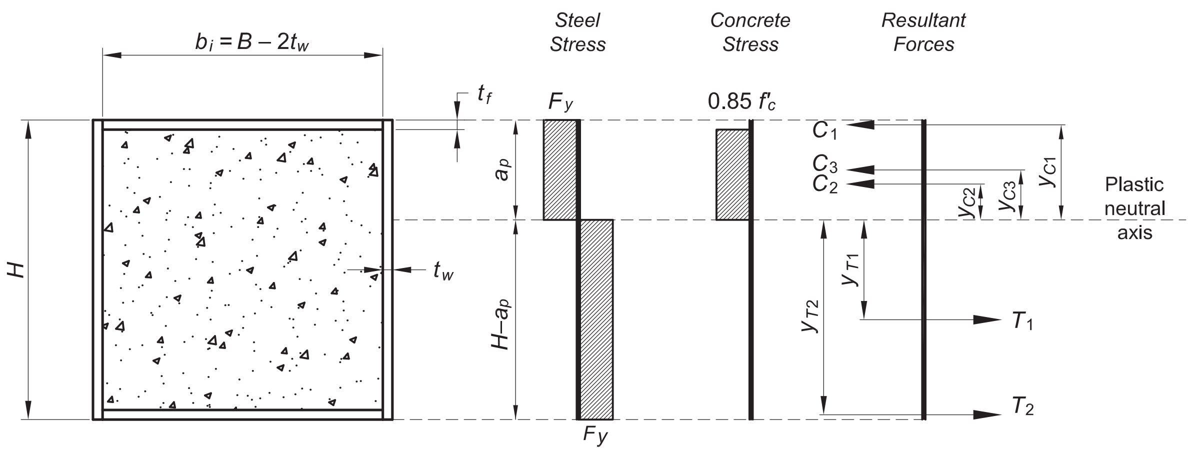


Figure I.7-2. Plastic moment stress blocks and force distribution.

Table I.7-1. Plastic Moment Equations

Component	Force	Moment Arm
Compression in steel flange	$C_1 = b_i t_f F_y$	$y_{C1} = a_p - \dfrac{t_f}{2}$
Compression in concrete	$C_2 = 0.85 f_c'\left(a_p - t_f\right)b_i$	$y_{C2} = \dfrac{a_p - t_f}{2}$
Compression in steel web	$C_3 = a_p 2t_w F_y$	$y_{C3} = \dfrac{a_p}{2}$
Tension in steel web	$T_1 = \left(H - a_p\right)2t_w F_y$	$y_{T1} = \dfrac{H - a_p}{2}$
Tension in steel flange	$T_2 = b_i t_f F_y$	$y_{T2} = H - a_p - \dfrac{t_f}{2}$
where: $a_p = \dfrac{2F_y H t_w + 0.85 f_c' b_i t_f}{4t_w F_y + 0.85 f_c' b_i}$ $M_p = \sum\left(\text{force}\right)\left(\text{moment arm}\right)$		

Using the equations provided in Table I.7-1 for the section in question results in the following:

$$a_p = \frac{2\left(36\text{ ksi}\right)\left(30.0\text{ in.}\right)\left(\tfrac{3}{8}\text{ in.}\right) + 0.85\left(7\text{ ksi}\right)\left(29.25\text{ in.}\right)\left(\tfrac{3}{8}\text{ in.}\right)}{4\left(\tfrac{3}{8}\text{ in.}\right)\left(36\text{ ksi}\right) + 0.85\left(7\text{ ksi}\right)\left(29.25\text{ in.}\right)}$$
$$= 3.84\text{ in.}$$

Force	Moment Arm	Force × Moment Arm
$C_1 = (29.25 \text{ in.})(\tfrac{3}{8} \text{ in.})(36 \text{ ksi})$ $= 395 \text{ kips}$	$y_{C1} = 3.84 \text{ in.} - \dfrac{\tfrac{3}{8} \text{ in.}}{2}$ $= 3.65 \text{ in.}$	$C_1 y_{C1} = 1{,}440 \text{ kip-in.}$
$C_2 = 0.85(7 \text{ ksi})(3.84 \text{ in.} - \tfrac{3}{8} \text{ in.})(29.25 \text{ in.})$ $= 603 \text{ kips}$	$y_{C2} = \dfrac{3.84 \text{ in.} - \tfrac{3}{8} \text{ in.}}{2}$ $= 1.73 \text{ in.}$	$C_2 y_{C2} = 1{,}040 \text{ kip-in.}$
$C_3 = (3.84 \text{ in.})(2)(\tfrac{3}{8} \text{ in.})(36 \text{ ksi})$ $= 104 \text{ kips}$	$y_{C3} = \dfrac{3.84 \text{ in.}}{2}$ $= 1.92 \text{ in.}$	$C_3 y_{C3} = 200 \text{ kip-in.}$
$T_1 = (30.0 \text{ in.} - 3.84 \text{ in.})(2)(\tfrac{3}{8} \text{ in.})(36 \text{ ksi})$ $= 706 \text{ kips}$	$y_{T1} = \dfrac{30.0 \text{ in.} - 3.84 \text{ in.}}{2}$ $= 13.1 \text{ in.}$	$T_1 y_{T1} = 9{,}250 \text{ kip-in.}$
$T_2 = (29.25 \text{ in.})(\tfrac{3}{8} \text{ in.})(36 \text{ ksi})$ $= 395 \text{ kips}$	$y_{T2} = 30.0 \text{ in.} - 3.84 \text{ in.} - \dfrac{\tfrac{3}{8} \text{ in.}}{2}$ $= 26.0 \text{ in.}$	$T_2 y_{T2} = 10{,}300 \text{ kip-in.}$

$$
\begin{aligned}
M_p &= \sum(\text{force})(\text{moment arm}) \\
&= \frac{1{,}440 \text{ kip-in.} + 1{,}040 \text{ kip-in.} + 200 \text{ kip-in.} + 9{,}250 \text{ kip-in.} + 10{,}300 \text{ kip-in.}}{12 \text{ in./ft}} \\
&= 1{,}850 \text{ kip-ft}
\end{aligned}
$$

Yield Moment Strength

The next step in determining the available flexural strength of a noncompact filled member is to determine the yield moment strength. The yield moment is defined in AISC *Specification* Section I3.4b(b) as the moment corresponding to first yield of the compression flange calculated using a linear elastic stress distribution with a maximum concrete compressive stress of $0.7 f_c'$. This concept is illustrated diagrammatically in *Specification* Commentary Figure C-I3.7(b) and follows the force distribution depicted in Figure I.7-3 and detailed in Table I.7-2.

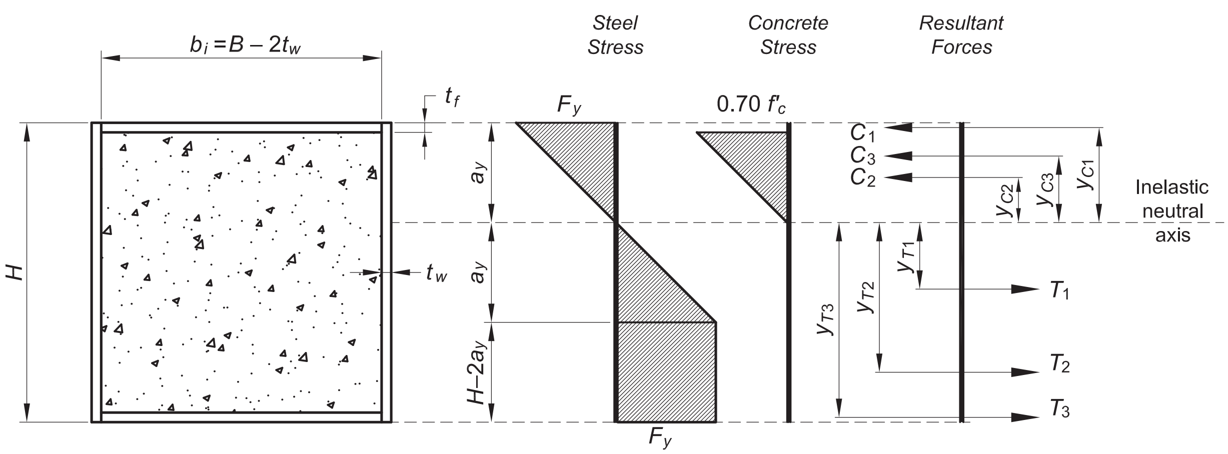


Figure I.7-3. Yield moment stress blocks and force distribution.

Table I.7-2. Yield Moment Equations

Component	Force	Moment Arm
Compression in steel flange	$C_1 = b_i t_f F_y$	$y_{C1} = a_y - \dfrac{t_f}{2}$
Compression in concrete	$C_2 = 0.35 f_c' (a_y - t_f) b_i$	$y_{C2} = \dfrac{2(a_y - t_f)}{3}$
Compression in steel web	$C_3 = a_y 2t_w 0.5F_y$	$y_{C3} = \dfrac{2a_y}{3}$
Tension in steel web	$T_1 = a_y 2t_w 0.5F_y$ $T_2 = (H - 2a_y) 2t_w F_y$	$y_{T1} = \dfrac{2a_y}{3}$ $y_{T2} = \dfrac{H}{2}$
Tension in steel flange	$T_3 = b_i t_f F_y$	$y_{T3} = H - a_y - \dfrac{t_f}{2}$

where:

$$a_y = \frac{2F_y H t_w + 0.35 f_c' b_i t_f}{4 t_w F_y + 0.35 f_c' b_i}$$

$$M_y = \sum(\text{force})(\text{moment arm})$$

Using the equations provided in Table I.7-2 for the section in question results in the following:

$$a_y = \frac{2(36 \text{ ksi})(30.0 \text{ in.})(\tfrac{3}{8} \text{ in.}) + 0.35(7 \text{ ksi})(29.25 \text{ in.})(\tfrac{3}{8} \text{ in.})}{4(\tfrac{3}{8} \text{ in.})(36 \text{ ksi}) + 0.35(7 \text{ ksi})(29.25 \text{ in.})}$$
$$= 6.66 \text{ in.}$$

Force	Moment Arm	Force × Moment Arm
$C_1 = (29.25 \text{ in.})(\tfrac{3}{8} \text{ in.})(36 \text{ ksi})$ $= 395 \text{ kips}$	$y_{C1} = 6.66 \text{ in.} - \dfrac{\tfrac{3}{8} \text{ in.}}{2}$ $= 6.47 \text{ in.}$	$C_1 y_{C1} = 2{,}560 \text{ kip-in.}$
$C_2 = 0.35(7 \text{ ksi})(6.66 \text{ in.} - \tfrac{3}{8} \text{ in.})(29.25 \text{ in.})$ $= 450 \text{ kips}$	$y_{C2} = \dfrac{2(6.66 \text{ in.} - \tfrac{3}{8} \text{ in.})}{3}$ $= 4.19 \text{ in.}$	$C_2 y_{C2} = 1{,}890 \text{ kip-in.}$
$C_3 = (6.66 \text{ in.})(2)(\tfrac{3}{8} \text{ in.})(0.5)(36 \text{ ksi})$ $= 89.9 \text{ kips}$	$y_{C3} = \dfrac{2(6.66 \text{ in.})}{3}$ $= 4.44 \text{ in.}$	$C_3 y_{C3} = 399 \text{ kip-in.}$
$T_1 = (6.66 \text{ in.})(2)(\tfrac{3}{8} \text{ in.})(0.5)(36 \text{ ksi})$ $= 89.9 \text{ kips}$	$y_{T1} = \dfrac{2(6.66 \text{ in.})}{3}$ $= 4.44 \text{ in.}$	$T_1 y_{T1} = 399 \text{ kip-in.}$
$T_2 = [30.0 - 2(6.66 \text{ in.})](2)(\tfrac{3}{8} \text{ in.})(36 \text{ ksi})$ $= 450 \text{ kips}$	$y_{T2} = \dfrac{30.0 \text{ in.}}{2}$ $= 15.0 \text{ in.}$	$T_2 y_{T2} = 6{,}750 \text{ kip-in.}$
$T_3 = (29.25 \text{ in.})(\tfrac{3}{8} \text{ in.})(36 \text{ ksi})$ $= 395 \text{ kips}$	$y_{T3} = 30.0 \text{ in.} - 6.66 \text{ in.} - \dfrac{\tfrac{3}{8} \text{ in.}}{2}$ $= 23.2 \text{ in.}$	$T_3 y_{T3} = 9{,}160 \text{ kip-in.}$

$$M_y = \sum(\text{force})(\text{moment arm})$$
$$= \frac{2,560 \text{ kip-in.} + 1,890 \text{ kip-in.} + 399 \text{ kip-in.} + 399 \text{ kip-in.} + 6,750 \text{ kip-in.} + 9,160 \text{ kip-in.}}{12 \text{ in./ft}}$$
$$= 1,760 \text{ kip-ft}$$

Now that both M_p and M_y have been determined, Equation I3-3b may be used in conjunction with the flexural slenderness values previously calculated to determine the nominal flexural strength of the composite section as follows:

$$M_n = M_p - (M_p - M_y)\frac{(\lambda - \lambda_p)}{(\lambda_r - \lambda_p)} \qquad \text{(}\textit{Spec}\text{. Eq. I3-3b)}$$

$$M_n = 1,850 \text{ kip-ft} - (1,850 \text{ kip-ft} - 1,760 \text{ kip-ft})\frac{(78.0 - 64.1)}{(85.1 - 64.1)}$$
$$= 1,790 \text{ kip-ft}$$

The available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(1,790 \text{ kip-ft})$ $= 1,610 \text{ kip-ft}$	$\Omega_b = 1.67$ $M_n/\Omega_b = 1,790 \text{ kip-ft}/1.67$ $= 1,070 \text{ kip-ft}$

Interaction of Flexure and Compression

Design of members for combined forces is performed in accordance with AISC *Specification* Section I5. For filled composite members with noncompact or slender sections, interaction is determined in accordance with Section H1.1 as follows:

LRFD	ASD
$P_u = 1,310 \text{ kips}$ $M_u = 552 \text{ kip-ft}$ $\frac{P_r}{P_c} = \frac{P_u}{\phi_c P_n}$ $= \frac{1,310 \text{ kips}}{4,320 \text{ kips}}$ $= 0.303 \geq 0.2$ Use *Specification* Equation H1-1a. $\frac{P_u}{\phi_c P_n} + \frac{8}{9}\left(\frac{M_u}{\phi_b M_n}\right) \leq 1.0$ $\frac{1,310 \text{ kips}}{4,320 \text{ kips}} + \frac{8}{9}\left(\frac{552 \text{ kip-ft}}{1,610 \text{ kip-ft}}\right) \leq 1.0$ $0.608 < 1.0$ **o.k.**	$P_a = 1,370 \text{ kips}$ $M_a = 248 \text{ kip-ft}$ $\frac{P_r}{P_c} = \frac{P_a}{P_n / \Omega_c}$ $= \frac{1,370 \text{ kips}}{2,880 \text{ kips}}$ $= 0.476 \geq 0.2$ Use *Specification* Equation H1-1a. $\frac{P_a}{P_n / \Omega_c} + \frac{8}{9}\left(\frac{M_a}{M_n / \Omega_b}\right) \leq 1.0$ $\frac{1,370 \text{ kips}}{2,880 \text{ kips}} + \frac{8}{9}\left(\frac{248 \text{ kip-ft}}{1,070 \text{ kip-ft}}\right) \leq 1.0$ $0.682 < 1.0$ **o.k.**

The composite section is adequate; however, as there is available strength remaining for the trial plate thickness chosen, re-analyze the section to determine the adequacy of a reduced plate thickness.

Trial Size 2 (Slender)

The calculated geometric section properties using a reduced plate thickness of t = ¼ in. are:

$$B = 30.0 \text{ in.}$$
$$H = 30.0 \text{ in.}$$
$$b_i = B - 2t = 29.50 \text{ in.}$$
$$h_i = H - 2t = 29.50 \text{ in.}$$

$$A_g = 900 \text{ in.}^2$$
$$A_c = 870 \text{ in.}^2$$
$$A_s = 29.8 \text{ in.}^2$$

$$\begin{aligned} E_c &= w_c^{1.5}\sqrt{f_c'} \\ &= \left(145 \text{ lb/ft}^3\right)^{1.5}\sqrt{7 \text{ ksi}} \\ &= 4{,}620 \text{ ksi} \end{aligned}$$

$$\begin{aligned} I_{gx} &= BH^3/12 \\ &= 67{,}500 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{cx} &= b_i h_i^3/12 \\ &= 63{,}100 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{sx} &= I_{gx} - I_{cx} \\ &= 4{,}400 \text{ in.}^4 \end{aligned}$$

Limitations of AISC Specification Sections I1.3 and I2.2a

(1) Concrete Strength: $3 \text{ ksi} \le f_c' \le 10 \text{ ksi}$

$f_c' = 7 \text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75 \text{ ksi}$

$F_y = 36 \text{ ksi}$ **o.k.**

(3) Cross sectional area of steel section: $A_s \ge 0.01A_g$

$$\begin{aligned} 29.8 \text{ in.}^2 &\ge (0.01)\left(900 \text{ in.}^2\right) \\ &> 9.00 \text{ in.}^2 \quad \textbf{o.k.} \end{aligned}$$

Classify Section for Local Buckling

As noted previously, the definitions of width, depth and thickness used in the evaluation of slenderness are provided in AISC *Specification* Tables B4.1a and B4.1b.

For a box column, the slenderness ratio is determined as the ratio of clear distance to wall thickness:

$$\begin{aligned} \lambda &= \frac{b_i}{t} \\ &= \frac{h_i}{t} \\ &= \frac{29.5 \text{ in.}}{\text{¼ in.}} \\ &= 118 \end{aligned}$$

Classify section for local buckling in steel elements subject to axial compression from AISC *Specification* Table I1.1A. As determined previously, $\lambda_r = 85.1$.

$$\lambda_{max} = 5.00\sqrt{\frac{E}{F_y}}$$
$$= 5.00\sqrt{\frac{29{,}000\text{ ksi}}{36\text{ ksi}}}$$
$$= 142$$

$\lambda_r \le \lambda \le \lambda_{max}$

$85.1 \le 118 \le 142$; therefore, the section is slender for compression

Classification of the section for local buckling in elements subject to flexure occurs separately per AISC *Specification* Table I1.1B. Because the flange limitations for bending are the same as those for compression,

$\lambda_r \le \lambda \le \lambda_{max}$

$85.1 \le 118 \le 142$; therefore, the section is slender for flexure

Available Compressive Strength

Compressive strength for a slender filled member is determined in accordance with AISC *Specification* Section I2.2b(c).

$$F_{cr} = \frac{9E_s}{\left(\frac{b}{t}\right)^2} \quad \text{(Spec. Eq. I2-10)}$$
$$= \frac{9(29{,}000\text{ ksi})}{(118)^2}$$
$$= 18.7\text{ ksi}$$

$$P_{no} = F_{cr}A_s + 0.7f'_c\left(A_c + A_{sr}\frac{E_s}{E_c}\right) \quad \text{(Spec. Eq. I2-9e)}$$
$$= (18.7\text{ ksi})(29.8\text{ in.}^2) + 0.7(7\text{ ksi})(870\text{ in.}^2 + 0)$$
$$= 4{,}820\text{ kips}$$

$$C_3 = 0.6 + 2\left(\frac{A_s}{A_c + A_s}\right) \le 0.9 \quad \text{(Spec. Eq. I2-13)}$$
$$= 0.6 + 2\left(\frac{29.8\text{ in.}^2}{870\text{ in.}^2 + 29.8\text{ in.}^2}\right) \le 0.9$$
$$= 0.666 < 0.9$$

$$EI_{eff} = E_sI_s + E_sI_{sr} + C_3E_cI_c \quad \text{(Spec. Eq. I2-12)}$$
$$= (29{,}000\text{ ksi})(4{,}400\text{ in.}^4) + 0 + 0.666(4{,}620\text{ ksi})(63{,}100\text{ in.}^4)$$
$$= 322{,}000{,}000\text{ ksi}$$

$$P_e = \pi^2(EI_{eff})/(KL)^2 \text{ where } K = 1.0 \text{ in accordance with the direct analysis method} \quad \text{(Spec. Eq. I2-5)}$$
$$= \frac{\pi^2(322{,}000{,}000\text{ ksi})}{[(30.0\text{ ft})(12\text{ in./ft})]^2}$$
$$= 24{,}500\text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{4{,}820 \text{ kips}}{24{,}500 \text{ kips}}$$
$$= 0.197 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] \qquad \text{(Spec. Eq. I2-2)}$$
$$= 4{,}820 \text{ kips}(0.658)^{0.197}$$
$$= 4{,}440 \text{ kips}$$

According to AISC *Specification* Section I2.2b the compression strength need not be less than that determined for the bare steel member using *Specification* Chapter E. It can be shown that the compression strength of the bare steel for this section is equal to 450 kips, thus the strength of the composite section controls.

The available compressive strength is:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n = 0.75(4{,}440 \text{ kips})$ $= 3{,}330 \text{ kips}$	$\Omega_c = 2.00$ $P_n/\Omega_c = 4{,}440 \text{ kips}/2.00$ $= 2{,}220 \text{ kips}$

Available Flexural Strength

Flexural strength of slender filled composite members is determined in accordance with AISC *Specification* Section I3.4b(c). The nominal flexural strength is determined as the first yield moment, M_{cr}, corresponding to a flange compression stress of F_{cr} using a linear elastic stress distribution with a maximum concrete compressive stress of $0.7 f_c'$. This concept is illustrated diagrammatically in *Specification* Commentary Figure C-I3.7(c) and follows the force distribution depicted in Figure I.7-4 and detailed in Table I.7-3.

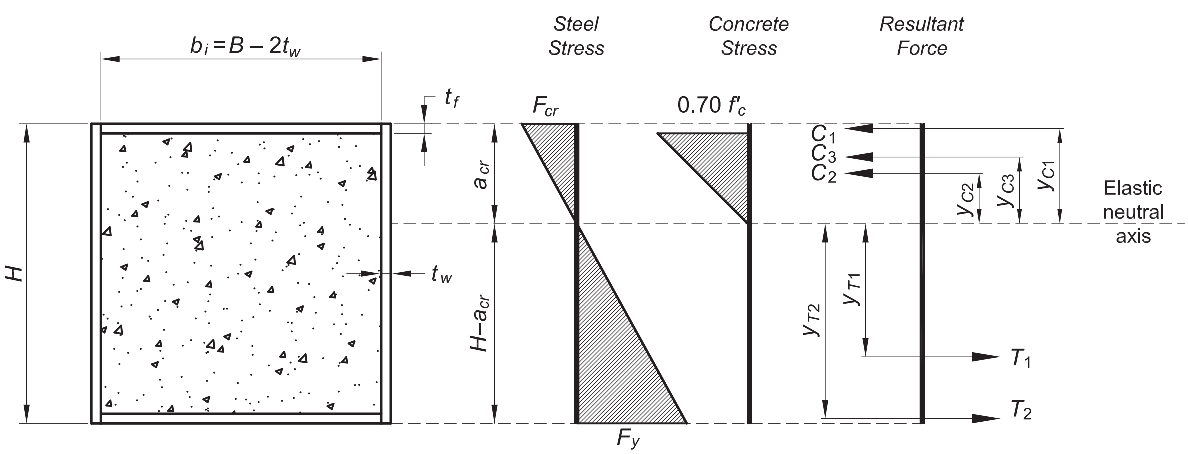


Figure I.7-4. First yield moment stress blocks and force distribution.

Table I.7-3. First Yield Moment Equations		
Component	Force	Moment arm
Compression in steel flange	$C_1 = b_i t_f F_{cr}$	$y_{C1} = a_{cr} - \frac{t_f}{2}$
Compression in concrete	$C_2 = 0.35 f_c'(a_{cr} - t_f) b_i$	$y_{C2} = \frac{2(a_{cr} - t_f)}{3}$
Compression in steel web	$C_3 = a_{cr} 2t_w 0.5F_{cr}$	$y_{C3} = \frac{2a_{cr}}{3}$
Tension in steel web	$T_1 = (H - a_{cr}) 2t_w 0.5F_y$	$y_{T1} = \frac{2(H - a_{cr})}{3}$
Tension in steel flange	$T_2 = b_i t_f F_y$	$y_{T2} = H - a_{cr} - \frac{t_f}{2}$
where: $a_{cr} = \frac{F_y H t_w + (0.35 f_c' + F_y - F_{cr}) b_i t_f}{t_w (F_{cr} + F_y) + 0.35 f_c' b_i}$ $M_{cr} = \sum(\text{force})(\text{moment arm})$		

Using the equations provided in Table I.7-3 for the section in question results in the following:

$$a_{cr} = \frac{(36\text{ ksi})(30.0\text{ in.})(\tfrac{1}{4}\text{ in.}) + [0.35(7\text{ ksi}) + 36\text{ ksi} - 18.7\text{ ksi}](29.5\text{ in.})(\tfrac{1}{4}\text{ in.})}{(\tfrac{1}{4}\text{ in.})(18.7\text{ ksi} + 36\text{ ksi}) + 0.35(7\text{ ksi})(29.5\text{ in.})}$$
$$= 4.84\text{ in.}$$

Force	Moment Arm	Force × Moment Arm
$C_1 = (29.5\text{ in.})(\tfrac{1}{4}\text{ in.})(18.7\text{ ksi})$ $= 138\text{ kips}$	$y_{C1} = 4.84\text{ in.} - \frac{\tfrac{1}{4}\text{ in.}}{2}$ $= 4.72\text{ in.}$	$C_1 y_{C1} = 651\text{ kip-in.}$
$C_2 = 0.35(7\text{ ksi})(4.84\text{ in.} - \tfrac{1}{4}\text{ in.})(29.5\text{ in.})$ $= 332\text{ kips}$	$y_{C2} = \frac{2(4.84\text{ in.} - \tfrac{1}{4}\text{ in.})}{3}$ $= 3.06\text{ in.}$	$C_2 y_{C2} = 1{,}020\text{ kip-in.}$
$C_3 = (4.84\text{ in.})(2)(\tfrac{1}{4}\text{ in.})(0.5)(18.7\text{ ksi})$ $= 22.6\text{ kips}$	$y_{C3} = \frac{2(4.84\text{ in.})}{3}$ $= 3.23\text{ in.}$	$C_3 y_{C3} = 73.0\text{ kip-in.}$
$T_1 = (30.0\text{ in.} - 4.84\text{ in.})(2)(\tfrac{1}{4}\text{ in.})(0.5)(36\text{ ksi})$ $= 226\text{ kips}$	$y_{T1} = \frac{2(30.0\text{ in.} - 4.84\text{ in.})}{3}$ $= 16.8\text{ in.}$	$T_1 y_{T1} = 3{,}800\text{ kip-in.}$
$T_2 = (29.5\text{ in.})(\tfrac{1}{4}\text{ in.})(36\text{ ksi})$ $= 266\text{ kips}$	$y_{T2} = 30.0\text{ in.} - 4.84\text{ in.} - \frac{\tfrac{1}{4}\text{ in}}{2}$ $= 25.0\text{ in.}$	$T_2 y_{T2} = 6{,}650\text{ kip-in.}$

$$M_{cr} = \sum(\text{force component})(\text{moment arm})$$
$$= \frac{651\text{ kip-in.} + 1{,}020\text{ kip-in.} + 73.0\text{ kip-in.} + 3{,}800\text{ kip-in.} + 6{,}650\text{ kip-in.}}{12\text{ in./ft}}$$
$$= 1{,}020\text{ kip-ft}$$

The available flexural strength is:

LRFD	ASD
$\phi_b = 0.90$ $M_n = 0.90(1{,}020 \text{ kip-ft})$ $= 918 \text{ kip-ft}$	$\Omega_b = 1.67$ $M_n/\Omega_b = 1{,}020 \text{ kip-ft}/1.67$ $= 611 \text{ kip-ft}$

Interaction of Flexure and Compression

The interaction of flexure and compression is determined in accordance with AISC *Specification* Section H1.1 as follows:

LRFD	ASD
$P_u = 1{,}310 \text{ kips}$ $M_u = 552 \text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{1{,}310 \text{ kips}}{3{,}330 \text{ kips}}$ $= 0.393 \geq 0.2$ Use AISC *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \leq 1.0$ $\dfrac{1{,}310 \text{ kips}}{3{,}330 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{552 \text{ kip-ft}}{918 \text{ kip-ft}}\right) \leq 1.0$ $0.928 < 1.0$ **o.k.**	$P_a = 1{,}370 \text{ kips}$ $M_a = 248 \text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{1{,}370 \text{ kips}}{2{,}220 \text{ kips}}$ $= 0.617 \geq 0.2$ Use AISC *Specification* Equation H1-1a. $\dfrac{P_a}{P_n / \Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n / \Omega_c}\right) \leq 1.0$ $\dfrac{1{,}370 \text{ kips}}{2{,}220 \text{ kips}} + \dfrac{8}{9}\left(\dfrac{248 \text{ kip-ft}}{611 \text{ kip-ft}}\right) \leq 1.0$ $0.978 < 1.0$ **o.k.**

Thus, a plate thickness of ¼ in. is adequate.

Note that in addition to the design checks performed for the composite condition, design checks for other load stages should be performed as required by AISC *Specification* Section I1. These checks should take into account the effect of hydrostatic loads from concrete placement as well as the strength of the steel section alone prior to composite action.

Available Shear Strength

According to AISC *Specification* Section I4.1 there are three acceptable methods for determining the available shear strength of the member: available shear strength of the steel section alone in accordance with Chapter G, available shear strength of the reinforced concrete portion alone per ACI 318, or available shear strength of the steel section in addition to the reinforcing steel ignoring the contribution of the concrete. Considering that the member in question does not have longitudinal reinforcing, it is determined by inspection that the shear strength will be controlled by the steel section alone using the provisions of Chapter G.

From AISC *Specification* Section G5 the nominal shear strength, V_n, of box members is determined using the provisions of Section G2.1 with $k_v = 5$. As opposed to HSS sections which require the use of a reduced web area to take into account the corner radii, the full web area of a box section may be used as follows:

$$A_w = 2dt_w \text{ where } d = \text{full depth of section parallel to the required shear force}$$
$$= 2(30.0 \text{ in.})(\tfrac{1}{4} \text{ in.})$$
$$= 15.0 \text{ in.}^2$$

The slenderness value, h/t_w, for the web used in *Specification* Section G2.1(b) is the same as that calculated previously for use in local buckling classification, $\lambda = 118$.

$$\frac{h}{t_w} > 1.37\sqrt{k_v E/F_y}$$
$$\frac{h}{t_w} > 1.37\sqrt{5\left(\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}\right)}$$
$$118 > 86.9$$

Therefore, use AISC *Specification* Equation G2-5.

The web shear coefficient and nominal shear strength are calculated as:

$$C_v = \frac{1.51k_v E}{(h/t_w)^2 F_y} \qquad \text{(}\textit{Spec}\text{. Eq. G2-5)}$$
$$= \frac{1.51(5)(29{,}000 \text{ ksi})}{(118)^2 (36 \text{ ksi})}$$
$$= 0.437$$
$$V_n = 0.6F_y A_w C_v \qquad \text{(}\textit{Spec}\text{. Eq. G2-1)}$$
$$= 0.6(36 \text{ ksi})(15.0 \text{ in.}^2)(0.437)$$
$$= 142 \text{ kips}$$

The available shear strength is checked as follows:

LRFD	ASD
$V_u = 36.8$ kips $\phi_v = 0.9$ $\phi_v V_n \geq V_u$ $\phi_v V_n = 0.9(142 \text{ kips})$ $= 128 \text{ kips} > 36.8 \text{ kips}$ **o.k.**	$V_a = 22.1$ kips $\Omega_v = 1.67$ $V_n / \Omega_v \geq V_a$ $V_n / \Omega_v = \dfrac{142 \text{ kips}}{1.67}$ $= 85.0 \text{ kips} > 22.1 \text{ kips}$ **o.k.**

Force allocation and load transfer

Load transfer calculations for applied axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions is provided in Design Example I.3.

Summary

It has been determined that a 30 in. × 30 in. composite box column composed of ¼-in.-thick plate is adequate for the imposed loads.

EXAMPLE I.8 ENCASED COMPOSITE MEMBER FORCE ALLOCATION AND LOAD TRANSFER

Given:

Refer to Figure I.8-1.

Part I: For each loading condition (a) through (c) determine the required longitudinal shear force, V_r', to be transferred between the embedded steel section and concrete encasement.

Part II: For loading condition (b), investigate the force transfer mechanisms of direct bearing and shear connection.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, f_c' = 5 ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Applied loading, P_r, for each condition illustrated in Figure I.8-1 is composed of the following loads:

P_D = 260 kips
P_L = 780 kips

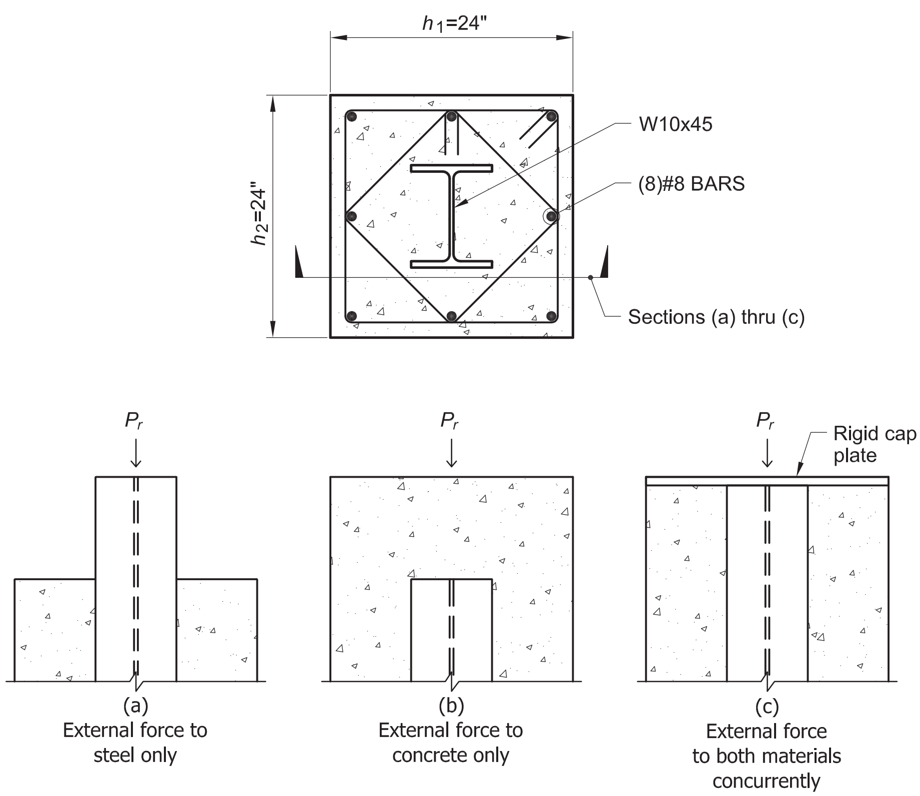


Fig. I.8-1. Encased composite member in compression.

Solution:

Part I—Force Allocation

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1 and Figure I.8-1, the geometric properties of the encased W10×45 are as follows:

$A_s = 13.3 \text{ in.}^2$ $\quad t_w = 0.350 \text{ in.}$ $\quad h_1 = 24.0 \text{ in.}$
$b_f = 8.02 \text{ in.}$ $\quad d = 10.1 \text{ in.}$ $\quad h_2 = 24.0 \text{ in.}$
$t_f = 0.620 \text{ in.}$

Additional geometric properties of the composite section used for force allocation and load transfer are calculated as follows:

$$\begin{aligned} A_g &= h_1 h_2 \\ &= (24.0 \text{ in.})(24.0 \text{ in.}) \\ &= 576 \text{ in.}^2 \end{aligned}$$

$A_{sri} = 0.79 \text{ in.}^2$ for a No. 8 bar

$$\begin{aligned} A_{sr} &= \sum_{i=1}^{n} A_{sri} \\ &= 8\left(0.79 \text{ in.}^2\right) \\ &= 6.32 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_c &= A_g - A_s - A_{sr} \\ &= 576 \text{ in.}^2 - 13.3 \text{ in.}^2 - 6.32 \text{ in.}^2 \\ &= 556 \text{ in.}^2 \end{aligned}$$

where
A_c = cross-sectional area of concrete encasement, in.^{2}
A_g = gross cross-sectional area of composite section, in.^{2}
A_{sri} = cross-sectional area of reinforcing bar *i*, in.^{2}
A_{sr} = cross-sectional area of continuous reinforcing bars, in.^{2}
n = number of continuous reinforcing bars in composite section

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(260 \text{ kips}) + 1.6(780 \text{ kips})$ $= 1{,}560 \text{ kips}$	$P_r = P_a$ $= 260 \text{ kips} + 780 \text{ kips}$ $= 1{,}040 \text{ kips}$

Composite Section Strength for Force Allocation

In accordance with AISC *Specification* Section I6, force allocation calculations are based on the nominal axial compressive strength of the encased composite member without length effects, P_{no}. This section strength is defined in Section I2.1b as:

$$\begin{aligned} P_{no} &= F_y A_s + F_{ysr} A_{sr} + 0.85 f_c' A_c \qquad (\textit{Spec}.\ \text{Eq. I2-4}) \\ &= (50 \text{ ksi})\left(13.3 \text{ in.}^2\right) + (60 \text{ ksi})\left(6.32 \text{ in.}^2\right) + 0.85(5 \text{ ksi})\left(556 \text{ in.}^2\right) \\ &= 3{,}410 \text{ kips} \end{aligned}$$

Transfer Force for Condition (a)

Refer to Figure I.8-1(a). For this condition, the entire external force is applied to the steel section only, and the provisions of AISC *Specification* Section I6.2a apply.

$$V_r' = P_r\left(1-\frac{F_yA_s}{P_{no}}\right) \qquad (Spec.\ \text{Eq. I6-1})$$

$$= P_r\left[1-\frac{(50\text{ ksi})(13.3\text{ in.}^2)}{3,410\text{ kips}}\right]$$

$$= 0.805P_r$$

LRFD	ASD
$V_r' = 0.805(1,560\text{ kips})$ $= 1,260\text{ kips}$	$V_r' = 0.805(1,040\text{ kips})$ $= 837\text{ kips}$

Transfer Force for Condition (b)

Refer to Figure I.8-1(b). For this condition, the entire external force is applied to the concrete encasement only, and the provisions of AISC *Specification* Section I6.2b apply.

$$V_r' = P_r\left(\frac{F_yA_s}{P_{no}}\right) \qquad (Spec.\ \text{Eq. I6-2})$$

$$= P_r\left[\frac{(50\text{ ksi})(13.3\text{ in.}^2)}{3,410\text{ kips}}\right]$$

$$= 0.195P_r$$

LRFD	ASD
$V_r' = 0.195(1,560\text{ kips})$ $= 304\text{ kips}$	$V_r' = 0.195(1,040\text{ kips})$ $= 203\text{ kips}$

Transfer Force for Condition (c)

Refer to Figure I.8-1(c). For this condition, external force is applied to the steel section and concrete encasement concurrently, and the provisions of AISC *Specification* Section I6.2c apply.

AISC *Specification Commentary* Section I6.2 states that when loads are applied to both the steel section and concrete encasement concurrently, V_r' can be taken as the difference in magnitudes between the portion of the external force applied directly to the steel section and that required by Equation I6-2. This concept can be written in equation form as follows:

$$V_r' = \left|P_{rs} - P_r\left(\frac{F_yA_s}{P_{no}}\right)\right| \qquad \text{(Eq. 1)}$$

where

P_{rs} = portion of external force applied directly to the steel section (kips)

Currently the *Specification* provides no specific requirements for determining the distribution of the applied force for the determination of P_{rs}, so it is left to engineering judgment. For a bearing plate condition such as the one represented in Figure I.8-1(c), one possible method for determining the distribution of applied forces is to use an elastic distribution based on the material axial stiffness ratios as follows:

Return to Table of Contents

$$E_c = w_c^{1.5}\sqrt{f'_c}$$
$$= \left(145\text{ lb/ft}^3\right)^{1.5}\sqrt{5\text{ ksi}}$$
$$= 3,900\text{ ksi}$$

$$P_{rs} = \left(\frac{E_s A_s}{E_s A_s + E_c A_c + E_{sr} A_{sr}}\right) P_r$$
$$= \left[\frac{(29,000\text{ ksi})(13.3\text{ in.}^2)}{(29,000\text{ ksi})(13.3\text{ in.}^2)+(3,900\text{ ksi})(556\text{ in.}^2)+(29,000\text{ ksi})(6.32\text{ in.}^2)}\right] P_r$$
$$= 0.141P_r$$

Substituting the results into Equation 1 yields:

$$V'_r = \left|0.141P_r - P_r\left(\frac{F_y A_s}{P_{no}}\right)\right|$$
$$= \left|0.141P_r - P_r\left[\frac{(50\text{ ksi})(13.3\text{ in.}^2)}{3,410\text{ kips}}\right]\right|$$
$$= 0.0540P_r$$

LRFD	ASD
$V'_r = 0.0540(1,560\text{ kips})$ $= 84.2\text{ kips}$	$V'_r = 0.0540(1,040\text{ kips})$ $= 56.2\text{ kips}$

An alternate approach would be use of a plastic distribution method whereby the load is partitioned to each material in accordance with their contribution to the composite section strength given in Equation I2-4. This method eliminates the need for longitudinal shear transfer provided the local bearing strength of the concrete and steel are adequate to resist the forces resulting from this distribution.

Additional Discussion

- The design and detailing of the connections required to deliver external forces to the composite member should be performed according to the applicable sections of AISC *Specification* Chapters J and K.
- The connection cases illustrated by Figure I.8-1 are idealized conditions representative of the mechanics of actual connections. For instance, an extended single plate connection welded to the flange of the W10 and extending out beyond the face of concrete to attach to a steel beam is an example of a condition where it may be assumed that all external force is applied directly to the steel section only.

Solution:

Part II—Load Transfer

The required longitudinal force to be transferred, V'_r, determined in Part I condition (b) is used to investigate the applicable force transfer mechanisms of AISC *Specification* Section I6.3: direct bearing and shear connection. As indicated in the *Specification*, these force transfer mechanisms may not be superimposed; however, the mechanism providing the greatest nominal strength may be used. Note that direct bond interaction is not applicable to encased composite members as the variability of column sections and connection configurations makes confinement and bond strength more difficult to quantify than in filled HSS.

Direct Bearing

Determine Layout of Bearing Plates

One method of utilizing direct bearing as a load transfer mechanism is through the use of internal bearing plates welded between the flanges of the encased W-shape as indicated in Figure I.8-2.

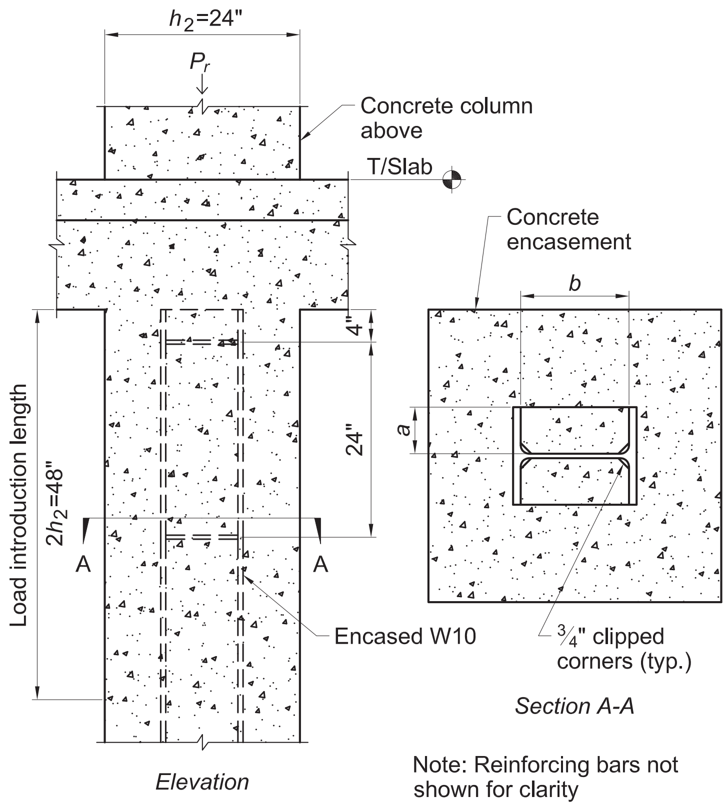


Fig. I.8-2. Composite member with internal bearing plates.

When using bearing plates in this manner, it is essential that concrete mix proportions and installation techniques produce full bearing at the plates. Where multiple sets of bearing plates are used as illustrated in Figure I.8-2, it is recommended that the minimum spacing between plates be equal to the depth of the encased steel member to enhance constructability and concrete consolidation. For the configuration under consideration, this guideline is met with a plate spacing of $24 \text{ in.} \geq d = 10.1 \text{ in.}$

Bearing plates should be located within the load introduction length given in AISC *Specification* Section I6.4a. The load introduction length is defined as two times the minimum transverse dimension of the composite member both above and below the load transfer region. The load transfer region is defined in *Specification* Commentary Section I6.4 as the depth of the connection. For the connection configuration under consideration, where the majority of the required force is being applied from the concrete column above, the depth of connection is conservatively taken as zero. Because the composite member only extends to one side of the point of force transfer, the bearing plates should be located within $2h_2$ = 48 in. of the top of the composite member as indicated in Figure I.8-2.

Available Strength for the Limit State of Direct Bearing

Assuming two sets of bearing plates are to be used as indicated in Figure I.8-2, the total contact area between the bearing plates and the concrete, A_1, may be determined as follows:

$$a = \frac{b_f - t_w}{2}$$
$$= \frac{8.02\text{ in.} - 0.350\text{ in.}}{2}$$
$$= 3.84\text{ in.}$$

$$b = d - 2t_f$$
$$= 10.1\text{ in.} - 2(0.620\text{ in.})$$
$$= 8.86\text{ in.}$$

$$c = \text{width of clipped corners}$$
$$= \tfrac{3}{4}\text{ in.}$$

$$A_1 = \left(2ab - 2c^2\right)\left(\text{number of bearing plate sets}\right)$$
$$= \left[2(3.84\text{ in.})(8.86\text{ in.}) - 2(\tfrac{3}{4}\text{ in.})^2\right](2)$$
$$= 134\text{ in.}^2$$

The available strength for the direct bearing force transfer mechanism is:

$$R_n = 1.7 f_c' A_1 \qquad (\textit{Spec}.\ \text{Eq. I6-3})$$
$$= 1.7(5\text{ ksi})(134\text{ in.}^2)$$
$$= 1{,}140\text{ kips}$$

LRFD	ASD
$\phi_B = 0.65$ $\phi_B R_n \geq V_r'$ $\phi_B R_n = 0.65(1{,}140\text{ kips})$ $= 741\text{ kips} > 304\text{ kips}$ **o.k.**	$\Omega_B = 2.31$ $R_n / \Omega_B \geq V_r'$ $R_n / \Omega_B = \dfrac{1{,}140\text{ kips}}{2.31}$ $= 494\text{ kips} > 203\text{ kips}$ **o.k.**

Thus two sets of bearing plates are adequate. From these calculations it can be seen that one set of bearing plates are adequate for force transfer purposes; however, the use of two sets of bearing plates serves to reduce the bearing plate thickness calculated in the following section.

Required Bearing Plate Thickness

There are several methods available for determining the bearing plate thickness. For rectangular plates supported on three sides, elastic solutions for plate stresses such as those found in *Roark's Formulas for Stress and Strain* (Young and Budynas, 2002) may be used in conjunction with AISC *Specification* Section F12 for thickness calculations. Alternately, yield line theory or computational methods such as finite element analysis may be employed.

For this example, yield line theory is employed. Results of the yield line analysis depend on an assumption of column flange strength versus bearing plate strength in order to estimate the fixity of the bearing plate to column flange connection. In general, if the thickness of the bearing plate is less than the column flange thickness, fixity and plastic hinging can occur at this interface; otherwise, the use of a pinned condition is conservative. Ignoring the fillets of the W-shape and clipped corners of the bearing plate, the yield line pattern chosen for the fixed condition is depicted in Figure I.8-3. Note that the simplifying assumption of 45° yield lines illustrated in Figure I.8-3 has been shown to provide reasonably accurate results (Park and Gamble, 2000), and that this yield line pattern is only valid where $b \geq 2a$.

Return to Table of Contents

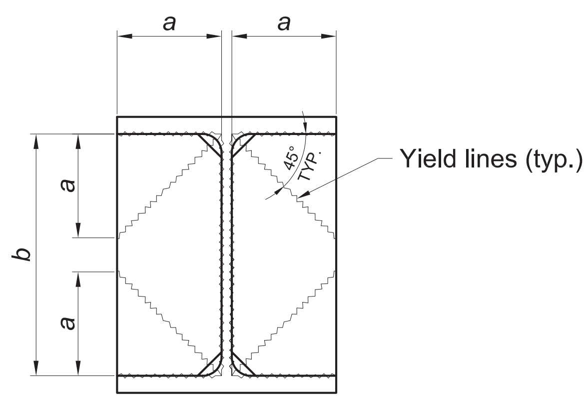


Fig. I.8-3. Internal bearing plate yield line pattern (fixed condition).

The plate thickness using $F_y = 36$ ksi material may be determined as:

LRFD	ASD
$\phi = 0.90$ If $t_p \ge t_f$: $t_p = \sqrt{\dfrac{2a^2 w_u (3b - 2a)}{3\phi F_y (4a + b)}}$ If $t_p < t_f$: $t_p = \sqrt{\dfrac{2a^2 w_u (3b - 2a)}{3\phi F_y (6a + b)}}$ where w_u = bearing pressure on plate determined using LRFD load combinations $= \dfrac{V_r'}{A_1}$ $= \dfrac{304 \text{ kips}}{134 \text{ in.}^2}$ $= 2.27$ ksi Assuming $t_p \ge t_f$ $t_p = \sqrt{\dfrac{2(3.84 \text{ in.})^2 (2.27 \text{ ksi})[3(8.86 \text{ in.}) - 2(3.84 \text{ in.})]}{3(0.90)(36 \text{ ksi})[4(3.84 \text{ in.}) + 8.86 \text{ in.}]}}$ $= 0.733$ in. Select ¾-in. plate. $t_p = ¾ \text{ in.} > t_f = 0.620 \text{ in.}$ **assumption o.k.**	$\Omega = 1.67$ If $t_p \ge t_f$: $t_p = \sqrt{\left(\dfrac{2\Omega}{3F_y}\right)\left[\dfrac{a^2 w_u (3b - 2a)}{(4a + b)}\right]}$ If $t_p < t_f$: $t_p = \sqrt{\left(\dfrac{2\Omega}{3F_y}\right)\left[\dfrac{a^2 w_u (3b - 2a)}{(6a + b)}\right]}$ where w_u = bearing pressure on plate determined using ASD load combinations $= \dfrac{V_r'}{A_1}$ $= \dfrac{203 \text{ kips}}{134 \text{ in.}^2}$ $= 1.51$ ksi Assuming $t_p \ge t_f$ $t_p = \sqrt{\dfrac{2(1.67)(3.84 \text{ in.})^2 (1.51 \text{ksi})[3(8.86 \text{ in.}) - 2(3.84 \text{ in.})]}{3(36 \text{ ksi})[4(3.84 \text{ in.}) + 8.86 \text{ in.}]}}$ $= 0.733$ in. Select ¾-in. plate $t_p = ¾ \text{ in.} > t_f = 0.620 \text{ in.}$ **assumption o.k.**

Thus, select ¾-in.-thick bearing plates.

Bearing Plate to Encased Steel Member Weld

The bearing plates should be connected to the encased steel member using welds designed in accordance with AISC *Specification* Chapter J to develop the full strength of the plate. For fillet welds, a weld size of $\frac{5}{8}t_p$ will serve to develop the strength of either a 36- or 50-ksi plate as discussed in AISC *Manual* Part 10.

Shear Connection

Shear connection involves the use of steel headed stud or channel anchors placed on at least two faces of the steel shape in a generally symmetric configuration to transfer the required longitudinal shear force. For this example, ¾-in.-diameter × 4³⁄₁₆-in.-long steel headed stud anchors composed of ASTM A108 material are selected. From AISC *Manual* Table 2-6, the specified minimum tensile strength, F_u, of ASTM A108 material is 65 ksi.

Available Shear Strength of Steel Headed Stud Anchors

The available shear strength of an individual steel headed stud anchor is determined in accordance with the composite component provisions of AISC *Specification* Section I8.3 as directed by Section I6.3b.

$$Q_{nv} = F_u A_{sa} \quad \text{(Spec. Eq. I8-3)}$$

$$A_{sa} = \frac{\pi(\text{¾ in.})^2}{4} = 0.442 \text{ in.}^2$$

LRFD	ASD
$\phi_v = 0.65$ $\phi_v Q_{nv} = 0.65(65 \text{ ksi})(0.442 \text{ in.}^2)$ $= 18.7$ kips per steel headed stud anchor	$\Omega_v = 2.31$ $Q_{nv} / \Omega_v = \dfrac{(65 \text{ ksi})(0.442 \text{ in.}^2)}{2.31}$ $= 12.4$ kips per steel headed stud anchor

Required Number of Steel Headed Stud Anchors

The number of steel headed stud anchors required to transfer the longitudinal shear is calculated as follows:

LRFD	ASD
$n_{anchors} = \dfrac{V_r'}{\phi_v Q_{nv}}$ $= \dfrac{304 \text{ kips}}{18.7 \text{ kips}}$ $= 16.3$ steel headed stud anchors	$n_{anchors} = \dfrac{V_r'}{Q_{nv}/\Omega_v}$ $= \dfrac{203 \text{ kips}}{12.4 \text{ kips}}$ $= 16.4$ steel headed stud anchors

With anchors placed in pairs on each flange, select 20 anchors to satisfy the symmetry provisions of AISC *Specification* Section I6.4a.

Placement of Steel Headed Stud Anchors

Steel headed stud anchors are placed within the load introduction length in accordance with AISC *Specification* Section I6.4a. Since the composite member only extends to one side of the point of force transfer, the steel anchors are located within $2h_2 = 48$ in. of the top of the composite member.

Placing two anchors on each flange provides four anchors per group, and maximum stud spacing within the load introduction length is determined as:

$$s_{max} = \frac{\text{load introduction length} - \text{distance to first anchor group from upper end of encased shape}}{\left[(\text{total number of anchors}) \big/ (\text{number of anchors per group}) \right] - 1}$$

$$= \frac{48 \text{ in.} - 6 \text{ in.}}{\left[(20 \text{ anchors}) \big/ (4 \text{ anchors per group}) \right] - 1}$$

$$= 10.5 \text{ in.}$$

Use 10.0 in. spacing beginning 6 in. from top of encased member.

In addition to anchors placed within the load introduction length, anchors must also be placed along the remainder of the composite member at a maximum spacing of 32 times the anchor shank diameter = 24 in. in accordance with AISC *Specification* Sections I6.4a and I8.3e.

The chosen anchor layout and spacing is illustrated in Figure I.8-4.

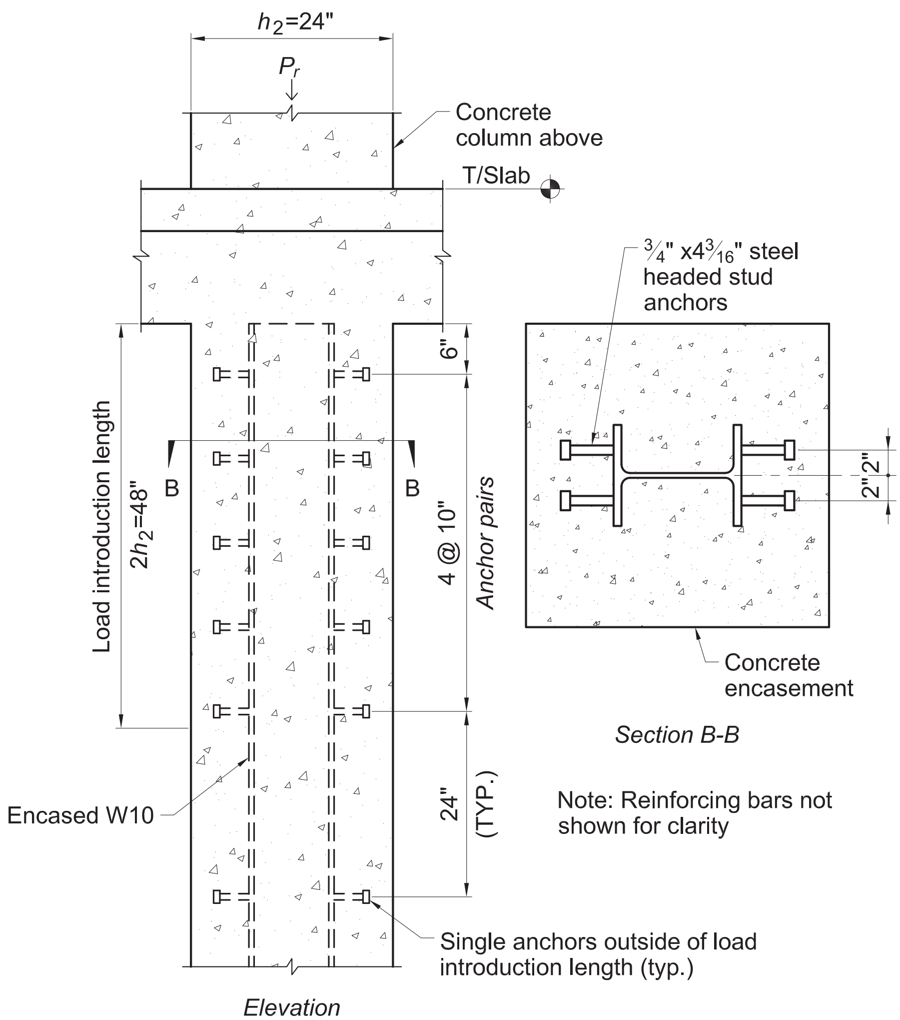


Fig. I.8-4. Composite member with steel anchors.

Steel Headed Stud Anchor Detailing Limitations of AISC *Specification* Sections I6.4a, I8.1 and I8.3

Steel headed stud anchor detailing limitations are reviewed in this section with reference to the anchor configuration provided in Figure I.8-4 for anchors having a shank diameter, d_{sa}, of ¾ in. Note that these provisions are specific to the detailing of the anchors themselves and that additional limitations for the structural steel, concrete and reinforcing components of composite members should be reviewed as demonstrated in Design Example I.9.

(1) Anchors must be placed on at least two faces of the steel shape in a generally symmetric configuration:

Anchors are located in pairs on both faces. **o.k.**

(2) Maximum anchor diameter: $d_{sa} \le 2.5\left(t_f\right)$

$$\text{¾ in.} < 2.5\left(0.620 \text{ in.}\right) = 1.55 \text{ in.} \quad \textbf{o.k.}$$

(3) Minimum steel headed stud anchor height-to-diameter ratio: $h / d_{sa} \ge 5$

The minimum ratio of installed anchor height (base to top of head), h, to shank diameter, d_{sa}, must meet the provisions of AISC *Specification* Section I8.3 as summarized in the User Note table at the end of the section. For shear in normal weight concrete the limiting ratio is five. As previously discussed, a 4³⁄₁₆-in.-long anchor was selected from anchor manufacturer's data. As the h/d_{sa} ratio is based on the installed length, a length reduction for burn off during installation of ³⁄₁₆ in. is taken to yield the final installed length of 4 in.

$$\frac{h}{d_{sa}} = \frac{4 \text{ in.}}{\text{¾ in.}} = 5.33 > 5 \quad \textbf{o.k.}$$

(4) Minimum lateral clear concrete cover = 1 in.

From AWS D1.1 Figure 7.1, the head diameter of a ¾-in.-diameter stud anchor is equal to 1.25 in.

$$\begin{aligned} \text{lateral clear cover} &= \left(\frac{h_1}{2}\right) - \left(\frac{\text{lateral spacing between anchor centerlines}}{2}\right) - \left(\frac{\text{anchor head diameter}}{2}\right) \\ &= \left(\frac{24 \text{ in.}}{2}\right) - \left(\frac{4 \text{ in.}}{2}\right) - \left(\frac{1.25 \text{ in.}}{2}\right) \\ &= 9.38 \text{ in.} > 1.0 \text{ in.} \quad \textbf{o.k.} \end{aligned}$$

(5) Minimum anchor spacing:

$$\begin{aligned} s_{min} &= 4d_{sa} \\ &= 4\left(\text{¾ in.}\right) \\ &= 3.00 \text{ in.} \end{aligned}$$

In accordance with AISC *Specification* Section I8.3e, this spacing limit applies in any direction.

$$s_{transverse} = 4 \text{ in.} \ge s_{min} \quad \textbf{o.k.}$$
$$s_{longitudinal} = 10 \text{ in.} \ge s_{min} \quad \textbf{o.k.}$$

(6) Maximum anchor spacing:

$$\begin{aligned} s_{max} &= 32d_{sa} \\ &= 32\left(\text{¾ in.}\right) \\ &= 24.0 \text{ in.} \end{aligned}$$

In accordance with AISC *Specification* Section I6.4a, the spacing limits of Section I8.1 apply to steel anchor spacing both within and outside of the load introduction region.

$s = 24.0 \text{ in.} \leq s_{max}$ **o.k.**

(7) Clear cover above the top of the steel headed stud anchors:

Minimum clear cover over the top of the steel headed stud anchors is not explicitly specified for steel anchors in composite components; however, in keeping with the intent of AISC *Specification* Section I1.1, it is recommended that the clear cover over the top of the anchor head follow the cover requirements of ACI 318 Section 7.7. For concrete columns, ACI 318 specifies a clear cover of 1½ in.

$$\begin{aligned}\text{clear cover above anchor} &= \frac{h_2}{2} - \frac{d}{2} - \text{installed anchor length} \\ &= \frac{24 \text{ in.}}{2} - \frac{10.1 \text{ in.}}{2} - 4 \text{ in.} \\ &= 2.95 \text{ in.} > 1\tfrac{1}{2} \text{ in.} \quad \textbf{o.k.}\end{aligned}$$

Concrete Breakout

AISC *Specification* Section I8.3a states that in order to use Equation I8-3 for shear strength calculations as previously demonstrated, concrete breakout strength in shear must not be an applicable limit state. If concrete breakout is deemed to be an applicable limit state, the *Specification* provides two alternatives: either the concrete breakout strength can be determine explicitly using ACI 318 Appendix D in accordance with *Specification* Section I8.3a(2), or anchor reinforcement can be provided to resist the breakout force as discussed in *Specification* Section I8.3a(1).

Determining whether concrete breakout is a viable failure mode is left to the engineer. According to AISC *Specification* Commentary Section I8.3, "it is important that it be deemed by the engineer that a concrete breakout failure mode in shear is directly avoided through having the edges perpendicular to the line of force supported, and the edges parallel to the line of force sufficiently distant that concrete breakout through a side edge is not deemed viable."

For the composite member being designed, no free edge exists in the direction of shear transfer along the length of the column, and concrete breakout in this direction is not an applicable limit state. However, it is still incumbent upon the engineer to review the possibility of concrete breakout through a side edge parallel to the line of force.

One method for explicitly performing this check is through the use of the provisions of ACI 318 Appendix D as follows:

ACI 318 Section D.6.2.1(c) specifies that concrete breakout shall be checked for shear force parallel to the edge of a group of anchors using twice the value for the nominal breakout strength provided by ACI 318 Equation D-22 when the shear force in question acts perpendicular to the edge.

For the composite member being designed, symmetrical concrete breakout planes form to each side of the encased shape, one of which is illustrated in Figure I.8-5.

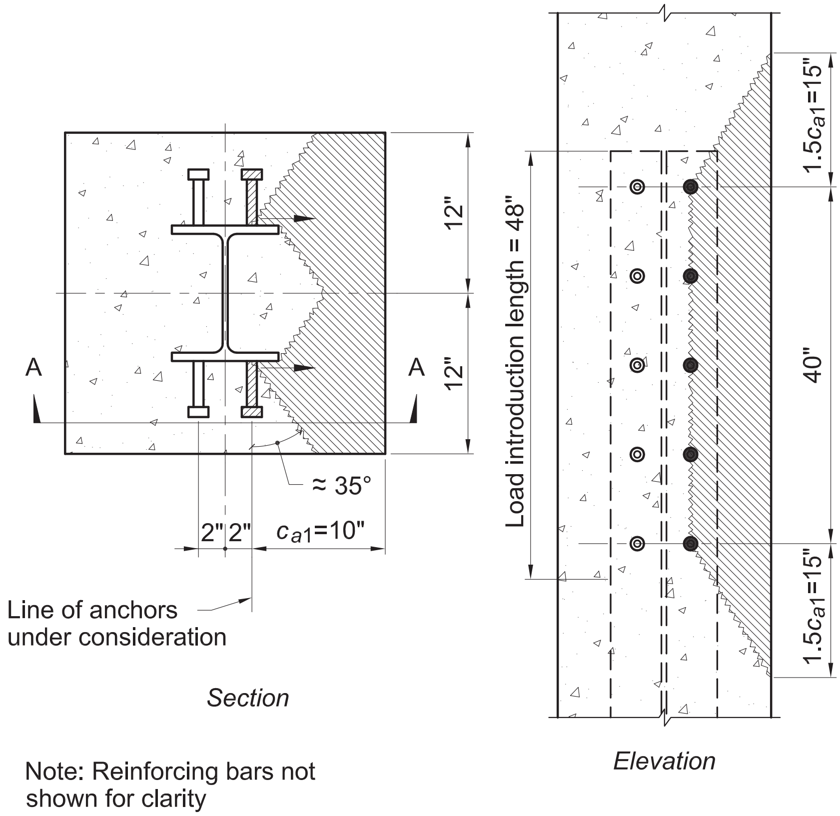


Fig. I.8-5. Concrete breakout check for shear force parallel to an edge.

ϕ = 0.75 for anchors governed by concrete breakout with supplemental reinforcement (provided by tie reinforcement) in accordance with ACI 318 Section D.4.4(c).

$$V_{cbg} = 2\left[\frac{A_{Vc}}{A_{Vco}}\Psi_{ec,V}\Psi_{ed,V}\Psi_{c,V}\Psi_{h,V}V_b\right] \text{ for shear force parallel to an edge} \quad \text{(ACI 318 Eq. D-22)}$$

$$A_{Vco} = 4.5(c_{a1})^2 \quad \text{(ACI 318 Eq. D-23)}$$

$$= 4.5(10 \text{ in.})^2$$

$$= 450 \text{ in.}^2$$

$$A_{Vc} = (15 \text{ in.} + 40 \text{ in.} + 15 \text{ in.})(24 \text{ in.}) \quad \text{from Figure I.8-5}$$

$$= 1{,}680 \text{ in.}^2$$

$\Psi_{ec,V} = 1.0$ no eccentricity

$\Psi_{ed,V} = 1.0$ in accordance with ACI 318 Section D.6.2.1(c)

$\Psi_{c,V} = 1.4$ compression-only member assumed uncracked

$\Psi_{h,V} = 1.0$

$$V_b = \left[8\left(\frac{l_e}{d_{sa}}\right)^{0.2}\sqrt{d_{sa}}\right]\lambda\sqrt{f'_c}(c_{a1})^{1.5} \quad \text{(ACI 318 Eq. D-25)}$$

where

l_e = 4 in. − ⅜-in. anchor head thickness from AWS D1.1, Figure 7.1
= 3.63 in.

d_{sa} = ¾-in. anchor diameter

λ = 1.0 from ACI 318 Section 8.6.1 for normal weight concrete

$$V_b = \left[8\left(\frac{3.63\text{ in.}}{\tfrac{3}{4}\text{ in.}}\right)^{0.2}\sqrt{\tfrac{3}{4}\text{ in.}}\right](1.0)\frac{\sqrt{5{,}000\text{ psi}}}{1{,}000\text{ lb/kip}}(10\text{ in.})^{1.5}$$
$$= 21.2\text{ kips}$$

$$V_{cbg} = 2\left[\frac{1{,}680\text{ in.}^2}{450\text{ in.}^2}(1.0)(1.0)(1.4)(1.0)(21.2\text{ kips})\right]$$
$$= 222\text{ kips}$$

$$\phi V_{cbg} = 0.75(222\text{ kips})$$
$$= 167\text{ kips per breakout plane}$$

$$\phi V_{cbg} = (2\text{ breakout planes})(167\text{ kips/plane})$$
$$= 334\text{ kips}$$

$\phi V_{cbg} \geq V_r' = 304$ kips **o.k.**

Thus, concrete breakout along an edge parallel to the direction of the longitudinal shear transfer is not a controlling limit state, and Equation I8-3 is appropriate for determining available anchor strength.

Encased beam-column members with reinforcing detailed in accordance with the AISC *Specification* have demonstrated adequate confinement in tests to prevent concrete breakout along a parallel edge from occurring; however, it is still incumbent upon the engineer to review the project-specific detailing used for susceptibility to this limit state.

If concrete breakout was determined to be a controlling limit state, transverse reinforcing ties could be analyzed as anchor reinforcement in accordance with AISC *Specification* Section I8.3a(1), and tie spacing through the load introduction length adjusted as required to prevent breakout. Alternately, the steel headed stud anchors could be relocated to the web of the encased member where breakout is prevented by confinement between the column flanges.

Return to Table of Contents

EXAMPLE I.9 ENCASED COMPOSITE MEMBER IN AXIAL COMPRESSION

Given:

Determine if the 14 ft long, encased composite member illustrated in Figure I.9-1 is adequate for the indicated dead and live loads.

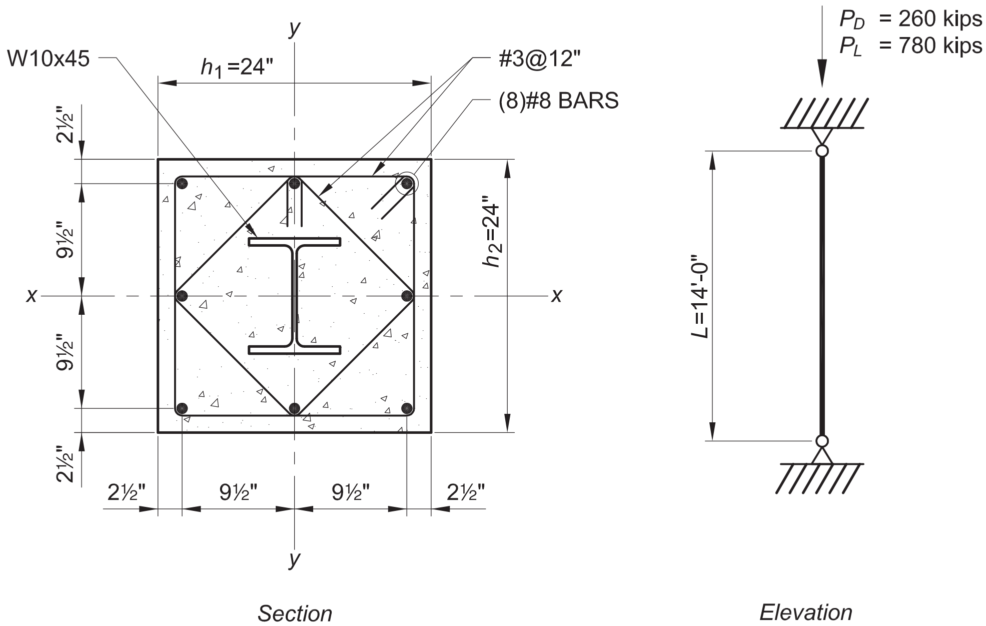


Fig. I.9-1. Encased composite member section and applied loading.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, $f_c' = 5$ ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Solution:

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, Figure I.9-1, and Design Example I.8, geometric and material properties of the composite section are:

$$
\begin{aligned}
A_s &= 13.3 \text{ in.}^2 & I_{sx} &= 248 \text{ in.}^4 & A_g &= 576 \text{ in.}^2 \\
b_f &= 8.02 \text{ in.} & I_{sy} &= 53.4 \text{ in.}^4 & A_{sri} &= 0.79 \text{ in.}^2 \\
t_f &= 0.620 \text{ in.} & h_1 &= 24.0 \text{ in.} & A_{sr} &= 6.32 \text{ in.}^2 \\
t_w &= 0.350 \text{ in.} & h_2 &= 24.0 \text{ in.} & A_c &= 556 \text{ in.}^2 \\
E_c &= 3{,}900 \text{ ksi}
\end{aligned}
$$

The moment of inertia of the reinforcing bars about the elastic neutral axis of the composite section, I_{sr}, is required for composite member design and is calculated as follows:

$$d_b = 1 \text{ in. for the diameter of a No. 8 bar}$$

$$\begin{aligned} I_{sri} &= \frac{\pi d_b^4}{64} \\ &= \frac{\pi(1 \text{ in.})^4}{64} \\ &= 0.0491 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{sr} &= \sum_{i=1}^{n} I_{sri} + \sum_{i=1}^{n} A_{sri} e_i^2 \\ &= 8\left(0.0491 \text{ in.}^4\right) + 6\left(0.79 \text{ in.}^2\right)\left(9.50 \text{ in.}\right)^2 + 2\left(0.79 \text{ in.}^2\right)\left(0 \text{ in.}\right)^2 \\ &= 428 \text{ in.}^4 \end{aligned}$$

where

A_{sri} = cross-sectional area of reinforcing bar i, in.^{2}
I_{sri} = moment of inertia of reinforcing bar i about its elastic neutral axis, in.^{4}
I_{sr} = moment of inertia of the reinforcing bars about the elastic neutral axis of the composite section, in.^{4}
d_b = nominal diameter of reinforcing bar, in.
e_i = eccentricity of reinforcing bar i with respect to the elastic neutral axis of the composite section, in.
n = number of reinforcing bars in composite section

Note that the elastic neutral axis for each direction of the section in question is located at the x-x and y-y axes illustrated in Figure I.9-1, and that the moment of inertia calculated for the longitudinal reinforcement is valid about either axis due to symmetry.

The moment of inertia values for the concrete about each axis are determined as:

$$\begin{aligned} I_{cx} &= I_{gx} - I_{sx} - I_{srx} \\ &= \frac{(24.0 \text{ in.})^4}{12} - 248 \text{ in.}^4 - 428 \text{ in.}^4 \\ &= 27{,}000 \text{ in.}^4 \end{aligned}$$

$$\begin{aligned} I_{cy} &= I_{gy} - I_{sy} - I_{sry} \\ &= \frac{(24.0 \text{ in.})^4}{12} - 53.4 \text{ in.}^4 - 428 \text{ in.}^4 \\ &= 27{,}200 \text{ in.}^4 \end{aligned}$$

Classify Section for Local Buckling

In accordance with AISC *Specification* Section I1.2, local buckling effects need not be considered for encased composite members, thus all encased sections are treated as compact sections for strength calculations.

Material and Detailing Limitations

According to the User Note at the end of AISC *Specification* Section I1.1, the intent of the *Specification* is to implement the noncomposite detailing provisions of ACI 318 in conjunction with the composite-specific provisions of *Specification* Chapter I. Detailing provisions may be grouped into material related limits, transverse reinforcement provisions, and longitudinal and structural steel reinforcement provisions as illustrated in the following discussion.

Material limits are provided in AISC *Specification* Sections I1.1(2) and I1.3 as follows:

(1) Concrete strength: $3\text{ ksi} \le f_c' \le 10\text{ ksi}$

$f_c' = 5\text{ ksi}$ **o.k.**

(2) Specified minimum yield stress of structural steel: $F_y \le 75\text{ ksi}$

$F_y = 50\text{ ksi}$ **o.k.**

(3) Specified minimum yield stress of reinforcing bars: $F_{yr} \le 75\text{ ksi}$

$F_{yr} = 60\text{ ksi}$ **o.k.**

Transverse reinforcement limitations are provided in AISC *Specification* Section I1.1(3), I2.1a(2) and ACI 318 as follows:

(1) Tie size and spacing limitations:

The AISC *Specification* requires that either lateral ties or spirals be used for transverse reinforcement. Where lateral ties are used, a minimum of either No. 3 bars spaced at a maximum of 12 in. on center or No. 4 bars or larger spaced at a maximum of 16 in. on center are required.

No. 3 lateral ties at 12 in. o.c. are provided. **o.k.**

Note that AISC *Specification* Section I1.1(1) specifically excludes the composite column provisions of ACI 318 Section 10.13, so it is unnecessary to meet the tie reinforcement provisions of ACI 318 Section 10.13.8 when designing composite columns using the provisions of AISC *Specification* Chapter I.

If spirals are used, the requirements of ACI 318 Sections 7.10 and 10.9.3 should be met according to the User Note at the end of AISC *Specification* Section I2.1a.

(2) Additional tie size limitation:

No. 4 ties or larger are required where No. 11 or larger bars are used as longitudinal reinforcement in accordance with ACI 318 Section 7.10.5.1.

No. 3 lateral ties are provided for No. 8 longitudinal bars. **o.k.**

(3) Maximum tie spacing should not exceed 0.5 times the least column dimension:

$$s_{max} = 0.5\min\begin{Bmatrix} h_1 = 24.0\text{ in.} \\ h_2 = 24.0\text{ in.} \end{Bmatrix}$$
$$= 12.0\text{ in.}$$
$$s = 12.0\text{ in.} \le s_{max} \quad \textbf{o.k.}$$

(4) Concrete cover:

ACI 318 Section 7.7 contains concrete cover requirements. For concrete not exposed to weather or in contact with ground, the required cover for column ties is 1½ in.

$$\text{cover} = 2.5\text{ in.} - \frac{d_b}{2} - \text{diameter of No. 3 tie}$$
$$= 2.5\text{ in.} - \tfrac{1}{2}\text{ in.} - \tfrac{3}{8}\text{ in.}$$
$$= 1.63\text{ in.} > 1\tfrac{1}{2}\text{ in.} \quad \textbf{o.k.}$$

(5) Provide ties as required for lateral support of longitudinal bars:

AISC *Specification* Commentary Section I2.1a references Chapter 7 of ACI 318 for additional transverse tie requirements. In accordance with ACI 318 Section 7.10.5.3 and Figure R7.10.5, ties are required to support longitudinal bars located farther than 6 in. clear on each side from a laterally supported bar. For corner bars, support is typically provided by the main perimeter ties. For intermediate bars, Figure I.9-1 illustrates one method for providing support through the use of a diamond-shaped tie.

Longitudinal and structural steel reinforcement limits are provided in AISC *Specification* Sections I1.1(4), I2.1 and ACI 318 as follows:

(1) Structural steel minimum reinforcement ratio: $A_s/A_g \geq 0.01$

$$\frac{13.3 \text{ in.}^2}{576 \text{ in.}^2} = 0.0231 \quad \textbf{o.k.}$$

An explicit maximum reinforcement ratio for the encased steel shape is not provided in the AISC *Specification*; however, a range of 8 to 12% has been noted in the literature to result in economic composite members for the resistance of gravity loads (Leon and Hajjar, 2008).

(2) Minimum longitudinal reinforcement ratio: $A_{sr}/A_g \geq 0.004$

$$\frac{6.32 \text{ in.}^2}{576 \text{ in.}^2} = 0.0110 \quad \textbf{o.k.}$$

As discussed in AISC *Specification Commentary* Section I2.1a(3), only continuously developed longitudinal reinforcement is included in the minimum reinforcement ratio, so longitudinal restraining bars and other discontinuous longitudinal reinforcement is excluded. Note that this limitation is used in lieu of the minimum ratio provided in ACI 318 as discussed in *Specification* Commentary Section I1.1(4).

(3) Maximum longitudinal reinforcement ratio: $A_{sr}/A_g \leq 0.08$

$$\frac{6.32 \text{ in.}^2}{576 \text{ in.}^2} = 0.0110 \quad \textbf{o.k.}$$

This longitudinal reinforcement limitation is provided in ACI 318 Section 10.9.1. It is recommended that all longitudinal reinforcement, including discontinuous reinforcement not used in strength calculations, be included in this ratio as it is considered a practical limitation to mitigate congestion of reinforcement. If longitudinal reinforcement is lap spliced as opposed to mechanically coupled, this limit is effectively reduced to 4% in areas away from the splice location.

(4) Minimum number of longitudinal bars:

ACI 318 Section 10.9.2 requires a minimum of four longitudinal bars within rectangular or circular members with ties and six bars for columns utilizing spiral ties. The intent for rectangular sections is to provide a minimum of one bar in each corner, so irregular geometries with multiple corners require additional longitudinal bars.

8 bars provided. **o.k.**

(5) Clear spacing between longitudinal bars:

ACI 318 Section 7.6.3 requires a clear distance between bars of $1.5d_b$ or 1½ in.

$$s_{min} = \max\begin{Bmatrix} 1.5d_b = 1\tfrac{1}{2}\text{ in.} \\ 1\tfrac{1}{2}\text{ in.} \end{Bmatrix}$$
$$= 1\tfrac{1}{2}\text{ in. clear}$$
$$s = 9.5\text{ in.} - 1.0\text{ in.}$$
$$= 8.5\text{ in.} > 1\tfrac{1}{2}\text{ in.} \quad \textbf{o.k.}$$

(6) Clear spacing between longitudinal bars and the steel core:

AISC *Specification* Section I2.1e requires a minimum clear spacing between the steel core and longitudinal reinforcement of 1.5 reinforcing bar diameters, but not less than 1½ in.

$$s_{min} = \max\begin{Bmatrix} 1.5d_b = 1\tfrac{1}{2}\text{ in.} \\ 1\tfrac{1}{2}\text{ in.} \end{Bmatrix}$$
$$= 1\tfrac{1}{2}\text{ in. clear}$$

Closest reinforcing bars to the encased section are the center bars adjacent to each flange:

$$s = \frac{h_2}{2} - \frac{d}{2} - 2.5\text{ in.} - \frac{d_b}{2}$$
$$= \frac{24\text{ in.}}{2} - \frac{10.1\text{ in.}}{2} - 2.5\text{ in.} - \frac{1\text{ in.}}{2}$$
$$= 3.95\text{ in.}$$
$$s = 3.95\text{ in.} \geq s_{min} = 1\tfrac{1}{2}\text{ in.} \quad \textbf{o.k.}$$

(7) Concrete cover for longitudinal reinforcement:

ACI 318 Section 7.7 provides concrete cover requirements for reinforcement. The cover requirements for column ties and primary reinforcement are the same, and the tie cover was previously determined to be acceptable, thus the longitudinal reinforcement cover is acceptable by inspection.

From Chapter 2 of ASCE/SEI, the required compressive strength is:

LRFD	ASD
$P_r = P_u$ $= 1.2(260\text{ kips}) + 1.6(780\text{ kips})$ $= 1,560\text{ kips}$	$P_r = P_a$ $= 260\text{ kips} + 780\text{ kips}$ $= 1,040\text{ kips}$

Available Compressive Strength

The nominal axial compressive strength without consideration of length effects, P_{no}, is determined from AISC *Specification* Section I2.1b as:

$$P_{no} = F_y A_s + F_{ysr} A_{sr} + 0.85 f_c' A_c \qquad (\textit{Spec}.\text{ Eq. I2-4})$$
$$= (50\text{ ksi})(13.3\text{ in.}^2) + (60\text{ ksi})(6.32\text{ in.}^2) + 0.85(5\text{ ksi})(556\text{ in.}^2)$$
$$= 3,410\text{ kips}$$

Because the unbraced length is the same in both the *x-x* and *y-y* directions, the column will buckle about the axis having the smaller effective composite section stiffness, EI_{eff}. Noting the moment of inertia values determined previously for the concrete and reinforcing steel are similar about each axis, the column will buckle about the weak axis of the steel shape by inspection. I_{cy}, I_{sy} and I_{sry} are therefore used for calculation of length effects in accordance with AISC *Specification* Section I2.1b as follows:

$$C_1 = 0.1 + 2\left(\frac{A_s}{A_c + A_s}\right) \le 0.3 \qquad (\textit{Spec}.\ \text{Eq. I2-7})$$

$$= 0.1 + 2\left(\frac{13.3 \text{ in.}^2}{556 \text{ in.}^2 + 13.3 \text{ in.}^2}\right) \le 0.3$$

$$= 0.147 < 0.3 \qquad \mathbf{0.147\ controls}$$

$$EI_{eff} = E_s I_{sy} + 0.5 E_s I_{sry} + C_1 E_c I_{cy} \qquad (\text{from } \textit{Spec}.\ \text{Eq. I2-6})$$

$$= (29{,}000 \text{ ksi})(53.4 \text{ in.}^4) + 0.5(29{,}000 \text{ ksi})(428 \text{ in.}^4)$$

$$+ 0.147(3{,}900 \text{ ksi})(27{,}200 \text{ in.}^4)$$

$$= 23{,}300{,}000 \text{ ksi}$$

$$P_e = \pi^2 (EI_{eff})/(KL)^2 \text{ where } K = 1.0 \text{ for a pin-ended member} \qquad (\textit{Spec}.\ \text{Eq. I2-5})$$

$$= \frac{\pi^2 (23{,}300{,}000 \text{ ksi})}{\left[(1.0)(14 \text{ ft})(12 \text{ in./ft})\right]^2}$$

$$= 8{,}150 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{3{,}410 \text{ kips}}{8{,}150 \text{ kips}}$$

$$= 0.418 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] \qquad (\textit{Spec}.\ \text{Eq. I2-2})$$

$$= (3{,}410 \text{ kips})(0.658)^{0.418}$$

$$= 2{,}860 \text{ kips}$$

Check adequacy of the composite column for the required axial compressive strength:

LRFD	ASD
$\phi_c = 0.75$ $\phi_c P_n \ge P_u$ $\phi_c P_n = 0.75(2{,}860 \text{ kips})$ $= 2{,}150 \text{ kips} > 1{,}560 \text{ kips}$ **o.k.**	$\Omega_c = 2.00$ $P_n / \Omega_c \ge P_a$ $P_n / \Omega_c = \dfrac{2{,}860 \text{ kips}}{2.00}$ $= 1{,}430 \text{ kips} > 1{,}040 \text{ kips}$ **o.k.**

Available Compressive Strength of Composite Section Versus Bare Steel Section

Due to the differences in resistance and safety factors between composite and noncomposite column provisions, it is possible in rare instances to calculate a lower available compressive strength for an encased composite column than one would calculate for the corresponding bare steel section. However, in accordance with AISC *Specification* Section I2.1b, the available compressive strength need not be less than that calculated for the bare steel member in accordance with Chapter E.

From AISC *Manual* Table 4-1:

LRFD	ASD
$\phi_c P_n = 359\text{ kips}$ $359\text{ kips} < 2{,}150\text{ kips}$	$P_n / \Omega_c = 239\text{ kips}$ $239\text{ kips} < 1{,}430\text{ kips}$

Thus, the composite section strength controls and is adequate for the required axial compressive strength as previously demonstrated.

Force Allocation and Load Transfer

Load transfer calculations for external axial forces should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions for encased composite members is provided in Design Example I.8.

Typical Detailing Convention

Designers are directed to AISC Design Guide 6 (Griffis, 1992) for additional discussion and typical details of encased composite columns not explicitly covered in this example.

Return to Table of Contents

EXAMPLE I.10 ENCASED COMPOSITE MEMBER IN AXIAL TENSION

Given:

Determine if the 14 ft long, encased composite member illustrated in Figure I.10-1 is adequate for the indicated dead load compression and wind load tension. The entire load is applied to the encased steel section.

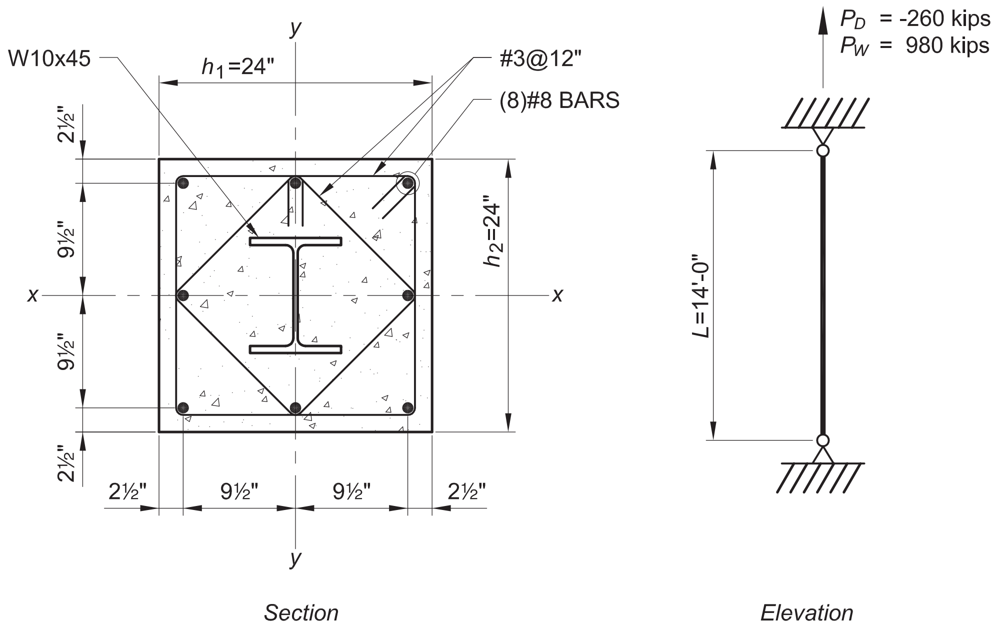


Fig. I.10-1. Encased composite member section and applied loading.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, $f_c' = 5$ ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Solution:

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1 and Figure I.10-1, the relevant properties of the composite section are:

$$A_s = 13.3 \text{ in.}^2$$
$$A_{sr} = 6.32 \text{ in.}^2 \quad \text{(area of eight No. 8 bars)}$$

Material and Detailing Limitations

Refer to Design Example I.9 for a check of material and detailing limitations specified in AISC *Specification* Chapter I for encased composite members.

Taking compression as negative and tension as positive, from Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
Governing Uplift Load Combination $= 0.9D + 1.0W$ $P_r = P_u$ $= 0.9(-260 \text{ kips}) + 1.0(980 \text{ kips})$ $= 746 \text{ kips}$	Governing Uplift Load Combination $= 0.6D + 0.6W$ $P_r = P_a$ $= 0.6(-260 \text{ kips}) + 0.6(980 \text{ kips})$ $= 432 \text{ kips}$

Available Tensile Strength

Available tensile strength for an encased composite member is determined in accordance with AISC *Specification* Section I2.1c.

$$P_n = F_y A_s + F_{ysr} A_{sr} \qquad (\textit{Spec}. \text{ Eq. I2-8})$$
$$= (50 \text{ ksi})(13.3 \text{ in.}^2) + (60 \text{ ksi})(6.32 \text{ in.}^2)$$
$$= 1{,}040 \text{ kips}$$

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n \geq P_u$ $\phi_t P_n = 0.90(1{,}040 \text{ kips})$ $= 936 \text{ kips} > 746 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $P_n / \Omega_t \geq P_a$ $P_n / \Omega_t = \dfrac{1{,}040 \text{ kips}}{1.67}$ $= 623 \text{ kips} > 432 \text{ kips}$ **o.k.**

Force Allocation and Load Transfer

In cases where all of the tension is applied to either the reinforcing steel or the encased steel shape, and the available strength of the reinforcing steel or encased steel shape by itself is adequate, no additional load transfer calculations are required.

In cases such as the one under consideration, where the available strength of both the reinforcing steel and the encased steel shape are needed to provide adequate tension resistance, AISC *Specification* Section I6 can be modified for tensile load transfer requirements by replacing the P_{no} term in Equations I6-1 and I6-2 with the nominal tensile strength, P_n, determined from Equation I2-8.

For external tensile force applied to the encased steel section:

$$V_r' = P_r\left(1 - \frac{F_y A_s}{P_n}\right) \qquad \text{(Eq. 1)}$$

For external tensile force applied to the longitudinal reinforcement of the concrete encasement:

$$V_r' = P_r\left(\frac{F_y A_s}{P_n}\right) \qquad \text{(Eq. 2)}$$

where

P_r = required external tensile force applied to the composite member, kips

P_n = nominal tensile strength of encased composite member from Equation I2-8, kips

Per the problem statement, the entire external force is applied to the encased steel section, thus Equation 1 is used as follows:

$$V_r' = P_r\left[1 - \frac{(50\text{ ksi})(13.3\text{ in.}^2)}{1{,}040\text{ kips}}\right]$$
$$= 0.361P_r$$

LRFD	ASD
$V_r' = 0.361(746\text{ kips})$ $= 269\text{ kips}$	$V_r' = 0.361(432\text{ kips})$ $= 156\text{ kips}$

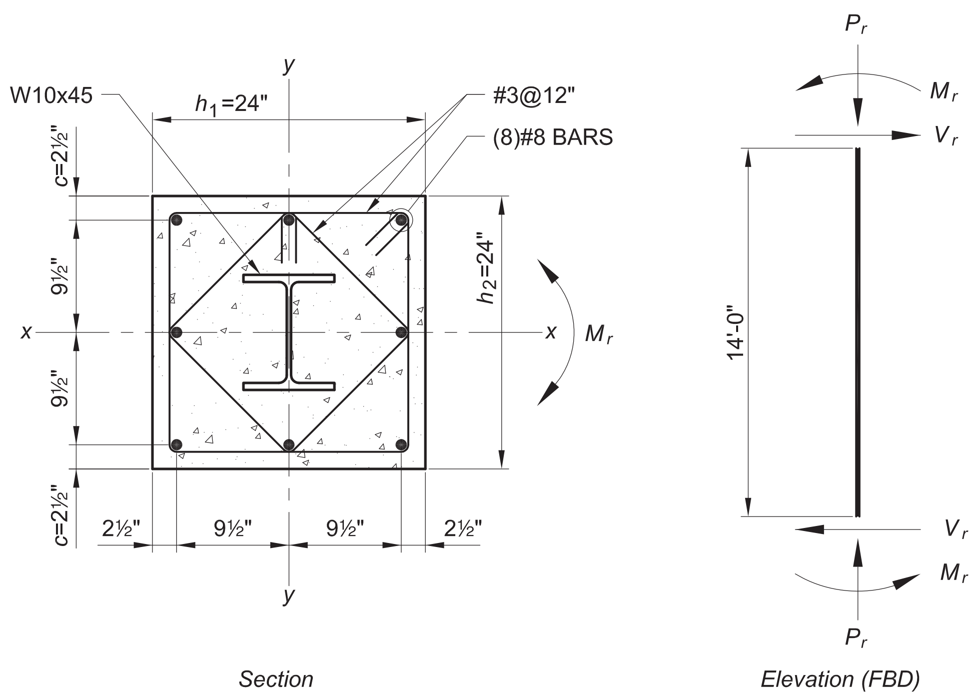
The longitudinal shear force must be transferred between the encased steel shape and longitudinal reinforcing using the force transfer mechanisms of direct bearing or shear connection in accordance with AISC *Specification* Section I6.3 as illustrated in Design Example I.8.

Return to Table of Contents

EXAMPLE I.11 ENCASED COMPOSITE MEMBER IN COMBINED AXIAL COMPRESSION, FLEXURE AND SHEAR

Given:

Determine if the 14 ft long, encased composite member illustrated in Figure I.11-1 is adequate for the indicated axial forces, shears and moments that have been determined in accordance with the direct analysis method of AISC *Specification* Chapter C for the controlling ASCE/SEI 7-10 load combinations.



	LRFD	ASD
P_r (kips)	1,170	879
M_r (kip-ft)	670	302
V_r (kips)	95.7	57.4

Fig. I.11-1. Encased composite member section and member forces.

The composite member consists of an ASTM A992 W-shape encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, $f_c' = 5$ ksi.

Deformed reinforcing bars conform to ASTM A615 with a minimum yield stress, F_{yr}, of 60 ksi.

Solution:

From AISC *Manual* Table 2-4, the steel material properties are:

ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

From AISC *Manual* Table 1-1, Figure I.11-1, and Design Examples I.8 and I.9, the geometric and material properties of the composite section are:

$A_s = 13.3 \text{ in.}^2$ | $I_{sy} = 53.4 \text{ in.}^4$ | $A_g = 576 \text{ in.}^2$ | $h_1 = 24.0 \text{ in.}$

$d = 10.1 \text{ in.}$ | $Z_{sx} = 54.9 \text{ in.}^3$ | $A_{sr} = 6.32 \text{ in.}^2$ | $h_2 = 24.0 \text{ in.}$

$b_f = 8.02 \text{ in.}$ | $S_{sx} = 49.1 \text{ in.}^3$ | $A_c = 556 \text{ in.}^2$ | $c = 2\frac{1}{2} \text{ in.}$

$t_f = 0.620 \text{ in.}$ | $E_c = 3{,}900 \text{ ksi}$ | $I_{sr} = 428 \text{ in.}^4$ | $I_{cx} = 27{,}000 \text{ in.}^4$

$t_w = 0.350 \text{ in.}$ | $I_{cy} = 27{,}200 \text{ in.}^4$

The area of continuous reinforcing located at the centerline of the composite section, A_{srs}, is determined from Figure I.11-1 as follows:

$$A_{srs} = 2(A_{srsi})$$
$$= 2(0.79 \text{ in.}^2)$$
$$= 1.58 \text{ in.}^2$$

where

A_{srsi} = area of reinforcing bar i at centerline of composite section
= 0.79 in.^{2} for a No. 8 bar

For the section under consideration, A_{srs} is equal about both the x-x and y-y axis.

Classify Section for Local Buckling

In accordance with AISC *Specification* Section I1.2, local buckling effects need not be considered for encased composite members, thus all encased sections are treated as compact sections for strength calculations.

Material and Detailing Limitations

Refer to Design Example I.9 for a check of material and detailing limitations.

Interaction of Axial Force and Flexure

Interaction between axial forces and flexure in composite members is governed by AISC *Specification* Section I5 which permits the use of a strain compatibility method or plastic stress distribution method.

The strain compatibility method is a generalized approach that allows for the construction of an interaction diagram based upon the same concepts used for reinforced concrete design. Application of the strain compatibility method is required for irregular/nonsymmetrical sections, and its general implementation may be found in reinforced concrete design texts and will not be discussed further here.

Plastic stress distribution methods are discussed in AISC *Specification* Commentary Section I5 which provides four procedures. The first procedure, Method 1, invokes the interaction equations of Section H1. The second procedure, Method 2, involves the construction of a piecewise-linear interaction curve using the plastic strength equations provided in Figure I-1 located within the front matter of the Chapter I Design Examples. The third procedure, Method 2—Simplified, is a reduction of the piecewise-linear interaction curve that allows for the use of less conservative interaction equations than those presented in Chapter H. The fourth and final procedure, Method 3, utilizes AISC *Design Guide 6* (Griffis, 1992).

For this design example, three of the available plastic stress distribution procedures are reviewed and compared. Method 3 is not demonstrated as it is not applicable to the section under consideration due to the area of the encased steel section being smaller than the minimum limit of 4% of the gross area of the composite section provided in the earlier *Specification* upon which Design Guide 6 is based.

Method 1—Interaction Equations of Section H1

The most direct and conservative method of assessing interaction effects is through the use of the interaction equations of AISC *Specification* Section H1. Unlike concrete filled HSS shapes, the available compressive and flexural strengths of encased members are not tabulated in the AISC *Manual* due to the large variety of possible combinations. Calculations must therefore be performed explicitly using the provisions of Chapter I.

Available Compressive Strength

The available compressive strength is calculated as illustrated in Design Example I.9.

LRFD	ASD
$\phi_c P_n = 2{,}150 \text{ kips}$	$P_n / \Omega_c = 1{,}430 \text{ kips}$

Nominal Flexural Strength

The applied moment illustrated in Figure I.11-1 is resisted by the flexural strength of the composite section about its strong (x-x) axis. The strength of the section in pure flexure is calculated using the equations of Figure I-1a found in the front matter of the Chapter I Design Examples for Point B. Note that the calculation of the flexural strength at Point B first requires calculation of the flexural strength at Point D as follows:

$$\begin{aligned}
Z_r &= \left(A_{sr} - A_{srs}\right)\left(\frac{h_2}{2} - c\right) \\
&= \left(6.32 \text{ in.}^2 - 1.58 \text{ in.}^2\right)\left(\frac{24.0 \text{ in.}}{2} - 2.5 \text{ in.}\right) \\
&= 45.0 \text{ in.}^3
\end{aligned}$$

$$\begin{aligned}
Z_c &= \frac{h_1 h_2^2}{4} - Z_s - Z_r \\
&= \frac{(24.0 \text{ in.})(24.0 \text{ in.})^2}{4} - 54.9 \text{ in.}^3 - 45.0 \text{ in.}^3 \\
&= 3{,}360 \text{ in.}^3
\end{aligned}$$

$$\begin{aligned}
M_D &= Z_s F_y + Z_r F_{yr} + \frac{Z_c}{2}\left(0.85 f_c'\right) \\
&= \left(54.9 \text{ in.}^3\right)(50 \text{ ksi}) + \left(45.0 \text{ in.}^3\right)(60 \text{ ksi}) + \frac{3{,}360 \text{ in.}^3}{2}(0.85)(5 \text{ ksi}) \\
&= \frac{12{,}600 \text{ kip-in.}}{12 \text{ in./ft}} \\
&= 1{,}050 \text{ kip-ft}
\end{aligned}$$

Assuming h_n is within the flange $\left(\frac{d}{2} - t_f < h_n \le \frac{d}{2}\right)$:

$$h_n = \frac{0.85 f_c'\left(A_c + A_s - d b_f + A_{srs}\right) - 2F_y\left(A_s - d b_f\right) - 2F_{yr} A_{srs}}{2\left[0.85 f_c'\left(h_1 - b_f\right) + 2F_y b_f\right]}$$

$$\begin{aligned}
&= \Big\{0.85(5 \text{ ksi})\left[556 \text{ in.}^2 + 13.3 \text{ in.}^2 - (10.1 \text{ in.})(8.02 \text{ in.}) + 1.58 \text{ in.}^2\right] - 2(50 \text{ ksi})\left[13.3 \text{ in.}^2 - (10.1 \text{ in.})(8.02 \text{ in.})\right] \\
&\quad - 2(60 \text{ ksi})\left(1.58 \text{ in.}^2\right)\Big\} \Big/ 2\left[0.85(5 \text{ ksi})(24.0 \text{ in.} - 8.02 \text{ in.}) + 2(50 \text{ ksi})(8.02 \text{ in.})\right] \\
&= 4.98 \text{ in.}
\end{aligned}$$

Check assumption:

$$\left(\frac{10.1\text{ in.}}{2} - 0.620\text{ in.}\right) \le h_n \le \frac{10.1\text{ in.}}{2}$$

$4.43\text{ in.} < h_n = 4.98\text{ in.} < 5.05\text{ in.}$ assumption **o.k.**

$$Z_{sn} = Z_s - b_f\left(\frac{d}{2} - h_n\right)\left(\frac{d}{2} + h_n\right)$$
$$= 54.9\text{ in.}^3 - (8.02\text{ in.})\left(\frac{10.1\text{ in.}}{2} - 4.98\text{ in.}\right)\left(\frac{10.1\text{ in.}}{2} + 4.98\text{ in.}\right)$$
$$= 49.3\text{ in.}^3$$
$$Z_{cn} = h_1 h_n^2 - Z_{sn}$$
$$= (24.0\text{ in.})(4.98\text{ in.})^2 - 49.3\text{ in.}^3$$
$$= 546\text{ in.}^3$$
$$M_B = M_D - Z_{sn}F_y - \frac{Z_{cn}(0.85 f_c')}{2}$$
$$= 12{,}600\text{ kip-in.} - (49.3\text{ in.}^3)(50\text{ ksi}) - \frac{(546\text{ in.}^3)(0.85)(5\text{ ksi})}{2}$$
$$= \frac{8{,}970\text{ kip-in.}}{12\text{ in./ft}}$$
$$= 748\text{ kip-ft}$$

Available Flexural Strength

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90(748\text{ kip-ft})$ $= 673\text{ kip-ft}$	$\Omega_b = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{748\text{ kip-ft}}{1.67}$ $= 448\text{ kip-ft}$

Interaction of Axial Compression and Flexure

LRFD	ASD
$\phi_c P_n = 2{,}150\text{ kips}$ $\phi_b M_n = 673\text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_u}{\phi_c P_n}$ $= \dfrac{1{,}170\text{ kips}}{2{,}150\text{ kips}}$ $= 0.544 > 0.2$ Use AISC *Specification* Equation H1-1a. $\dfrac{P_u}{\phi_c P_n} + \dfrac{8}{9}\left(\dfrac{M_u}{\phi_b M_n}\right) \le 1.0$	$P_n / \Omega_c = 1{,}430\text{ kips}$ $M_n / \Omega_c = 448\text{ kip-ft}$ $\dfrac{P_r}{P_c} = \dfrac{P_a}{P_n / \Omega_c}$ $= \dfrac{879\text{ kips}}{1{,}430\text{ kips}}$ $= 0.615 > 0.2$ Use AISC *Specification* Equation H1-1a. $\dfrac{P_a}{P_n / \Omega_c} + \dfrac{8}{9}\left(\dfrac{M_a}{M_n / \Omega_b}\right) \le 1.0$

Return to Table of Contents

LRFD	ASD
$\dfrac{1{,}170 \text{ kips}}{2{,}150 \text{ kips}}+\dfrac{8}{9}\left(\dfrac{670 \text{ kip-ft}}{673 \text{ kip-ft}}\right)\leq 1.0$ $1.43>1.0$ **n.g.**	$\dfrac{879 \text{ kips}}{1{,}430 \text{ kips}}+\dfrac{8}{9}\left(\dfrac{302 \text{ kip-ft}}{448 \text{ kip-ft}}\right)\leq 1.0$ $1.21>1.0$ **n.g.**

Method 1 indicates that the section is inadequate for the applied loads. The designer can elect to choose a new section that passes the interaction check or re-analyze the current section using a less conservative design method such as Method 2. The use of Method 2 is illustrated in the following section.

Method 2—Interaction Curves from the Plastic Stress Distribution Model

The procedure for creating an interaction curve using the plastic stress distribution model is illustrated graphically in AISC *Specification* Commentary Figure C-I5.2, and repeated here.

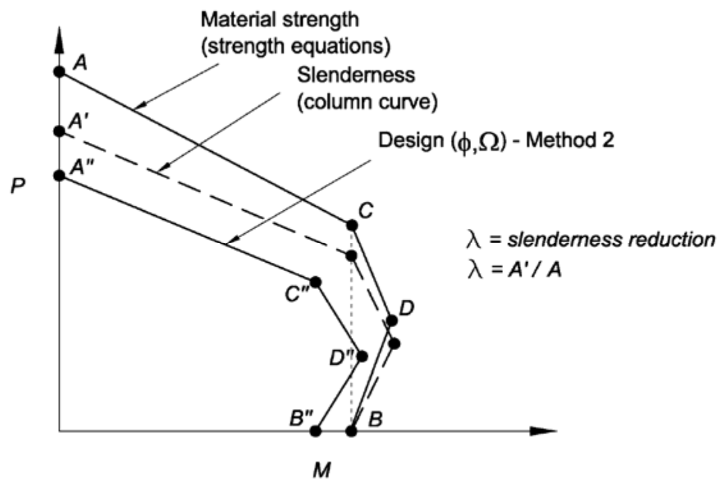


Fig. C-I5.2. Interaction diagram for composite beam-columns – Method 2.

Referencing Figure C.I5.2, the nominal strength interaction surface A, B, C, D is first determined using the equations of Figure I-1a found in the front matter of the Chapter I Design Examples. This curve is representative of the short column member strength without consideration of length effects. A slenderness reduction factor, λ, is then calculated and applied to each point to create surface A′, B′, C′, D′ . The appropriate resistance or safety factors are then applied to create the design surface A′′ ,B′′ ,C′′ ,D′′ . Finally, the required axial and flexural strengths from the applicable load combinations of ASCE/SEI 7-10 are plotted on the design surface. The member is then deemed acceptable for the applied loading if all points fall within the design surface. These steps are illustrated in detail by the following calculations.

Step 1: Construct nominal strength interaction surface A, B, C, D without length effects

Using the equations provided in Figure I-1a for bending about the *x-x* axis yields:

Point A (pure axial compression):

$$\begin{aligned} P_A &= A_sF_y + A_{sr}F_{yr} + 0.85f_c'A_c \\ &= \left(13.3 \text{ in.}^2\right)\left(50 \text{ ksi}\right)+\left(6.32 \text{ in.}^2\right)\left(60 \text{ ksi}\right)+0.85\left(5 \text{ ksi}\right)\left(556 \text{ in.}^2\right) \\ &= 3{,}410 \text{ kips} \end{aligned}$$

$$M_A = 0 \text{ kip-ft}$$

Point D (maximum nominal moment strength):

$$P_D = \frac{0.85 f_c' A_c}{2}$$
$$= \frac{0.85(5 \text{ ksi})(556 \text{ in.}^2)}{2}$$
$$= 1{,}180 \text{ kips}$$

Calculation of M_D was demonstrated previously in Method 1.

$$M_D = 1{,}050 \text{ kip-ft}$$

Point B (pure flexure):

$$P_B = 0 \text{ kips}$$

Calculation of M_B was demonstrated previously in Method 1.

$$M_B = 748 \text{ kip-ft}$$

Point C (intermediate point):

$$P_C = 0.85 f_c' A_c$$
$$= 0.85(5 \text{ ksi})(556 \text{ in.}^2)$$
$$= 2{,}360 \text{ kips}$$
$$M_C = M_B$$
$$= 748 \text{ kip-ft}$$

The calculated points are plotted to construct the nominal strength interaction surface without length effects as depicted in Figure I.11-2.

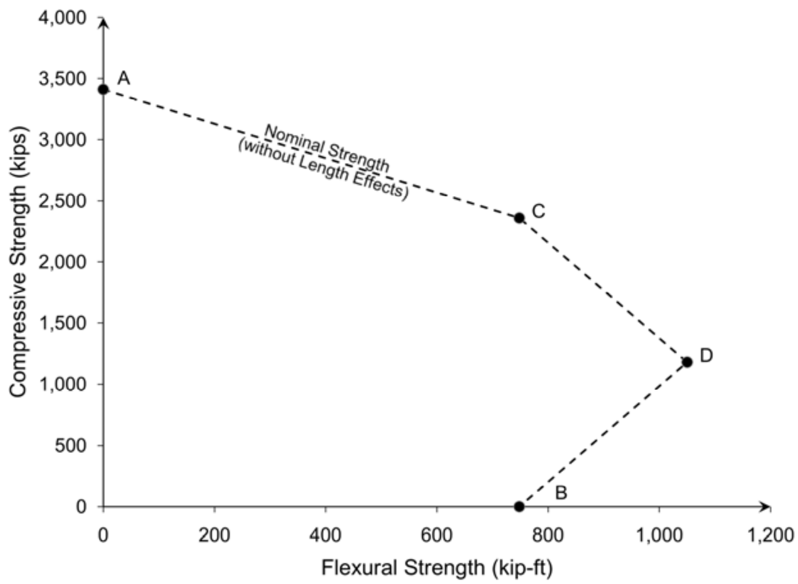


Fig. I.11-2. Nominal strength interaction surface without length effects.

Step 2: Construct nominal strength interaction surface A′, B′, C′, D′ with length effects

The slenderness reduction factor, λ, is calculated for Point A using AISC *Specification* Section I2.1 in accordance with *Specification* Commentary Section I5.

Because the unbraced length is the same in both the *x-x* and *y-y* directions, the column will buckle about the axis having the smaller effective composite section stiffness, EI_{eff}. Noting the moment of inertia values for the concrete and reinforcing steel are similar about each axis, the column will buckle about the weak axis of the steel shape by inspection. I_{cy}, I_{sy} and I_{sry} are therefore used for calculation of length effects in accordance with AISC *Specification* Section I2.1b.

$$P_{no} = P_A$$
$$= 3{,}410 \text{ kips}$$

$$C_1 = 0.1 + 2\left(\frac{A_s}{A_c + A_s}\right) \leq 0.3 \qquad (\textit{Spec}. \text{ Eq. I2-7})$$
$$= 0.1 + 2\left(\frac{13.3 \text{ in.}^2}{556 \text{ in.}^2 + 13.3 \text{ in.}^2}\right) \leq 0.3$$
$$= 0.147 < 0.3; \text{ therefore } C_1 = 0.147.$$

$$EI_{eff} = E_s I_{sy} + 0.5 E_s I_{sry} + C_1 E_c I_{cy} \qquad (\textit{Spec}. \text{ Eq. I2-6})$$
$$= (29{,}000 \text{ ksi})(53.4 \text{ in.}^4) + 0.5(29{,}000 \text{ ksi})(428 \text{ in.}^4)$$
$$+ 0.147(3{,}900 \text{ ksi})(27{,}200 \text{ in.}^4)$$
$$= 23{,}300{,}000 \text{ ksi}$$

$$P_e = \pi^2 (EI_{eff}) / (KL)^2 \text{ where } K = 1.0 \text{ in accordance with the direct analysis method} \qquad (\textit{Spec}. \text{ Eq. I2-5})$$
$$= \frac{\pi^2 (23{,}300{,}000 \text{ ksi})}{\left[(1.0)(14 \text{ ft})(12 \text{ in./ft})\right]^2}$$
$$= 8{,}150 \text{ kips}$$

$$\frac{P_{no}}{P_e} = \frac{3{,}410 \text{ kips}}{8{,}150 \text{ kips}}$$
$$= 0.418 < 2.25$$

Therefore, use AISC *Specification* Equation I2-2.

$$P_n = P_{no}\left[0.658^{\frac{P_{no}}{P_e}}\right] \qquad (\textit{Spec}. \text{ Eq. I2-2})$$
$$= 3{,}410 \text{ kips}(0.658)^{0.418}$$
$$= 2{,}860 \text{ kips}$$

$$\lambda = \frac{P_n}{P_{no}}$$
$$= \frac{2{,}860 \text{ kips}}{3{,}410 \text{ kips}}$$
$$= 0.839$$

In accordance with AISC *Specification* Commentary Section I5, the same slenderness reduction is applied to each of the remaining points on the interaction surface as follows:

Return to Table of Contents

$$P_{A'} = \lambda P_A$$
$$= 0.839(3,410 \text{ kips})$$
$$= 2,860 \text{ kips}$$
$$P_{B'} = \lambda P_B$$
$$= 0.839(0 \text{ kips})$$
$$= 0 \text{ kips}$$
$$P_{C'} = \lambda P_C$$
$$= 0.839(2,360 \text{ kips})$$
$$= 1,980 \text{ kips}$$
$$P_{D'} = \lambda P_D$$
$$= 0.839(1,180 \text{ kips})$$
$$= 990 \text{ kips}$$

The modified axial strength values are plotted with the flexural strength values previously calculated to construct the nominal strength interaction surface including length effects. These values are superimposed on the nominal strength surface not including length effects for comparison purposes in Figure I.11-3.

The consideration of length effects results in a vertical reduction of the nominal strength curve as illustrated by Figure I.11-3. This vertical movement creates an unsafe zone within the shaded area of the figure where flexural capacities of the nominal strength (with length effects) curve exceed the section capacity. Application of resistance or safety factors reduces this unsafe zone as illustrated in the following step; however, designers should be cognizant of the potential for unsafe designs with loads approaching the predicted flexural capacity of the section. Alternately, the use of Method 2—Simplified eliminates this possibility altogether.

Step 3: Construct design interaction surface A″, B″, C″, D″ and verify member adequacy

The final step in the Method 2 procedure is to reduce the interaction surface for design using the appropriate resistance or safety factors.

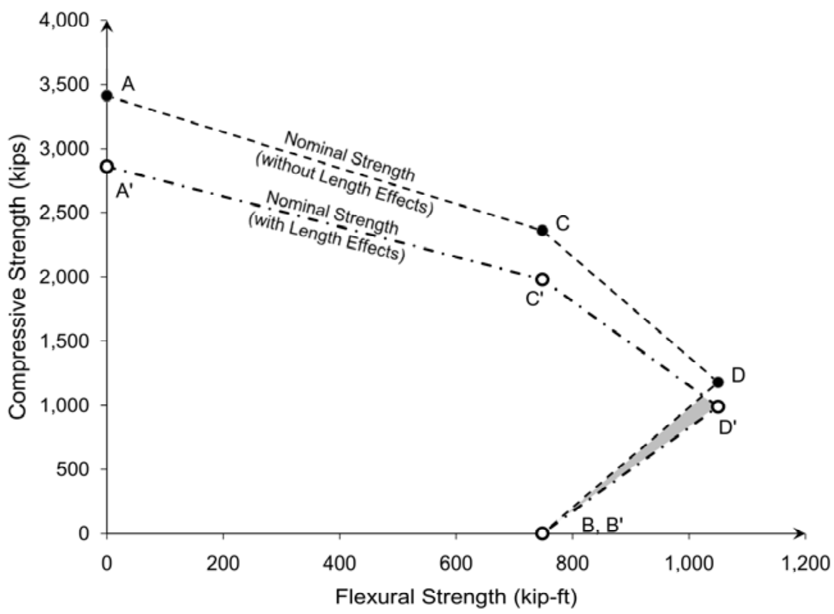


Fig. I.11-3. Nominal strength interaction surfaces (with and without length effects).

The available compressive and flexural strengths are determined as follows:

LRFD	ASD
Design compressive strength: $\phi_c = 0.75$ $P_{X''} = \phi_c P_{X'}$ where $X = A,\ B,\ C$ or D $P_{A''} = 0.75(2{,}860 \text{ kips})$ $= 2{,}150 \text{ kips}$ $P_{B''} = 0.75(0 \text{ kips})$ $= 0 \text{ kips}$ $P_{C''} = 0.75(1{,}980 \text{ kips})$ $= 1{,}490 \text{ kips}$ $P_{D''} = 0.75(990 \text{ kips})$ $= 743 \text{ kips}$	Allowable compressive strength: $\Omega_c = 2.00$ $P_{X''} = P_{X'} / \Omega_c$ where $X = A,\ B,\ C$ or D $P_{A''} = 2{,}860 \text{ kips} / 2.00$ $= 1{,}430 \text{ kips}$ $P_{B''} = 0 \text{ kips} / 2.00$ $= 0 \text{ kips}$ $P_{C''} = 1{,}980 \text{ kips} / 2.00$ $= 990 \text{ kips}$ $P_{D''} = 990 \text{ kips} / 2.00$ $= 495 \text{ kips}$
Design flexural strength: $\phi_b = 0.90$ $M_{X''} = \phi_b M_{X'}$ where $X = A,\ B,\ C$ or D $M_{A''} = 0.90(0 \text{ kip-ft})$ $= 0 \text{ kip-ft}$ $M_{B''} = 0.90(748 \text{ kip-ft})$ $= 673 \text{ kip-ft}$ $M_{C''} = 0.90(748 \text{ kip-ft})$ $= 673 \text{ kip-ft}$ $M_{D''} = 0.90(1{,}050 \text{ kip-ft})$ $= 945 \text{ kip-ft}$	Allowable flexural strength: $\Omega_b = 1.67$ $M_{X''} = M_{X'} / \Omega_b$ where $X = A,\ B,\ C$ or D $M_{A''} = 0 \text{ kip-ft} / 1.67$ $= 0 \text{ kip-ft}$ $M_{B''} = 748 \text{ kip-ft} / 1.67$ $= 448 \text{ kip-ft}$ $M_{C''} = 748 \text{ kip-ft} / 1.67$ $= 448 \text{ kip-ft}$ $M_{D''} = 1{,}050 \text{ kip-ft} / 1.67$ $= 629 \text{ kip-ft}$

The available strength values for each design method can now be plotted. These values are superimposed on the nominal strength surfaces (with and without length effects) previously calculated for comparison purposes in Figure I.11-4.

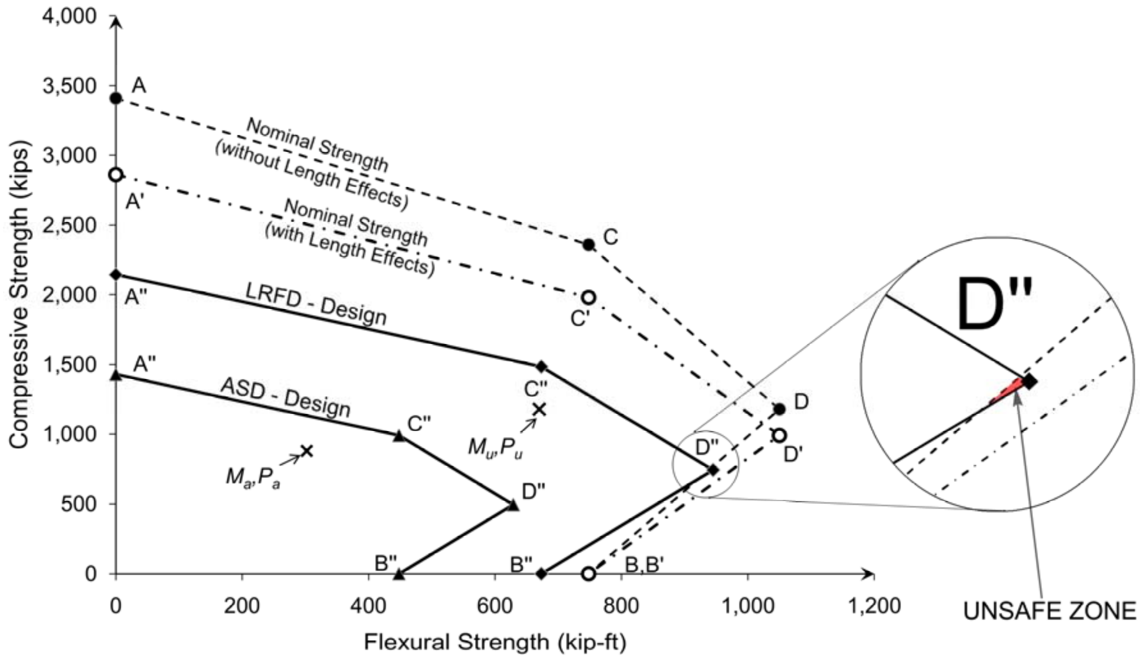


Fig. I.11-4. Available and nominal interaction surfaces.

By plotting the required axial and flexural strength values on the available strength surfaces indicated in Figure I.11-4, it can be seen that both ASD (M_a,P_a) and LRFD (M_u,P_u) points lie within their respective design surfaces. The member in question is therefore adequate for the applied loads.

As discussed previously in Step 2 as well as in AISC *Specification* Commentary Section I5, when reducing the flexural strength of Point D for length effects and resistance or safety factors, an unsafe situation could result whereby additional flexural strength is permitted at a lower axial compressive strength than predicted by the cross section strength of the member. This effect is highlighted by the magnified portion of Figure I.11-4, where LRFD design point D″ falls slightly below the nominal strength curve. Designs falling within this zone are unsafe and not permitted.

Method 2—Simplified

The unsafe zone discussed in the previous section for Method 2 is avoided in the Method 2—Simplified procedure by the removal of Point D″ from the Method 2 interaction surface leaving only points A″, B″ and C″ as illustrated in Figure I.11-5. Reducing the number of interaction points also allows for a bilinear interaction check defined by AISC *Specification* Commentary Equations C-I5-1a and C-I5-1b to be performed.

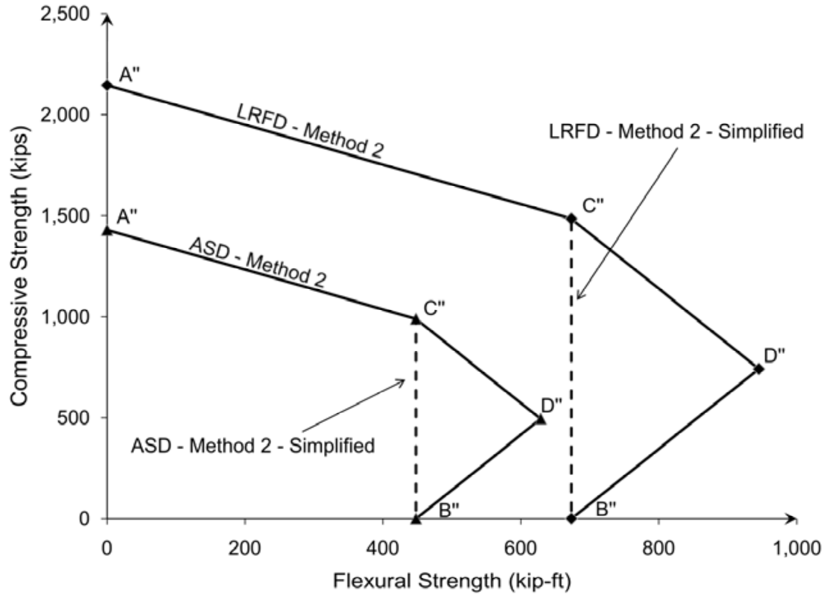


Fig. I.11-5. Comparison of Method 2 and Method 2 —Simplified.

Using the available strength values previously calculated in conjunction with the Commentary equations, interaction ratios are determined as follows:

LRFD	ASD
$P_r = P_u$ $= 1{,}170$ kips $P_r < P_{C''}$ $< 1{,}490$ kips Therefore, use Commentary Equation C-I5-1a. $\dfrac{M_r}{M_C} = \dfrac{M_u}{M_{C''}} \leq 1.0$ $\dfrac{670 \text{ kip-ft}}{673 \text{ kip-ft}} \leq 1.0$ $1.0 = 1.0$ **o.k.**	$P_r = P_a$ $= 879$ kips $P_r < P_{C''}$ < 990 kips Therefore, use Commentary Equation C-I5-1a. $\dfrac{M_r}{M_C} = \dfrac{M_a}{M_{C''}} \leq 1.0$ $\dfrac{302 \text{ kip-ft}}{448 \text{ kip-ft}} \leq 1.0$ $0.67 < 1.0$ **o.k.**

Thus, the member is adequate for the applied loads.

Comparison of Methods

The composite member was found to be inadequate using Method 1—Chapter H interaction equations, but was found to be adequate using both Method 2 and Method 2—Simplified procedures. A comparison between the methods is most easily made by overlaying the design curves from each method as illustrated in Figure I.11-6 for LRFD design.

Return to Table of Contents

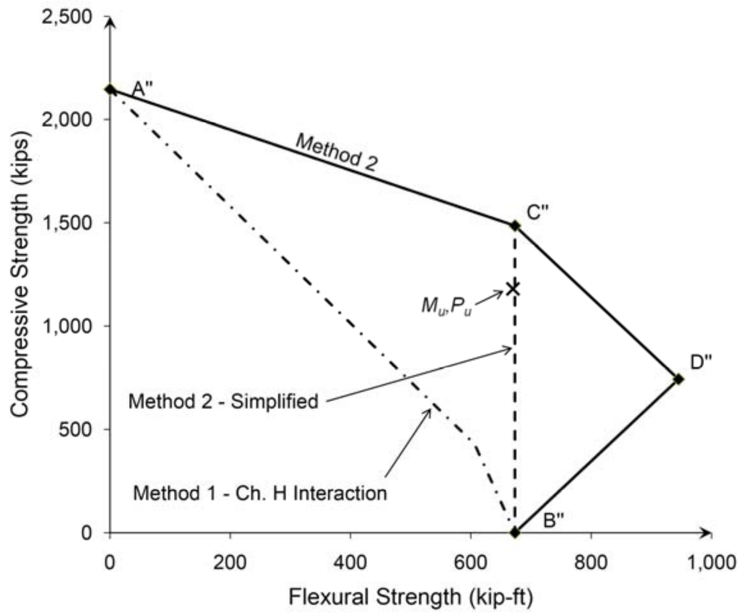


Fig. I.11-6. Comparison of interaction methods (LRFD).

From Figure I.11-6, the conservative nature of the Chapter H interaction equations can be seen. Method 2 provides the highest available strength; however, the Method 2—Simplified procedure also provides a good representation of the design curve. The procedure in Figure I-1 for calculating the flexural strength of Point C″ first requires the calculation of the flexural strength for Point D″. The design effort required for the Method 2—Simplified procedure, which utilizes Point C″, is therefore not greatly reduced from Method 2.

Available Shear Strength

According to AISC *Specification* Section I4.1, there are three acceptable options for determining the available shear strength of an encased composite member:

- Option 1—Available shear strength of the steel section alone in accordance with AISC *Specification* Chapter G.
- Option 2—Available shear strength of the reinforced concrete portion alone per ACI 318.
- Option 3—Available shear strength of the steel section in addition to the reinforcing steel ignoring the contribution of the concrete.

Option 1—Available Shear Strength of Steel Section

A W10×45 member meets the criteria of AISC *Specification* Section G2.1(a) according to the User Note at the end of the section. As demonstrated in Design Example I.9, No. 3 ties at 12 in. on center as illustrated in Figure I.11-1 satisfy the minimum detailing requirements of the *Specification*. The nominal shear strength may therefore be determined as:

$$C_v = 1.0 \qquad \text{(Spec. Eq. G2-2)}$$

$$A_w = dt_w$$
$$= (10.1 \text{ in.})(0.350 \text{ in.})$$
$$= 3.54 \text{ in.}^2$$

$$V_n = 0.6F_yA_wC_v \qquad \text{(Spec. Eq. G2-1)}$$
$$= 0.6(50 \text{ ksi})(3.54 \text{ in.}^2)(1.0)$$
$$= 106 \text{ kips}$$

The available shear strength of the steel section is:

LRFD	ASD
$V_u = 95.7$ kips $\phi_v = 1.0$ $\phi_v V_n \geq V_u$ $\phi_v V_n = 1.0(106 \text{ kips})$ $= 106 \text{ kips} > 95.7 \text{ kips}$ **o.k.**	$V_a = 57.4$ kips $\Omega_v = 1.50$ $V_n / \Omega_v \geq V_a$ $V_n / \Omega_v = \dfrac{106 \text{ kips}}{1.50}$ $= 70.7 \text{ kips} > 57.4 \text{ kips}$ **o.k.**

Option 2—Available Shear Strength of the Reinforced Concrete (Concrete and Transverse Steel Reinforcement)

The available shear strength of the steel section alone has been shown to be sufficient; however, the amount of transverse reinforcement required for shear resistance in accordance with AISC *Specification* Section I4.1(b) will be determined for demonstration purposes.

Tie Requirements for Shear Resistance

The nominal concrete shear strength is:

$$V_c = 2\lambda\sqrt{f'_c}b_wd \qquad \text{(ACI 318 Eq. 11-3)}$$

where

$\lambda = 1.0$ for normal weight concrete from ACI 318 Section 8.6.1

$b_w = h_1$

d = distance from extreme compression fiber to centroid of longitudinal tension reinforcement

$= 24 \text{ in.} - 2\tfrac{1}{2} \text{ in.}$

$= 21.5$ in.

$$V_c = 2(1.0)\sqrt{5{,}000 \text{ psi}}(24 \text{ in.})(21.5 \text{ in.})\left(\frac{1 \text{ kip}}{1{,}000 \text{ lb}}\right)$$
$$= 73.0 \text{ kips}$$

The tie requirements for shear resistance are determined from ACI 318 Chapter 11 and AISC *Specification* Section I4.1(b), as follows:

LRFD	ASD
$V_u = 95.7$ kips $\phi_v = 0.75$ $\dfrac{A_v}{s} = \dfrac{V_u - \phi_v V_c}{\phi_v f_{yr} d}$ $= \dfrac{95.7 \text{ kips} - 0.75(73.0 \text{ kips})}{0.75(60 \text{ ksi})(21.5 \text{ in.})}$ $= 0.0423$ in. Using two legs of No. 3 ties with $A_v = 0.11$ in.^{2} from ACI 318 Appendix E: $\dfrac{2(0.11 \text{ in.}^2)}{s} = 0.0423$ in. $s = 5.20$ in. Using two legs of the No. 4 ties with $A_v = 0.20$ in.^{2}: $\dfrac{2(0.20 \text{ in.}^2)}{s} = 0.0423$ in. $s = 9.46$ in. From ACI 318 Section 11.4.5.1, the maximum spacing is: $s_{max} = \dfrac{d}{2}$ $= \dfrac{21.5 \text{ in.}}{2}$ $= 10.8$ in. **Use No. 3 ties at 5 in. o.c. or No. 4 ties at 9 in. o.c.**	$V_a = 57.4$ kips $\Omega_v = 2.00$ $\dfrac{A_v}{s} = \dfrac{V_a - (V_c/\Omega_v)}{f_{yr} d/\Omega_v}$ $= \dfrac{57.4 \text{ kips} - \left(\dfrac{73.0 \text{ kips}}{2.00}\right)}{\dfrac{(60 \text{ ksi})(21.5 \text{ in.})}{2.00}}$ $= 0.0324$ in. Using two legs of No. 3 ties with $A_v = 0.11$ in.^{2} from ACI 318 Appendix E: $\dfrac{2(0.11 \text{ in.}^2)}{s} = 0.0324$ in. $s = 6.79$ in. Using two legs of the No. 4 ties with $A_v = 0.20$ in.^{2}: $\dfrac{2(0.20 \text{ in.}^2)}{s} = 0.0324$ in. $s = 12.3$ in. From ACI 318 Section 11.4.5.1, the maximum spacing is: $s_{max} = \dfrac{d}{2}$ $= \dfrac{21.5 \text{ in.}}{2}$ $= 10.8$ in. **Use No. 3 ties at 6 in. o.c. or No. 4 ties at 10 in. o.c.**

Minimum Reinforcing Limits

Check that the minimum shear reinforcement is provided as required by ACI 318, Section 11.4.6.3.

$$A_{v,min} = 0.75\sqrt{f_c'}\left(\frac{b_w s}{f_{yr}}\right) \geq \frac{50 b_w s}{f_{yr}} \qquad \text{(from ACI 318 Eq. 11-13)}$$

$$\frac{A_{v,min}}{s} = \frac{0.75\sqrt{5{,}000 \text{ psi}}(24 \text{ in.})}{60{,}000 \text{ psi}} \geq \frac{50(24 \text{ in.})}{60{,}000 \text{ psi}}$$
$$= 0.0212 \geq 0.0200$$

LRFD	ASD
$\dfrac{A_v}{s} = 0.0423 \text{ in.} > 0.0212$ **o.k.**	$\dfrac{A_v}{s} = 0.0324 \text{ in.} > 0.0212$ **o.k.**

Maximum Reinforcing Limits

From ACI 318 Section 11.4.5.3, maximum stirrup spacing is reduced to d/4 if $V_s \geq 4\sqrt{f_c'}b_w d$. If No. 4 ties at 9 in. on center are selected:

$$V_s = \frac{A_v f_{yr} d}{s} \qquad \text{(ACI 318 Eq. 11-15)}$$
$$= \frac{2\left(0.20 \text{ in.}^2\right)\left(60 \text{ ksi}\right)\left(21.5 \text{ in.}\right)}{9 \text{ in.}}$$
$$= 57.3 \text{ kips}$$
$$V_{s,max} = 4\sqrt{f_c'}b_w d$$
$$= 4\sqrt{5{,}000 \text{ psi}}\left(24 \text{ in.}\right)\left(21.5 \text{ in.}\right)\left(\frac{1 \text{ kip}}{1{,}000 \text{ lb}}\right)$$
$$= 146 \text{ kips} > 57.3 \text{ kips}$$

Therefore, the stirrup spacing is acceptable.

Option 3—Determine Available Shear Strength of the Steel Section Plus Reinforcing Steel

The third procedure combines the shear strength of the reinforcing steel with that of the encased steel section, ignoring the contribution of the concrete. AISC *Specification* Section I4.1(c) provides a combined resistance and safety factor for this procedure. Note that the combined resistance and safety factor takes precedence over the factors in Chapter G used for the encased steel section alone in Option 1. The amount of transverse reinforcement required for shear resistance is determined as follows:

Tie Requirements for Shear Resistance

The nominal shear strength of the encased steel section was previously determined to be:

$$V_{n,steel} = 106 \text{ kips}$$

The tie requirements for shear resistance are determined from ACI 318 Chapter 11 and AISC *Specification* Section I4.1(c), as follows:

LRFD	ASD
$V_u = 95.7$ kips as $\phi_v = 0.75$ $\frac{A_v}{s} = \frac{V_u - \phi_v V_{n,steel}}{\phi_v f_{yr} d}$ $= \frac{95.7 \text{ kips} - 0.75\left(106 \text{ kips}\right)}{0.75\left(60 \text{ ksi}\right)\left(21.5 \text{ in.}\right)}$ $= 0.0167$ in.	$V_a = 57.4$ kips $\Omega_v = 2.00$ $\frac{A_v}{s} = \frac{V_a - \left(V_{n,steel}/\Omega_v\right)}{f_{yr} d/\Omega_v}$ $= \frac{57.4 \text{ kips} - \left(106 \text{ kips}/2.00\right)}{\left[\frac{\left(60 \text{ ksi}\right)\left(21.5 \text{ in.}\right)}{2.00}\right]}$ $= 0.00682$ in.

As determined in Option 2, the minimum value of $A_v/s = 0.0212$, and the maximum tie spacing for shear resistance is 10.8 in. Using two legs of No. 3 ties for A_v:

$$\frac{2\left(0.11\text{ in.}^2\right)}{s} = 0.0212\text{ in.}$$
$$s = 10.4\text{ in.} < s_{max} = 10.8\text{ in.}$$

Use No. 3 ties at 10 in. o.c.

Summary and Comparison of Available Shear Strength Calculations

The use of the steel section alone is the most expedient method for calculating available shear strength and allows the use of a tie spacing which may be greater than that required for shear resistance by ACI 318. Where the strength of the steel section alone is not adequate, Option 3 will generally result in reduced tie reinforcement requirements as compared to Option 2.

Force Allocation and Load Transfer

Load transfer calculations should be performed in accordance with AISC *Specification* Section I6. The specific application of the load transfer provisions is dependent upon the configuration and detailing of the connecting elements. Expanded treatment of the application of load transfer provisions for encased composite members is provided in Design Example I.8 and AISC Design Guide 6.

EXAMPLE I.12 STEEL ANCHORS IN COMPOSITE COMPONENTS

Given:

Select an appropriate ¾-in.-diameter, Type B steel headed stud anchor to resist the dead and live loads indicated in Figure I.12-1. The anchor is part of a composite system that may be designed using the steel anchor in composite components provisions of AISC *Specification* Section I8.3.

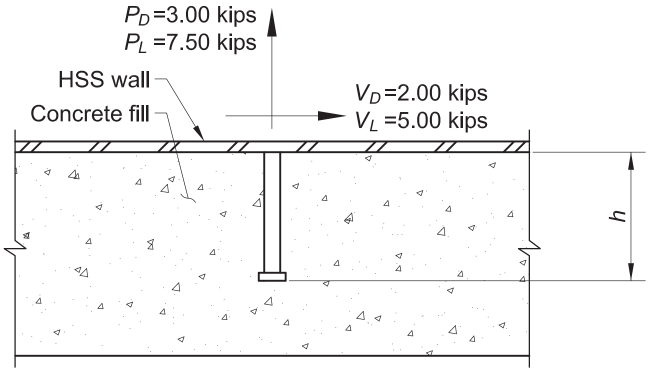


Fig. I.12-1. Steel headed stud anchor and applied loading.

The steel headed stud anchor is encased by normal weight (145 lb/ft^3) reinforced concrete having a specified concrete compressive strength, $f_c' = 5$ ksi. In accordance with AWS D1.1, steel headed stud anchors shall be made from material conforming to the requirements of ASTM A108. From AISC *Manual* Table 2-6, the specified minimum tensile stress, F_u, for ASTM A108 material is 65 ksi.

The anchor is located away from edges such that concrete breakout in shear is not a viable limit state, and the nearest anchor is located 24 in. away. The concrete is considered to be uncracked.

Solution:

Minimum Anchor Length

AISC *Specification* Section I8.3 provides minimum length to shank diameter ratios for anchors subjected to shear, tension, and interaction of shear and tension in both normal weight and lightweight concrete. These ratios are also summarized in the User Note provided within Section I8.3. For normal weight concrete subject to shear and tension, $h / d \geq 8$, thus:

$$h \geq 8d$$
$$\geq 8(\text{¾ in.})$$
$$\geq 6.00\text{ in.}$$

This length is measured from the base of the steel headed stud anchor to the top of the head after installation. From anchor manufacturer's data, a standard stock length of 6³⁄16 in. is selected. Using a ³⁄16-in. length reduction to account for burn off during installation yields a final installed length of 6.00 in.

$$6.00\text{ in.} = 6.00\text{ in.} \quad \textbf{o.k.}$$

Select a ¾-in.-diameter × 6³⁄16-in.-long headed stud anchor.

Required Shear and Tensile Strength

From Chapter 2 of ASCE/SEI 7, the required shear and tensile strengths are:

LRFD	ASD
Governing Load Combination for interaction: $= 1.2D + 1.6L$ $Q_{uv} = 1.2(2.00 \text{ kips}) + 1.6(5.00 \text{ kips})$ $= 10.4 \text{ kips (shear)}$ $Q_{ut} = 1.2(3.00 \text{ kips}) + 1.6(7.50 \text{ kips})$ $= 15.6 \text{ kips (tension)}$	Governing Load Combination for interaction: $= D + L$ $Q_{av} = 2.00 \text{ kips} + 5.00 \text{ kips}$ $= 7.00 \text{ kips (shear)}$ $Q_{at} = 3.00 \text{ kips} + 7.50 \text{ kips}$ $= 10.5 \text{ kips (tension)}$

Available Shear Strength

Per the problem statement, concrete breakout is not considered to be an applicable limit state. AISC Equation I8-3 may therefore be used to determine the available shear strength of the steel headed stud anchor as follows:

$$Q_{nv} = F_u A_{sa} \qquad \text{(Spec. Eq. I8-3)}$$

where

$$A_{sa} = \text{cross-sectional area of steel headed stud anchor}$$
$$= \frac{\pi(\text{3/4 in.})^2}{4}$$
$$= 0.442 \text{ in.}^2$$

$$Q_{nv} = (65 \text{ ksi})(0.442 \text{ in.}^2)$$
$$= 28.7 \text{ kips}$$

LRFD	ASD
$\phi_v = 0.65$ $\phi_v Q_{nv} = 0.65(28.7 \text{ kips})$ $= 18.7 \text{ kips}$	$\Omega_v = 2.31$ $Q_{nv} / \Omega_v = \frac{28.7 \text{ kips}}{2.31}$ $= 12.4 \text{ kips}$

Alternately, available shear strengths can be selected directly from Table I.12-1 located at the end of this example.

Available Tensile Strength

The nominal tensile strength of a steel headed stud anchor is determined using AISC *Specification* Equation I8-4 provided the edge and spacing limitations of AISC *Specification* Section I8.3b are met as follows:

(1) Minimum distance from centerline of anchor to free edge: $1.5h = 1.5(6.00 \text{ in.}) = 9.00 \text{ in.}$

There are no free edges, therefore this limitation does not apply.

(2) Minimum distance between centerlines of adjacent anchors: $3h = 3(6.00 \text{ in.}) = 18.0 \text{ in.}$

$18.0 \text{ in.} < 24 \text{ in.}$ **o.k.**

Return to Table of Contents

Equation I8-4 may therefore be used as follows:

$$Q_{nt} = F_u A_{sa} \qquad (Spec.\ Eq.\ I8\text{-}4)$$

$$Q_{nt} = (65\text{ ksi})(0.442\text{ in.}^2)$$
$$= 28.7\text{ kips}$$

LRFD	ASD
$\phi_t = 0.75$ $\phi_t Q_{nt} = 0.75(28.7\text{ kips})$ $= 21.5\text{ kips}$	$\Omega_t = 2.00$ $\dfrac{Q_{nt}}{\Omega_t} = \dfrac{28.7\text{ kips}}{2.00}$ $= 14.4\text{ kips}$

Alternately, available tension strengths can be selected directly from Table I.12-1 located at the end of this example.

Interaction of Shear and Tension

The detailing limits on edge distances and spacing imposed by AISC *Specification* Section I8.3c for shear and tension interaction are the same as those previously reviewed separately for tension and shear alone. Tension and shear interaction is checked using *Specification* Equation I8-5 which can be written in terms of LRFD and ASD design as follows:

LRFD	ASD
$\left[\left(\dfrac{Q_{ut}}{\phi_t Q_{nt}}\right)^{5/3} + \left(\dfrac{Q_{uv}}{\phi_v Q_{nv}}\right)^{5/3}\right] \le 1.0$ $\left[\left(\dfrac{15.6\text{ kips}}{21.5\text{ kips}}\right)^{5/3} + \left(\dfrac{10.4\text{ kips}}{18.7\text{ kips}}\right)^{5/3}\right] = 0.96$ $0.96 < 1.0$ **o.k.**	$\left[\left(\dfrac{Q_{at}}{Q_{nt}/\Omega_t}\right)^{5/3} + \left(\dfrac{Q_{av}}{Q_{nv}/\Omega_v}\right)^{5/3}\right] \le 1.0$ $\left[\left(\dfrac{10.5\text{ kips}}{14.4\text{ kips}}\right)^{5/3} + \left(\dfrac{7.00\text{ kips}}{12.4\text{ kips}}\right)^{5/3}\right] = 0.98$ $0.98 < 1.0$ **o.k.**

Thus, a ¾-in.-diameter × 6³⁄₁₆-in.-long headed stud anchor is adequate for the applied loads.

Limits of Application

The application of the steel anchors in composite component provisions have strict limitations as summarized in the User Note provided at the beginning of AISC *Specification* Section I8.3. These provisions do not apply to typical composite beam designs nor do they apply to hybrid construction where the steel and concrete do not resist loads together via composite action such as in embed plates. This design example is intended solely to illustrate the calculations associated with an isolated anchor that is part of an applicable composite system.

Available Strength Table

Table I.12-1 provides available shear and tension strengths for standard Type B steel headed stud anchors conforming to the requirements of AWS D1.1 for use in composite components.

Table I.12-1. Steel Headed Stud Anchor Available Strengths

Anchor Shank Diameter	A_{sa}	Q_{nv}/Ω_v	$\phi_v Q_{nv}$	Q_{nt}/Ω_t	$\phi_t Q_{nt}$
		kips	kips	kips	kips
in.	in.²	ASD	LRFD	ASD	LRFD
½	0.196	5.52	8.30	6.38	9.57
⅝	0.307	8.63	13.0	9.97	15.0
¾	0.442	12.4	18.7	14.4	21.5
⅞	0.601	16.9	25.4	N/A[a]	N/A[a]
1	0.785	22.1	33.2	25.5	38.3
ASD	LRFD	[a] ⅞-in.-diameter anchors conforming to AWS D1.1 Figure 7.1 do not meet the minimum head-to-shank diameter ratio of 1.6 as required for tensile resistance per AISC *Specification* Section I8.3.			
$\Omega_v = 2.31$ $\Omega_t = 2.00$	$\phi_v = 0.65$ $\phi_t = 0.75$				

CHAPTER I DESIGN EXAMPLE REFERENCES

ASCE (2002), *Design Loads on Structures During Construction*, SEI/ASCE 37-02, American Society of Civil Engineers, Reston, VA.

Griffis, L.G. (1992), *Load and Resistance Factor Design of W-Shapes*, Design Guide 6, AISC, Chicago, IL.

ICC (2009), *International Building Code*, International Code Council, Falls Church, VA.

Leon, R.T. and Hajjar, J.F. (2008), "Limit State Response of Composite Columns and Beam-Columns Part 2: Application of Design Provisions for the 2005 AISC Specification," *Engineering Journal*, AISC, Vol. 45, No. 1, 1st Quarter, pp. 21–46.

Murray, T.M., Allen, D.E. and Ungar, E.E. (1997), *Floor Vibrations Due to Human Activity*, Design Guide 11, AISC, Chicago, IL.

Park, R. and Gamble, W.L. (2000), *Reinforced Concrete Slabs,* 2nd Ed., John Wiley & Sons, New York, NY.

SDI (2006), *Standard for Composite Steel Floor Deck*, ANSI/SDI C1.0-2006, Fox River Grove, IL.

West, M.A. and Fisher, J.M. (2003), *Serviceability Design Consideration for Steel Buildings*, Design Guide 3, 2nd Ed., AISC, Chicago, IL.

Young, W.C. and Budynas, R.C. (2002), *Roark's Formulas for Stress and Strain,* 7th Ed., McGraw-Hill, New York, NY.

Chapter J
Design of Connections

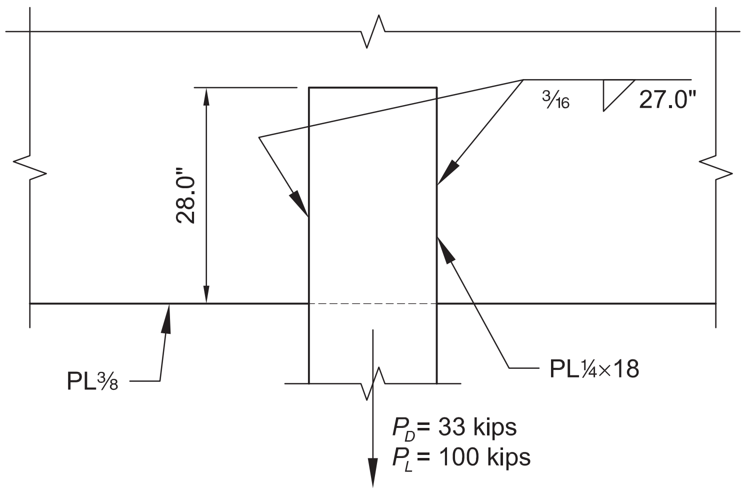
Chapter J of the AISC *Specification* addresses the design and review of connections. The chapter's primary focus is the design of welded and bolted connections. Design requirements for fillers, splices, column bases, concentrated forces, anchors rods and other threaded parts are also covered. Special requirements for connections subject to fatigue are not covered in this chapter.

EXAMPLE J.1 FILLET WELD IN LONGITUDINAL SHEAR

Given:

A ¼-in. × 18-in. wide plate is fillet welded to a ⅜-in. plate. The plates are ASTM A572 Grade 50 and have been properly sized. Use 70-ksi electrodes. Note that the plates would normally be specified as ASTM A36, but F_y = 50 ksi plate has been used here to demonstrate the requirements for long welds.

Verify the welds for the loads shown.



Solution:

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
P_u = 1.2(33.0 kips) + 1.6(100 kips) = 200 kips	P_a = 33.0 kips + 100 kips = 133 kips

Maximum and Minimum Weld Size

Because the thickness of the overlapping plate is ¼ in., the maximum fillet weld size that can be used without special notation per AISC *Specification* Section J2.2b, is a 3⁄16-in. fillet weld. A 3⁄16-in. fillet weld can be deposited in the flat or horizontal position in a single pass (true up to 5⁄16-in.).

From AISC *Specification* Table J2.4, the minimum size of fillet weld, based on a material thickness of ¼ in. is ⅛ in.

Length of Weld Required

The nominal weld strength per inch of 3⁄16-in. weld, determined from AISC *Specification* Section J2.4(a) is:

$$R_n = F_{nw}A_{we} \qquad (Spec.\ \text{Eq. J2-4})$$
$$= (0.60\,F_{EXX})(A_{we})$$
$$= 0.60(70\text{ ksi})\left(\tfrac{3}{16}\text{ in.}/\sqrt{2}\right)$$
$$= 5.57\text{ kips/in.}$$

LRFD	ASD
$\dfrac{P_u}{\phi R_n} = \dfrac{200\text{ kips}}{0.75\left(5.57\text{ kips/in.}\right)}$ $= 47.9$ in. or 24 in. of weld on each side	$\dfrac{P_a\Omega}{R_n} = \dfrac{133\text{ kips}\left(2.00\right)}{5.57\text{ kips/in.}}$ $= 47.8$ in. or 24 in. of weld on each side

From AISC *Specification* Section J2.2b, for longitudinal fillet welds used alone in end connections of flat-bar tension members, the length of each fillet weld shall be not less than the perpendicular distance between them.

$24\text{ in.} \geq 18\text{ in.}$ **o.k.**

From AISC *Specification* Section J2.2b, check the weld length to weld size ratio, because this is an end loaded fillet weld.

$$\frac{L}{w} = \frac{24\text{ in.}}{\tfrac{3}{16}\text{ in.}}$$

$= 128 > 100$. therefore, AISC *Specification* Equation J2-1 must be applied, and the length of weld increased, because the resulting β will reduce the available strength below the required strength.

Try a weld length of 27 in.

The new length to weld size ratio is:

$$\frac{27.0\text{ in.}}{\tfrac{3}{16}\text{ in.}} = 144$$

For this ratio:

$$\beta = 1.2 - 0.002(l/w) \leq 1.0 \qquad (\textit{Spec}.\text{ Eq. J2-1})$$
$$= 1.2 - 0.002(144)$$
$$= 0.912$$

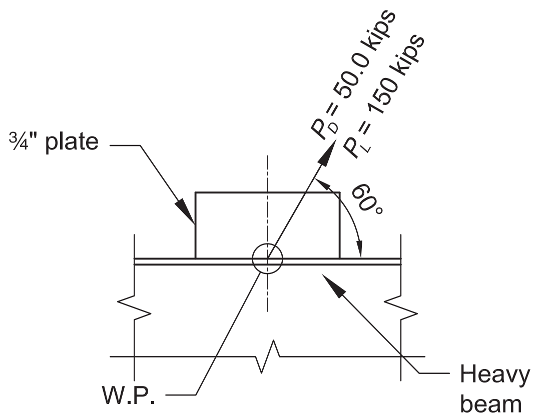
Recheck the weld at its reduced strength.

LRFD	ASD
$\phi R_n = \left(0.912\right)\left(0.75\right)\left(5.57\text{ kips/in.}\right)\left(54.0\text{ in.}\right)$ $= 206\text{ kips} > P_u = 200\text{ kips}$ **o.k.** Therefore, use 27 in. of weld on each side.	$\dfrac{R_n}{\Omega} = \dfrac{(0.912)\left(5.57\text{ kips/in.}\right)\left(54.0\text{ in.}\right)}{2.00}$ $= 137\text{ kips} > P_a = 133\text{ kips}$ **o.k.** Therefore, use 27 in. of weld on each side.

EXAMPLE J.2 FILLET WELD LOADED AT AN ANGLE

Given:

Design a fillet weld at the edge of a gusset plate to carry a force of 50.0 kips due to dead load and 150 kips due to live load, at an angle of 60° relative to the weld. Assume the beam and the gusset plate thickness and length have been properly sized. Use a 70-ksi electrode.



Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(50.0\text{ kips}) + 1.6(150\text{ kips})$ $= 300\text{ kips}$	$P_a = 50.0\text{ kips} + 150\text{ kips}$ $= 200\text{ kips}$

Assume a 5⁄16-in. fillet weld is used on each side of the plate.

Note that from AISC *Specification* Table J2.4, the minimum size of fillet weld, based on a material thickness of ¾ in. is ¼ in.

Available Shear Strength of the Fillet Weld Per Inch of Length

From AISC *Specification* Section J2.4(a), the nominal strength of the fillet weld is determined as follows:

$$A_{we} = \frac{5/16\text{ in.}}{\sqrt{2}}$$
$$= 0.221\text{ in.}$$

$$F_{nw} = 0.60F_{EXX}\left(1.0 + 0.5\sin^{1.5}\theta\right) \qquad (Spec.\text{ Eq. J2-5})$$
$$= 0.60(70\text{ ksi})\left(1.0 + 0.5\sin^{1.5}60^\circ\right)$$
$$= 58.9\text{ ksi}$$

$$R_n = F_{nw}A_{we} \qquad (Spec.\text{ Eq. J2-4})$$
$$= 58.9\text{ ksi}(0.221\text{ in.})$$
$$= 13.0\text{ kip/in.}$$

From AISC *Specification* Section J2.4(a), the available shear strength per inch of weld length is:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = 0.75(13.0 \text{ kip/in.})$ $= 9.75 \text{ kip/in.}$	$\dfrac{R_n}{\Omega} = \dfrac{13.0 \text{ kip/in.}}{2.00}$ $= 6.50 \text{ kip/in.}$
For 2 sides:	For 2 sides:
$\phi R_n = 2(0.75)(13.0 \text{ kip/in.})$ $= 19.5 \text{ kip/in.}$	$\dfrac{R_n}{\Omega} = \dfrac{2(13.0 \text{ kip/in.})}{2.00}$ $= 13.0 \text{ kip/in.}$

Required Length of Weld

LRFD	ASD
$l = \dfrac{300 \text{ kips}}{19.5 \text{ kip/in.}}$ $= 15.4 \text{ in.}$	$l = \dfrac{200 \text{ kips}}{13.0 \text{ kip/in.}}$ $= 15.4 \text{ in.}$
Use 16 in. on each side of the plate.	Use 16 in. on each side of the plate.

EXAMPLE J.3 COMBINED TENSION AND SHEAR IN BEARING TYPE CONNECTIONS

Given:

A ¾-in.-diameter ASTM A325-N bolt is subjected to a tension force of 3.5 kips due to dead load and 12 kips due to live load, and a shear force of 1.33 kips due to dead load and 4 kips due to live load. Check the combined stresses according to AISC *Specification* Equations J3-3a and J3-3b.

Solution:

From Chapter 2 of ASCE/SEI 7, the required tensile and shear strengths are:

LRFD	ASD
Tension: $T_u = 1.2(3.50\text{ kips}) + 1.6(12.0\text{ kips})$ $= 23.4\text{ kips}$ Shear: $V_u = 1.2(1.33\text{ kips}) + 1.6(4.00\text{ kips})$ $= 8.00\text{ kips}$	Tension: $T_a = 3.50\text{ kips} + 12.0\text{ kips}$ $= 15.5\text{ kips}$ Shear: $V_a = 1.33\text{ kips} + 4.00\text{ kips}$ $= 5.33\text{ kips}$

Available Tensile Strength

When a bolt is subject to combined tension and shear, the available tensile strength is determined according to the limit states of tension and shear rupture, from AISC *Specification* Section J3.7 as follows.

From AISC *Specification* Table J3.2,

$F_{nt} = 90\text{ ksi}, F_{nv} = 54\text{ ksi}$

From AISC *Manual* Table 7-1, for a ¾-in.-diameter bolt,

$A_b = 0.442\text{ in.}^2$

The available shear stress is determined as follows and must equal or exceed the required shear stress.

LRFD	ASD
$\phi = 0.75$ $\phi F_{nv} = 0.75(54\text{ ksi})$ $= 40.5\text{ ksi}$ $f_{rv} = \dfrac{V_u}{A_b}$ $= \dfrac{8.00\text{ kips}}{0.442\text{ in.}^2}$ $= 18.1\text{ ksi} \le 40.5\text{ ksi}$	$\Omega = 2.00$ $\dfrac{F_{nv}}{\Omega} = \dfrac{54\text{ ksi}}{2.00}$ $= 27.0$ $f_{rv} = \dfrac{V_a}{A_b}$ $= \dfrac{5.33\text{ kips}}{0.442\text{ in.}^2}$ $= 12.1\text{ ksi} \le 27.0\text{ ksi}$ **o.k.**

The available tensile strength of a bolt subject to combined tension and shear is as follows:

LRFD	ASD
$F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3.3a) $= 1.3(90 \text{ ksi}) - \dfrac{90 \text{ ksi}}{0.75(54 \text{ ksi})}(18.1 \text{ ksi})$ $= 76.8 \text{ ksi} \le 90 \text{ ksi}$ $R_n = F'_{nt} A_b$ (*Spec*. Eq. J3-2) $= 76.8 \text{ ksi}(0.442 \text{ in.}^2)$ $= 33.9 \text{ kips}$ For combined tension and shear, $\phi = 0.75$ from AISC *Specification* Section J3.7 Design tensile strength: $\phi R_n = 0.75(33.9 \text{ kips})$ $= 25.4 \text{ kips} > 23.4 \text{ kips}$ **o.k.**	$F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3.3b) $= 1.3(90 \text{ ksi}) - \dfrac{2.00(90 \text{ ksi})}{54 \text{ ksi}}(12.1 \text{ ksi})$ $= 76.7 \text{ ksi} \le 90 \text{ ksi}$ $R_n = F'_{nt} A_b$ (*Spec*. Eq. J3-2) $= 76.7 \text{ ksi}(0.442 \text{ in.}^2)$ $= 33.9 \text{ kips}$ For combined tension and shear, $\Omega = 2.00$ from AISC *Specification* Section J3.7 Allowable tensile strength: $\dfrac{R_n}{\Omega} = \dfrac{33.9 \text{ kips}}{2.00}$ $= 17.0 \text{ kips} > 15.5 \text{ kips}$ **o.k.**

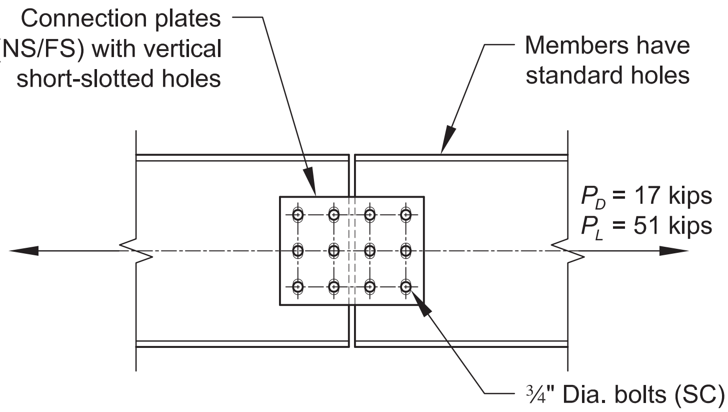
Return to Table of Contents

EXAMPLE J.4A SLIP-CRITICAL CONNECTION WITH SHORT-SLOTTED HOLES

Slip-critical connections shall be designed to prevent slip and for the limit states of bearing-type connections.

Given:

Select the number of ¾-in.-diameter ASTM A325 slip-critical bolts with a Class A faying surface that are required to support the loads shown when the connection plates have short slots transverse to the load and no fillers are provided. Select the number of bolts required for slip resistance only.



Solution:

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$P_u = 1.2(17.0 \text{ kips}) + 1.6(51.0 \text{ kips})$ $= 102 \text{ kips}$	$P_a = 17.0 \text{ kips} + 51.0 \text{ kips}$ $= 68.0 \text{ kips}$

From AISC *Specification* Section J3.8(a), the available slip resistance for the limit state of slip for standard size and short-slotted holes perpendicular to the direction of the load is determined as follows:

$\phi = 1.00 \qquad \Omega = 1.50$

μ = 0.30 for Class A surface
$D_u = 1.13$
h_f = 1.0, factor for fillers, assuming no more than one filler
T_b = 28 kips, from AISC *Specification* Table J3.1
n_s = 2, number of slip planes

$$R_n = \mu D_u h_f T_b n_s \qquad \text{(Spec. Eq. J3-4)}$$
$$= 0.30(1.13)(1.0)(28 \text{ kips})(2)$$
$$= 19.0 \text{ kips/bolt}$$

The available slip resistance is:

LRFD	ASD
$\phi R_n = 1.00(19.0 \text{ kips/bolt})$ $= 19.0 \text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = \dfrac{19.0 \text{ kips/bolt}}{1.50}$ $= 12.7 \text{ kips/bolt}$

Required Number of Bolts

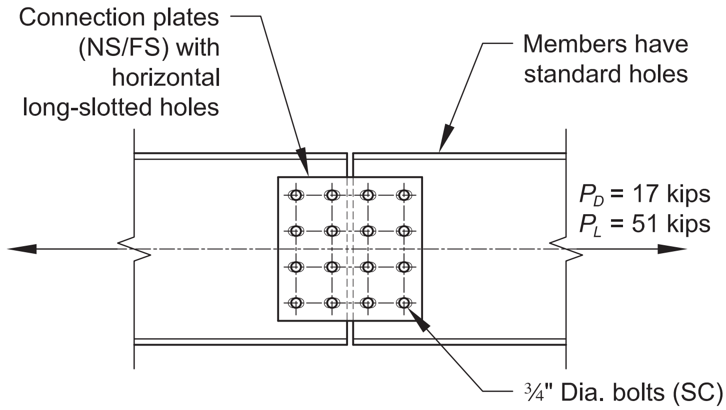
LRFD	ASD
$n_b = \dfrac{P_u}{\phi R_n}$ $= \dfrac{102 \text{ kips}}{19.0 \text{ kips/bolt}}$ $= 5.37 \text{ bolts}$ Use 6 bolts	$n_b = \dfrac{P_a}{\left(\dfrac{R_n}{\Omega}\right)}$ $= \dfrac{68.0 \text{ kips}}{(12.7 \text{ kips/bolt})}$ $= 5.37 \text{ bolts}$ Use 6 bolts

Note: To complete the design of this connection, the limit states of bolt shear, bolt bearing, tensile yielding, tensile rupture, and block shear rupture must be determined.

EXAMPLE J.4B SLIP-CRITICAL CONNECTION WITH LONG-SLOTTED HOLES

Given:

Repeat Example J.4A with the same loads, but assuming that the connected pieces have long-slotted holes in the direction of the load.



Solution:

The required strength from Example J.4A is:

LRFD	ASD
P_u = 102 kips	P_a = 68.0 kips

From AISC *Specification* Section J3.8(c), the available slip resistance for the limit state of slip for long-slotted holes is determined as follows:

$\phi = 0.70 \qquad \Omega = 2.14$

μ = 0.30 for Class A surface
D_u = 1.13
h_f = 1.0, factor for fillers, assuming no more than one filler
T_b = 28 kips, from AISC *Specification* Table J3.1
n_s = 2, number of slip planes

$$R_n = \mu D_u h_f T_b n_s \qquad \text{(Spec. Eq. J3-4)}$$
$$= 0.30(1.13)(1.0)(28 \text{ kips})(2)$$
$$= 19.0 \text{ kips/bolt}$$

The available slip resistance is:

LRFD	ASD
$\phi R_n = 0.70(19.0 \text{ kips/bolt})$ $= 13.3 \text{ kips/bolt}$	$\dfrac{R_n}{\Omega} = \dfrac{19.0 \text{ kips/bolt}}{2.14}$ $= 8.88 \text{ kips/bolt}$

Required Number of Bolts

LRFD	ASD
$n_b = \dfrac{P_u}{\phi R_n}$ $= \dfrac{102 \text{ kips}}{13.3 \text{ kips/bolt}}$ $= 7.67 \text{ bolts}$ Use 8 bolts	$n_b = \dfrac{P_a}{\left(\dfrac{R_n}{\Omega}\right)}$ $= \dfrac{68.0 \text{ kips}}{8.88 \text{ kips/bolt}}$ $= 7.66 \text{ bolts}$ Use 8 bolts

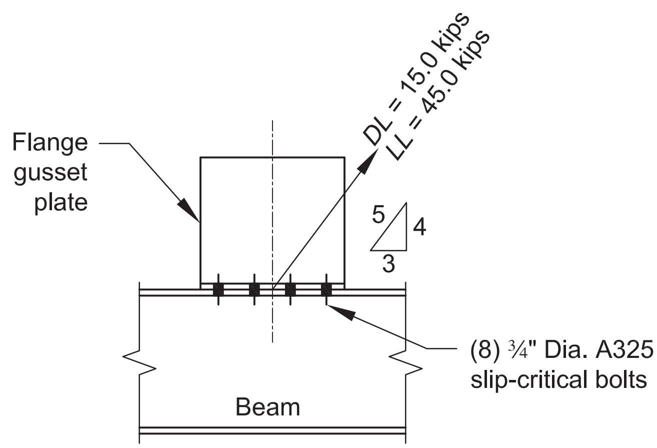
Note: To complete the design of this connection, the limit states of bolt shear, bolt bearing, tensile yielding, tensile rupture, and block shear rupture must be determined.

EXAMPLE J.5 COMBINED TENSION AND SHEAR IN A SLIP-CRITICAL CONNECTION

Because the pretension of a bolt in a slip-critical connection is used to create the clamping force that produces the shear strength of the connection, the available shear strength must be reduced for any load that produces tension in the connection.

Given:

The slip-critical bolt group shown as follows is subjected to tension and shear. Use ¾-in.-diameter ASTM A325 slip-critical Class A bolts in standard holes. This example shows the design for bolt slip resistance only, and assumes that the beams and plates are adequate to transmit the loads. Determine if the bolts are adequate.



Solution:

μ = 0.30 for Class A surface
D_u = 1.13
n_b = 8, number of bolts carrying the applied tension
h_f = 1.0, factor for fillers, assuming no more than one filler
T_b = 28 kips, from AISC *Specification* Table J3.1
n_s = 1, number of slip planes

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$P_u = 1.2(15.0\text{ kips})+1.6(45.0\text{ kips})$ $= 90.0\text{ kips}$ By geometry, $T_u = \frac{4}{5}(90.0\text{ kips}) = 72.0\text{ kips}$ $V_u = \frac{3}{5}(90.0\text{ kips}) = 54.0\text{ kips}$	$P_a = 15.0\text{ kips} + 45.0\text{ kips}$ $= 60.0\text{ kips}$ By geometry, $T_a = \frac{4}{5}(60.0\text{ kips}) = 48.0\text{ kips}$ $V_a = \frac{3}{5}(60.0\text{ kips}) = 36.0\text{ kips}$

Available Bolt Tensile Strength

The available tensile strength is determined from AISC *Specification* Section J3.6.

From AISC *Specification* Table J3.2 for Group A bolts, the nominal tensile strength in ksi is, $F_{nt} = 90$ ksi . From AISC *Manual* Table 7-1, $A_b = 0.442$ in.^{2}

$$A_b = \frac{\pi(\text{¾ in.})^2}{4}$$
$$= 0.442 \text{ in.}^2$$

The nominal tensile strength in kips is,

$$R_n = F_{nt} A_b \qquad \text{(from } Spec. \text{ Eq. J3-1)}$$
$$= 90 \text{ ksi}(0.442 \text{ in.}^2)$$
$$= 39.8 \text{ kips}$$

The available tensile strength is,

LRFD	ASD
$\phi R_n = 0.75\left(\frac{39.8 \text{ kips}}{\text{bolt}}\right) > \frac{72.0 \text{ kips}}{8 \text{ bolts}}$ $= 29.9 \text{ kips/bolt} > 9.00 \text{ kips/bolt}$ **o.k.**	$\frac{R_n}{\Omega} = \left(\frac{39.8 \text{ kips/bolt}}{2.00}\right) > \frac{48.0 \text{ kips}}{8 \text{ bolts}}$ $= 19.9 \text{ kips/bolt} > 6.00 \text{ kips/bolt}$ **o.k.**

Available Slip Resistance Per Bolt

The available slip resistance of one bolt is determined using AISC *Specification* Equation J3-4 and Section J3.8.

LRFD	ASD
Determine the available slip resistance $(T_u = 0)$ of a bolt. $\phi = 1.00$ $\phi R_n = \phi\mu D_u h_f T_b n_s$ $= 1.00(0.30)(1.13)(1.0)(28 \text{ kips})(1)$ $= 9.49 \text{ kips/bolt}$	Determine the available slip resistance $(T_a = 0)$ of a bolt. $\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{\mu D_u h_f T_b n_s}{\Omega}$ $= \frac{0.30(1.13)(1.0)(28 \text{ kips})(1)}{1.50}$ $= 6.33 \text{ kips/bolt}$

Available Slip Resistance of the Connection

Because the clip-critical connection is subject to combined tension and shear, the available slip resistance is multiplied by a reduction factor provided in AISC *Specification* Section J3.9.

LRFD	ASD
Slip-critical combined tension and shear coefficient: $k_{sc} = 1 - \frac{T_u}{D_u T_b n_b}$ (*Spec*. Eq. J3-5a) $= 1 - \frac{72.0 \text{ kips}}{1.13(28 \text{ kips})(8)}$ $= 0.716$ $\phi = 1.00$	Slip-critical combined tension and shear coefficient: $k_{sc} = 1 - \frac{1.5T_a}{D_u T_b n_b}$ (*Spec*. Eq. J3-5b) $= 1 - \frac{1.5(48.0 \text{ kips})}{1.13(28 \text{ kips})(8)}$ $= 0.716$ $\Omega = 1.50$

LRFD	ASD
$\phi R_n = \phi R_n k_s n_b$ $= 9.49 \text{ kips/bolt}(0.716)(8 \text{ bolts})$ $= 54.4 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{R_n}{\Omega} k_s n_b$ $= 6.33 \text{ kips/bolt}(0.716)(8 \text{ bolts})$ $= 36.3 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Note: The bolt group must still be checked for all applicable strength limit states for a bearing-type connection.

EXAMPLE J.6 BEARING STRENGTH OF A PIN IN A DRILLED HOLE

Given:

A 1-in.-diameter pin is placed in a drilled hole in a 1½-in. ASTM A36 plate. Determine the available bearing strength of the pinned connection, assuming the pin is stronger than the plate.

Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

The available bearing strength is determined from AISC *Specification* Section J7, as follows:

The projected bearing area is,

$$\begin{aligned} A_{pb} &= dt_p \\ &= 1.00 \text{ in.}(1.50 \text{ in.}) \\ &= 1.50 \text{ in.}^2 \end{aligned}$$

The nominal bearing strength is,

$$\begin{aligned} R_n &= 1.8F_yA_{pb} \\ &= 1.8(36 \text{ ksi})(1.50 \text{ in.}^2) \\ &= 97.2 \text{ kips} \end{aligned}$$ (*Spec*. Eq. J7-1)

The available bearing strength of the plate is:

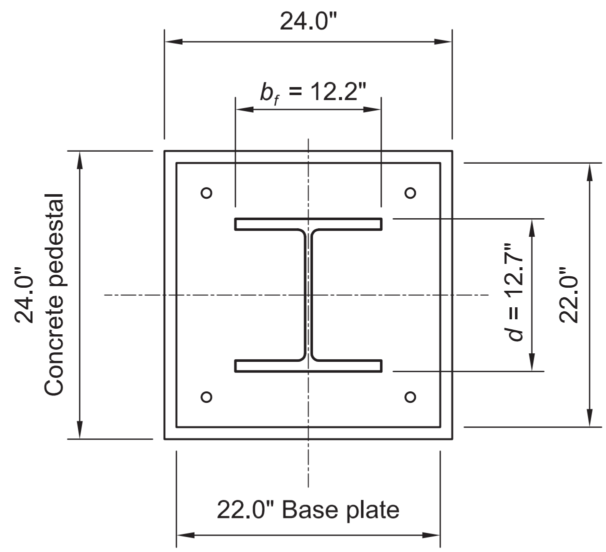
LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(97.2 \text{ kips})$ $= 72.9$ kips	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{97.2 \text{ kips}}{2.00}$ $= 48.6$ kips

EXAMPLE J.7 BASE PLATE BEARING ON CONCRETE

Given:

An ASTM A992 W12×96 column bears on a 24-in. × 24-in. concrete pedestal with f'_c = 3 ksi. The space between the base plate and the concrete pedestal has grout with f'_c = 4 ksi. Design the ASTM A36 base plate to support the following loads in axial compression:

P_D = 115 kips
P_L = 345 kips



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Base Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Column
W12×96
d = 12.7 in.
b_f = 12.2 in.
t_f = 0.900 in.
t_w = 0.550 in.

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
$P_u = 1.2(115 \text{ kips}) + 1.6(345 \text{ kips})$ $= 690 \text{ kips}$	$P_a = 115 \text{ kips} + 345 \text{ kips}$ $= 460 \text{ kips}$

Base Plate Dimensions

Determine the required base plate area from AISC *Specification* Section J8 assuming bearing on the full area of the concrete support.

LRFD	ASD
$\phi_c = 0.65$ $A_{1(req)} = \dfrac{P_u}{\phi_c 0.85 f_c'}$ $= \dfrac{690 \text{ kips}}{0.65(0.85)(3 \text{ ksi})}$ $= 416 \text{ in.}^2$	$\Omega_c = 2.31$ $A_{1(req)} = \dfrac{\Omega_c P_a}{0.85 f_c'}$ $= \dfrac{2.31(460 \text{ kips})}{0.85(3 \text{ ksi})}$ $= 417 \text{ in.}^2$

Note: The strength of the grout has conservatively been neglected, as its strength is greater than that of the concrete pedestal.

Try a 22.0 in. × 22.0 in. base plate.

Verify $N \geq d + 2(3.00 \text{ in.})$ and $B \geq b_f + 2(3.00 \text{ in.})$ for anchor rod pattern shown in diagram:

$$d + 2(3.00 \text{ in.}) = 12.7 \text{ in.} + 2(3.00 \text{ in.})$$
$$= 18.7 \text{ in.} < 22 \text{ in.} \quad \textbf{o.k.}$$

$$b_f + 2(3.00 \text{ in.}) = 12.2 \text{ in.} + 2(3.00 \text{ in.})$$
$$= 18.2 \text{ in.} < 22 \text{ in.} \quad \textbf{o.k.}$$

Base plate area:

$$A_1 = NB$$
$$= 22.0 \text{ in.}(22.0 \text{ in.})$$
$$= 484 \text{ in.}^2 > 417 \text{ in.}^2 \quad \textbf{o.k.}$$

Note: A square base plate with a square anchor rod pattern will be used to minimize the chance for field and shop problems.

Concrete Bearing Strength

Use AISC *Specification* Equation J8-2 because the base plate covers less than the full area of the concrete support.

Because the pedestal is square and the base plate is a c oncentrically located square, the full pedestal area is also the geometrically similar area. Therefore,

$$A_2 = 24.0 \text{ in.}(24.0 \text{ in.})$$
$$= 576 \text{ in.}^2$$

The available bearing strength is,

LRFD	ASD
$\phi_c = 0.65$ $\phi_c P_p = \phi_c 0.85 f'_c A_1 \sqrt{\dfrac{A_2}{A_1}} \le \phi_c 1.7 f'_c A_1$ $= 0.65(0.85)(3\text{ ksi})(484\text{ in.}^2)\sqrt{\dfrac{576\text{ in.}^2}{484\text{ in.}^2}}$ $\le 0.65(1.7)(3\text{ ksi})(484\text{ in.}^2)$ $= 875\text{ kips} \le 1{,}600\text{ kips, use 875 kips}$ 875 kips > 690 kips **o.k.**	$\Omega_c = 2.31$ $\dfrac{P_p}{\Omega_c} = \dfrac{0.85 f'_c A_1}{\Omega_c}\sqrt{\dfrac{A_2}{A_1}} \le \dfrac{1.7 f'_c A_1}{\Omega_c}$ $= \dfrac{0.85(3\text{ ksi})(484\text{ in.}^2)}{2.31}\sqrt{\dfrac{576\text{ in.}^2}{484\text{ in.}^2}}$ $\le \dfrac{1.7(3\text{ ksi})(484\text{ in.}^2)}{2.31}$ $= 583\text{ kips} \le 1{,}070\text{ kips, use 583 kips}$ 583 kips > 460 kips **o.k.**

Notes:

1. $A_2/A_1 \le 4$; therefore, the upper limit in AISC *Specification* Equation J8-2 does not control.
2. As the area of the base plate approaches the area of concrete, the modifying ratio, $\sqrt{A_2/A_1}$, approaches unity and AISC *Specification* Equation J8-2 converges to AISC *Specification* Equation J8-1.

Required Base Plate Thickness

The base plate thickness is determined in accordance with AISC *Manual* Part 14.

$$m = \frac{N - 0.95d}{2} \qquad \text{(\textit{Manual} Eq. 14-2)}$$
$$= \frac{22.0\text{ in.} - 0.95(12.7\text{ in.})}{2}$$
$$= 4.97\text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} \qquad \text{(\textit{Manual} Eq. 14-3)}$$
$$= \frac{22.0\text{ in.} - 0.8(12.2\text{ in.})}{2}$$
$$= 6.12\text{ in.}$$

$$n' = \frac{\sqrt{db_f}}{4} \qquad \text{(\textit{Manual} Eq. 14-4)}$$
$$= \frac{\sqrt{12.7\text{ in.}(12.2\text{ in.})}}{4}$$
$$= 3.11\text{ in.}$$

LRFD	ASD
$X=\left[\dfrac{4db_f}{(d+b_f)^2}\right]\dfrac{P_u}{\phi_c P_p}$ (*Manual* Eq. 14-6a) $=\left[\dfrac{4(12.7\text{ in.})(12.2\text{ in.})}{(12.7\text{ in.}+12.2\text{ in.})^2}\right]\dfrac{690\text{ kips}}{875\text{ kips}}$ $=0.788$	$X=\left[\dfrac{4db_f}{(d+b_f)^2}\right]\dfrac{\Omega_c P_a}{P_p}$ (*Manual* Eq. 14-6b) $=\left[\dfrac{4(12.7\text{ in.})(12.2\text{ in.})}{(12.7\text{ in.}+12.2\text{ in.})^2}\right]\dfrac{460\text{ kips}}{583\text{ kips}}$ $=0.789$

$$\lambda=\frac{2\sqrt{X}}{1+\sqrt{1-X}}\leq 1$$ (*Manual* Eq. 14-5)

$$=\frac{2\sqrt{0.788}}{1+\sqrt{1-0.788}}$$
$$=1.22>1,\text{ use }\lambda=1$$

Note: λ can always be conservatively taken equal to 1.

$$\lambda n'=(1)(3.11\text{ in.})$$
$$=3.11\text{ in.}$$

$$l=\max(m,\ n,\ \lambda n')$$
$$=\max(4.97\text{ in., }6.12\text{ in., }3.11\text{ in.})$$
$$=6.12\text{ in.}$$

LRFD	ASD
$f_{pu}=\dfrac{P_u}{BN}$ $=\dfrac{690\text{ kips}}{22.0\text{ in.}(22.0\text{ in.})}$ $=1.43\text{ ksi}$ From AISC *Manual* Equation 14-7a: $t_{min}=l\sqrt{\dfrac{2f_{pu}}{0.9F_y}}$ $=6.12\text{ in.}\sqrt{\dfrac{2(1.43\text{ ksi})}{0.9(36\text{ ksi})}}$ $=1.82\text{ in.}$	$f_{pa}=\dfrac{P_a}{BN}$ $=\dfrac{460\text{ kips}}{22.0\text{ in.}(22.0\text{ in.})}$ $=0.950\text{ ksi}$ From AISC *Manual* Equation 14-7b: $t_{min}=l\sqrt{\dfrac{3.33f_{pa}}{F_y}}$ $=6.12\text{ in.}\sqrt{\dfrac{3.33(0.950\text{ ksi})}{36\text{ ksi}}}$ $=1.81\text{ in.}$

Use a 2.00-in.-thick base plate.

Chapter K
Design of HSS and Box Member Connections

Examples K.1 through K.6 illustrate common beam to column shear connections that have been adapted for use with HSS columns. Example K.7 illustrates a through-plate shear connection, which is unique to HSS columns. Calculations for transverse and longitudinal forces applied to HSS are illustrated in Examples K.8 and K.9. An example of an HSS truss connection is given in Example K.10. Examples of HSS cap plate, base plate and end plate connections are given in Examples K.11 through K.13.

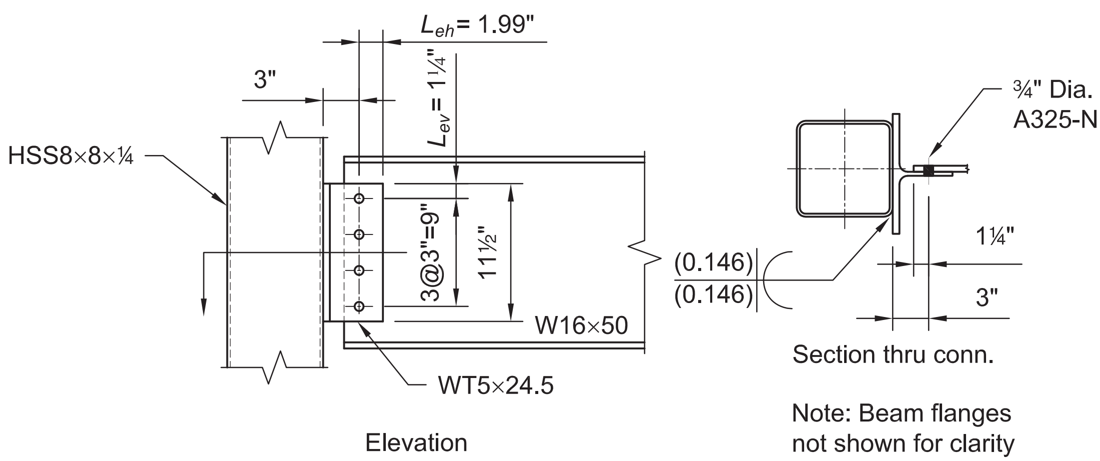
EXAMPLE K.1 WELDED/BOLTED WIDE TEE CONNECTION TO AN HSS COLUMN

Given:

Design a connection between an ASTM A992 W16×50 beam and an ASTM A500 Grade B HSS8×8×¼ column using an ASTM A992 WT5×24.5. Use ¾-in.-diameter ASTM A325-N bolts in standard holes with a bolt spacing, *s*, of 3 in., vertical edge distance L_{ev} of 1¼ in. and 3 in. from the weld line to the bolt line. Design as a flexible connection for the following vertical shear loads:

P_D = 6.20 kips
P_L = 18.5 kips

Note: A tee with a flange width wider than 8 in. was selected to provide sufficient surface for flare bevel groove welds on both sides of the column, because the tee will be slightly offset from the column centerline.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Tee
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1, 1-8 and 1-12, the geometric properties are as follows:

W16×50
t_w = 0.380 in.
d = 16.3 in.
t_f = 0.630 in.
T = 13⅝ in.

WT5×24.5
$t_s = t_w$ = 0.340 in.
d = 4.99 in.
t_f = 0.560 in.
b_f = 10.0 in.
k_1 = 13⁄16 in.

HSS8×8×¼
t = 0.233 in.
B = 8.00 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
P_u = 1.2(6.20 kips) + 1.6(18.5 kips) = 37.0 kips	P_a = 6.20 kips + 18.5 kips = 24.7 kips

Calculate the available strength assuming the connection is flexible.

Required Number of Bolts

The required number of bolts will ultimately be determined using the coefficient, C, from AISC *Manual* Table 7-6. First, the available strength per bolt must be determined.

Determine the available shear strength of a single bolt.

LRFD	ASD
ϕr_n = 17.9 kips from AISC *Manual* Table 7-1	$\frac{r_n}{\Omega}$ = 11.9 kips from AISC *Manual* Table 7-1

Determine single bolt bearing strength based on edge distance.

LRFD	ASD
L_{ev} = 1¼ in. ≥ 1 in. from AISC *Specification* Table J3.4 From AISC *Manual* Table 7-5, ϕr_n = 49.4 kips/in.(0.340 in.) = 16.8 kips	L_{ev} = 1¼ in. ≥ 1 in. from AISC *Specification* Table J3.4 From AISC *Manual* Table 7-5, $\frac{r_n}{\Omega}$ = 32.9 kips/in.(0.340 in.) = 11.2 kips

Determine single bolt bearing capacity based on spacing and AISC *Specification* Section J3.3.

LRFD	ASD
s = 3.00 in. ≥ 2⅔(¾ in.) = 2.00 in. From AISC *Manual* Table 7-4, ϕr_n = 87.8 kips/in.(0.340 in.) = 29.9 kips	s = 3.00 in. ≥ 2⅔(¾ in.) = 2.00 in. From AISC *Manual* Table 7-4, $\dfrac{r_n}{\Omega}$ = 58.5 kips/in.(0.340 in.) = 19.9 kips

Bolt bearing strength based on edge distance controls over available shear strength of the bolt.

Determine the coefficient for the eccentrically loaded bolt group.

LRFD	ASD
$C_{min} = \dfrac{P_u}{\phi r_n} = \dfrac{37.0 \text{ kips}}{16.8 \text{ kips}} = 2.20$ Using e = 3.00 in. and s = 3.00 in., determine C from AISC *Manual* Table 7-6. Try four rows of bolts, C = 2.81 > 2.20 **o.k.**	$C_{min} = \dfrac{P_a}{r_n / \Omega} = \dfrac{24.7 \text{ kips}}{11.2 \text{ kips}} = 2.21$ Using e = 3.00 in. and s = 3.00 in., determine C from AISC *Manual* Table 7-6. Try four rows of bolts, C = 2.81 > 2.21 **o.k.**

WT *Stem Thickness and Length*

AISC *Manual* Part 9 stipulates a maximum tee stem thickness that should be provided for rotational ductility as follows:

$$t_{s\,max} = \frac{d_b}{2} + \tfrac{1}{16} \text{ in.} \qquad (\textit{Manual} \text{ Eq. 9-38})$$

$$= \frac{(\tfrac{3}{4} \text{ in.})}{2} + \tfrac{1}{16} \text{ in.}$$

$$= 0.438 \text{ in.} > 0.340 \text{ in.} \qquad \textbf{o.k.}$$

Note: The beam web thickness is greater than the WT stem thickness. If the beam web were thinner than the WT stem, this check could be satisfied by checking the thickness of the beam web.

Determine the length of the WT required as follows:

A W16×50 has a T-dimension of 13⅝ in.

$$L_{min} = T/2 \text{ from AISC } \textit{Manual} \text{ Part 10}$$
$$= (13\tfrac{5}{8} \text{ in.})/2$$
$$= 6.81 \text{ in.}$$

Determine WT length required for bolt spacing and edge distances.

$L = 3(3.00 \text{ in.}) + 2(1\tfrac{1}{4} \text{ in.})$
$= 11.5 \text{ in.} < T = 13\tfrac{5}{8} \text{ in.}$ **o.k.**

Try L = 11.5 in.

Stem Shear Yielding Strength

Determine the available shear strength of the tee stem based on the limit state of shear yielding from AISC *Specification* Section J4.2.

$$R_n = 0.6F_y A_{gv} \qquad (\textit{Spec}.\ \text{Eq. J4-3})$$
$$= 0.6(50 \text{ ksi})(11.5 \text{ in.})(0.340 \text{ in.})$$
$$= 117 \text{ kips}$$

LRFD	ASD
$\phi R_n = 1.00(117 \text{ kips})$ $= 117 \text{ kips}$ 117 kips > 37.0 kips **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{117 \text{ kips}}{1.50}$ $= 78.0 \text{ kips}$ 78.0 kips > 24.7 kips **o.k.**

Because of the geometry of the WT and because the WT flange is thicker than the stem and carries only half of the beam reaction, flexural yielding and shear yielding of the flange are not critical limit states.

Stem Shear Rupture Strength

Determine the available shear strength of the tee stem based on the limit state of shear rupture from AISC *Specification* Section J4.2.

$$R_n = 0.6F_u A_{nv} \qquad (\textit{Spec}.\ \text{Eq. J4-4})$$
$$= 0.6F_u \left[L - n(d_h + \tfrac{1}{16} \text{ in.})\right](t_s)$$
$$= 0.6(65 \text{ ksi})[11.5 \text{ in.} - 4(\tfrac{13}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})](0.340 \text{ in.})$$
$$= 106 \text{ kips}$$

LRFD	ASD
$\phi R_n = 0.75(106 \text{ kips})$ $= 79.5 \text{ kips}$ 79.5 kips > 37.0 kips **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{106 \text{ kips}}{2.00}$ $= 53.0 \text{ kips}$ 53.0 kips > 24.7 kips **o.k.**

Stem Block Shear Rupture Strength

Determine the available strength for the limit state of block shear rupture from AISC *Specification* Section J4.3.

For this case U_{bs} = 1.0.

Use AISC *Manual* Tables 9-3a, 9-3b and 9-3c. Assume L_{eh} = 1.99 in. ≈ 2.00 in.

LRFD	ASD
$\dfrac{\phi F_u A_{nt}}{t} = 76.2$ kips/in. $\dfrac{\phi 0.60 F_y A_{gv}}{t} = 231$ kips/in. $\dfrac{\phi 0.60 F_u A_{nv}}{t} = 210$ kips/in. $\phi R_n = \phi 0.60 F_u A_{nv} + \phi U_{bs} F_u A_{nt}$ $\leq \phi 0.60 F_y A_{gv} + \phi U_{bs} F_u A_{nt}$ (from *Spec*. Eq. J4-5) $\phi R_n = 0.340 \text{ in.}(210 \text{ kips/in.} + 76.2 \text{ kips/in.})$ $\leq 0.340 \text{ in.}(231 \text{ kips/in.} + 76.2 \text{ kips/in.})$ $= 97.3 \text{ kips} \leq 104 \text{ kips}$ 97.3 kips > 37.0 kips **o.k.**	$\dfrac{F_u A_{nt}}{t\Omega} = 50.8$ kips/in. $\dfrac{0.60 F_y A_{gv}}{t\Omega} = 154$ kips/in. $\dfrac{0.60 F_u A_{nv}}{t\Omega} = 140$ kips/in. $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ $\leq \dfrac{0.60 F_y A_{gv}}{\Omega} + \dfrac{U_{bs} F_u A_{nt}}{\Omega}$ (from *Spec*. Eq. J4-5) $\dfrac{R_n}{\Omega} = 0.340 \text{ in.}(140 \text{ kips/in.} + 50.8 \text{ kips/in.})$ $\leq 0.340 \text{ in.}(154 \text{ kips/in.} + 50.8 \text{ kips/in.})$ $= 64.9 \text{ kips} \leq 69.6 \text{ kips}$ 64.9 kips > 24.7 kips **o.k.**

Stem Flexural Strength

The required flexural strength for the tee stem is,

LRFD	ASD
$M_u = P_u e$ $= 37.0 \text{ kips}(3.00 \text{ in.})$ $= 111$ kip-in.	$M_a = P_a e$ $= 24.7 \text{ kips}(3.00 \text{ in.})$ $= 74.1$ kip-in.

The tee stem available flexural strength due to yielding is determined as follows, from AISC *Specification* Section F11.1. The stem, in this case, is treated as a rectangular bar.

$$Z = \frac{t_s d^2}{4}$$
$$= \frac{0.340 \text{ in.}(11.5 \text{ in.})^2}{4}$$
$$= 11.2 \text{ in.}^3$$

$$S = \frac{t_s d^2}{6}$$
$$= \frac{0.340 \text{ in.}(11.5 \text{ in.})^2}{6}$$
$$= 7.49 \text{ in.}^3$$

$$M_n = M_p = F_y Z \leq 1.6 M_y \qquad (\textit{Spec}.\text{ Eq. F11-1})$$
$$= 50 \text{ ksi}\left(11.2 \text{ in.}^3\right) \leq 1.6(50 \text{ ksi})\left(7.49 \text{ in.}^3\right)$$
$$= 560 \text{ kip-in.} \leq 599 \text{ kip-in.}$$

Therefore, use M_n = 560 kip-in.

Note: The 1.6 limit will never control for a plate, because the shape factor for a plate is 1.5.

The tee stem available flexural yielding strength is:

LRFD	ASD
$\phi M_n = 0.90(560 \text{ kip-in.})$ $= 504 \text{ kip-in.} > 111 \text{ kip-in.}$ **o.k.**	$\dfrac{M_n}{\Omega} = \dfrac{560 \text{ kip-in.}}{1.67}$ $= 335 \text{ kip-in.} > 74.1 \text{ kip-in.}$ **o.k.**

The tee stem available flexural strength due to lateral-torsional buckling is determined from Section F11.2.

$$\frac{L_b d}{t_s^2} = \frac{(3.00\text{ in.})(11.5\text{ in.})}{(0.340\text{ in.})^2}$$
$$= 298$$
$$\frac{0.08E}{F_y} = \frac{0.08(29{,}000\text{ ksi})}{50\text{ ksi}}$$
$$= 46.4$$
$$\frac{1.9E}{F_y} = \frac{1.9(29{,}000\text{ ksi})}{50\text{ ksi}}$$
$$= 1{,}102$$

Because 46.4 < 298 < 1,102, Equation F11-2 is applicable.

$$M_n = C_b\left[1.52 - 0.274\left(\frac{L_b d}{t^2}\right)\frac{F_y}{E}\right]M_y \le M_p \qquad (\textit{Spec.}\text{ Eq. F11-2})$$
$$= 1.00\left[1.52 - 0.274(298)\frac{50\text{ ksi}}{29{,}000\text{ ksi}}\right](50\text{ ksi})(7.49\text{ ksi}) \le (50\text{ ksi})(11.2\text{ in.}^3)$$
$$= 517\text{ kip-in.} \le 560\text{ kip-in.}$$

LRFD	ASD
$\phi = 0.90$ $\phi M_n = 0.90(517 \text{ kip-in.})$ $= 465 \text{ kip-in.} > 111 \text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega_b} = \dfrac{517\text{ kip-in.}}{1.67}$ $= 310\text{ kip-in.} > 74.1\text{ kip-in.}$ **o.k.**

The tee stem available flexural rupture strength is determined from Part 9 of the AISC *Manual* as follows:

$$Z_{net} = \frac{td^2}{4} - 2t\left(d_h + \tfrac{1}{16}\text{ in.}\right)(1.5\text{ in.} + 4.5\text{ in.})$$
$$= \frac{0.340\text{ in.}(11.5\text{ in.})^2}{4} - 2(0.340\text{ in.})\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)(1.5\text{ in.} + 4.5\text{ in.})$$
$$= 7.67\text{ in.}^3$$

$$M_n = F_u Z_{net}$$ (*Manual* Eq. 9-4)

$$= 65 \text{ ksi}\left(7.67 \text{ in.}^3\right)$$

$$= 499 \text{ kip-in.}$$

LRFD	ASD
$\phi M_n = 0.75(499 \text{ kip-in.})$ $= 374 \text{ kip-in.} > 111 \text{ kip-in.}$ **o.k.**	$\dfrac{M_n}{\Omega} = \dfrac{499 \text{ kip-in.}}{2.00}$ $= 250 \text{ kip-in.} > 74.1 \text{ kip-in.}$ **o.k.**

Beam Web Bearing

$$t_w > t_s$$

$$0.380 \text{ in.} > 0.340 \text{ in.}$$

Beam web is satisfactory for bearing by comparison with the WT.

Weld Size

Because the flange width of the WT is larger than the width of the HSS, a flare bevel groove weld is required. Taking the outside radius as 2(0.233 in.) = 0.466 in. and using AISC *Specification* Table J2.2, the effective throat thickness of the flare bevel weld is E = 5⁄16(0.466 in.) = 0.146 in.

Using AISC *Specification* Table J2.3, the minimum effective throat thickness of the flare bevel weld, based on the 0.233 in. thickness of the HSS column, is 1⁄8 in.

$$E = 0.146 \text{ in.} > 1/8 \text{ in.}$$

The equivalent fillet weld that provides the same throat dimension is:

$$\left(\frac{D}{16}\right)\left(\frac{1}{\sqrt{2}}\right) = 0.146$$

$$D = 16\sqrt{2}\left(0.146\right)$$

$$= 3.30 \text{ sixteenths of an inch}$$

The equivalent fillet weld size is used in the following calculations.

Weld Ductility

Let $b_f = B = 8.00$ in.

$$b = \frac{b_f - 2k_1}{2}$$
$$= \frac{8.00 \text{ in.} - 2\left(\tfrac{13}{16} \text{ in.}\right)}{2}$$
$$= 3.19 \text{ in.}$$

$$w_{min} = 0.0155 \frac{F_y t_f^{\,2}}{b}\left(\frac{b^2}{L^2} + 2\right) \le \left(\tfrac{5}{8}\right) t_s$$ (*Manual* Eq. 9-36)

$$= 0.0155 \frac{50 \text{ ksi}\left(0.560 \text{ in.}\right)^2}{3.19 \text{ in.}}\left[\frac{\left(3.19 \text{ in.}\right)^2}{\left(11.5 \text{ in.}\right)^2} + 2\right] \le \left(\tfrac{5}{8}\right)\left(0.340 \text{ in.}\right)$$
$$= 0.158 \text{ in.} \le 0.213 \text{ in.}$$

0.158 in. = 2.53 sixteenths of an inch

D_{min} = 2.53 < 3.30 sixteenths of an inch **o.k.**

Nominal Weld Shear Strength

The load is assumed to act concentrically with the weld group (flexible connection).

a = 0, therefore, C = 3.71 from AISC *Manual* Table 8-4

$$R_n = CC_1Dl$$
$$= 3.71(1.00)(3.30 \text{ sixteenths of an inch})(11.5 \text{ in.})$$
$$= 141 \text{ kips}$$

Shear Rupture of the HSS at the Weld

$$t_{min} = \frac{3.09D}{F_u}$$ (*Manual* Eq. 9-2)

$$= \frac{3.09\left(3.30 \text{ sixteenths}\right)}{58 \text{ ksi}}$$
$$= 0.176 \text{ in.} < 0.233 \text{ in.}$$

By inspection, shear rupture of the WT flange at the welds will not control.

Therefore, the weld controls.

From AISC *Specification* Section J2.4, the available weld strength is:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(141 \text{ kips})$ $= 106 \text{ kips} > 37.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{141 \text{ kips}}{2.00}$ $= 70.5 \text{ kips} > 24.7 \text{ kips}$ **o.k.**

Return to Table of Contents

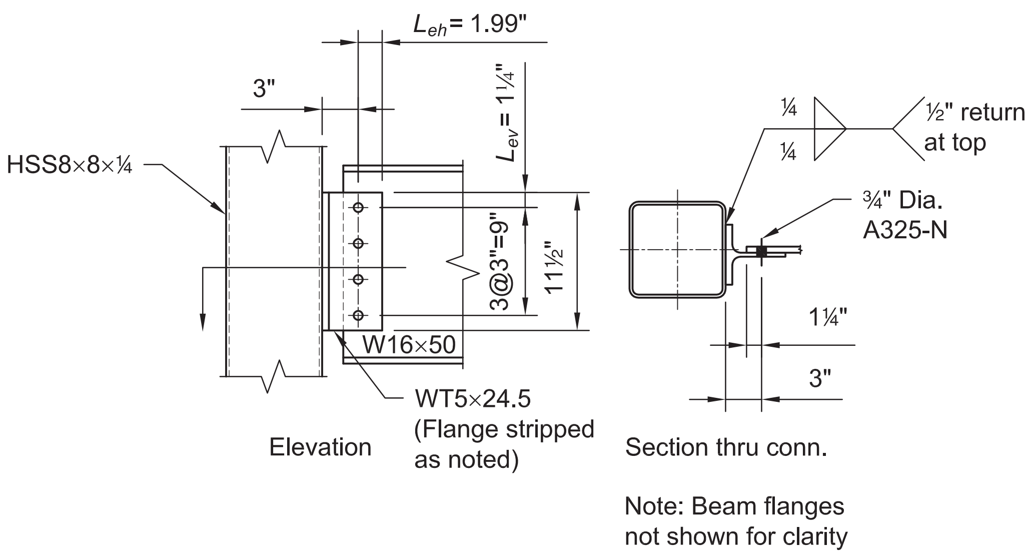
EXAMPLE K.2 WELDED/BOLTED NARROW TEE CONNECTION TO AN HSS COLUMN

Given:

Design a connection for an ASTM A992 W16×50 beam to an ASTM A500 Grade B HSS8×8×¼ column using a tee with fillet welds against the flat width of the HSS. Use ¾-in.-diameter A325-N bolts in standard holes with a bolt spacing, s, of 3.00 in., vertical edge distance L_{ev} of 1¼ in. and 3.00 in. from the weld line to the center of the bolt line. Use 70-ksi electrodes. Assume that, for architectural purposes, the flanges of the WT from the previous example have been stripped down to a width of 5 in. Design as a flexible connection for the following vertical shear loads:

P_D = 6.20 kips
P_L = 18.5 kips

Note: This is the same problem as Example K.1 with the exception that a narrow tee will be selected which will permit fillet welds on the flat of the column. The beam will still be centered on the column centerline; therefore, the tee will be slightly offset.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Tee
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1, 1-8 and 1-12, the geometric properties are as follows:

W16×50
t_w = 0.380 in.
d = 16.3 in.
t_f = 0.630 in.

HSS8×8×¼
t = 0.233 in.
B = 8.00 in.

WT5×24.5
t_s = t_w = 0.340 in.
d = 4.99 in.
t_f = 0.560 in.
k_1 = 13⁄16 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
P_u = 1.2(6.20 kips) + 1.6(18.5 kips) = 37.0 kips	P_a = 6.20 kips + 18.5 kips = 24.7 kips

The WT stem thickness, WT length, WT stem strength and beam web bearing strength are verified in Example K.1. The required number of bolts is also determined in Example K.1.

Maximum WT Flange Width

Assume ¼-in. welds and HSS corner radius equal to 2.25 times the nominal thickness 2.25(¼ in.) = 9⁄16 in.

The recommended minimum shelf dimension for ¼-in. fillet welds from AISC *Manual* Figure 8-11 is ½ in.

Connection offset:

$$\frac{0.380\text{ in.}}{2}+\frac{0.340\text{ in.}}{2}=0.360\text{ in.}$$

$$b_f \le 8.00\text{ in.} - 2\left(\tfrac{9}{16}\text{ in.}\right) - 2\left(\tfrac{1}{2}\text{ in.}\right) - 2\left(0.360\text{ in.}\right)$$
$$5.00\text{ in.} \le 5.16\text{ in.} \qquad \textbf{o.k.}$$

Minimum Fillet Weld Size

From AISC *Specification* Table J2.4, the minimum fillet weld size = ⅛ in. (D = 2) for welding to 0.233-in. material.

Weld Ductility

As defined in Figure 9-5 of the AISC *Manual* Part 9,

$$b=\frac{b_f-2k_1}{2}$$

$$= \frac{5.00 \text{ in.} - 2\left(^{13}\!/_{16} \text{ in.}\right)}{2}$$
$$= 1.69 \text{ in.}$$

$$w_{min} = 0.0155 \frac{F_y t_f^{\,2}}{b}\left(\frac{b^2}{L^2} + 2\right) \le \left(^5\!/_8\right) t_s \qquad (\textit{Manual} \text{ Eq. 9-36})$$

$$= 0.0155 \frac{50 \text{ ksi}\left(0.560 \text{ in.}\right)^2}{1.69 \text{ in.}}\left[\frac{\left(1.69 \text{ in.}\right)^2}{\left(11.5 \text{ in.}\right)^2} + 2\right] \le \left(^5\!/_8\right)\left(0.340 \text{ in.}\right)$$
$$= 0.291 \text{ in.} \le 0.213 \text{ in., use } 0.213 \text{ in.}$$

D_{min} = 0.213 in.(16)
= 3.41 sixteenths of an inch.

Try a ¼-in. fillet weld as a practical minimum, which is less than the maximum permitted weld size of t_f – 1⁄16 in. = 0.560 in. – 1⁄16 in. = 0.498 in. Provide ½-in. return welds at the top of the WT to meet the criteria listed in AISC *Specification* Section J2.2b.

Minimum HSS Wall Thickness to Match Weld Strength

$$t_{min} = \frac{3.09D}{F_u} \qquad (\textit{Manual} \text{ Eq. 9-2})$$
$$= \frac{3.09\left(4\right)}{58 \text{ ksi}}$$
$$= 0.213 \text{ in.} < 0.233 \text{ in.}$$

By inspection, shear rupture of the flange of the WT at the welds will not control.

Therefore, the weld controls.

Available Weld Shear Strength

The load is assumed to act concentrically with the weld group (flexible connection).

a = 0, therefore, C = 3.71 from AISC *Manual* Table 8-4

$R_n = CC_1Dl$
= 3.71(1.00)(4 sixteenths of an inch)(11.5 in.)
= 171 kips

From AISC *Specification* Section J2.4, the available fillet weld shear strength is:

LRFD	ASD
$\phi = 0.75$ ϕR_n = 0.75(171 kips) = 128 kips 128 kips > 37.0 kips **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{171 \text{ kips}}{2.00}$ = 85.5 kips 85.5 kips > 24.7 kips **o.k.**

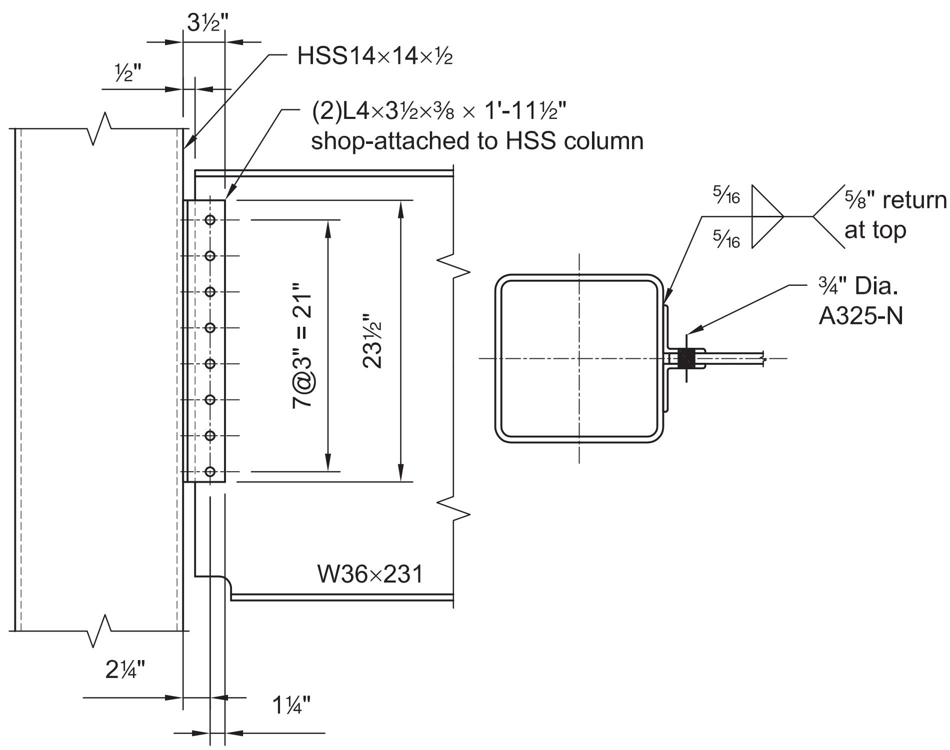
Return to Table of Contents

EXAMPLE K.3 DOUBLE ANGLE CONNECTION TO AN HSS COLUMN

Given:

Use AISC *Manual* Tables 10-1 and 10-2 to design a double-angle connection for an ASTM A992 W36×231 beam to an ASTM A500 Grade B HSS14×14×½ column. Use ¾-in.-diameter ASTM A325-N bolts in standard holes. The angles are ASTM A36 material. Use 70-ksi electrodes. The bottom flange cope is required for erection. Use the following vertical shear loads:

P_D = 37.5 kips
P_L = 113 kips



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

W36×231
t_w = 0.760 in.
T = 31⅜ in.

HSS14×14×½
t = 0.465 in.
B = 14.0 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips})$ $= 226 \text{ kips}$	$R_a = 37.5 \text{ kips} + 113 \text{ kips}$ $= 151 \text{ kips}$

Bolt and Weld Design

Try 8 rows of bolts and 5⁄16-in. welds.

Obtain the bolt group and angle available strength from AISC *Manual* Table 10-1.

LRFD	ASD
$\phi R_n = 286 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 191 \text{ kips} > 151 \text{ kips}$ **o.k.**

Obtain the available weld strength from AISC *Manual* Table 10-2 (welds B).

LRFD	ASD
$\phi R_n = 279 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 186 \text{ kips} > 151 \text{ kips}$ **o.k.**

Minimum Support Thickness

The minimum required support thickness using Table 10-2 is determined as follow for F_u = 58 ksi material.

$$0.238 \text{ in.}\left(\frac{65 \text{ ksi}}{58 \text{ ksi}}\right) = 0.267 \text{ in.} < 0.465 \text{ in.} \qquad \textbf{o.k.}$$

Minimum Angle Thickness

$t_{min} = w + \frac{1}{16}$ in., from AISC *Specification* Section J2.2b
$= \frac{5}{16}$ in. $+ \frac{1}{16}$ in.
$= \frac{3}{8}$ in.

Use ⅜-in. angle thickness to accommodate the welded legs of the double angle connection.

Use 2L4×32×⅜×1′-11½″.

Minimum Angle Length

$L = 23.5 \text{ in.} > T/2$

$> 31\frac{3}{8}$ in./2
> 15.7 in. **o.k.**

Minimum Column Width

The workable flat for the HSS column is 14.0 in. – 2(2.25)(½ in.) = 11.8 in.

The recommended minimum shelf dimension for 5⁄16-in. fillet welds from AISC *Manual* Figure 8-11 is 9⁄16 in.

The minimum acceptable width to accommodate the connection is:

$$2(4.00 \text{ in.}) + 0.760 \text{ in.} + 2(\tfrac{9}{16} \text{ in.}) = 9.89 \text{ in.} < 11.8 \text{ in.}$$ **o.k.**

Available Beam Web Strength

The available beam web strength, from AISC *Manual* Table 10-1 for an uncoped beam with L_{eh}= 1¾ in., is:

LRFD	ASD
$\phi R_n = 702$ kips/in.(0.760 in.) $= 534$ kips 534 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega} = 468$ kips/in.(0.760 in.) $= 356$ kips 356 kips > 151 kips **o.k.**

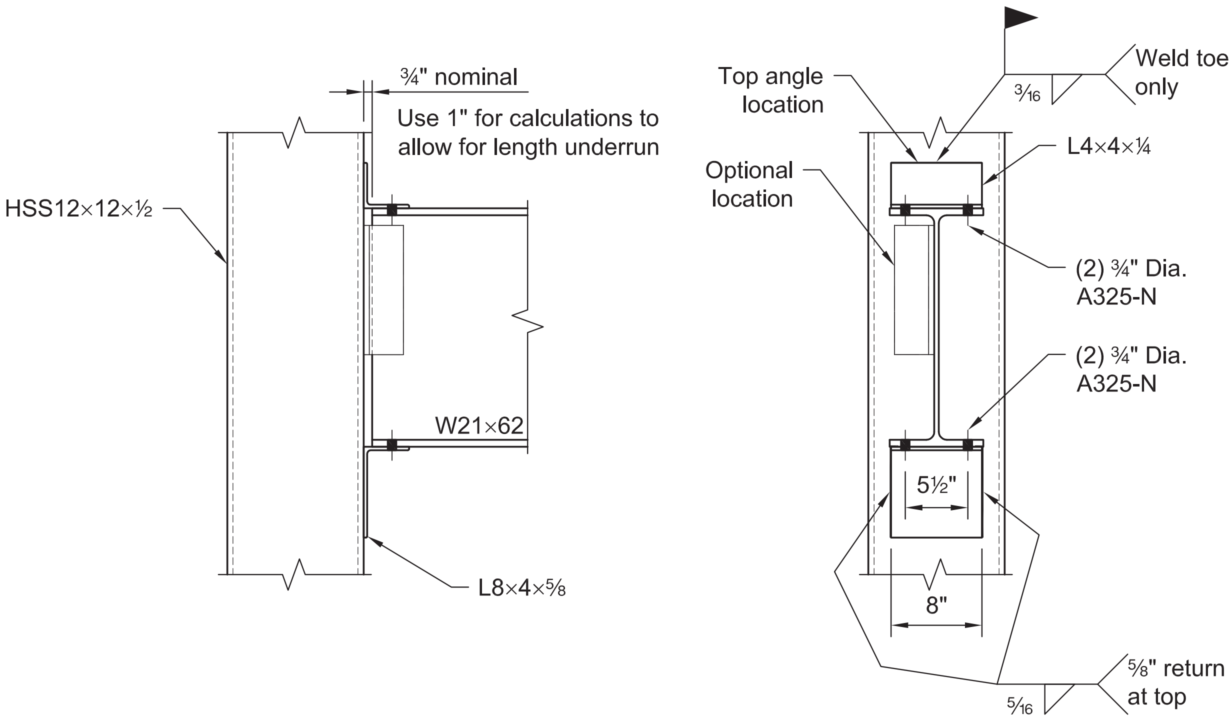
Return to Table of Contents

EXAMPLE K.4 UNSTIFFENED SEATED CONNECTION TO AN HSS COLUMN

Given:

Use AISC *Manual* Table 10-6 to design an unstiffened seated connection for an ASTM A992 W21×62 beam to an ASTM A500 Grade B HSS12×12×½ column. The angles are ASTM A36 material. Use 70-ksi electrodes. Use the following vertical shear loads:

P_D = 9.00 kips
P_L = 27.0 kips



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

W21×62
t_w = 0.400 in.
d = 21.0 in.
k_{des} = 1.12 in.

HSS12×12×½
t = 0.465 in.
B = 12.0 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(9.00 \text{ kips}) + 1.6(27.0 \text{ kips})$ $= 54.0 \text{ kips}$	$R_a = 9.00 \text{ kips} + 27.0 \text{ kips}$ $= 36.0 \text{ kips}$

Seat Angle and Weld Design

Check local web yielding of the W21×62 using AISC *Manual* Table 9-4 and Part 10.

LRFD	ASD
From AISC *Manual* Equation 9-45a and Table 9-4, $l_{b\,min} = \dfrac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ $= \dfrac{54.0 \text{ kips} - 56.0 \text{ kips}}{20.0 \text{ kips/in.}} \geq 1.12 \text{ in.}$ Use $l_{b\,min}$ = 1.12 in. Check web crippling when $l_b/d \leq 0.2$. From AISC *Manual* Equation 9-47a, $l_{b\,min} = \dfrac{R_u - \phi R_3}{\phi R_4}$ $= \dfrac{54.0 \text{ kips} - 71.7 \text{ kips}}{5.37 \text{ kips/in.}}$ which results in a negative quantity. Check web crippling when $l_b/d > 0.2$. From AISC *Manual* Equation 9-48a, $l_{b\,min} = \dfrac{R_u - \phi R_5}{\phi R_6}$ $= \dfrac{54.0 \text{ kips} - 64.2 \text{ kips}}{7.16 \text{ kips/in.}}$ which results in a negative quantity.	From AISC *Manual* Equation 9-45b and Table 9-4, $l_{b\,min} = \dfrac{R_a - R_1/\Omega}{R_2/\Omega} \geq k_{des}$ $= \dfrac{36.0 \text{ kips} - 37.3 \text{ kips}}{13.3 \text{ kips/in.}} \geq 1.12 \text{ in.}$ Use $l_{b\,min}$ = 1.12 in. Check web crippling when $l_b/d \leq 0.2$. From AISC *Manual* Equation 9-47b, $l_{b\,min} = \dfrac{R_a - R_3/\Omega}{R_4/\Omega}$ $= \dfrac{36.0 \text{ kips} - 47.8 \text{ kips}}{3.58 \text{ kips/in.}}$ which results in a negative quantity. Check web crippling when $l_b/d > 0.2$. From AISC *Manual* Equation 9-48b, $l_{b\,min} = \dfrac{R_a - R_5/\Omega}{R_6/\Omega}$ $= \dfrac{36.0 \text{ kips} - 42.8 \text{ kips}}{4.77 \text{ kips/in.}}$ which results in a negative quantity.

Note: Generally, the value of l_b/d is not initially known and the larger value determined from the web crippling equations in the preceding text can be used conservatively to determine the bearing length required for web crippling.

For this beam and end reaction, the beam web strength exceeds the required strength (hence the negative bearing lengths) and the lower-bound bearing length controls ($l_{b\,req} = k_{des}$ = 1.12 in.). Thus, $l_{b\,min}$ = 1.12 in.

Try an L8×4×⅝ seat with 5⁄16-in. fillet welds.

Outstanding Angle Leg Available Strength

From AISC *Manual* Table 10-6 for an 8-in. angle length and $l_{b,req}$ = 1.12 in.≈1⅛ in., the outstanding angle leg available strength is:

LRFD	ASD
$\phi R_n = 81.0 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 53.9 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Available Weld Strength

From AISC *Manual* Table 10-6, for an 8 in. x 4 in. angle and 5⁄16-in. weld size, the available weld strength is:

LRFD	ASD
$\phi R_n = 66.7 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 44.5 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Minimum HSS Wall Thickness to Match Weld Strength

$$t_{min} = \frac{3.09D}{F_u} \quad \text{(}\textit{Manual}\text{ Eq. 9-2)}$$

$$= \frac{3.09(5)}{58 \text{ ksi}}$$

$$= 0.266 \text{ in.} < 0.465 \text{ in.}$$

Because t of the HSS is greater than t_{min} for the 5⁄16-in. weld, no reduction in the weld strength is required to account for the shear in the HSS.

Connection to Beam and Top Angle (AISC Manual Part 10)

Use a L4×4×¼ top angle. Use a 3⁄16-in. fillet weld across the toe of the angle for attachment to the HSS. Attach both the seat and top angles to the beam flanges with two ¾-in.-diameter ASTM A325-N bolts.

Return to Table of Contents

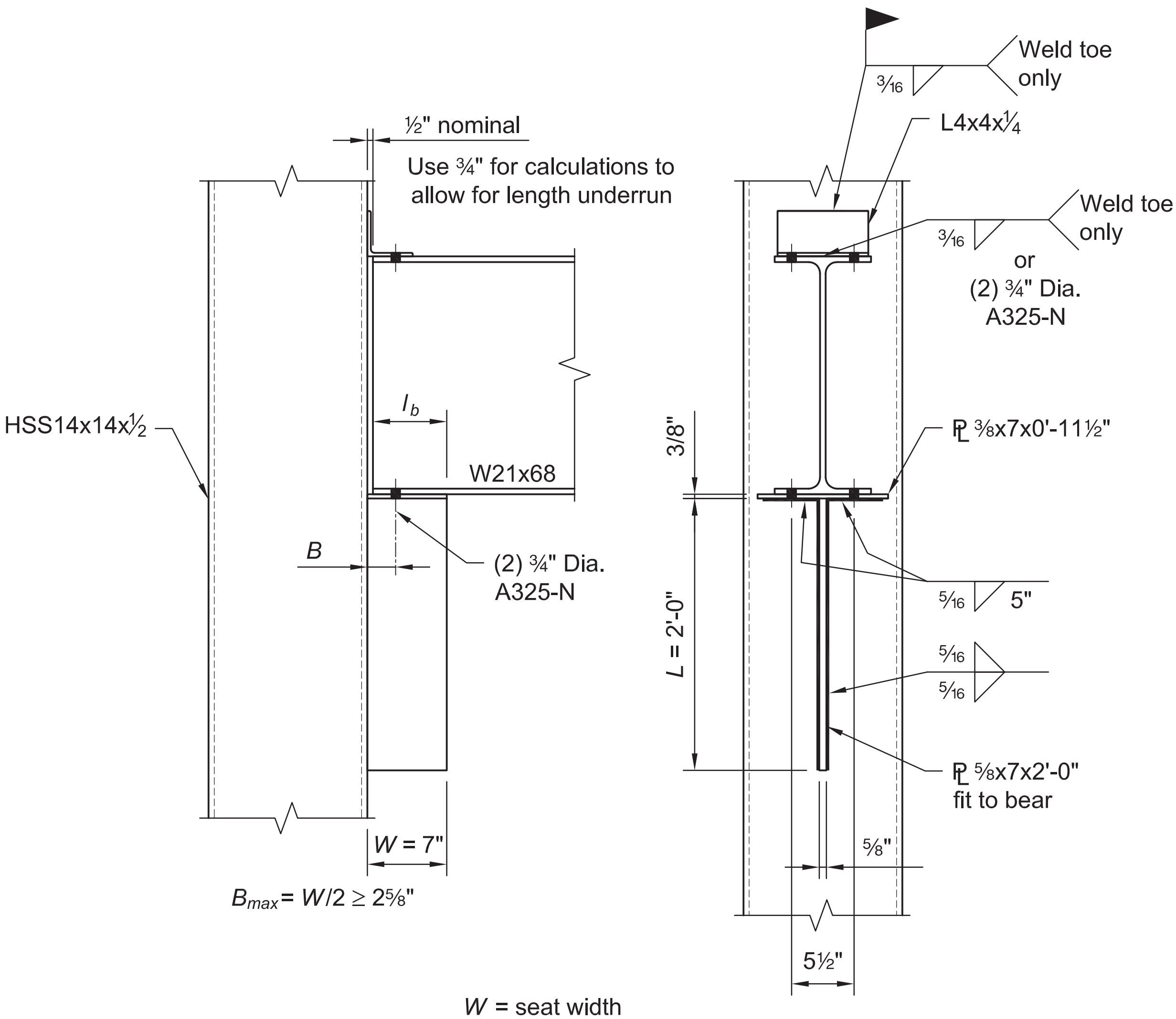
EXAMPLE K.5 STIFFENED SEATED CONNECTION TO AN HSS COLUMN

Given:

Use AISC *Manual* Tables 10-8 and 10-15 to design a stiffened seated connection for an ASTM A992 W21×68 beam to an ASTM A500 Grade B HSS14×14×½ column. Use the following vertical shear loads:

P_D = 20.0 kips
P_L = 60.0 kips

Use ¾-in.-diameter ASTM A325-N bolts in standard holes to connect the beam to the seat plate. Use 70-ksi electrode welds to connect the stiffener, seat plate and top angle to the HSS. The angle and plate material is ASTM A36.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Angles and Plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

W21×68
t_w = 0.430 in.
d = 21.1 in.
k_{des} = 1.19 in.

HSS14×14×½
t = 0.465 in.
B = 14.0 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$P_u = 1.2(20.0\text{ kips}) + 1.6(60.0\text{ kips})$ $= 120\text{ kips}$	$P_a = 20.0\text{ kips} + 60.0\text{ kips}$ $= 80.0\text{ kips}$

Limits of Applicability for AISC Specification Section K1.3

AISC *Specification* Table K1.2A gives the limits of applicability for plate-to-rectangular HSS connections. The limits that are applicable here are:

Strength: $F_y = 46\text{ ksi} \leq 52\text{ ksi}$ **o.k.**

Ductility: $\dfrac{F_y}{F_u} = \dfrac{46\text{ ksi}}{58\text{ ksi}}$
$= 0.793 \leq 0.8$ **o.k.**

Stiffener Width, W, Required for Web Local Crippling and Web Local Yielding

The stiffener width is determined based on web local crippling and web local yielding of the beam.

For web local crippling, assume l_b/d > 0.2 and use constants R_5 and R_6 from AISC *Manual* Table 9-4. Assume a ¾-in. setback for the beam end.

LRFD	ASD
$\phi R_5 = 75.9$ kips $\phi R_6 = 7.95$ kips $W_{min} = \dfrac{R_u - \phi R_5}{\phi R_6} + \text{setback} \geq k_{des} + \text{setback}$ $= \dfrac{120 \text{ kips} - 75.9 \text{ kips}}{7.95 \text{ kips/in.}} + \text{¾ in.} \geq 1.19 \text{ in.} + \text{¾ in.}$ $= 6.30 \text{ in.} \geq 1.94 \text{ in.}$	$R_5/\Omega = 50.6$ kips $R_6/\Omega = 5.30$ kips $W_{min} = \dfrac{R_a - R_5/\Omega}{R_6/\Omega} + \text{setback} \geq k_{des} + \text{setback}$ $= \dfrac{80.0 \text{ kips} - 50.6 \text{ kips}}{5.30 \text{ kips/in.}} + \text{¾ in.} \geq 1.19 \text{ in.} + \text{¾ in.}$ $= 6.30 \text{ in.} \geq 1.94 \text{ in.}$

For web local yielding, use constants R_1 and R_2 from AISC *Manual* Table 9-4. Assume a ¾-in. setback for the beam end.

LRFD	ASD
$\phi R_1 = 64.0$ kips $\phi R_2 = 21.5$ kips $W_{min} = \dfrac{R_u - \phi R_1}{\phi R_2} + \text{setback}$ $= \dfrac{120 \text{ kips} - 64.0 \text{ kips}}{21.5 \text{ kips/in.}} + \text{¾ in.}$ $= 3.35 \text{ in.}$	$R_1/\Omega = 42.6$ kips $R_2/\Omega = 14.3$ kips $W_{min} = \dfrac{R_a - R_1/\Omega}{R_2/\Omega} + \text{setback}$ $= \dfrac{80.0 \text{ kips} - 42.6 \text{ kips}}{14.3 \text{ kips/in.}} + \text{¾ in.}$ $= 3.37 \text{ in.}$

The minimum stiffener width, W_{min}, for web local crippling controls. Use W = 7.00 in.

Check the assumption that $l_b/d > 0.2$.

$$l_b = 7.00 \text{ in.} - \text{¾ in.}$$
$$= 6.25 \text{ in.}$$
$$\frac{l_b}{d} = \frac{6.25 \text{ in.}}{21.1 \text{ in.}}$$
$$= 0.296 > 0.2, \text{ as assumed}$$

Weld Strength Requirements for the Seat Plate

Try a stiffener of length, L = 24 in. with 5⁄16-in. fillet welds. Enter AISC *Manual* Table 10-8 using W = 7 in. as determined in the preceding text.

LRFD	ASD
$\phi R_n = 293 \text{ kips} > 120 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 195 \text{ kips} > 80.0 \text{ kips}$ **o.k.**

From AISC *Manual* Part 10, Figure 10-10(b), the minimum length of the seat-plate-to-HSS weld on each side of the stiffener is $0.2L$ = 4.8 in. This establishes the minimum weld between the seat plate and stiffener; use 5 in. of 5⁄16-in. weld on each side of the stiffener.

Minimum HSS Wall Thickness to Match Weld Strength

The minimum HSS wall thickness required to match the shear rupture strength of the base metal to that of the weld is:

$$t_{min} = \frac{3.09D}{F_u}$$ (*Manual* Eq. 9-2)

$$= \frac{3.09(5)}{58 \text{ ksi}}$$

$$= 0.266 \text{ in.} < 0.465 \text{ in.}$$

Because t of the HSS is greater than t_{min} for the 5⁄16-in. fillet weld, no reduction in the weld strength to account for shear in the HSS is required.

Stiffener Plate Thickness

From AISC *Manual* Part 10, Design Table 10-8 discussion, to develop the stiffener-to-seat-plate welds, the minimum stiffener thickness is:

$$t_{p\ min} = 2w$$

$$= 2(\text{5⁄16 in.})$$

$$= \text{5⁄8 in.}$$

For a stiffener with F_y = 36 ksi and a beam with F_y = 50 ksi, the minimum stiffener thickness is:

$$t_{p\ min} = \left(\frac{F_{y\,beam}}{F_{y\,stiffener}}\right)t_w$$

$$= \left(\frac{50 \text{ ksi}}{36 \text{ ksi}}\right)(0.430 \text{ in.})$$

$$= 0.597 \text{ in.}$$

For a stiffener with F_y = 36 ksi and a column with F_u = 58 ksi, the maximum stiffener thickness is determined from AISC *Specification* Table K1.2 as follows:

$$t_{p\ max} = \frac{F_u t}{F_{yp}}$$ (from *Spec*. Eq. K1-3)

$$= \frac{58 \text{ ksi}(0.465 \text{ in.})}{36 \text{ ksi}}$$

$$= 0.749 \text{ in.}$$

Use stiffener thickness of 5⁄8 in.

Determine the stiffener length using AISC *Manual* Table 10-15.

LRFD	ASD
$\left(\frac{R_u W}{t^2}\right)_{req} = \frac{120 \text{ kips}(7.00 \text{ in.})}{(0.465 \text{ in.})^2}$ $= 3{,}880 \text{ kips/in.}$	$\left(\frac{R_a W}{t^2}\right)_{req} = \frac{80.0 \text{ kips}(7.00 \text{ in.})}{(0.465 \text{ in.})^2}$ $= 2{,}590 \text{ kips/in.}$

To satisfy the minimum, select a stiffener L = 24 in. from AISC *Manual* Table 10-15.

LRFD	ASD
$\dfrac{R_uW}{t^2}$ = 3,910 kips/in. > 3,880 kips/in. **o.k.**	$\dfrac{R_aW}{t^2}$ = 2,600 kips/in. > 2,590 kips/in. **o.k.**

Use PL⅝ in.×7 in.×2 ft-0 in. for the stiffener.

HSS Width Check

The minimum width is $0.4L + t_p + 2(2.25t)$

$$B = 14.0 \text{ in.} > 0.4(24.0 \text{ in.}) + \tfrac{5}{8} \text{ in.} + 2(2.25)(0.465 \text{ in.})$$
$$= 12.3 \text{ in.}$$

Seat Plate Dimensions

To accommodate two ¾-in.-diameter ASTM A325-N bolts on a 5½-in. gage connecting the beam flange to the seat plate, a width of 8 in. is required. To accommodate the seat-plate-to-HSS weld, the required width is:

$$2(5.00 \text{ in.}) + \tfrac{5}{8} \text{ in.} = 10.6 \text{ in.}$$

Note: To allow room to start and stop welds, an 11.5 in. width is used.

Use PL⅜ in.×7 in.×0 ft-11½ in. for the seat plate.

Top Angle, Bolts and Welds (AISC Manual Part 10)

The minimum weld size for the HSS thickness according to AISC *Specification* Table J2.4 is 3⁄16 in. The angle thickness should be 1⁄16 in. larger.

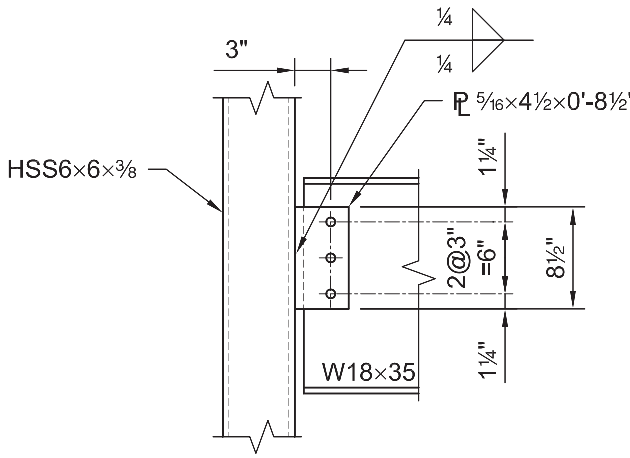
Use L4×4×¼ with 3⁄16-in. fillet welds along the toes of the angle to the beam flange and HSS. Alternatively, two ¾-in.-diameter ASTM A325-N bolts may be used to connect the leg of the angle to the beam flange.

EXAMPLE K.6 SINGLE-PLATE CONNECTION TO A RECTANGULAR HSS COLUMN

Given:

Use AISC *Manual* Table 10-10a to design a single-plate connection for an ASTM A992 W18×35 beam framing into an ASTM A500 Grade B HSS6×6×⅜ column. Use ¾-in.-diameter ASTM A325-N bolts in standard holes and 70-ksi weld electrodes. The plate material is ASTM A36. Use the following vertical shear loads:

P_D = 6.50 kips
P_L = 19.5 kips



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

W18×35
d = 17.7 in.
t_w = 0.300 in.
T = 15½ in.

HSS6×6×⅜
$B = H = 6.00$ in.
$t = 0.349$ in.
$b/t = 14.2$

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(6.50\text{ kips}) + 1.6(19.5\text{ kips})$ $= 39.0\text{ kips}$	$R_a = 6.50\text{ kips} + 19.5\text{ kips}$ $= 26.0\text{ kips}$

Limits of Applicability of AISC Specification Section K1.3

AISC *Specification* Table K1.2A gives the following limits of applicability for plate-to-rectangular HSS connections. The limits applicable here are:

HSS wall slenderness:

$$\frac{(B-3t)}{t} \le 1.40\sqrt{\frac{E}{F_y}}$$

$$\frac{[6.00\text{ in.} - 3(0.349\text{ in.})]}{0.349\text{ in.}} \le 1.40\sqrt{\frac{29{,}000\text{ ksi}}{46\text{ ksi}}}$$

$14.2 < 35.2$ **o.k.**

Material Strength:
$F_y = 46\text{ ksi} \le 52\text{ ksi}$ **o.k.**

Ductility:

$$\frac{F_y}{F_u} = \frac{46\text{ ksi}}{58\text{ ksi}}$$
$$= 0.793 \le 0.8$$ **o.k.**

Using AISC *Manual* Part 10, determine if a single-plate connection is suitable (the HSS wall is not slender).

Maximum Single-Plate Thickness

From AISC *Specification* Table K1.2, the maximum single-plate thickness is:

$$t_p \le \frac{F_u}{F_{yp}}t$$ (*Spec*. Eq. K1-3)

$$= \frac{58\text{ ksi}}{36\text{ ksi}}(0.349\text{ in.})$$
$$= 0.562\text{ in.}$$

Note: Limiting the single-plate thickness precludes a shear yielding failure of the HSS wall.

Single-Plate Connection

Try 3 bolts and a 5⁄16-in. plate thickness with ¼-in. fillet welds.

$t_p = \tfrac{5}{16}\text{ in.} < 0.562\text{ in.}$ **o.k.**

Note: From AISC *Manual* Table 10-9, either the plate or the beam web must satisfy:

$$t = \tfrac{5}{16}\text{ in.} \le d/2 + \tfrac{1}{16}\text{ in.}$$
$$\le \tfrac{3}{4}\text{ in.}/2 + \tfrac{1}{16}\text{ in.}$$
$$\le 0.438\text{ in.} \quad \textbf{o.k.}$$

Obtain the available single-plate connection strength from AISC *Manual* Table 10-10a.

LRFD	ASD
$\phi R_n = 43.4\text{ kips} > 39.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 28.8\text{ kips} > 26.0\text{ kips}$ **o.k.**

Use a PL5⁄16×4½×0′-8½″.

HSS Shear Rupture at Welds

The minimum HSS wall thickness required to match the shear rupture strength of the HSS wall to that of the weld is:

$$t_{min} = \frac{3.09D}{F_u} \qquad \text{(\textit{Manual} Eq. 9-2)}$$
$$= \frac{3.09(4)}{58\text{ ksi}}$$
$$= 0.213\text{ in.} < t = 0.349\text{ in.} \quad \textbf{o.k.}$$

Available Beam Web Bearing Strength (AISC Manual Table 10-1)

For three ¾-in.-diameter bolts and L_{eh} = 1½ in., the bottom of AISC *Manual* Table 10-1 gives the uncoped beam web available bearing strength per inch of thickness. The available beam web bearing strength can then be calculated as follows:

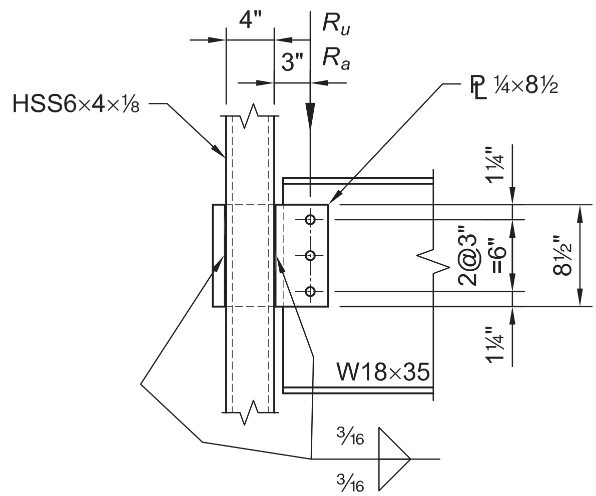
LRFD	ASD
$\phi R_n = 263\text{ kips/in.}(0.300\text{ in.})$ $= 78.9\text{ kips} > 39.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 176\text{ kips/in.}(0.300\text{ in.})$ $= 52.8\text{ kips} > 26.0\text{ kips}$ **o.k.**

EXAMPLE K.7 THROUGH-PLATE CONNECTION

Given:

Use AISC *Manual* Table 10-10a to check a through-plate connection between an ASTM A992 W18×35 beam and an ASTM A500 Grade B HSS6×4×⅛ with the connection to one of the 6 in. faces, as shown in the figure. A thin-walled column is used to illustrate the design of a through-plate connection. Use ¾-in.-diameter ASTM A325-N bolts in standard holes and 70-ksi weld electrodes. The plate is ASTM A36 material. Use the following vertical shear loads:

P_D = 3.30 kips
P_L = 9.90 kips



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-11, the geometric properties are as follows:

W18×35
d = 17.7 in.
t_w = 0.300 in.
T = 15½ in.

HSS6×4×⅛
B = 4.00 in.
H = 6.00 in.
t = 0.116 in.
h/t = 48.7

Limits of Applicability of AISC Specification Section K1.3

AISC *Specification* Table K1.2A gives the following limits of applicability for plate-to-rectangular HSS connections. The limits applicable here follow.

HSS wall slenderness: Check if a single-plate connection is allowed.

$$\frac{(H-3t)}{t} \leq 1.40\sqrt{\frac{E}{F_y}}$$

$$\frac{\left[6.00-3\left(0.116\text{ in.}\right)\right]}{0.116\text{ in.}} \leq 1.40\sqrt{\frac{29{,}000\text{ ksi}}{46\text{ ksi}}}$$

$48.7 > 35.2$ **n.g.**

Because the HSS6×4×⅛ is slender, a through-plate connection should be used instead of a single-plate connection. Through-plate connections are typically very expensive. When a single-plate connection is not adequate, another type of connection, such as a double-angle connection may be preferable to a through-plate connection.

AISC *Specification* Chapter K does not contain provisions for the design of through-plate shear connections. The following procedure treats the connection of the through-plate to the beam as a single-plate connection.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(3.30\text{ kips}) + 1.6(9.90\text{ kips})$ $= 19.8\text{ kips}$	$R_a = 3.30\text{ kips} + 9.90\text{ kips}$ $= 13.2\text{ kips}$

Portion of the Through-Plate Connection that Resembles a Single-Plate

Try three rows of bolts (L = 8½ in.) and a ¼-in. plate thickness with 3⁄16-in. fillet welds.

L = 8½ in. ≥ $T/2$
≥ (15½ in.)/2
≥ 7.75 in. **o.k.**

Note: From AISC *Manual* Table 10-9, either the plate or the beam web must satisfy:

t = ¼ in. ≤ $d_b/2$ + 1⁄16 in.
≤ ¾ in./2 + 1⁄16 in.
≤ 0.438 in. **o.k.**

Obtain the available single-plate connection strength from AISC *Manual* Table 10-10a.

LRFD	ASD
$\phi R_n = 38.3 \text{ kips} > 19.8 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 25.6 \text{ kips} > 13.2 \text{ kips}$ **o.k.**

Required Weld Strength

The available strength for the welds in this connection is checked at the location of the maximum reaction, which is along the weld line closest to the bolt line. The reaction at this weld line is determined by taking a moment about the weld line farthest from the bolt line.

a = 3.00 in. (distance from bolt line to nearest weld line)

LRFD	ASD
$V_{fu} = \dfrac{R_u(B+a)}{B}$ $= \dfrac{19.8 \text{ kips}(4.00 \text{ in.} + 3.00 \text{ in.})}{4.00 \text{ in.}}$ $= 34.7 \text{ kips}$	$V_{fa} = \dfrac{R_a(B+a)}{B}$ $= \dfrac{13.2 \text{ kips}(4.00 \text{ in.} + 3.00 \text{ in.})}{4.00 \text{ in.}}$ $= 23.1 \text{ kips}$

Available Weld Strength

The minimum required weld size is determined using AISC *Manual* Part 8.

LRFD	ASD
$D_{req} = \dfrac{\phi R_n}{1.392l}$ (from *Manual* Eq. 8-2a) $= \dfrac{34.7 \text{ kips}}{1.392(8.50 \text{ in.})(2)}$ $= 1.47 \text{ sixteenths} < 3 \text{ sixteenths}$ **o.k.**	$D_{req} = \dfrac{R_n/\Omega}{0.928l}$ (from *Manual* Eq. 8-2b) $= \dfrac{23.1 \text{ kips}}{0.928(8.50 \text{ in.})(2)}$ $= 1.46 \text{ sixteenths} < 3 \text{ sixteenths}$ **o.k.**

HSS Shear Yielding and Rupture Strength

The available shear strength of the HSS due to shear yielding and shear rupture is determined from AISC *Specification* Section J4.2.

LRFD	ASD
For shear yielding of HSS at the connection, $\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ (from *Spec.* Eq. J4-3) $= 1.00(0.60)(46 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)$ $= 54.4 \text{ kips}$ 54.4 kips > 34.7 kips **o.k.** For shear rupture of HSS at the connection, $\phi = 0.75$	For shear yielding of HSS at the connection, $\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ (from *Spec.* Eq. J4-3) $= \dfrac{(0.60)(46 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)}{1.50}$ $= 36.3 \text{ kips}$ 36.3 kips > 23.1 kips **o.k.** For shear rupture of HSS at the connection, $\Omega = 2.00$

$\phi R_n = \phi 0.60 F_u A_{nv}$ (from *Spec.* Eq. J4-4) $= 0.75(0.60)(58 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)$ $= 51.5 \text{ kips}$ 51.5 kips > 34.7 kips **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ (from *Spec.* Eq. J4-4) $= \dfrac{(0.60)(58 \text{ ksi})(0.116 \text{ in.})(8.50 \text{ in.})(2)}{2.00}$ $= 34.3 \text{ kips}$ 34.3 kips > 23.1 kips **o.k.**

Available Beam Web Bearing Strength

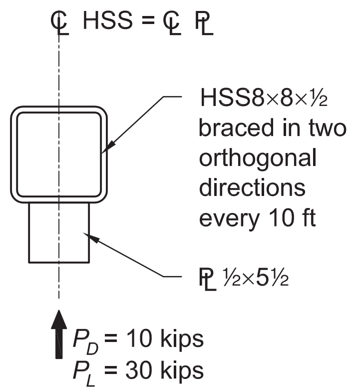
The available beam web bearing strength is determined from the bottom portion of Table 10-1, for three ¾-in.-diameter bolts and an uncoped beam. The table provides the available strength in kips/in. and the available beam web bearing strength is:

LRFD	ASD
$\phi R_n = 263 \text{ kips/in.}(0.300 \text{ in.})$ $= 78.9 \text{ kips} > 19.8 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 176 \text{ kips/in.}(0.300 \text{ in.})$ $= 52.8 \text{ kips} > 13.2 \text{ kips}$ **o.k.**

EXAMPLE K.8 TRANSVERSE PLATE LOADED PERPENDICULAR TO THE HSS AXIS ON A RECTANGULAR HSS

Given:

Verify the local strength of the ASTM A500 Grade B HSS column subject to the transverse loads given as follows, applied through a 5½-in.-wide ASTM A36 plate. The HSS8×8×½ is in compression with nominal axial loads of $P_{D\ column}$ = 54.0 kips and $P_{L\ column}$ = 162 kips. The HSS has negligible required flexural strength.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Plate
ASTM A36
F_{yp} = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-12, the geometric properties are as follows:

HSS8×8×½
B = 8.00 in.
t = 0.465 in.

Plate
B_p = 5½ in.
t_p = ½ in.

Limits of Applicability of AISC Specification Section K1.3

AISC *Specification* Table K1.2A provides the limits of applicability for plate-to-rectangular HSS connections. The following limits are applicable in this example.

HSS wall slenderness:
$B/t = 14.2 \leq 35$ **o.k.**

Width ratio:

$B_p/B = 5\frac{1}{2}$ in./8.00 in.
$= 0.688$

$0.25 \leq B_p/B \leq 1.0$ **o.k.**

Material strength:

$F_y = 46$ ksi ≤ 52 ksi for HSS **o.k.**

Ductility:

$F_y/F_u = 46$ ksi/58 ksi
$= 0.793 \leq 0.8$ for HSS **o.k.**

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
Transverse force from the plate: $P_u = 1.2(10.0\text{ kips}) + 1.6(30.0\text{ kips})$ $= 60.0$ kips Column axial force: $P_r = P_{u\ column}$ $= 1.2(54.0\text{ kips}) + 1.6(162\text{ kips})$ $= 324$ kips	Transverse force from the plate: $P_a = 10.0\text{ kips} + 30.0\text{ kips}$ $= 40.0$ kips Column axial force: $P_r = P_{a\ column}$ $= 54.0\text{ kips} + 162\text{ kips}$ $= 216$ kips

Available Local Yielding Strength of Plate from AISC Specification Table K1.2

The available local yielding strength of the plate is determined from AISC *Specification* Table K1.2.

$$R_n = \frac{10}{B/t} F_y t B_p \leq F_{yp} t_p B_p$$ (*Spec*. Eq. K1-7)

$$= \frac{10}{8.00\text{ in.}/0.465\text{ in.}}(46\text{ ksi})(0.465\text{ in.})(5\tfrac{1}{2}\text{ in.}) \leq 36\text{ ksi}(\tfrac{1}{2}\text{ in.})(5\tfrac{1}{2}\text{ in.})$$

$= 68.4$ kips ≤ 99.0 kips **o.k.**

LRFD	ASD
$\phi = 0.95$ $\phi R_n = 0.95(68.4\text{ kips})$ $= 65.0\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\Omega = 1.58$ $\dfrac{R_n}{\Omega} = \dfrac{68.4\text{ kips}}{1.58}$ $= 43.3\text{ kips} > 40.0\text{ kips}$ **o.k.**

HSS Shear Yielding (Punching)

The available shear yielding (punching) strength of the HSS is determined from AISC *Specification* Table K1.2.

This limit state need not be checked when $B_p > B - 2t$, nor when $B_p < 0.85B$.

$B - 2t = 8.00\text{ in.} - 2(0.465\text{ in.})$
$= 7.07$ in.

$0.85B = 0.85(8.00\text{ in.})$
$= 6.80$ in.

Therefore, because B_p < 6.80 in., this limit state does not control.

Other Limit States

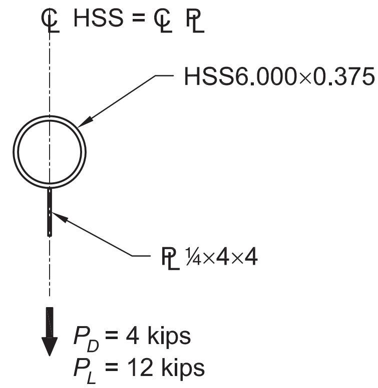
The other limit states listed in AISC *Specification* Table K1.2 apply only when β = 1.0. Because $B_p/B < 1.0$, these limit states do not apply.

Return to Table of Contents

EXAMPLE K.9 LONGITUDINAL PLATE LOADED PERPENDICULAR TO THE HSS AXIS ON A ROUND HSS

Given:

Verify the local strength of the ASTM A500 Grade B HSS6.000×0.375 tension chord subject to transverse loads, P_D = 4.00 kips and P_L = 12.0 kips, applied through a 4 in. wide ASTM A36 plate.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Chord
ASTM A500 Grade B
F_y = 42 ksi
F_u = 58 ksi

Plate
ASTM A36
F_{yp} = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-13, the geometric properties are as follows:

HSS6.000×0.375
D = 6.00 in.
t = 0.349 in.

Limits of Applicability of AISC Specification Section K1.2, Table K1.1A

AISC *Specification* Table K1.1A provides the limits of applicability for plate-to-round connections. The applicable limits for this example are:

HSS wall slenderness:
D/t = 6.00 in./0.349 in.
= 17.2 ≤ 50 for T-connections **o.k.**

Material strength:
F_y = 42 ksi ≤ 52 ksi for HSS **o.k.**

Ductility:

F_y/F_u = 42 ksi/58 ksi
= 0.724 ≤ 0.8 for HSS **o.k.**

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
P_u = 1.2(4.00 kips) + 1.6(12.0 kips) = 24.0 kips	P_a = 4.00 kips + 12.0 kips = 16.0 kips

HSS Plastification Limit State

The limit state of HSS plastification applies and is determined from AISC *Specification* Table K1.1.

$$R_n = 5.5F_y t^2\left(1+0.25\frac{l_b}{D}\right)Q_f$$ (*Spec*. Eq. K1-2)

From the AISC *Specification* Table K1.1 Functions listed at the bottom of the table, for an HSS connecting surface in tension, Q_f = 1.0.

$$R_n = 5.5(42 \text{ ksi})(0.349 \text{ in.})^2\left(1+0.25\frac{4.00 \text{ in.}}{6.00 \text{ in.}}\right)(1.0)$$
$$= 32.8 \text{ kips}$$

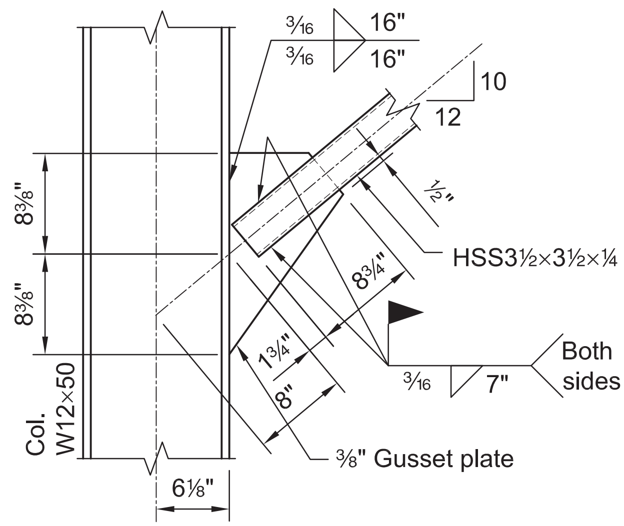
The available strength is:

LRFD	ASD
ϕR_n = 0.90(32.8 kips) = 29.5 kips > 24.0 kips **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{32.8 \text{ kips}}{1.67}$ = 19.6 kips > 16.0 kips **o.k.**

EXAMPLE K.10 HSS BRACE CONNECTION TO A W-SHAPE COLUMN

Given:

Verify the strength of an ASTM A500 Grade B HSS3½×3½×¼ brace for required axial forces of 80.0 kips (LRFD) and 52.0 kips (ASD). The axial force may be either tension or compression. The length of the brace is 6 ft. Design the connection of the HSS brace to the gusset plate. Allow 1⁄16 in. for fit of the slot over the gusset plate.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Brace
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Plate
ASTM A36
F_{yp} = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-12, the geometric properties are as follows:

HSS3½×3½×¼
A = 2.91 in.^{2}
r = 1.32 in.
t = 0.233 in.

Available Compressive Strength of Brace

Obtain the available axial compressive strength of the brace from AISC *Manual* Table 4-4.

K = 1.0
L_b = 6.00 ft

LRFD	ASD
$\phi_c P_n = 98.6 \text{ kips} > 80.0 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_c} = 65.6 \text{ kips} > 52.0 \text{ kips}$ **o.k.**

Available Tensile Yielding Strength of Brace

Obtain the available tensile yielding strength of the brace from AISC *Manual* Table 5-5.

LRFD	ASD
$\phi_t P_n = 120 \text{ kips} > 80.0 \text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega_t} = 80.2 \text{ kips} > 52.0 \text{ kips}$ **o.k.**

Available Tensile Rupture Strength of the Brace

Due to plate geometry, 8¾ in. of overlap occurs. Try four 3⁄16-in. fillet welds, each 7-in. long. Based on AISC *Specification* Table J2.4 and the HSS thickness of ¼ in., the minimum weld size is an ⅛-in. fillet weld.

Determine the available tensile strength of the brace from AISC *Specification* Section D2.

$$A_e = A_n U \qquad \text{(}\textit{Spec.}\text{ Eq. D3-1)}$$

where

$$\begin{aligned} A_n &= A_g - 2(t)(\text{gusset plate thickness} + \tfrac{1}{16}\text{ in.}) \\ &= 2.91 \text{ in.}^2 - 2(0.233 \text{ in.})(\tfrac{3}{8}\text{ in.} + \tfrac{1}{16}\text{ in.}) \\ &= 2.71 \text{ in.}^2 \end{aligned}$$

Because $l = 7$ in. > $H = 3\tfrac{1}{2}$ in., from AISC *Specification* Table D3.1, Case 6,

$$U = 1 - \frac{\bar{x}}{l}$$

$$\bar{x} = \frac{B^2 + 2BH}{4(B+H)} \text{ from AISC } \textit{Specification} \text{ Table D3.1}$$

$$\begin{aligned} &= \frac{(3\tfrac{1}{2}\text{ in.})^2 + 2(3\tfrac{1}{2}\text{ in.})(3\tfrac{1}{2}\text{ in.})}{4(3\tfrac{1}{2}\text{ in.} + 3\tfrac{1}{2}\text{ in.})} \\ &= 1.31 \text{ in.} \end{aligned}$$

Therefore,

$$\begin{aligned} U &= 1 - \frac{\bar{x}}{l} \\ &= 1 - \frac{1.31 \text{ in.}}{7.00 \text{ in.}} \\ &= 0.813 \end{aligned}$$

And,

$$\begin{aligned} A_e &= A_n U \qquad \text{(}\textit{Spec.}\text{ Eq. D3-1)} \\ &= 2.71 \text{ in.}^2(0.813) \\ &= 2.20 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} P_n &= F_u A_e \qquad \text{(}\textit{Spec.}\text{ Eq. D2-2)} \\ &= 58 \text{ ksi}(2.20 \text{ in.}^2) \end{aligned}$$

= 128 kips

Using AISC *Specification* Section D2, the available tensile rupture strength is:

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(128\text{ kips})$ $= 96.0\text{ kips} > 80.0\text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_c} = \dfrac{128\text{ kips}}{2.00}$ $= 64.0\text{ kips} > 52.0\text{ kips}$ **o.k.**

Available Strength of 3⁄16-in. Weld of Plate to HSS

From AISC *Manual* Part 8, the available strength of a 3⁄16-in. fillet weld is:

LRFD	ASD
$\phi R_n = 4(1.392Dl)$ (from *Manual* Eq. 8-2a) $= 4(1.392)(3\text{ sixteenths})(7.00\text{ in.})$ $= 117\text{ kips} > 80.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 4(0.928Dl)$ (from *Manual* Eq. 8-2b) $= 4(0.928)(3\text{ sixteenths})(7.00\text{ in.})$ $= 78.0\text{ kips} > 52.0\text{ kips}$ **o.k.**

HSS Shear Rupture Strength at Welds (weld on one side)

$$t_{min} = \frac{3.09D}{F_u} \qquad (\textit{Manual}\text{ Eq. 9-2})$$

$$= \frac{3.09(3)}{58}$$

$$= 0.160\text{ in.} < 0.233\text{ in.} \qquad \textbf{o.k.}$$

Gusset Plate Shear Rupture Strength at Welds (weld on two sides)

$$t_{min} = \frac{6.19D}{F_u} \qquad (\textit{Manual}\text{ Eq. 9-3})$$

$$= \frac{6.19(3)}{58}$$

$$= 0.320\text{ in.} < 3/8\text{ in.} \qquad \textbf{o.k.}$$

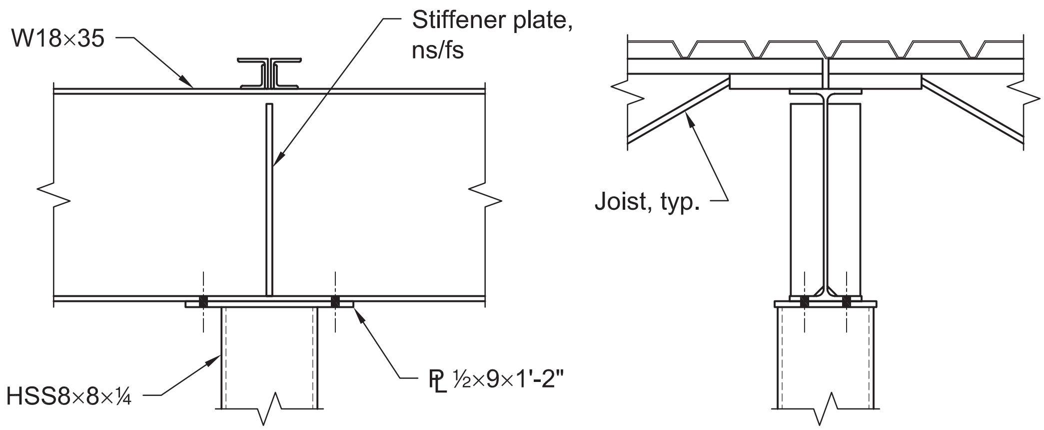
A complete check of the connection would also require consideration of the limit states of the other connection elements, such as:

- Whitmore buckling
- Local capacity of column web yielding and crippling
- Yielding of gusset plate at gusset-to-column intersection

EXAMPLE K.11 RECTANGULAR HSS COLUMN WITH A CAP PLATE, SUPPORTING A CONTINUOUS BEAM

Given:

Verify the local strength of the ASTM A500 Grade B HSS8×8×¼ column subject to the given ASTM A992 W18×35 beam reactions through the ASTM A36 cap plate. Out of plane stability of the column top is provided by the beam web stiffeners; however, the stiffeners will be neglected in the column strength calculations. The column axial forces are R_D = 24 kips and R_L = 30 kips.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Cap Plate
ASTM A36
F_{yp} = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

W18×35
d = 17.7 in.
b_f = 6.00 in.
t_w = 0.300 in.
t_f = 0.425 in.
k_1 = ¾ in.

HSS8×8×¼
t = 0.233 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(24.0 kips) + 1.6(30.0 kips) = 76.8 kips	R_a = 24.0 kips + 30.0 kips = 54.0 kips

Assume the vertical beam reaction is transmitted to the HSS through bearing of the cap plate at the two column faces perpendicular to the beam.

Bearing Length, l_b, at Bottom of W18×35

Assume the dispersed load width, $l_b = 5t_p + 2k_1$. With $t_p = t_f$.

$$\begin{aligned} l_b &= 5t_f + 2k_1 \\ &= 5(0.425 \text{ in.}) + 2(3/4 \text{ in.}) \\ &= 3.63 \text{ in.} \end{aligned}$$

Available Strength: Local Yielding of HSS Sidewalls

Determine the applicable equation from AISC *Specification* Table K1.2.

$$\begin{aligned} 5t_p + l_b &= 5(1/2 \text{ in.}) + 3.63 \text{ in.} \\ &= 6.13 \text{ in.} < 8.00 \text{ in.} \end{aligned}$$

Therefore, only two walls contribute and AISC *Specification* Equation K1-14a applies.

$$\begin{aligned} R_n &= 2F_y t(5t_p + l_b) \\ &= 2(46 \text{ ksi})(0.233 \text{ in.})(6.13 \text{ in.}) \\ &= 131 \text{ kips} \end{aligned}$$ (*Spec*. Eq. K1-14a)

The available wall local yielding strength of the HSS is:

LRFD	ASD
ϕ= 1.00 ϕR_n = 1.00(131 kips) = 131 kips > 76.8 kips **o.k.**	Ω = 1.50 $\dfrac{R_n}{\Omega} = \dfrac{131 \text{ kips}}{1.50}$ = 87.3 kips > 54.0 kips **o.k.**

Available Strength: Local Crippling of HSS Sidewalls

From AISC *Specification* Table K1.2, the available wall local crippling strength of the HSS is determined as follows:

$$R_n = 1.6t^2\left[1 + \frac{6l_b}{B}\left(\frac{t}{t_p}\right)^{1.5}\right]\sqrt{EF_y\frac{t_p}{t}}$$ (*Spec*. Eq. K1-15)

$$\begin{aligned} &= 1.6(0.233 \text{ in.})^2\left[1 + \left(\frac{6(6.13 \text{ in.})}{8.00 \text{ in.}}\right)\left(\frac{0.233 \text{ in.}}{1/2 \text{ in.}}\right)^{1.5}\right]\sqrt{29{,}000 \text{ ksi}(46 \text{ ksi})\frac{1/2 \text{ in.}}{0.233 \text{ in.}}} \\ &= 362 \text{ kips} \end{aligned}$$

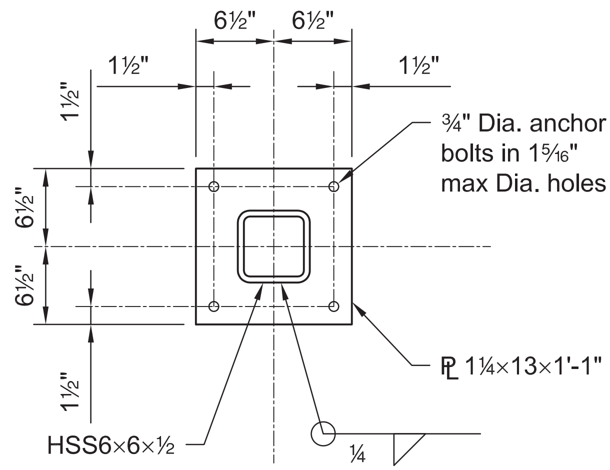
LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(362\text{ kips})$ $= 272\text{ kips} > 76.8\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{362\text{ kips}}{2.00}$ $= 181\text{ kips} > 54.0\text{ kips}$ **o.k.**

Note: This example illustrates the application of the relevant provisions of Chapter K of the AISC *Specification*. Other limit states should also be checked to complete the design.

EXAMPLE K.12 RECTANGULAR HSS COLUMN BASE PLATE

Given:

An ASTM A500 Grade B HSS6×6×½ column is supporting loads of 40.0 kips of dead load and 120 kips of live load. The column is supported by a 7 ft-6 in. × 7 ft-6 in. concrete spread footing with f_c' = 3,000 psi. Size an ASTM A36 base plate for this column.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Base Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-12, the geometric properties are as follows:

HSS6×6×½
$B = H$ = 6.00 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
P_u = 1.2(40.0 kips) + 1.6(120 kips) = 240 kips	P_a = 40.0 kips + 120 kips = 160 kips

Note: The procedure illustrated here is similar to that presented in AISC Design Guide 1, *Base Plate and Anchor Rod Design* (Fisher and Kloiber, 2006), and Part 14 of the AISC *Manual*.

Try a base plate which extends 3½ in. from each face of the HSS column, or 13 in. × 13 in.

Available Strength for the Limit State of Concrete Crushing

On less than the full area of a concrete support,

$$P_p = 0.85 f_c' A_1 \sqrt{A_2 / A_1} \;\leq\; 1.7 f_c' A_1 \qquad \text{(Spec. Eq. J8-2)}$$

$$\begin{aligned} A_1 &= BN \\ &= 13.0 \text{ in.}(13.0 \text{ in.}) \\ &= 169 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_2 &= 90.0 \text{ in.}(90.0 \text{ in.}) \\ &= 8{,}100 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} P_p &= 0.85(3 \text{ ksi})(169 \text{ in.}^2)\sqrt{\frac{8{,}100 \text{ in.}^2}{169 \text{ in.}^2}} \leq 1.7(3 \text{ ksi})(169 \text{ in.}^2) \\ &= 2{,}980 \text{ kips} \leq 862 \text{ kips} \end{aligned}$$

Use P_p = 862 kips.

Note: The limit on the right side of AISC *Specification* Equation J8-2 will control when A_2/A_1 exceeds 4.0.

LRFD	ASD
ϕ_c = 0.65 from AISC *Specification* Section J8 $\phi_c P_p = 0.65(862 \text{ kips})$ $= 560 \text{ kips} > 240 \text{ kips}$ **o.k.**	Ω_c = 2.31 from AISC *Specification* Section J8 $\dfrac{P_p}{\Omega_c} = \dfrac{862 \text{ kips}}{2.31}$ $= 373 \text{ kips} > 160 \text{ kips}$ **o.k.**

Pressure Under Bearing Plate and Required Thickness

For a rectangular HSS, the distance m or n is determined using 0.95 times the depth and width of the HSS.

$$\begin{aligned} m = n &= \frac{N - 0.95(\text{outside dimension})}{2} \qquad \text{(Manual Eq. 14-2)} \\ &= \frac{13.0 \text{ in.} - 0.95(6.00 \text{ in.})}{2} \\ &= 3.65 \text{ in.} \end{aligned}$$

The critical bending moment is the cantilever moment outside the HSS perimeter. Therefore, $m = n = l$.

LRFD	ASD
$f_{pu} = \dfrac{P_u}{A_1}$ $= \dfrac{240 \text{ kips}}{169 \text{ in.}^2}$ $= 1.42 \text{ ksi}$ $M_u = \dfrac{f_{pu} l^2}{2}$ $Z = \dfrac{t_p^2}{4}$	$f_{pa} = \dfrac{P_a}{A_1}$ $= \dfrac{160 \text{ kips}}{169 \text{ in.}^2}$ $= 0.947 \text{ ksi}$ $M_a = \dfrac{f_{pa} l^2}{2}$ $Z = \dfrac{t_p^2}{4}$

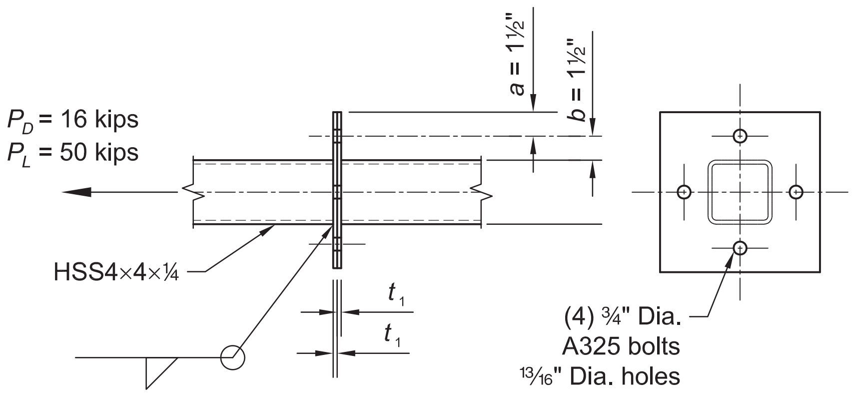
LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$M_n = M_p = F_yZ$ (*Spec*. Eq. F11-1)	$M_n = M_p = F_yZ$ (*Spec*. Eq. F11-1)
Equating:	Equating:
$M_u = \phi_b M_n$ and solving for t_p gives:	$M_a = M_n/\Omega_b$ and solving for t_p gives:
$t_{p(req)} = \sqrt{\dfrac{2f_{pu}l^2}{\phi_b F_y}}$	$t_{p(req)} = \sqrt{\dfrac{2f_{pa}l^2}{F_y/\Omega_b}}$
$= \sqrt{\dfrac{2(1.42\text{ ksi})(3.65\text{ in.})^2}{0.90(36\text{ ksi})}}$	$= \sqrt{\dfrac{2(0.947\text{ ksi})(3.65\text{ in.})^2}{(36\text{ ksi})/1.67}}$
$= 1.08$ in.	$= 1.08$ in.
Or use AISC *Manual* Equation 14-7a:	Or use AISC *Manual* Equation 14-7b:
$t_{p,(req)} = l\sqrt{\dfrac{2P_u}{0.9F_yBN}}$	$t_{p,(req)} = l\sqrt{\dfrac{3.33P_a}{F_yBN}}$
$= 3.65\sqrt{\dfrac{2(240\text{ kips})}{0.9(36\text{ ksi})(13.0\text{ in.})(13.0\text{ in.})}}$	$= 3.65\sqrt{\dfrac{3.33(160\text{ kips})}{(36\text{ ksi})(13.0\text{ in.})(13.0\text{ in.})}}$
$= 1.08$ in.	$= 1.08$ in.

Therefore, use a 1¼ in.-thick base plate.

EXAMPLE K.13 RECTANGULAR HSS STRUT END PLATE

Given:

Determine the weld leg size, end plate thickness, and the size of ASTM A325 bolts required to resist forces of 16 kips from dead load and 50 kips from live load on an ASTM A500 Grade B HSS4×4×¼ section. The end plate is ASTM A36. Use 70-ksi weld electrodes.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Strut
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

End Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-12, the geometric properties are as follows:

HSS4×4×¼
t = 0.233 in.
A = 3.37 in.^{2}

From Chapter 2 of ASCE/SEI 7, the required tensile strength is:

LRFD	ASD
P_u = 1.2(16.0 kips) + 1.6(50.0 kips) = 99.2 kips	P_a = 16.0 kips + 50.0 kips = 66.0 kips

Preliminary Size of the (4) ASTM A325 Bolts

LRFD	ASD
$r_{ut} = \frac{P_u}{n}$ $= \frac{99.2\text{ kips}}{4}$ $= 24.8\text{ kips}$ Using AISC *Manual* Table 7-2, try ¾-in.-diameter ASTM A325 bolts. $\phi r_n = 29.8\text{ kips}$	$r_{at} = \frac{P_a}{n}$ $= \frac{66.0\text{ kips}}{4}$ $= 16.5\text{ kips}$ Using AISC *Manual* Table 7-2, try ¾-in.-diameter ASTM A325 bolts. $\frac{r_n}{\Omega} = 19.9\text{ kips}$

End-Plate Thickness with Consideration of Prying Action (AISC Manual Part 9)

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right)$$ (*Manual* Eq. 9-27)

$$= 1.50\text{ in.} + \frac{3/4\text{ in.}}{2} \le 1.25(1.50\text{ in.}) + \frac{3/4\text{ in.}}{2}$$

$$= 1.88\text{ in.} \le 2.25\text{ in.}$$ **o.k.**

$$b' = b - \frac{d_b}{2}$$ (*Manual* Eq. 9-21)

$$= 1.50\text{ in.} - \frac{3/4\text{ in.}}{2}$$

$$= 1.13\text{ in.}$$

$$\rho = \frac{b'}{a'}$$ (*Manual* Eq. 9-26)

$$= \frac{1.13}{1.88}$$

$$= 0.601$$

$d' = 13/16$ in.

The tributary length per bolt,

p = (full plate width)/(number of bolts per side)
= 10.0 in./1
= 10.0 in.

$$\delta = 1 - \frac{d'}{p}$$ (*Manual* Eq. 9-24)

$$= 1 - \frac{13/16\text{ in.}}{10.0\text{ in.}}$$

$$= 0.919$$

LRFD	ASD
$\beta = \dfrac{1}{\rho}\left(\dfrac{\phi r_n}{r_{ut}} - 1\right)$ (*Manual* Eq. 9-25) $= \dfrac{1}{0.601}\left(\dfrac{29.8\text{ kips}}{24.8\text{ kips}} - 1\right)$ $= 0.335$ $\alpha' = \dfrac{1}{\delta}\left(\dfrac{\beta}{1-\beta}\right)$ $= \dfrac{1}{0.919}\left(\dfrac{0.335}{1-0.335}\right)$ $= 0.548 \le 1.0$	$\beta = \dfrac{1}{\rho}\left(\dfrac{r_n/\Omega}{r_{at}} - 1\right)$ (*Manual* Eq. 9-25) $= \dfrac{1}{0.601}\left(\dfrac{19.9\text{ kips}}{16.5\text{ kips}} - 1\right)$ $= 0.343$ $\alpha' = \dfrac{1}{\delta}\left(\dfrac{\beta}{1-\beta}\right)$ $= \dfrac{1}{0.919}\left(\dfrac{0.343}{1-0.343}\right)$ $= 0.568 \le 1.0$

Use Equation 9-23 for t_{req} in Chapter 9 of the AISC *Manual*, except that F_u is replaced by F_y per recommendation of Willibald, Packer and Puthli (2003) and Packer et al. (2010).

LRFD	ASD
$t_{req} = \sqrt{\dfrac{4r_{ut}b'}{\phi pF_y(1+\delta\alpha')}}$ $= \sqrt{\dfrac{4(24.8\text{ kips})(1.13\text{ in.})}{0.9(10.0\text{ in.})(36\text{ ksi})(1+0.919(0.548))}}$ $= 0.480\text{ in.}$ Use a ½-in. end plate, $t_1 > 0.480$ in., further bolt check for prying not required. Use (4) ¾-in.-diameter A325 bolts.	$t_{req} = \sqrt{\dfrac{\Omega 4r_{at}b'}{pF_y(1+\delta\alpha')}}$ $= \sqrt{\dfrac{1.67(4)(16.5\text{ kips})(1.13\text{ in.})}{10.0\text{ in.}(36\text{ ksi})(1+0.919(0.568))}}$ $= 0.477\text{ in.}$ Use a ½-in. end plate, $t_1 > 0.477$ in., further bolt check for prying not required. Use (4) ¾-in.-diameter A325 bolts.

Required Weld Size

$$R_n = F_{nw}A_{we} \quad \text{(Spec. Eq. J2-4)}$$

$$F_{nw} = 0.60F_{EXX}\left(1.0 + 0.50\sin^{1.5}\theta\right) \quad \text{(Spec. Eq. J2-5)}$$
$$= 0.60(70\text{ ksi})\left(1.0 + 0.50\sin^{1.5}0.90°\right)$$
$$= 63.0\text{ ksi}$$

$$l = 4(4.00\text{ in.})$$
$$= 16.0\text{ in.}$$

Note: This weld length is approximate. A more accurate length could be determined by taking into account the curved corners of the HSS.

From AISC *Specification* Table J2.5:

LRFD	ASD
For shear load on fillet welds $\phi = 0.75$ $w \geq \dfrac{P_u}{\phi F_{nw}(0.707)l}$ from AISC *Manual* Part 8 $\geq \dfrac{99.2 \text{ kips}}{0.75(63.0 \text{ ksi})(0.707)(16.0 \text{ in.})}$ ≥ 0.186 in.	For shear load on fillet welds $\Omega = 2.00$ $w \geq \dfrac{\Omega P_a}{F_{nw}(0.707)l}$ from AISC *Manual* Part 8 $\geq \dfrac{2.00(66.0 \text{ kips})}{(63.0 \text{ ksi})(0.707)(16.0 \text{ in.})}$ ≥ 0.185 in.

Try w = 3⁄16 in. > 0.186 in.

Minimum Weld Size Requirements

For t = ¼ in., the minimum weld size = ⅛ in. from AISC *Specification* Table J2.4.

Results:

Use 3⁄16-in. weld with ½-in. end plate and (4) ¾-in.-diameter ASTM A325 bolts, as required for strength in the previous calculation.

CHAPTER K DESIGN EXAMPLE REFERENCES

Fisher, J.M. and Kloiber, L.A. (2006), *Base Plate and Anchor Rod Design,* Design Guide 1, 2nd Ed., AISC, Chicago, IL

Packer, J.A., Sherman, D. and Lecce, M. (2010), *Hollow Structural Section Connections*, Design Guide 24, AISC, Chicago, IL.

Willibald, S., Packer, J.A. and Puthli, R.S. (2003), "Design Recommendations for Bolted Rectangular HSS Flange Plate Connections in Axial Tension," *Engineering Journal*, AISC, Vol. 40, No. 1, 1st Quarter, pp. 15-24.

APPENDIX 6

STABILITY BRACING FOR COLUMNS AND BEAMS

This Appendix contains provisions for evaluating column and beam braces.

The governing limit states for column and beam design may include flexural, torsional and flexural-torsional buckling for columns and lateral-torsional buckling for beams. In the absence of other intermediate bracing, column unbraced lengths are defined between points of obviously adequate lateral restraint, such as floor and roof diaphragms which are part of the building's lateral force resisting systems. Similarly, beams are often braced against lateral-torsional buckling by relatively strong and stiff bracing elements such as a continuously connected floor slab or roof diaphragm. However, at times, unbraced lengths are bounded by elements that may or may not possess adequate strength and stiffness to provide sufficient bracing. AISC *Specification* Appendix 6 provides equations for determining the required strength and stiffness of braces that have not been included in the second-order analysis of the structural system. It is not intended that the provisions of Appendix 6 apply to bracing that is part of the lateral force resisting system.

Background for the provisions can be found in references cited in the Commentary including "Fundamentals of Beam Bracing" (Yura, 2001) and the *Guide to Stability Design Criteria for Metal Structures* (Ziemian, 2010). AISC *Manual* Part 2 also provides information on member stability bracing.

6.1 GENERAL PROVISIONS

Lateral column and beam bracing may be either relative or nodal while torsional beam bracing may be nodal or continuous. The User Note in AISC *Specification* Appendix 6, Section 6.1 states "A relative brace controls the movement of the brace point with respect to adjacent braced points. A nodal brace controls the movement at the braced point without direct interaction with adjacent braced points. A continuous bracing system consists of bracing that is attached along the entire member length;…." Relative and nodal bracing systems are discussed further in AISC *Specification* Commentary Appendix 6, Section 6.1. Examples of each are shown in the Commentary Figure C-A-6.1.

A rigorous second-order analysis, including initial out-of-straightness, may be used in lieu of the requirements of this appendix.

6.2 COLUMN BRACING

The requirements in this section apply to bracing associated with the limit state of flexural buckling. For columns that could experience torsional or flexural-torsional buckling, the designer must ensure that sufficient bracing to resist the torsional component of buckling is provided. See Helwig and Yura (1999).

Column braces may be relative or nodal. The type of bracing must be determined before the requirements for strength and stiffness can be determined. The requirements are derived for an infinite number of braces along the column and are thus conservative for most columns as explained in the Commentary. Provision is made in this section for reducing the required brace stiffness for nodal bracing when the column required strength is less than the available strength of the member. The Commentary also provides an approach to reduce the requirements when a finite number of nodal braces are provided.

6.3 BEAM BRACING

The requirements in this section apply to bracing associated with the limit state of lateral-torsional buckling. Bracing to resist lateral-torsional buckling may be accomplished by a *lateral brace*, or a *torsional brace*, or a combination of the two to prevent twist of the section. Lateral bracing should normally be connected near the compression flange. The exception is for cantilevers and near inflection points. Torsional bracing may be connected anywhere on the cross section in a manner to prevent twist of the section.

According to AISC *Specification* Section F1(2), the design of members for flexure is based on the assumption that points of support are restrained against rotation about their longitudinal axis. The bracing requirements in Appendix 6 are for intermediate braces in addition to those at the support.

In members subject to double curvature, inflection points are not to be considered as braced points unless bracing is provided at that location. In addition, the bracing nearest the inflection point must be attached to prevent twist, either as a torsional brace or as lateral braces attached to both flanges as described in AISC *Specification* Appendix 6, Section 6.3.1(2).

6.3.1 Lateral Bracing. As with column bracing, beam bracing may be relative or nodal. In addition, it is permissible to provide torsional bracing. This section provides requirements for determining the required lateral brace strength and stiffness for relative and nodal braces.

For nodal braces, provision is made in this section to reduce the required brace stiffness when the actual unbraced length is less than the maximum unbraced length for the required flexural strength.

The Commentary provides alternative equations for required brace strength and stiffness that may result in a significantly smaller required strength and stiffness due to the conservative nature of the requirements of this section.

6.3.2 Torsional Bracing. This section provides requirements for determining the required bracing flexural strength and stiffness for nodal and continuous torsional bracing. Torsional bracing can be connected to the section at any location. However, if the beam has inadequate distortional (out-of-plane) bending stiffness, torsional bracing will be ineffective. Web stiffeners can be provided when necessary, to increase the web distortional stiffness for nodal torsional braces.

As is the case for columns and for lateral beam nodal braces, it is possible to reduce the required brace stiffness when the required strength of the member is less than the available strength for the provided location of bracing.

Provisions for continuous torsional bracing are also provided. A slab connected to the top flange of a beam in double curvature may provide sufficient continuous torsional bracing as discussed in the Commentary. For this condition there is no unbraced length between braces so the unbraced length used in the strength and stiffness equations is the maximum unbraced length permitted to provide the required strength in the beam. In addition, for continuous torsional bracing, stiffeners are not permitted to be used to increase web distortional stiffness.

6.4 BEAM-COLUMN BRACING

For bracing of beam-columns, the required strength and stiffness are to be determined for the column and beam independently as specified in AISC *Specification* Appendix 6, Sections 6.2 and 6.3. These values are then to be combined, depending on the type of bracing provided.

APPENDIX 6 REFERENCES

Helwig, T. and Yura, J.A. (1999), "Torsional Bracing of Columns," *Journal of Structural Engineering*, ASCE, Vol. 125, No. 5, pp. 547–555.

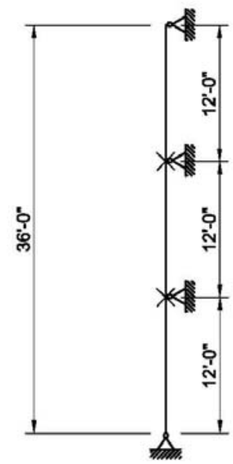
Yura, J.A. (2001), "Fundamentals of Beam Bracing," *Engineering Journal*, AISC, Vol. 38, No. 1, 1st Quarter, pp. 11–26.

Ziemian, R.D. (ed.) (2010), *Guide to Stability Design Criteria for Metal Structures,* 6th Ed., John Wiley & Sons, Inc., Hoboken, NJ.

EXAMPLE A-6.1 NODAL STABILITY BRACING OF A COLUMN

Given:

An ASTM A992 W12×72 column carries a dead load of 105 kips and a live load of 315 kips. The column is 36 ft long and is braced laterally and torsionally at its ends. Intermediate lateral braces for the *x*- and *y*-axis are provided at the one-third points as shown. Thus, the unbraced length for the limit state of flexural-torsional buckling is 36 ft and the unbraced length for flexural buckling is 12 ft. The column has sufficient strength to support the applied loads with this bracing. Find the strength and the stiffness requirements for the intermediate nodal braces, such that the unbraced length for the column can be taken as 12 ft.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Column
ASTM A992
$F_y = 50$ ksi

Required Compressive Strength

The required strengths from the governing load combinations are:

LRFD	ASD
LRFD Load Combination 2 from ASCE/SEI 7 Section 2.3.2 $P_u = 1.2(105 \text{ kips}) + 1.6(315 \text{ kips})$ $= 630 \text{ kips}$	ASD Load Combination 2 from ASCE/SEI 7 Section 2.4.1 $P_a = 105 \text{ kips} + 315 \text{ kips}$ $= 420 \text{ kips}$

From AISC *Manual* Table 4-1 at $KL_y = 12$ ft, the available strength of the W12×72 is:

LRFD	ASD
$\phi_c P_n = 806$ kips 806 kips > 630 kips **o.k.**	$\dfrac{P_n}{\Omega_c} = 536$ kips 536 kips > 420 kips **o.k.**

Calculate the required nodal brace strength

From AISC *Specification* Equation A-6-3, the required nodal brace strength is:

LRFD	ASD
$P_{rb} = 0.01P_u$ $= 0.01\text{ (630 kips)}$ $= 6.30\text{ kips}$	$P_{rb} = 0.01P_a$ $= 0.01\text{ (420 kips)}$ $= 4.20\text{ kips}$

Calculate the required nodal brace stiffness

From AISC *Specification* Equation A-6-4, the required nodal brace stiffness is:

LRFD	ASD
$\phi = 0.75$ $\beta_{br} = \dfrac{1}{\phi}\left(\dfrac{8P_u}{L_b}\right)$ $= \dfrac{1}{0.75}\left[\dfrac{8(630\text{ kips})}{(12\text{ ft})(12\text{ in./ft})}\right]$ $= 46.7\text{ kip/in.}$	$\Omega = 2.00$ $\beta_{br} = \Omega\left(\dfrac{8P_a}{L_b}\right)$ $= 2.00\left[\dfrac{8(420\text{ kips})}{(12\text{ ft})(12\text{ in./ft})}\right]$ $= 46.7\text{ kip/in.}$

Determine the maximum permitted unbraced length for the required strength

From AISC *Manual* Table 4-1:

LRFD	ASD
$KL = 18.9$ ft for $P_u = 632$ kips	$KL = 18.9$ ft for $P_a = 421$ kips

Calculate the required nodal brace stiffness for this increased unbraced length

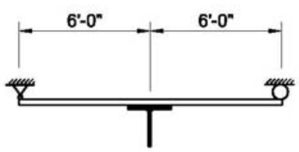
It is permissible to size the braces to provide the lower stiffness determined using the maximum unbraced length permitted to carry the required strength according to AISC *Specification* Appendix 6, Section 6.2.2. From AISC *Specification* Equation A-6-4:

LRFD	ASD
$\phi = 0.75$ (LRFD) $\beta_{br} = \dfrac{1}{\phi}\left(\dfrac{8P_u}{L_b}\right)$ $= \dfrac{1}{0.75}\left[\dfrac{8(630\text{ kips})}{(18.9\text{ ft})(12\text{ in./ft})}\right]$ $= 29.6\text{ kip/in.}$	$\Omega = 2.00$ (ASD) $\beta_{br} = \Omega\left(\dfrac{8P_a}{L_b}\right)$ $= 2.00\left(\dfrac{8(420\text{ kips})}{(18.9\text{ ft})(12\text{ in./ft})}\right)$ $= 29.6\text{ kip/in.}$

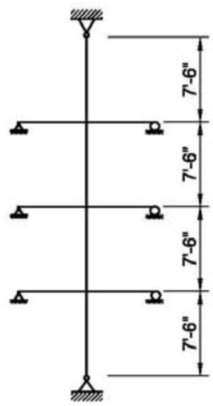
EXAMPLE A-6.2 NODAL STABILITY BRACING OF A COLUMN

Given:

An ASTM A992 WT7×34 column carries a dead load of 25 kips and a live load of 75 kips. The column is 30 ft long as shown. The unbraced length for this column is 7.5 ft. Bracing about the *y*-axis is provided by the axial resistance of a W-shape connected to the flange of the WT, while bracing about the *x*-axis is provided by the flexural resistance of the same W-shape loaded at the midpoint of a 12-ft simple span. Find the strength and stiffness requirements for the nodal braces and select an appropriate W-shape, based on *x*-axis flexural buckling of the WT. Assume that the axial strength and stiffness of the W-shape are adequate to brace the *y*-axis of the WT. Also, assume the column is braced laterally and torsionally at its ends and torsionally at each brace point.



Plan



Elevation

Solution:

This column is braced at each end by the supports and at the one-quarter points by a W-shape as shown.

From AISC *Manual* Table 2-4, the material properties are as follows:

Column and brace
ASTM A992
F_y =50 ksi
F_u =65 ksi

Required Compressive Strength

The required strength is determined from the following governing load combinations:

LRFD	ASD
LRFD Load Combination 2 from ASCE/SEI 7 Section 2.3.2 $P_u = 1.2(25 \text{ kips}) + 1.6(75 \text{ kips})$ $= 150 \text{ kips}$	ASD Load Combination 2 from ASCE/SEI 7 Section 2.4.1 $P_a = 25 \text{ kips} + 75 \text{ kips}$ $= 100 \text{ kips}$

Available Compressive Strength

From AISC *Manual* Table 4-7 at KL_x = 7.5 ft, the available axial compressive strength of the WT7×34 is:

LRFD		ASD	
$\phi_c P_n$ = 357 kips		P_n/Ω_c = 238 kips	
357 kips > 150 kips	**o.k.**	238 kips > 100 kips	**o.k.**

Required Nodal Brace Size

The required nodal brace strength from AISC *Specification* Equation A-6-3 is:

LRFD	ASD
$P_{rb} = 0.01P_u$ $= 0.01(150\text{ kips})$ $= 1.50\text{ kips}$	$P_{rb} = 0.01P_a$ $= 0.01(100\text{ kips})$ $= 1.00\text{ kips}$

The required nodal brace stiffness from AISC *Specification* Equation A-6-4 is:

LRFD	ASD
$\phi = 0.75$ (LRFD) $\beta_{br} = \frac{1}{\phi}\left(\frac{8P_u}{L_b}\right)$ $= \frac{1}{0.75}\left[\frac{8(150\text{ kips})}{(7.50\text{ ft})(12\text{ in./ft})}\right]$ $= 17.8\text{ kip/in.}$	$\Omega = 2.00$ (ASD) $\beta_{br} = \Omega\left(\frac{8P_a}{L_b}\right)$ $= 2.00\left[\frac{8(100\text{ kips})}{(7.50\text{ ft})(12\text{ in./ft})}\right]$ $= 17.8\text{ kip/in.}$

The brace is a beam loaded in bending at its midspan. Thus, its flexural stiffness can be determined from Case 7 of AISC *Manual* Table 3-23 to be $48EI/L^3$, which must be greater than the required nodal brace stiffness, β_{br}. Also, the flexural strength of the beam, $\phi_b M_p$, for a compact laterally supported beam, must be greater than the moment resulting from the required brace strength over the beam's simple span, $M_{rb} = P_{rb}L/4$.

LRFD	ASD
Based on brace stiffness, the minimum required moment of inertia of the beam is: $I_{br} = \frac{\beta_{br}L^3}{48E}$ $= \frac{(17.8\text{ kip/in.})(12.0\text{ ft})^3(12\text{ in./ft})^3}{48(29,000\text{ ksi})}$ $= 38.2\text{ in.}^4$ Based on moment strength for a compact laterally supported beam, the minimum required plastic section modulus is:	Based on brace stiffness, the minimum required moment of inertia of the beam is: $I_{br} = \frac{\beta_{br}L^3}{48E}$ $= \frac{17.8\text{ kip/in.}(12.0\text{ ft})^3(12\text{ in./ft})^3}{48(29,000\text{ ksi})}$ $= 38.2\text{ in.}^4$ Based on moment strength for a compact laterally supported beam, the minimum required plastic section modulus is:

$Z_{req} = \dfrac{M_{rb}}{\phi F_y}$ $= \dfrac{(1.50\text{ kips})(12.0\text{ ft})(12\text{ in./ft})/4}{0.90(50\text{ ksi})}$ $= 1.20\text{ in.}^3$ Select a W8×13 compact member with $I_x = 39.6\text{ in.}^4$ $Z_x = 11.4\text{ in.}^3$	$Z_{req} = \dfrac{\Omega M_{rb}}{F_y}$ $= \dfrac{1.67(1.00\text{ kips})(12.0\text{ ft})(12\text{ in./ft})/4}{50\text{ ksi}}$ $= 1.20\text{ in.}^3$ Select a W8×13 compact member with $I_x = 39.6\text{ in.}^4$ $Z_x = 11.4\text{ in.}^3$

Note that because the live-to-dead load ratio is 3, the LRFD and ASD results are identical.

The required stiffness can be reduced if the maximum permitted unbraced length is used as described in AISC *Specification* Appendix 6, Section 6.2, and also if the actual number of braces are considered, as discussed in the commentary. The following demonstrates how this affects the design.

From AISC *Manual* Table 4-7, the maximum permitted unbraced length for the required strength is as follows:

LRFD	ASD
$KL = 18.6$ ft for $P_u = 150$ kips	$KL = 18.6$ ft for $P_a = 100$ kips

From AISC *Specification* Commentary Equation C-A-6-4, determine the reduction factor for three intermediate braces:

LRFD	ASD
$\dfrac{2n-1}{2n} = \dfrac{2(3)-1}{2(3)}$ $= 0.833$	$\dfrac{2n-1}{2n} = \dfrac{2(3)-1}{2(3)}$ $= 0.833$

Determine the required nodal brace stiffness for the increased unbraced length and number of braces using AISC *Specification* Equation A-6-4.

LRFD	ASD
$\phi = 0.75$ $\beta_{br} = 0.833\left[\dfrac{1}{\phi}\left(\dfrac{8P_u}{L_b}\right)\right]$ $= 0.833\left\{\dfrac{1}{0.75}\left[\dfrac{8(150\text{ kips})}{(18.6\text{ ft})(12\text{ in./ft})}\right]\right\}$ $= 5.97\text{ kip/in.}$	$\Omega = 2.00$ $\beta_{br} = 0.833\left[\Omega\left(\dfrac{8P_a}{L_b}\right)\right]$ $= 0.833\left\{2.00\left[\dfrac{8(100\text{ kips})}{(18.6\text{ ft})(12\text{ in./ft})}\right]\right\}$ $= 5.97\text{ kip/in.}$

Determine the required brace size based on this new stiffness requirement.

LRFD	ASD
Based on brace stiffness, the minimum required moment of inertia of the beam is:	Based on brace stiffness, the minimum required moment of inertia of the beam is:

$$I_{br} \geq \frac{\beta_{br} L^3}{48E}$$

$$\geq \frac{(5.97\text{ kip/in.})(12.0\text{ ft})^3(12\text{ in./ft})^3}{48(29{,}000\text{ ksi})}$$

$$\geq 12.8\text{ in.}^4$$

Based on the unchanged flexural strength for a compact laterally supported beam, the minimum required plastic section modulus is:

$$Z_{req} = \frac{M_{rb}}{\phi F_y}$$

$$= \frac{1.50\text{ kips}(12.0\text{ ft})(12\text{ in./ft})/4}{0.90(50\text{ ksi})}$$

$$= 1.20\text{ in.}^3$$

Select a W6×8.5 noncompact member with $I_x = 14.9$ in.^{4} and $Z_x = 5.73$ in.^{4}

$$I_{br} \geq \frac{\beta_{br} L^3}{48E}$$

$$\geq \frac{(5.97\text{ kip/in.})(12.0\text{ ft})^3(12\text{ in./ft})^3}{48(29{,}000\text{ ksi})}$$

$$\geq 12.8\text{ in.}^4$$

Based on the unchanged flexural strength for a compact laterally supported beam, the minimum required plastic section modulus is:

$$Z_{req} = \frac{\Omega M_{rb}}{F_y}$$

$$= \frac{1.67(1.00\text{ kips})(12.0\text{ ft})(12\text{ in./ft})/4}{50\text{ ksi}}$$

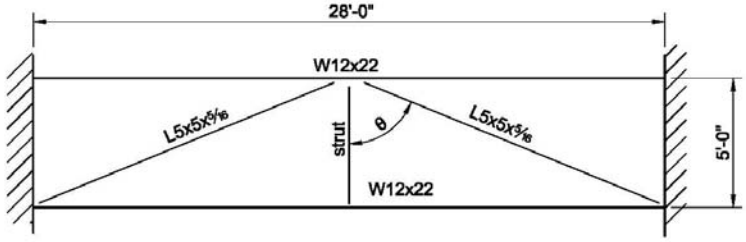
$$= 1.20\text{ in.}^3$$

Select a W6×8.5 noncompact member with $I_x = 14.9$ in.^{4} and $Z_x = 5.73$ in.^{4}

EXAMPLE A-6.3 NODAL STABILITY BRACING OF A BEAM—CASE I

Given:

A walkway in an industrial facility has a span of 28 ft as shown in the given plan view. The walkway has a deck of grating which is not sufficient to brace the beams. The ASTM A992 W12×22 beams along walkway edges are braced against twist at the ends as required by AISC *Specification* Section F1(2) and are connected by an L3x3x¼ strut at midspan. The two diagonal ASTM A36 L5x5x5⁄16 braces are connected to the top flange of the beams at the supports and at the strut at the middle. The strut and the brace connections are welded; therefore bolt slippage does not need to be accounted for in the stiffness calculation. The dead load on each beam is 0.0500 kip/ft and the live load is 0.125 kip/ft. Determine if the diagonal braces are strong enough and stiff enough to brace this walkway.



Plan View

Solution:

Because the diagonal braces are connected directly to an unyielding support that is independent of the midspan brace point, they are designed as nodal braces. The strut will be assumed to be sufficiently strong and stiff to force the two beams to buckle together.

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Diagonal braces
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-7, the geometric properties are as follows:

Beam
W12×22
h_o = 11.9 in.

Diagonal braces
L5×5×5⁄16
A = 3.07 in.^{2}

Required Strength

Determine the required strength for each beam using the governing load combination.

LRFD	ASD
From LRFD Load Combination 2 of ASCE/SEI 7 Section 2.3.2 w_u = 1.2(0.0500 kip/ft) +1.6(0.125 kip/ft) = 0.260 kip/ft	From ASD Load Combination 2 of ASCE/SEI 7 Section 2.4.1 w_a = 0.0500 kip/ft + 0.125 kip/ft = 0.175 kip/ft

Determine the required flexural strength for a uniformly loaded simply supported beam.

LRFD	ASD
M_u = (0.260 kip/ft)(28.0 ft)^2/8 = 25.5 kip-ft	M_a = (0.175 kip/ft)(28.0 ft)^2/8 = 17.2 kip-ft

It can be shown that the W12×22 beams are adequate with the unbraced length of 14.0 ft. Both beams need bracing in the same direction simultaneously.

Required Brace Strength and Stiffness

From AISC *Specification* Appendix 6, Section 6.3, and Equation A-6-7, determine the required nodal brace strength for each beam as follows:

LRFD	ASD
$P_{rb} = 0.02 M_u C_d / h_o$ C_d = 1.0 for bending in single curvature $P_{rb} = 0.02(25.5 \text{ kip-ft})(12 \text{ in./ft})(1.0)/(11.9 \text{ in.})$ $= 0.514$ kips	$P_{rb} = 0.02 M_a C_d / h_o$ C_d = 1.0 for bending in single curvature $P_{rb} = 0.02(17.2 \text{ kip-ft})(12 \text{ in./ft})(1.0)/(11.9 \text{ in.})$ $= 0.347$ kips

Because there are two beams to be braced, the total required brace strength is:

LRFD	ASD
P_{rb} = 2(0.514 kips) = 1.03 kips	P_{rb} = 2(0.347 kips) = 0.694 kips

There are two beams to brace and two braces to share the load. The worst case for design of the braces will be when they are in compression.

By geometry, the diagonal bracing length is $l = \sqrt{(14.0 \text{ ft})^2 + (5.00 \text{ ft})^2}$ = 14.9 ft, and the required brace strength is:

LRFD	ASD
$F_b \cos\theta = F_b (5.00 \text{ ft}/14.9 \text{ ft})$ $= 1.03$ kips Because there are two braces, the required brace strength is: $F_b = \dfrac{1.03 \text{ kips}}{2(5.00 \text{ ft}/14.9 \text{ ft})}$ $= 1.53$ kips	$F_b \cos\theta = F_b (5.00 \text{ ft}/14.9 \text{ ft})$ $= 0.694$ kips Because there are two braces, the required brace strength is: $F_b = \dfrac{0.694 \text{ kips}}{2(5.00 \text{ ft}/14.9 \text{ ft})}$ $= 1.03$

The required nodal brace stiffness is determined from AISC *Specification* Equation A-6-8 as follows:

LRFD	ASD
$\phi = 0.75$ $\beta_{br} = \frac{1}{\phi}\left(\frac{10M_u C_d}{L_b h_o}\right)$ $= \frac{1}{0.75}\left[\frac{10(25.5\text{ kip-ft})(12\text{ in./ft})(1.0)}{(14.0\text{ ft})(12\text{ in./ft})(11.9\text{ in.})}\right]$ $= 2.04\text{ kip/in.}$	$\Omega = 2.00$ $\beta_{br} = \Omega\left(\frac{10M_a C_d}{L_b h_o}\right)$ $= 2.00\left[\frac{10(17.2\text{ kip-ft})(12\text{ in./ft})(1.0)}{(14.0\text{ ft})(12\text{ in./ft})(11.9\text{ in.})}\right]$ $= 2.06\text{ kip/in.}$

Because there are two beams to be braced, the total required nodal brace stiffness is:

LRFD	ASD
$\beta_{br} = 2(2.04\text{ kip/in.})$ $= 4.08\text{ kip/in.}$	$\beta_{br} = 2(2.06\text{ kip/in.})$ $= 4.12\text{ kip/in.}$

The beams require bracing in order to have sufficient strength to carry the given load. However, locating that brace at the midspan provides flexural strength greater than the required strength. The maximum unbraced length permitted for the required flexural strength is 17.8 ft from AISC *Specification* Section F2. Thus, according to AISC *Specification* Appendix 6, Section 6.3.1b, this length could be used in place of 14.0 ft to determine the required stiffness. However, because the required stiffness is so small, the 14.0 ft length will be used here.

For a single brace located as shown previously, the stiffness is:

$$\beta = \frac{AE\cos^2\theta}{L}$$
$$= \frac{(3.07\text{ in.}^2)(29{,}000\text{ ksi})(5.00\text{ ft}/14.9\text{ ft})^2}{(14.9\text{ ft})(12\text{ in./ft})}$$
$$= 56.1\text{ kip/in.}$$

Because there are two braces, the system stiffness is twice this. Thus,

$$\beta = 2(56.1\text{ kip/in.})$$
$$= 112\text{ kip/in.}$$

Determine if the braces provide sufficient stiffness.

LRFD	ASD
$\beta = 112\text{ kip/in.} > 4.08\text{ kip/in.}$ **o.k.**	$\beta = 112\text{ kip/in.} > 4.12\text{ kip/in.}$ **o.k.**

Available Strength of Braces

The braces may be called upon to act in either tension or compression, depending on which transverse direction the system tries to buckle. Brace compression buckling will control over tension yielding. Therefore, determine the compressive strength of the braces assuming they are eccentrically loaded using AISC *Manual* Table 4-12.

LRFD	ASD

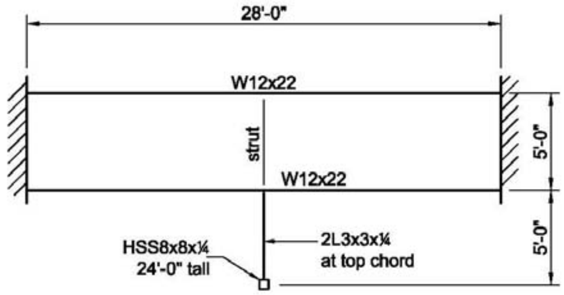
For KL = 14.9 ft, ϕP_n = 17.2 kips 1.53 kips < 17.2 kips **o.k.** Therefore the L5x5x5⁄16 has sufficient strength.	For KL = 14.9 ft, $\dfrac{P_n}{\Omega}$ = 11.2 kips 1.03 kips <11.2 kips **o.k.** Therefore the L5x5x5⁄16 has sufficient strength.

The L5x5x5⁄16 braces have sufficient strength and stiffness to act as the nodal braces for this system.

EXAMPLE A-6.4 NODAL STABILITY BRACING OF A BEAM—CASE II

Given:

A walkway in an industrial facility has a span of 28 ft as shown. The walkway has a deck of grating which is not sufficient to brace the beams. The ASTM A992 W12×22 beams are braced against twist at the ends, and they are connected by a strut connected at midspan. At that same point they are braced to an adjacent ASTM A500 Grade B HSS 8×8×¼ column by the attachment of a 5-ft-long ASTM A36 2L3x3x¼. The brace connections are all welded; therefore, bolt slippage does not need to be accounted for in the stiffness calculation. The adjacent column is not braced at the walkway level, but is adequately braced 12 ft below and 12 ft above the walkway level. The dead load on each beam is 0.050 kip/ft and the live load is 0.125 kip/ft. Determine if the bracing system has adequate strength and stiffness to brace this walkway.



Solution:

Because the bracing system does not interact directly with any other braced point on the beam, the double angle and column constitute a nodal brace system. The strut will be assumed to be sufficiently strong and stiff to force the two beams to buckle together.

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

HSS column
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Double-angle brace
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 1-7, the geometric properties are as follows:

Beam
W12×22
h_o = 11.9 in.

HSS column
HSS8×8×¼

I = 70.7 in.^{4}

Double-angle brace
2L3x3x4
A = 2.88 in.^{2}

Required Strength

Determine the required strength in kip/ft for each beam using the governing load combination.

LRFD	ASD
From LRFD Load Combination 2 of ASCE/SEI 7 Section 2.3.2 w_u = 1.2(0.0500 kip/ft) + 1.6(0.125 kip/ft) = 0.260 kip/ft	From ASD Load Combination 2 of ASCE/SEI 7 Section 2.4.1 w_a = 0.0500 kip/ft + 0.125 kip/ft = 0.175 kip/ft

Determine the required flexural strength for a uniformly distributed load on the simply supported beam as follows:

LRFD	ASD
M_u = (0.260 kip/ft)(28 ft)^2/8 = 25.5 kip-ft	M_a = (0.175 kips/ft)(28 ft)^2/8 = 17.2 kip-ft

It can be shown that the W12x22 beams are adequate with this unbraced length of 14.0 ft. Both beams need bracing in the same direction simultaneously.

Required Brace Strength and Stiffness

The required brace force for each beam is determined from AISC *Specification* Equation A-6-7 as follows:

LRFD	ASD
$P_{rb} = 0.02M_uC_d / h_o$ C_d = 1.0 for bending in single curvature. P_{rb} = 0.02(25.5 kip-ft)(12 in./ft)(1.0)/ (11.9 in.) = 0.514 kips	$P_{rb} = 0.02M_aC_d / h_o$ C_d = 1.0 for bending in single curvature. P_{rb} = 0.02(17.2 kip-ft)(12 in./ft)(1.0)/ (11.9 in.) = 0.347 kips

Because there are two beams, the total required brace force is:

LRFD	ASD
P_{rb} = 2(0.514 kips) = 1.03 kips	P_{rb} = 2(0.347 kips) = 0.694 kips

By inspection, the 2L3×3×¼ can carry the required bracing force. The HSS column can also carry the bracing force through bending on a 24-ft-long span. It will be shown that the change in length of the 2L3×3×¼ is negligible, so the available brace stiffness will come from the flexural stiffness of the column only.

Determine the required brace stiffness using AISC *Specification* Equation A-6-8.

LRFD	ASD
ϕ = 0.75 (LRFD)	Ω = 2.00 (ASD)

LRFD	ASD
$\beta_{br} = \frac{1}{\phi}\left(\frac{10M_uC_d}{L_bh_o}\right)$ $= \frac{1}{0.75}\left[\frac{10(25.5\text{ kip-ft})(12\text{ in./ft})(1.0)}{(14.0\text{ ft})(12\text{ in./ft})(11.9\text{ in.})}\right]$ $= 2.04\text{ kip/in.}$	$\beta_{br} = \Omega\left(\frac{10M_aC_d}{L_bh_o}\right)$ $= 2.00\left[\frac{10(17.2\text{ kip-ft})(12\text{ in./ft})(1.0)}{(14.0\text{ ft})(12\text{ in./ft})(11.9\text{ in.})}\right]$ $= 2.06\text{ kip/in.}$

The beams require one brace in order to have sufficient strength to carry the given load. However, locating that brace at midspan provides flexural strength greater than the required strength. The maximum unbraced length permitted for the required flexural strength is 17.8 ft from AISC *Specification* Section F2. Thus, according to AISC *Specification* Appendix 6, Section 6.3.1b, this length could be used in place of 14.0 ft to determine the required stiffness.

Available Stiffness

Because the brace stiffness comes from the combination of the axial stiffness of the double-angle member and the flexural stiffness of the column loaded at its midheight, the individual element stiffness will be determined and then combined.

The axial stiffness of the double angle is:

$$\begin{aligned}\beta &= \frac{AE}{L}\\ &= \frac{2.88\text{ in.}^2(29{,}000\text{ ksi})}{(5.00\text{ ft})(12\text{ in./ft})}\\ &= 1{,}390\text{ kip/in.}\end{aligned}$$

The available flexural stiffness of the HSS column with a point load at midspan is:

$$\begin{aligned}\beta &= \frac{48EI}{L^3}\\ &= \frac{48(29{,}000\text{ ksi})(70.7\text{ in.}^4)}{(24.0\text{ ft})^3(12\text{ in./ft})^3}\\ &= 4.12\text{ kip/in.}\end{aligned}$$

The combined stiffness is:

$$\begin{aligned}\frac{1}{\beta} &= \frac{1}{\beta_{angles}} + \frac{1}{\beta_{column}}\\ &= \frac{1}{1{,}390\text{ kip/in.}} + \frac{1}{4.12\text{ kip/in.}}\\ &= 0.243\text{ in./kip}\end{aligned}$$

Thus, the system stiffness is:

$$\beta = 4.12\text{ kip/in.}$$

The stiffness of the double-angle member could have reasonably been ignored.

Because the double-angle brace is ultimately bracing two beams, the required stiffness is multiplied by 2:

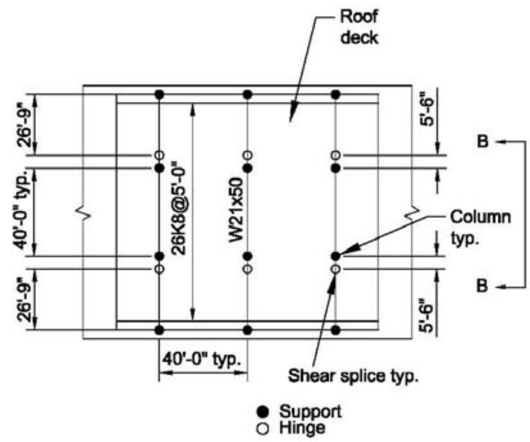
LRFD	ASD
4.12 kip/in. < (2.04 kip/in.)2 = 4.08 kip/in. **o.k.** The HSS8×8×¼ column is an adequate brace for the beams. However, if the column also carries an axial force, it must be checked for combined forces.	4.12 kip/in. < (2.06 kip/in.)2 = 4.12 kip/in. **o.k.** The HSS8×8×¼ column is an adequate brace for the beams. However, if the column also carries an axial force, it must be checked for combined forces.

Return to Table of Contents

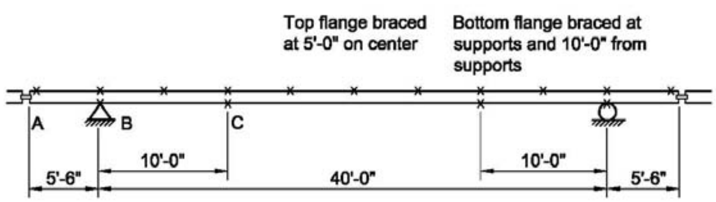
EXAMPLE A-6.5 NODAL STABILITY BRACING OF A BEAM WITH REVERSE CURVATURE BENDING

Given:

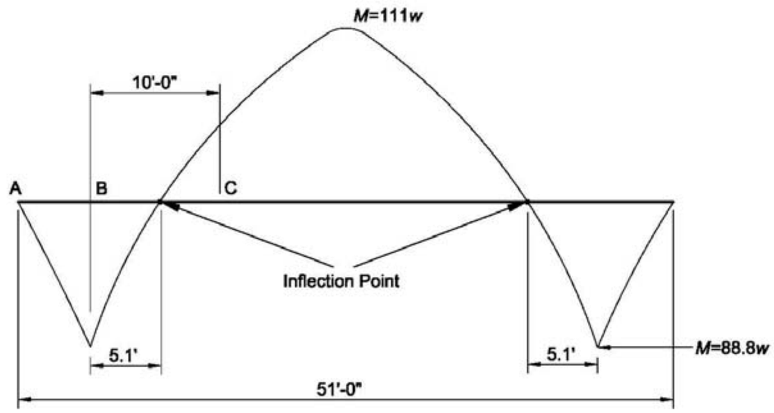
A roof system is composed of 26K8 steel joists spaced at 5-ft intervals and supported on ASTM A992 W21×50 girders as shown in Figure A-6.5-1. The roof dead load is 33 psf and the roof live load is 25 psf. Determine the required strength and stiffness of the braces needed to brace the girder at the support and near the inflection point. Determine the size of single-angle kickers connected to the bottom flange of the girder and the top chord of the joist, as shown, where the brace force will be taken by a connected rigid diaphragm.



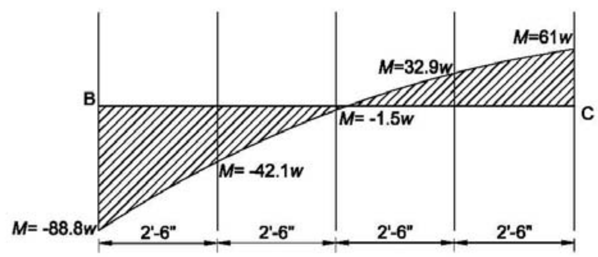
(a) Plan



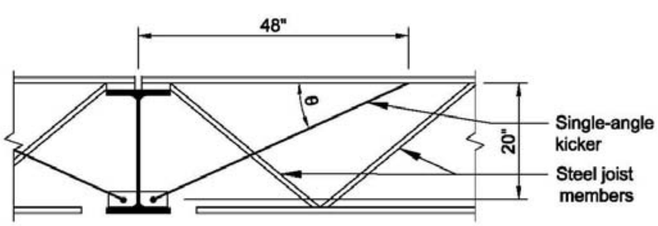
(b) Section B-B: Beam with bracing and moment diagram



(c) Moment diagram of beam



(d) Moment diagram between points B and C



(e) Bracing configuration

Fig. A-6.5-1. Example A-6.5 configuration.

Solution:

Since the braces will transfer their force to a rigid roof diaphragm, they will be treated as nodal braces.

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Single-angle brace
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From the Steel Joist Institute:

Joist
K-Series
$F_y = 50$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W21×50
$h_o = 20.3$ in.

Required Flexural Strength

The required flexural strength of the beam is determined as follows.

LRFD	ASD
From LRFD Load Combination 2 of ASCE/SEI 7 Section 2.3.2 $w_u = 1.2(33.0 \text{ psf}) + 1.6(25.0 \text{ psf})$ $= 79.6$ psf $w_u = 79.6 \text{ psf } (40.0 \text{ ft})/(1{,}000 \text{ lb/kip})$ $= 3.18$ kip/ft From Figure A-6.5-1(d), $M_{uB} = 88.8\ (3.18 \text{ kip/ft})$ $= 282$ kip-ft	From ASD Load Combination 2 of ASCE/SEI 7 Section 2.4.1 $w_a = 33.0 \text{ psf} + 25.0 \text{ psf}$ $= 58.0$ psf $w_a = 58.0 \text{ psf } (40.0 \text{ ft})/(1{,}000 \text{ lb/kip})$ $= 2.32$ kip/ft From Figure A-6.5-1(d), $M_{aB} = 88.8\ (2.32 \text{ kip/ft})$ $= 206$ kip-ft

Required Brace Strength and Stiffness

Determine the required force to brace the bottom flange of the girder with a nodal brace. The braces at points B and C will be determined based on the moment at B. However, because the brace at C is the closest to the inflection point, its strength and stiffness requirements are greater since they are influenced by the variable C_d which will be equal to 2.0.

The required brace force is determined from AISC *Specification* Equation A-6-7 as follows:

LRFD	ASD
$P_{rb} = 0.02M_rC_d/h_o$ $= 0.02(282 \text{ kip-ft})(12 \text{ in./ft})(2.0)/(20.3 \text{ in.})$ $= 6.67$ kips	$P_{rb} = 0.02M_rC_d/h_o$ $= 0.02(206 \text{ kip-ft})(12 \text{ in./ft})(2.0)/(20.3 \text{ in.})$ $= 4.87$ kips

Determine the required stiffness of the nodal brace at point C. The required brace stiffness is a function of the unbraced length. It is permitted to use the maximum unbraced length permitted for the beam based upon the required flexural strength. Thus, determine the maximum unbraced length permitted.

Based on AISC *Specification* Section F1, Equation F1-1, and the moment diagram shown in Figure A-6-6(d), for the beam between points B and C, the lateral-torsional buckling modification factor, C_b, is:

LRFD	ASD
$C_b = \dfrac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$ $= \dfrac{12.5\lvert -88.8w \rvert}{2.5\lvert -88.8w \rvert + 3\lvert -42.1w \rvert + 4\lvert -1.5w \rvert + 3\lvert 32.9w \rvert}$ $= 2.45$	$C_b = \dfrac{12.5M_{max}}{2.5M_{max} + 3M_A + 4M_B + 3M_C}$ $= \dfrac{12.5\lvert -88.8w \rvert}{2.5\lvert -88.8w \rvert + 3\lvert -42.1w \rvert + 4\lvert -1.5w \rvert + 3\lvert 32.9w \rvert}$ $= 2.45$

The maximum unbraced length for the required flexural strength can be determined by setting the available flexural strength based on AISC *Specification* Equation F2-3 (lateral-torsional buckling) equal to the required strength and solving for L_b (this is assuming that $L_b > L_r$).

LRFD	ASD
For a required flexural strength, M_u = 282 kip-ft, with C_b = 2.45, the unbraced length may be taken as L_b = 22.0 ft	For a required flexural strength, M_a = 206 kip-ft, with C_b = 2.45, the unbraced length may be taken as L_b = 20.6 ft

From AISC *Specification* Appendix 6, Section 6.3.1b, Equation A-6-8:

LRFD	ASD
$\phi = 0.75$ $\beta_{br} = \dfrac{1}{\phi}\left(\dfrac{10M_uC_d}{L_bh_o}\right)$ $= \dfrac{1}{0.75}\left[\dfrac{10(282 \text{ kip-ft})(12 \text{ in./ft})(2.0)}{(22.0 \text{ ft})(12 \text{ in./ft})(20.3 \text{ in.})}\right]$ $= 16.8 \text{ kip/in.}$	$\Omega = 2.00$ $\beta_{br} = \Omega\left(\dfrac{10M_aC_d}{L_bh_o}\right)$ $= 2.00\left[\dfrac{10(206 \text{ kip-ft})(12 \text{ in./ft})(2.0)}{(20.6 \text{ ft})(12 \text{ in./ft})(20.3 \text{ in.})}\right]$ $= 19.7 \text{ kip/in.}$

Because no deformation will be considered in the connections, only the brace itself will be used to provide the required stiffness. The brace is oriented with the geometry as shown in Figure A-6.5-1(e). Thus, the force in the brace is $F_{br} = P_{br}/(\cos\theta)$ and the stiffness of the brace is $AE(\cos^2\theta)/L$. There are two braces at each brace point. One would be in tension and one in compression, depending on the direction that the girder attempts to buckle. For simplicity in design, a single brace will be selected that will be assumed to be in tension. Only the limit state of yielding will be considered.

Select a single angle to meet the requirements of strength and stiffness, with a length of:

$$\sqrt{(48.0 \text{ in.})^2 + (20.0 \text{ in.})^2} = 52 \text{ in.}$$

LRFD	ASD
Required brace force $F_{br} = P_{rb}/(\cos\theta)$ $= (6.67 \text{ kips})/(48.0 \text{ ft}/52.0 \text{ ft})$ $= 7.23 \text{ kips}$	Required brace force $F_{br} = P_{rb}/(\cos\theta)$ $= (4.87 \text{ kips})/(48.0 \text{ ft}/52.0 \text{ ft})$ $= 5.28 \text{ kips}$

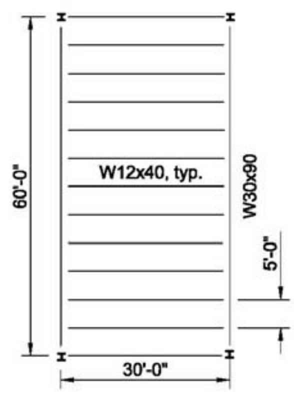
From AISC *Specification* Section D2(a), the required area based on available tensile strength is determined as follows: $A_g = F_{br} / \phi F_y$ $= 7.23\text{ kips}/[0.90(36\text{ ksi})]$ $= 0.223\text{ in.}^2$ The required area based on stiffness is: $A_g = \dfrac{\beta_{br} L}{E\cos^2\theta}$ $= \dfrac{(16.8\text{ kip/in.})(52.0\text{ in.})}{(29{,}000\text{ ksi})(48.0\text{ ft}/52.0\text{ ft})^2}$ $= 0.0354\text{ in.}^2$ The strength requirement controls, therefore select L2×2×⅛ with $A = 0.491\text{ in.}^2$	From AISC *Specification* Section D2(a), the required area based on available tensile strength is determined as follows: $A_g = \Omega F_{br} / F_y$ $= 1.67(5.28\text{ kips})/(36\text{ ksi})$ $= 0.245\text{ in.}^2$ The required area based on stiffness is: $A_g = \dfrac{\beta_{br} L}{E\cos^2\theta}$ $= \dfrac{(19.7\text{ kip/in.})(52.0\text{ in.})}{(29{,}000\text{ ksi})(48.0\text{ in.}/52.0\text{ ft})^2}$ $= 0.0415\text{ in.}^2$ The strength requirement controls, therefore select L2×2×⅛ with $A = 0.491\text{ in.}^2$

At the column at point B, the required strength would be one-half of that at point C, because $C_d = 1.0$ at point B instead of 2.0. However, since the smallest angle available has been selected for the brace, there is no reason to check further at the column and the same angle will be used there.

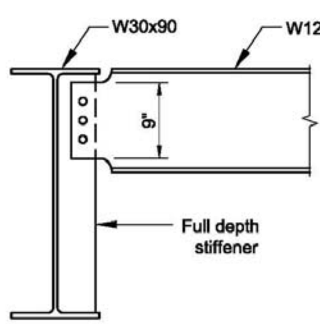
EXAMPLE A-6.6 NODAL TORSIONAL STABILITY BRACING OF A BEAM

Given:

A roof system is composed of W12×40 intermediate beams spaced 5 ft on center supporting a connected panel roof system that cannot be used as a diaphragm. The beams span 30 ft and are supported on W30×90 girders spanning 60 ft. This is an isolated roof structure with no connections to other structures that could provide lateral support to the girder compression flanges. Thus, the flexural resistance of the attached beams must be used to provide torsional stability bracing of the girders. The roof dead load is 40 psf and the roof live load is 24 psf. Determine if the beams are sufficient to provide nodal torsional stability bracing.



(a) Plan



(b) Nodal torsional brace connection

Fig. A-6.6-1. Roof system configuration for Example A-6.6.

Solution:

Because the bracing beams are not connected in a way that would permit them to transfer an axial bracing force, they must behave as nodal torsional braces if they are to effectively brace the girders.

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam and girder
ASTM A992
F_y = 50 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W12×40
t_w = 0.295 in.
I_x = 307 in.^{4}

Girder
W30×90
t_w = 0.470 in.
h_o = 28.9 in.
I_y = 115 in.^{4}

Required Flexural Strength

The required flexural strength of the girder is:

LRFD	ASD
From LRFD Load Combination 3 of ASCE/SEI 7 Section 2.3.2 w_u = 1.2(40 psf)+1.6(24 psf) = 86.4 psf w_u = 86.4 psf (15 ft)/(1,000 lb/kip) = 1.30 kip/ft M_u = (1.30 kip/ft)(60.0 ft)^2/8 = 585 kip-ft	From ASD Load Combination 3 of ASCE/SEI 7 Section 2.4.1 w_a = 40 psf + 24 psf = 64.0 psf w_a = 64.0 psf (15 ft)/(1,000 lb/kip) = 0.960 kip/ft M_a =(0.960 kip/ft)(60.0 ft)^2/8 = 432 kip-ft

With C_b = 1.0, from AISC *Manual* Table 3-10, the maximum unbraced length permitted for the W30×90 based upon required flexural strength is:

LRFD	ASD
For M_{uB} = 585 kip-ft, L_b = 22.0 ft	For M_{aB} = 432 kip-ft, L_b = 20.7 ft

Nodal Torsional Brace Design

The required flexural strength for a nodal torsional brace for the girder is determined from AISC *Specification* Appendix 6, Section 6.3.2a , Equation A-6-9, with braces every 5 ft, n = 11, and assuming C_b = 1.

LRFD	ASD
$M_{rb} = \dfrac{0.024 M_r L}{n C_b L_b}$ $= \dfrac{0.024(585\text{ kip-ft})(60.0\text{ ft})}{11(1.0)(22.0\text{ ft})}$ $= 3.48$ kip-ft	$M_{rb} = \dfrac{0.024 M_r L}{n C_b L_b}$ $= \dfrac{0.024(432\text{ kip-ft})(60.0\text{ ft})}{11(1.0)(20.7\text{ ft})}$ $= 2.73$ kip-ft

The required overall nodal torsional brace stiffness is determined from AISC *Specification* Appendix 6, Section 6.3.2a, Equation A-6-11, as follows:

LRFD	ASD
$\phi = 0.75$ $\beta_T = \dfrac{1}{\phi}\left(\dfrac{2.4LM_r^2}{nEI_yC_b^2}\right)$ $= \dfrac{1}{0.75}\left[\dfrac{2.4(60.0\text{ ft})(585\text{ kip-ft})^2(12\text{ in./ft})^3}{11(29{,}000\text{ ksi})(115\text{ in.}^4)(1.0)^2}\right]$ $= 3{,}100\text{ kip-in./rad}$	$\Omega = 3.00$ $\beta_T = \Omega\left(\dfrac{2.4LM_r^2}{nEI_yC_b^2}\right)$ $= 3.00\left(\dfrac{2.4(60.0\text{ ft})(432\text{ kip-ft})^2(12\text{ in./ft})^3}{11(29{,}000\text{ ksi})(115\text{ in.}^4)(1.0)^2}\right)$ $= 3800\text{ kip-in./rad}$

The distortional buckling stiffness of the girder web is a function of the web slenderness and the presence of any stiffeners, using AISC *Specification* Equation A-6-12. The web distortional stiffness is:

$$\beta_{sec} = \frac{3.3E}{h_o}\left(\frac{1.5h_o t_w^3}{12} + \frac{t_{st} b_s^3}{12}\right) \qquad \text{(Spec. Eq. A-6-12)}$$

The distortional stiffness of the girder web alone is:

$$\beta_{sec} = \frac{3.3E}{h_o}\left(\frac{1.5h_o t_w^3}{12}\right)$$
$$= \frac{3.3(29{,}000\text{ ksi})}{28.9\text{ in.}}\left[\frac{1.5(28.9\text{ in.})(0.470\text{ in.})^3}{12}\right]$$
$$= 1{,}240\text{ kip-in./rad}$$

For AISC *Specification* Equation A-6-10 to give a nonnegative result, the web distortional stiffness given by Equation A-6-12 must be greater than the required nodal torsional stiffness given by Equation A-6-11. Because the web distortional stiffness of the girder is less than the required nodal torsional stiffness for both LRFD and ASD, web stiffeners will be required.

Determine the torsional stiffness contributed by the beams. Both girders will buckle in the same direction forcing the beams to bend in reverse curvature. Thus, the flexural stiffness of the beam is:

$$\beta_{Tb} = \frac{6EI}{L}$$
$$= \frac{6(29{,}000\text{ ksi})(307\text{ in.}^4)}{(30.0\text{ ft})(12\text{ in./ft})}$$
$$= 148{,}000\text{ kip-in./rad}$$

Determining the required distortional stiffness of the girder will permit determination of the required stiffener size. The total stiffness is determined by summing the inverse of the distortional and flexural stiffnesses. Thus:

$$\frac{1}{\beta_T} = \frac{1}{\beta_{Tb}} + \frac{1}{\beta_{sec}}$$

Determine the minimum web distortional stiffness required to provide bracing for the girder.

LRFD	ASD

$\frac{1}{\beta_T} = \frac{1}{\beta_{Tb}} + \frac{1}{\beta_{sec}}$ $\frac{1}{3{,}100} = \frac{1}{148{,}000} + \frac{1}{\beta_{sec}}$ $\beta_{sec} = 3{,}170 \text{ kip-in./rad}$	$\frac{1}{\beta_T} = \frac{1}{\beta_{Tb}} + \frac{1}{\beta_{sec}}$ $\frac{1}{3{,}800} = \frac{1}{148{,}000} + \frac{1}{\beta_{sec}}$ $\beta_{sec} = 3{,}900 \text{ kip-in./rad}$

Using AISC *Specification* Equation A-6-12, determine the required width, b_s, of ⅜-in.-thick stiffeners.

LRFD	ASD
$\beta_{sec} = \frac{3.3E}{h_o}\left(\frac{1.5h_o t_w^3}{12} + \frac{t_{st} b_s^3}{12}\right)$ Using the total required girder web distortional stiffness and the contribution of the girder web distortional stiffness calculated previously, solve for the required width for ⅜-in.-thick stiffeners: $\beta_{sec} = 3{,}170 \text{ kip-in./rad}$ $3{,}170 \text{ kip-in./rad} = 1{,}240 \text{ kip-in./rad} + \frac{3.3(29{,}000 \text{ ksi})}{28.9 \text{ in.}}\left[\frac{(⅜ \text{ in.})b_s^3}{12}\right]$ and $b_s = 2.65$ in. Therefore, use a 4 in. x ⅜ in. full depth one-sided stiffener at the connection of each beam.	$\beta_{sec} = \frac{3.3E}{h_o}\left(\frac{1.5h_o t_w^3}{12} + \frac{t_{st} b_s^3}{12}\right)$ Using the total required girder web distortional stiffness and the contribution of the girder web distortional stiffness calculated previously, solve for the required width for ⅜-in.-thick stiffeners: $\beta_{sec} = 3{,}900 \text{ kip-in./rad}$ $3{,}900 \text{ kip-in./rad} = 1{,}240 \text{ kip-in./rad} + \frac{3.3(29{,}000 \text{ ksi})}{28.9 \text{ in.}}\left[\frac{(⅜ \text{ in.})b_s^3}{12}\right]$ and $b_s = 2.95$ in. Therefore, use a 4 in. x ⅜ in. full depth one-sided stiffener at the connection of each beam.

Available Flexural Strength

Each beam is connected to a girder web stiffener. Thus, each beam will be coped at the top and bottom as shown in Figure A-6.6-1(b) with a depth at the coped section of 9 in. The available flexural strength of the coped beam is determined using the provisions of AISC *Specification* Sections J4.5 and F11.

From AISC *Specification* Equation F11-1:

LRFD	ASD
$M_n = M_p = F_y Z \le 1.6M_y$ For a rectangle, $Z < 1.6S$. Therefore strength will be controlled by $F_y Z$ and $Z = \frac{(0.295 \text{ in.})(9.00 \text{ in.})^2}{4}$ $= 5.97 \text{ in.}^3$ $M_n = (50 \text{ ksi})(5.97 \text{ in.}^3)/12$ $= 24.9 \text{ kip-ft}$ $\phi M_n = 0.90(24.9 \text{ kip-ft})$ $= 22.4 \text{ kip-ft} > 3.48 \text{ kip-ft}$ **o.k.**	$M_n = M_p = F_y Z \le 1.6M_y$ For a rectangle, Z<1.6S. Therefore strength will be controlled by $F_y Z$ and $Z = \frac{(0.295 \text{ in.})(9.00 \text{ in.})^2}{4}$ $= 5.97 \text{ in.}^3$ $M_n = (50 \text{ ksi})(5.97 \text{ in.}^3)/12$ $= 24.9 \text{ kip-ft}$ $\frac{M_n}{\Omega} = \frac{24.9 \text{ kip-ft}}{1.67}$ $= 14.9 \text{ kip-ft} > 2.73 \text{ kip-ft}$ **o.k.**

Neglecting any rotation due to the bolts moving in the holes or any influence of the end moments on the strength of the beams, this system has sufficient strength and stiffness to provide nodal torsional bracing to the girders.

Additional connection design limit states may also need to be checked.

Part II
Examples Based on the AISC *Steel Construction Manual*

This part contains design examples demonstrating design aids and concepts provided in the AISC *Steel Construction Manual.*

Chapter IIA
Simple Shear Connections

The design of simple shear connections is covered in Part 10 of the AISC *Steel Construction Manual*.

Return to Table of Contents

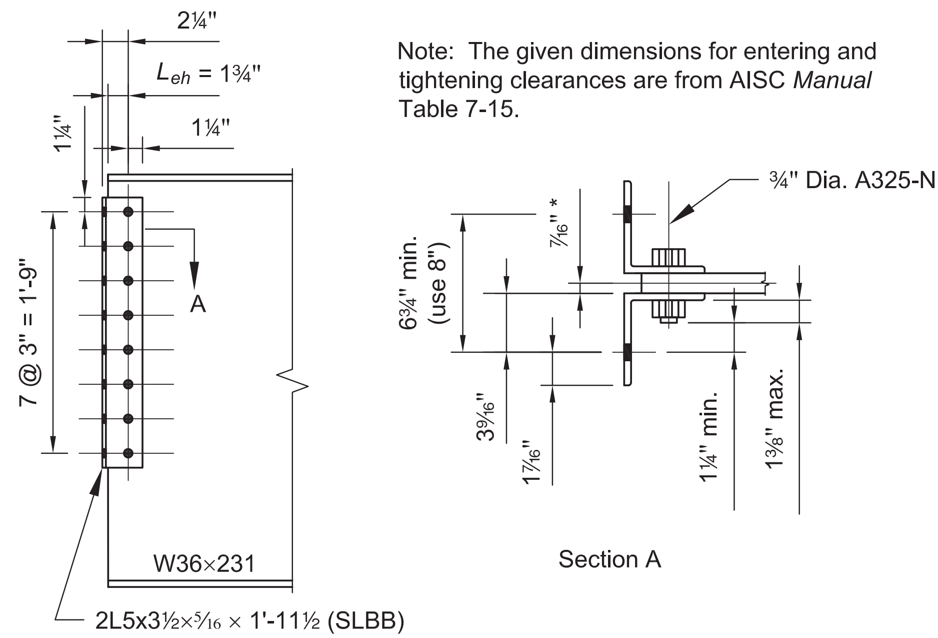
EXAMPLE II.A-1 ALL-BOLTED DOUBLE-ANGLE CONNECTION

Given:

Select an all-bolted double-angle connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange to support the following beam end reactions:

R_D = 37.5 kips
R_L = 113 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and ASTM A36 angles.



* This dimension (see sketch, Section A) is determined as one-half of the decimal web thickness rounded to the next higher 1⁄16 in. Example: 0.760"/2 = 0.380"; use 7⁄16 in. This will produce spacing of holes in the supporting beam slightly larger than detailed in the angles to permit spreading of angles (angles can be spread but not closed) at time of erection to supporting member. Alternatively, consider using horizontal short slots in the support legs of the angles.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(37.5 kips) + 1.6(113 kips) = 226 kips	R_a = 37.5 kips + 113 kips = 151 kips

Connection Design

AISC *Manual* Table 10-1 includes checks for the limit states of bearing, shear yielding, shear rupture, and block shear rupture on the angles, and shear on the bolts.

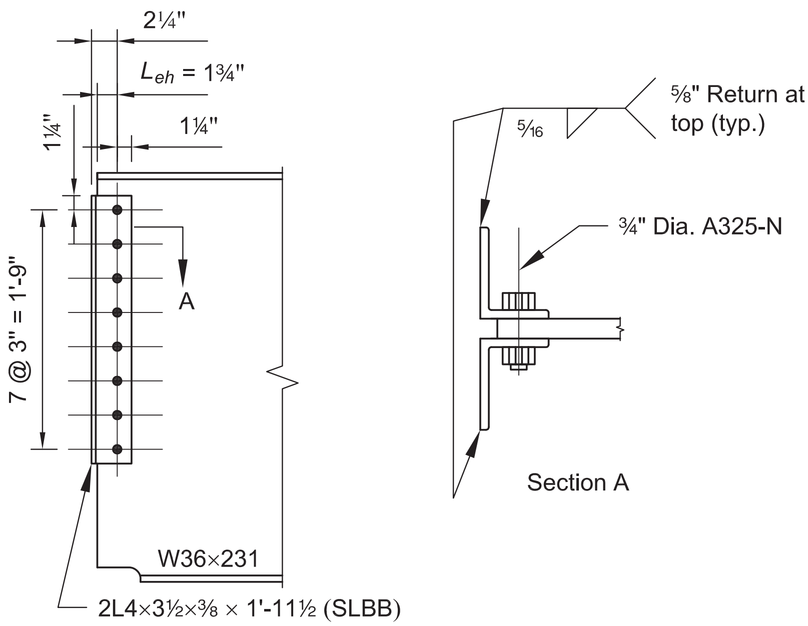
Try 8 rows of bolts and 2L5×3½×5⁄16 (SLBB).

LRFD	ASD
ϕR_n = 247 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 165 kips > 151 kips **o.k.**
Beam web strength from AISC *Manual* Table 10-1:	Beam web strength from AISC *Manual* Table 10-1:
Uncoped, L_{eh} = 1¾ in.	Uncoped, L_{eh} = 1¾ in.
ϕR_n = 702 kips/in.(0.760 in.) = 534 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 468 kips/in.(0.760 in.) = 356 kips > 151 kips **o.k.**
Bolt bearing on column flange from AISC *Manual* Table 10-1:	Bolt bearing on column flange from AISC *Manual* Table 10-1:
ϕR_n = 1,400 kips/in.(0.710 in.) = 994 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 936 kips/in.(0.710 in.) = 665 kips > 151 kips **o.k.**

EXAMPLE II.A-2 BOLTED/WELDED DOUBLE-ANGLE CONNECTION

Given:

Repeat Example II.A-1 using AISC *Manual* Table 10-2 to substitute welds for bolts in the support legs of the double-angle connection (welds B). Use 70-ksi electrodes.



Note: Bottom flange coped for erection.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(37.5 kips) + 1.6(113 kips) = 226 kips	R_a = 37.5 kips + 113 kips = 151 kips

Weld Design using AISC Manual Table 10-2 (welds B)

Try 5⁄16-in. weld size, L = 23 ½ in.

$t_{f\,min}$ = 0.238 in. < 0.710 in. **o.k.**

LRFD	ASD
ϕR_n = 279 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega} = 186$ kips > 151 kips **o.k.**

Angle Thickness

The minimum angle thickness for a fillet weld from AISC *Specification* Section J2.2b is:

t_{min} = w + 1⁄16 in.
= 5⁄16 in. + 1⁄16 in.
= 3⁄8 in.

Try 2L4×3½×3⁄8 (SLBB).

Angle and Bolt Design

AISC *Manual* Table 10-1 includes checks for the limit states of bearing, shear yielding, shear rupture, and block shear rupture on the angles, and shear on the bolts.

Check 8 rows of bolts and 3⁄8-in. angle thickness.

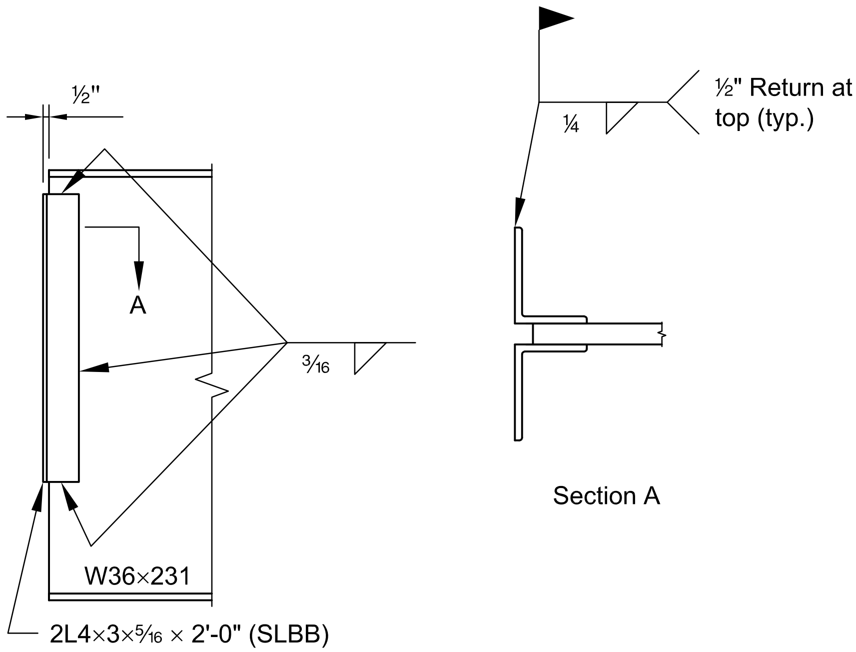
LRFD	ASD
ϕR_n = 286 kips > 226 kips **o.k.** Beam web strength: Uncoped, L_{eh} = 1¾ in. ϕR_n = 702 kips/in.(0.760 in.) = 534 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega} = 191$kips > 151 kips **o.k.** Beam web strength: Uncoped, L_{eh} = 1¾ in. $\dfrac{R_n}{\Omega}$ = 468 kips/in.(0.760 in.) = 356 kips > 151 kips **o.k.**

Note: In this example, because of the relative size of the cope to the overall beam size, the coped section does not control. When this cannot be determined by inspection, see AISC *Manual* Part 9 for the design of the coped section.

EXAMPLE II.A-3 ALL-WELDED DOUBLE-ANGLE CONNECTION

Given:

Repeat Example II.A-1 using AISC *Manual* Table 10-3 to design an all-welded double-angle connection between an ASTM A992 W36×231 beam and an ASTM A992 W14×90 column flange. Use 70-ksi electrodes and ASTM A36 angles.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W36×231
t_w = 0.760 in.

Column
W14×90
t_f = 0.710 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(37.5 kips) + 1.6(113 kips) = 226 kips	R_a = 37.5 kips + 113 kips = 151 kips

Design of Weld Between Beam Web and Angle (welds A)

Try 3⁄16-in. weld size, L = 24 in.

$t_{w\,min}$ = 0.286 in. < 0.760 in. **o.k.**

From AISC *Manual* Table 10-3:

LRFD	ASD
ϕR_n = 257 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 171 kips > 151 kips **o.k.**

Design of Weld Between Column Flange and Angle (welds B)

Try ¼-in. weld size, L = 24 in.

$t_{f\,min}$ = 0.190 in. < 0.710 in. **o.k.**

From AISC *Manual* Table 10-3:

LRFD	ASD
ϕR_n = 229 kips > 226 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 153 kips > 151 kips **o.k.**

Angle Thickness

Minimum angle thickness for weld from AISC *Specification* Section J2.2b:

$t_{min} = w$ + 1⁄16 in.
= ¼ in. + 1⁄16 in.
= 5⁄16 in.

Try 2L4×3×5⁄16 (SLBB).

Shear Yielding of Angles (AISC Specification Section J4.2)

$$A_{gv} = 2(24.0\text{ in.})(5/16\text{ in.}) = 15.0\text{ in.}^2$$

$$R_n = 0.60F_yA_{gv} \quad \text{(Spec. Eq. J4-3)}$$

$$= 0.60(36\text{ ksi})(15.0\text{ in.}^2) = 324\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(324 \text{ kips})$ $= 324 \text{ kips} > 226 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{324 \text{ kips}}{1.50}$ $= 216 \text{ kips} > 151 \text{ kips}$ **o.k.**

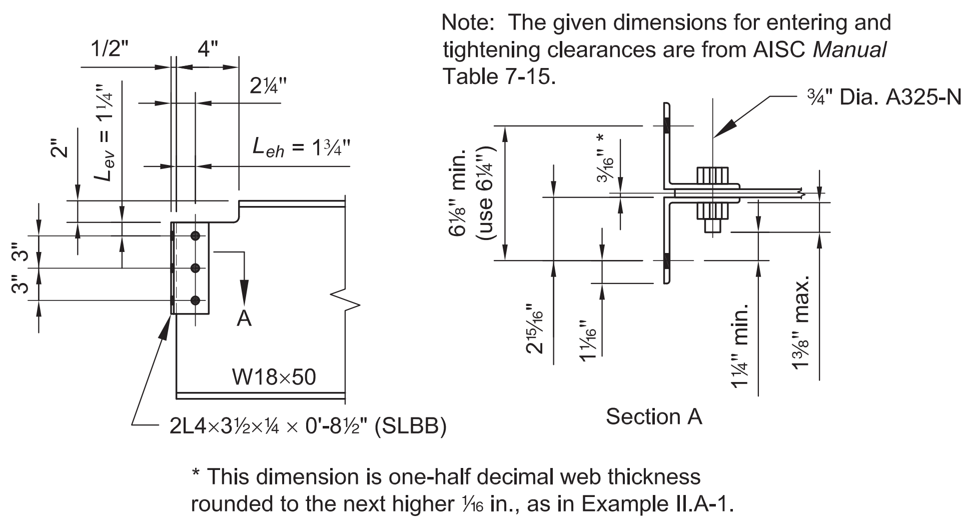
EXAMPLE II.A-4 ALL-BOLTED DOUBLE-ANGLE CONNECTION IN A COPED BEAM

Given:

Use AISC *Manual* Table 10-1 to select an all-bolted double-angle connection between an ASTM A992 W18×50 beam and an ASTM A992 W21×62 girder web to support the following beam end reactions:

R_D = 10 kips
R_L = 30 kips

The beam top flange is coped 2 in. deep by 4 in. long, L_{ev} = 1¼ in., L_{eh} = 1¾ in. Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and ASTM A36 angles.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
W18×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Girder
W21×62
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 9-2 and AISC *Manual* Figure 9-2, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
t_w = 0.355 in.
S_{net} = 23.4 in.^{3}
c = 4.00 in.
d_c = 2.00 in.
e = 4.00 in. + 0.500 in.
= 4.50 in.
h_o = 16.0 in.

Girder
W21×62
t_w = 0.400 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(10 kips) + 1.6(30 kips) = 60.0 kips	R_a = 10 kips + 30 kips = 40.0 kips

Connection Design

AISC *Manual* Table 10-1 includes checks for the limit states of bearing, shear yielding, shear rupture, and block shear rupture on the angles, and shear on the bolts.

Try 3 rows of bolts and 2L4×3½×¼ (SLBB).

LRFD	ASD
ϕR_n = 76.4 kips > 60.0 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 50.9 kips > 40.0 kips **o.k.**
Beam web strength from AISC *Manual* Table 10-1:	Beam web strength from AISC *Manual* Table 10-1:
Top flange coped, L_{ev} = 1¼ in., L_{eh} = 1¾ in.	Top flange coped, L_{ev} = 1¼ in., L_{eh} = 1¾ in.
ϕR_n = 200 kips/in.(0.355 in.) = 71.0 kips > 60.0 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 133 kips/in.(0.355 in.) = 47.2 kips > 40.0 kips **o.k.**
Bolt bearing on girder web from AISC *Manual* Table 10-1:	Bolt bearing on girder web from AISC *Manual* Table 10-1:
ϕR_n = 526 kips/in.(0.400 in.) = 210 kips > 60.0 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 351 kips/in.(0.400 in.) = 140 kips > 40.0 kips **o.k.**

Note: The middle portion of AISC *Manual* Table 10-1 includes checks of the limit-state of bolt bearing on the beam web and the limit-state of block shear rupture on coped beams. AISC *Manual* Tables 9-3a, 9-3b and 9-3c may be used to determine the available block shear strength for values of L_{ev} and L_{eh} beyond the limits of AISC *Manual* Table 10-1. For coped members, the limit states of flexural yielding and local buckling must be checked independently per AISC *Manual* Part 9.

Coped Beam Strength (AISC Manual Part 9)

Flexural Local Web Buckling

Verify $c \le 2d$ and $d_c \le \dfrac{d}{2}$.

$$c = 4.00 \text{ in.} \le 2(18.0 \text{ in.}) = 36.0 \text{ in.} \qquad \textbf{o.k.}$$

$$d_c = 2.00 \text{ in.} \le \frac{18.0 \text{ in.}}{2} = 9.00 \text{ in.} \qquad \textbf{o.k.}$$

$$\frac{c}{d} = \frac{4.00 \text{ in.}}{18.0 \text{ in.}} = 0.222$$

$$\frac{c}{h_o} = \frac{4.00 \text{ in.}}{16.0 \text{ in.}} = 0.250$$

Since $\dfrac{c}{d} \le 1.0$,

$$f = \frac{2c}{d} = 2(0.222) = 0.444 \qquad (\textit{Manual}\text{ Eq. 9-8})$$

Since $\dfrac{c}{h_o} \le 1.0$,

$$k = 2.2\left(\frac{h_o}{c}\right)^{1.65} \qquad (\textit{Manual}\text{ Eq. 9-10})$$

$$= 2.2\left(\frac{16.0 \text{ in.}}{4.00 \text{ in.}}\right)^{1.65} = 21.7$$

$$F_{cr} = 26{,}210\left(\frac{t_w}{h_o}\right)^2 fk \qquad (\textit{Manual}\text{ Eq. 9-7})$$

$$= 26{,}210\left(\frac{0.355 \text{ in.}}{16.0 \text{ in.}}\right)^2 (0.444)(21.7) = 124 \text{ ksi} \le 50 \text{ ksi}$$

Use F_{cr} = 50 ksi.

$$R_n = \frac{F_{cr} S_{net}}{e} \text{ from AISC } \textit{Manual} \text{ Equation 9-6}$$

$$= \frac{50 \text{ ksi}\left(23.4 \text{ in.}^3\right)}{4.50 \text{ in.}} = 260 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(260\text{ kips})$ $= 234\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{260\text{ kips}}{1.67}$ $= 156\text{ kips} > 40.0\text{ kips}$ **o.k.**

Shear Yielding of Beam Web (AISC Specification Section J4.2)

$$R_n = 0.60F_yA_{gv} \qquad (\textit{Spec}.\text{ Eq. J4-3})$$
$$= 0.60(50\text{ ksi})(0.355\text{ in.})(16.0\text{ in})$$
$$= 170\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(170\text{ kips})$ $= 170\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{170\text{ kips}}{1.50}$ $= 113\text{ kips} > 40.0\text{ kips}$ **o.k.**

Shear Rupture of Beam Web (AISC Specification Section J4.2)

$$A_{nv} = t_w\left[h_o - 3\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]$$
$$= 0.355\text{ in.}\left(16.0\text{ in.} - 2.63\text{ in.}\right)$$
$$= 4.75\text{ in.}^2$$

$$R_n = 0.60F_uA_{nv} \qquad (\textit{Spec}.\text{ Eq. J4-4})$$
$$= 0.60\left(65.0\text{ ksi}\right)\left(4.75\text{ in.}^2\right)$$
$$= 185\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(185\text{ kips})$ $= 139\text{ kips} > 60.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{185\text{ kips}}{2.00}$ $= 92.5\text{ kips} > 40.0\text{ kips}$ **o.k.**

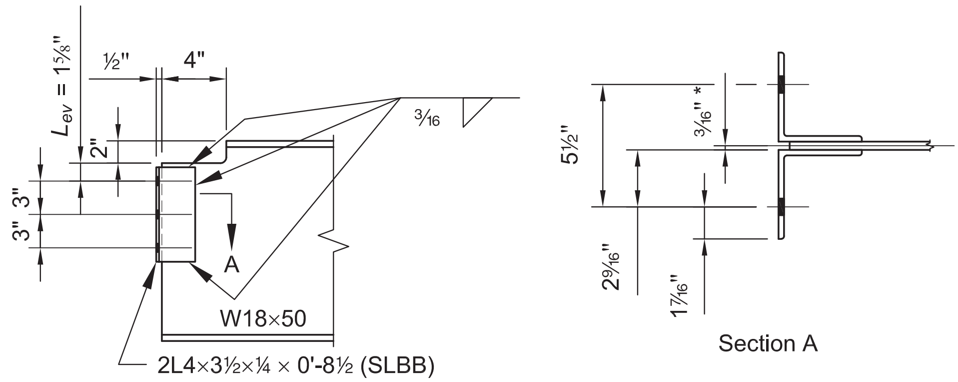
Because the cope is not greater than the length of the connection angle, it is assumed that other flexural limit states of rupture and lateral-torsional buckling do not control.

Return to Table of Contents

EXAMPLE II.A-5 WELDED/BOLTED DOUBLE-ANGLE CONNECTION IN A COPED BEAM

Given:

Repeat Example II.A-4 using AISC *Manual* Table 10-2 to substitute welds for bolts in the supported-beam-web legs of the double-angle connection (welds A). Use 70-ksi electrodes and ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and ASTM A36 angles..



* This dimension is one-half decimal web thickness rounded to the next higher 1⁄16 in., as in Example II.A-1.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
W18×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Girder
W21×62
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 9-2 and AISC *Manual* Figure 9-2, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
t_w = 0.355 in.
S_{net} = 23.4 in.^{3}
c = 4.00 in.
d_c = 2.00 in.
e = 4.00 in. + 0.500 in.

= 4.50 in.
h_o = 16.0 in.

Girder
W21×62
t_w = 0.400 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(10 kips) + 1.6(30 kips) = 60.0 kips	R_a = 10 kips + 30 kips =40.0 kips

Weld Design (welds A)

Try 3⁄16-in. weld size, L = 8½ in from AISC *Manual* Table 10-2.

$t_{w\,min}$ = 0.286 in. < 0.355 in. **o.k.**

From AISC *Manual* Table 10-2:

LRFD	ASD
ϕR_n = 110 kips > 60.0 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 73.5 kips > 40.0 kips **o.k.**

Minimum Angle Thickness for Weld

w = weld size

t_{min} = w + 1⁄16 in. from AISC *Specification* Section J2.2b
= 3⁄16 in. + 1⁄16 in.
= ¼ in.

Bolt Bearing on Supporting Member Web

From AISC *Manual* Table 10-1:

LRFD	ASD
ϕR_n = 526 kips/in.(0.400 in.) = 210 kips > 60.0 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 351 kips/in.(0.400 in.) = 140 kips > 40.0 kips **o.k.**

Bearing, Shear and Block Shear for Bolts and Angles

From AISC *Manual* Table 10-1:

LRFD	ASD
ϕR_n = 76.4 kips > 60.0 kips **o.k.**	ϕR_n = 50.9 kips > 40.0 kips **o.k.**

Note: The middle portion of AISC *Manual* Table 10-1 includes checks of the limit state of bolt bearing on the beam web and the limit state of block shear rupture on the beam web. AISC *Manual* Tables 9-3a, 9-3b and 9-3c may be used to determine the available block shear strength for values of L_{ev} and L_{eh} beyond the limits of AISC *Manual* Table 10-1. For coped members, the limit states of flexural yielding and local buckling must be checked independently per AISC *Manual* Part 9.

Coped Beam Strength (AISC Manual Part 9)

The coped beam strength is verified in Example II.A-4.

Shear Yielding of Beam Web

$$R_n = 0.60F_y A_{gv}$$ (*Spec*. Eq. J4-3)

$$= 0.60(50.0 \text{ ksi})(0.355 \text{ in.})(16.0 \text{ in.})$$

$$= 170 \text{ kips}$$

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(170 \text{ kips})$ $= 170 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{170 \text{ kips}}{1.50}$ $= 113 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Shear Rupture of Beam Web

$$R_n = 0.60F_u A_{nv}$$ (*Spec*. Eq. J4-4)

$$= 0.60(65.0 \text{ ksi})(0.355 \text{ in.})(16.0 \text{ in.})$$

$$= 222 \text{ kips}$$

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(222 \text{ kips})$ $= 167 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{222 \text{ kips}}{2.00}$ $= 111 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

EXAMPLE II.A-6 BEAM END COPED AT THE TOP FLANGE ONLY

Given:

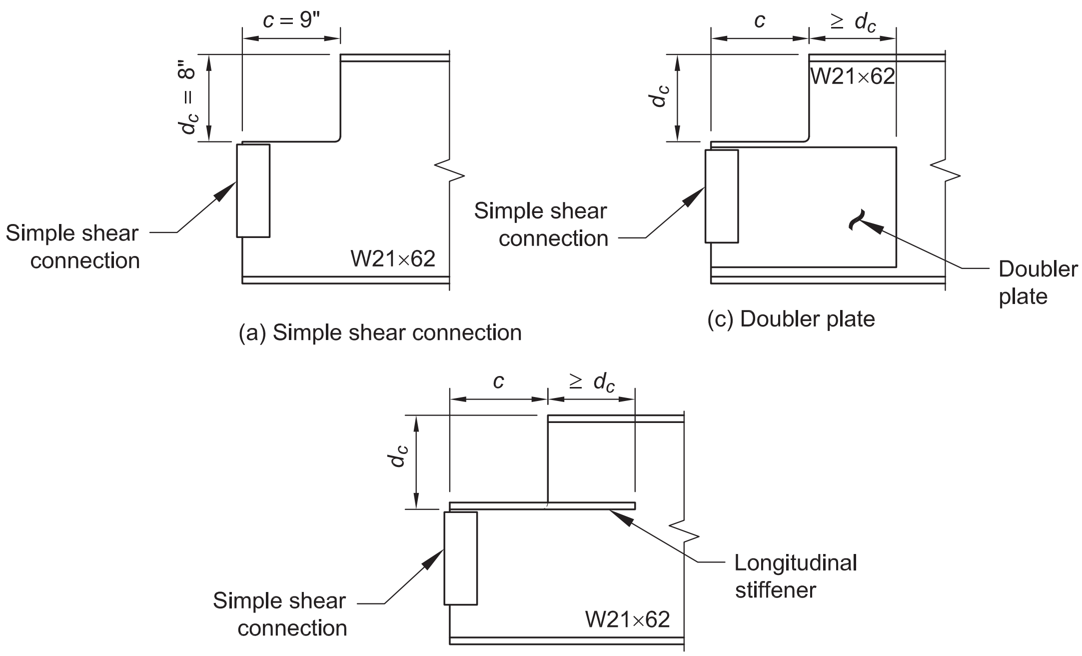
For an ASTM A992 W21×62 coped 8 in. deep by 9 in. long at the top flange only, assuming e = 9½ in. and using an ASTM A36 plate:

A. Calculate the available strength of the beam end, considering the limit states of flexural yielding, local buckling, shear yielding and shear rupture.

B. Choose an alternate ASTM A992 W21 shape to eliminate the need for stiffening for an end reaction of R_D = 16.5 kips and R_L = 47 kips.

C. Determine the size of doubler plate needed to stiffen the W21×62 for the given end reaction in Solution B.

D. Determine the size of longitudinal stiffeners needed to stiffen the W2 for the given end reaction in Solution B.



(a) Simple shear connection

(c) Doubler plate

(d) Longitudinal stiffener

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W21×62
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 9-2 and AISC *Manual* Figure 9-2, the geometric properties are as follows:

Beam
W21×62
d = 21.0 in.
t_w = 0.400 in.
b_f = 8.24 in.
t_w = 0.615 in.
S_{net} = 17.8 in.^{3}
c = 9.00 in.
d_c = 8.00 in.
e = 9.50 in.
h_o = 13.0 in.

Solution A:

Flexural Yielding and Local Web Buckling (AISC Manual Part 9)

Verify parameters.

$$c \le 2d$$
$$9.00 \text{ in.} \le 2(21.0 \text{ in.})$$
$$\le 42.0 \text{ in.} \quad \textbf{o.k.}$$
$$d_c \le \frac{d}{2}$$
$$8.00 \text{ in.} \le \frac{21.0 \text{ in.}}{2}$$
$$\le 10.5 \text{ in.} \quad \textbf{o.k.}$$
$$\frac{c}{d} = \frac{9.00 \text{ in.}}{21.0 \text{ in.}}$$
$$= 0.429$$

$$\frac{c}{h_o} = \frac{9.00 \text{ in.}}{13.0 \text{ in.}}$$
$$= 0.692$$

Because $\frac{c}{d} \le 1.0$,

$$f = 2\left(\frac{c}{d}\right) \quad \text{(Manual Eq. 9-8)}$$
$$= 2(0.429)$$
$$= 0.858$$

Because $\frac{c}{h_o} \le 1.0$,

$$k = 2.2\left(\frac{h_o}{c}\right)^{1.65} \quad \text{(Manual Eq. 9-10)}$$
$$= 2.2\left(\frac{13.0 \text{ in.}}{9.00 \text{ in.}}\right)^{1.65}$$
$$= 4.04$$

For a top cope only, the critical buckling stress is:

$$F_{cr} = 26,210\left(\frac{t_w}{h_o}\right)^2 fk \le F_y \qquad \text{(\textit{Manual} Eq. 9-7)}$$

$$= 26,210\left(\frac{0.400\text{ in.}}{13.0\text{ in.}}\right)^2 (0.858)(4.04) \le F_y$$

$$= 86.0\text{ ksi} \le F_y$$

Use $F_{cr} = F_y = 50$ ksi

$$R_n = \frac{F_{cr}S_{net}}{e} \text{ from AISC \textit{Manual} Equation 9-6}$$

$$= \frac{50\text{ ksi}\left(17.8\text{ in.}^3\right)}{9.50\text{ in.}}$$

$$= 93.7\text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(93.7\text{ kips})$ $= 84.3\text{ kips}$	$\Omega = 1.67$ $\frac{R_n}{\Omega} = \frac{93.7\text{ kips}}{1.67}$ $= 56.1\text{ kips}$

Shear Yielding of Beam Web

$$R_n = 0.60F_yA_{gv} \qquad \text{(\textit{Spec}. Eq. J4-3)}$$
$$= 0.60(50\text{ ksi})(0.400\text{ in.})(13.0\text{ in.})$$
$$= 156\text{ kips}$$

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(156\text{ kips})$ $= 156\text{ kips}$	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{156\text{ kips}}{1.50}$ $= 104\text{ kips}$

Shear Rupture of Beam Web

$$R_n = 0.60F_uA_{nv} \qquad \text{(\textit{Spec}. Eq. J4-4)}$$
$$= 0.60(65\text{ ksi})(0.400\text{ in.})(13.0\text{ in.})$$
$$= 203\text{ kips}$$

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(203 \text{ kips})$ $= 152 \text{ kips}$	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{203 \text{ kips}}{2.00}$ $= 102 \text{ kips}$

Thus, the available strength is controlled by local bucking.

LRFD	ASD
$\phi R_n = 84.3 \text{ kips}$	$\dfrac{R_n}{\Omega} = 56.1 \text{ kips}$

Solution B:

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(16.5 \text{ kips}) + 1.6(47 \text{ kips})$ $= 95.0 \text{ kips}$	$R_a = 16.5 \text{ kips} + 47 \text{ kips}$ $= 63.5 \text{ kips}$

As determined in Solution A, the available critical stress due to local buckling for a W21×62 with an 8-in.-deep cope is limited to the yield stress.

Required Section Modulus Based on Local Buckling

From AISC *Manual* Equations 9-5 and 9-6:

LRFD	ASD
$S_{req} = \dfrac{R_u e}{\phi F_y}$ $= \dfrac{95.0 \text{ kips}(9.50 \text{ in.})}{0.90(50.0 \text{ ksi})}$ $= 20.1 \text{ in.}^3$	$S_{req} = \dfrac{R_a e\,\Omega}{F_y}$ $= \dfrac{63.5 \text{ kips}(9.50 \text{ in.})(1.67)}{50.0 \text{ ksi}}$ $= 20.1 \text{ in.}^3$

Try a W21×73.

From AISC *Manual* Table 9-2:

$$S_{net} = 21.0 \text{ in.}^3 > 20.1 \text{ in.}^3 \quad \textbf{o.k.}$$

Note: By comparison to a W21×62, a W21×73 has sufficient shear strength.

Solution C:

Doubler Plate Design (AISC Manual Part 9)

LRFD	ASD
Doubler plate must provide a required strength of: 95.0 kips – 84.3 kips = 10.7 kips	Doubler plate must provide a required strength of: 63.5 kips – 56.1 kips = 7.40 kips
$S_{req} = \frac{(R_u - \phi R_{n\,beam})e}{\phi F_y}$ $= \frac{(95.0 \text{ kips} - 84.3 \text{ kips})(9.50 \text{ in.})}{0.90(50 \text{ ksi})}$ $= 2.26 \text{ in.}^3$ For an 8-in.-deep plate, $t_{req} = \frac{6S_{req}}{d^2}$ $= \frac{6(2.26 \text{ in.}^3)}{(8.00 \text{ in.})^2}$ $= 0.212 \text{ in.}$	$S_{req} = \frac{(R_a - R_{n\,beam}/\Omega)e\,\Omega}{F_y}$ $= \frac{(63.5 \text{ kips} - 56.1 \text{ kips})(9.50 \text{ in.})(1.67)}{50 \text{ ksi}}$ $= 2.35 \text{ in.}^3$ For an 8-in.-deep plate, $t_{req} = \frac{6S_{req}}{d^2}$ $= \frac{6(2.35 \text{ in.}^3)}{(8.00 \text{ in.})^2}$ $= 0.220 \text{ in.}$

Note: ASTM A572 Grade 50 plate is recommended in order to match the beam yield strength.

Thus, since the doubler plate must extend at least d_c beyond the cope, use a PL¼ in×8 in.×1ft 5 in. with 3⁄16-in. welds top and bottom.

Solution D:

Longitudinal Stiffener Design

Try PL¼ in.×4 in. slotted to fit over the beam web with F_y = 50 ksi.

From section property calculations for the neutral axis and moment of inertia, conservatively ignoring the beam fillets, the neutral axis is located 4.39 in. from the bottom flange (8.86 in. from the top of the stiffener).

	I_o (in.^{4})	Ad^2 (in.^{4})	$I_o + Ad^2$ (in.^{4})
Stiffener	0.00521	76.3	76.3
W21×62 web	63.3	28.9	92.2
W21×62 bottom flange	0.160	84.5	84.7
			$\Sigma = I_x = 253$ in.^{4}

Slenderness of the Longitudinal Stiffener

$\lambda_r = 0.95\sqrt{k_c E/F_L}$ from AISC *Specification* Table B4.1b Case 11

$$k_c = \frac{4}{\sqrt{h/t_w}} \text{ where } 0.35 \le k_c \le 0.76$$
$$= \frac{4}{\sqrt{11.9 \text{ in.}/0.400 \text{ in.}}}$$
$$= 0.733$$

use $k_c = 0.733$

$$S_{xc} = \frac{I_x}{c}$$
$$= \frac{253 \text{ in.}^4}{8.86 \text{ in.}}$$
$$= 28.6 \text{ in.}^3$$

$$S_{xt} = \frac{253 \text{ in.}^4}{4.39 \text{ in.}}$$
$$= 57.6 \text{ in.}^3$$

$$\frac{S_{xt}}{S_{xc}} = \frac{57.6 \text{ in.}^3}{28.6 \text{ in.}^3}$$
$$= 2.01 \ge 0.7, \text{ therefore,}$$

$$F_L = 0.7F_y$$
$$= 0.7(50 \text{ ksi})$$
$$= 35.0 \text{ ksi}$$

$$\lambda_r = 0.95\sqrt{0.733(29{,}000 \text{ ksi})/35.0 \text{ ksi}}$$
$$= 23.4$$

$$\frac{b}{t} = \frac{4.00 \text{ in.}}{2(\tfrac{1}{4} \text{ in.})}$$
$$= 8.00 < 23.4, \text{ therefore, the stiffener is not slender}$$

$$S_{net} = S_{xc}$$

The nominal strength of the reinforced section using AISC *Manual* Equation 9-6 is:

$$R_n = \frac{F_y S_{net}}{e}$$
$$= \frac{50 \text{ ksi}\left(28.6 \text{ in.}^3\right)}{9.50 \text{ in.}}$$
$$= 151 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(151 \text{ kips})$	$\Omega = 1.67$

$= 136 \text{ kips} > 95.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{151 \text{ kips}}{1.67}$ $= 90.4 \text{ kips} > 63.5 \text{ kips}$ **o.k.**

Note: ASTM A572 Grade 50 plate is recommended in order to match the beam yield strength.

Plate Dimensions

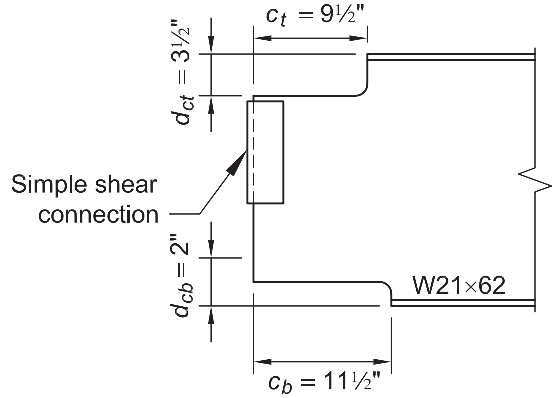
Since the longitudinal stiffening must extend at least d_c beyond the cope, use PL¼ in.×4 in.×1 ft 5 in. with ¼-in. welds.

Return to Table of Contents

EXAMPLE II.A-7 BEAM END COPED AT THE TOP AND BOTTOM FLANGES

Given:

For an ASTM A992 W16×40 coped 3½ in. deep by 9½ in. wide at the top flange and 2 in. deep by 11½ in. wide at the bottom flange calculate the available strength of the beam end, considering the limit states of flexural yielding and local buckling. Assume a ½-in. setback from the face of the support to the end of the beam.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Table 1-1 and AISC *Manual* Figure 9-3, the geometric properties are as follows:

d = 16.0 in.
t_w = 0.305 in.
t_f = 0.505 in.
b_f = 7.00 in.
c_t = 9.50 in.
d_{ct} = 3.50 in.
c_b = 11.5 in.
d_{cb} = 2.00 in.
e_b = 11.5 in. + 0.50 in.
= 12.0 in.
e_t = 9.50 in. + 0.50 in.
= 10.0 in.
h_o = 16.0 in. − 2.00 in. − 3.50 in.
= 10.5 in.

Local Buckling at the Compression (Top) Flange Cope

Because the bottom cope (tension) is longer than the top cope (compression) and $d_c > 0.2d$, the available buckling stress is calculated using AISC *Manual* Equation 9-14.

$$\lambda = \frac{h_o\sqrt{F_y}}{10t_w\sqrt{475+280\left(\frac{h_o}{c_t}\right)^2}}$$ (*Manual* Eq. 9-18)

$$= \frac{10.5\text{ in.}\sqrt{50\text{ ksi}}}{10(0.305\text{ in.})\sqrt{475+280\left(\frac{10.5\text{ in.}}{9.50\text{ in.}}\right)^2}}$$

$$= 0.852$$

Because, $0.7 < \lambda \leq 1.41$:

$$\begin{aligned} Q &= 1.34 - 0.486\lambda \\ &= 1.34 - 0.486(0.852) \\ &= 0.926 \end{aligned}$$ (*Manual* Eq. 9-16)

Available Buckling Stress

$$\begin{aligned} F_{cr} &= F_yQ \\ &= 50\text{ ksi}(0.926) \\ &= 46.3\text{ ksi} < 50\text{ ksi} \quad \text{(buckling controls)} \end{aligned}$$ (*Manual* Eq. 9-14)

Determine the net elastic section modulus:

$$S_{net} = \frac{t_wh_o^2}{6}$$

$$= \frac{(0.305\text{ in.})\left(10.5\text{ in.}^2\right)^2}{6}$$

$$= 5.60\text{ in.}^3$$

The strength based on flexural local buckling is determined as follows:

$$\begin{aligned} M_n &= F_{cr}S_{net} \\ &= 46.3\text{ ksi}(5.60\text{ in.}^3) \\ &= 259\text{ kip-in.} \end{aligned}$$ (*Manual* Eq. 9-6)

$$R_n = \frac{M_n}{e_t}$$

$$= \frac{259\text{ kip-in.}}{10.0\text{ in.}}$$

$$= 25.9\text{ kips}$$

LRFD	ASD
$\phi_b = 0.90$ $\phi_bR_n = 0.90(25.9\text{ kips})$ $= 23.3\text{ kips}$	$\Omega_b = 1.67$ $\frac{R_n}{\Omega_b} = \frac{25.9\text{ kips}}{1.67}$ $= 15.5\text{ kips}$

Check flexural yielding of the tension (bottom) flange cope.

From AISC *Manual* Table 9-2 the elastic section modulus of the remaining section is S_{net} = 15.6 in.^{3}

The strength based on flexural yielding is determined as follows:

$$\begin{aligned} M_n &= F_y S_{net} \\ &= 50 \text{ ksi}(15.6 \text{ in.}^3) \\ &= 780 \text{ kip-in.} \end{aligned}$$

$$\begin{aligned} R_n &= \frac{M_n}{e_b} \\ &= \frac{780 \text{ kip-in.}}{12.0 \text{ in.}} \\ &= 65.0 \text{ kips} \end{aligned}$$

LRFD	ASD
$\phi_b = 0.90$ $\phi_b R_n = 0.90(65.0 \text{ kips})$ $= 58.5 \text{ kips}$	$\Omega_b = 1.67$ $\dfrac{R_n}{\Omega_b} = \dfrac{65.0 \text{ kips}}{1.67}$ $= 38.9 \text{ kips}$

Thus, the available strength is controlled by local buckling in the top (compression) cope of the beam.

LRFD	ASD
$\phi_b = 0.90$ $\phi_b R_n = 23.3 \text{ kips}$	$\Omega_b = 1.67$ $\dfrac{R_n}{\Omega_b} = 15.5 \text{ kips}$

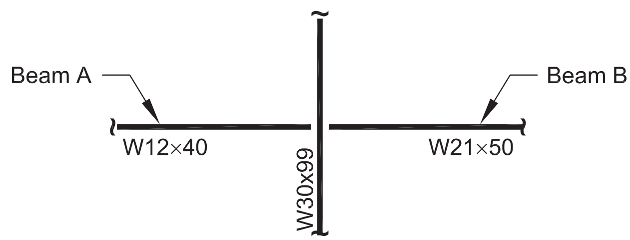
EXAMPLE II.A-8 ALL-BOLTED DOUBLE-ANGLE CONNECTIONS (BEAMS-TO-GIRDER WEB)

Given:

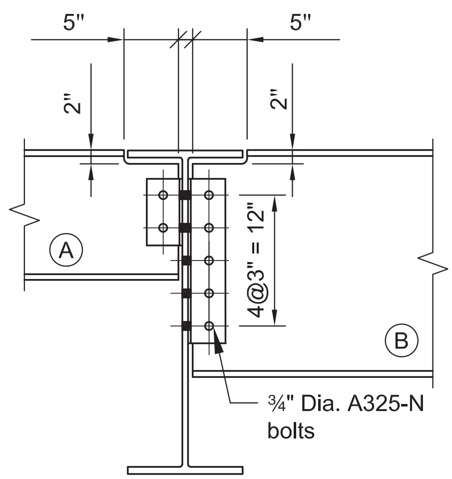
Design the all-bolted double-angle connections between the ASTM A992 W12×40 beam (A) and ASTM A992 W21×50 beam (B) and the ASTM A992 W30×99 girder-web to support the following beam end reactions:

Beam A	Beam B
R_{DA} = 4.17 kips	R_{DB} = 18.3 kips
R_{LA} = 12.5 kips	R_{LB} = 55.0 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and assume e = 5.50 in. Use ASTM A36 angles.



Part Plan



Section

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam A
W12×40
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Beam B
W21×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Girder
W30×99
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 9-2, the geometric properties are as follows:

Beam A
W12×40
t_w = 0.295 in.
d = 11.9 in.
h_o = 9.90 in.
S_{net} = 8.03 in.^{3}
d_c = 2.00 in.
c = 5.00 in.
e = 5.50 in.

Beam B
W21×50
t_w = 0.380 in.
d = 20.8 in.
h_o = 18.8 in.
S_{net} = 32.5 in.^{3}
d_c = 2.00 in.
c = 5.00 in.
e = 5.50 in.

Girder
W30×99
t_w = 0.520 in.
d = 29.7 in.

Beam A:

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_{Au} = 1.2(4.17 kips) + 1.6(12.5 kips) = 25.0 kips	R_{Aa} = 4.17 kips + 12.5 kips = 16.7 kips

Bolt Shear and Bolt Bearing, Shear Yielding, Shear Rupture and Block Shear Rupture of Angles

From AISC *Manual* Table 10-1, for two rows of bolts and ¼-in. angle thickness:

LRFD	ASD
ϕR_n = 48.9 kips > 25.0 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 32.6 kips > 16.7 kips **o.k.**

Bolt Bearing and Block Shear Rupture of Beam Web

From AISC *Manual* Table 10-1, for two rows of bolts and L_{ev} = 1¼ in. and L_{eh} = 1½ in.:

LRFD	ASD
ϕR_n = 126 kips/in.(0.295 in.) = 37.2 kips > 25.0 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 83.7 kips/in.(0.295 in.) = 24.7 kips > 16.7 kips **o.k.**

Coped Beam Strength (AISC Manual Part 9)

Flexural Yielding and Local Web Buckling

Verify parameters.

$$c \le 2d$$
$$5.00\text{ in.} \le 2(11.9\text{ in.})$$
$$\le 23.8\text{ in.} \qquad \textbf{o.k.}$$

$$d_c \le \frac{d}{2}$$
$$2.00\text{ in.} \le \frac{11.9\text{ in.}}{2}$$
$$\le 5.95\text{ in.} \qquad \textbf{o.k.}$$

$$\frac{c}{d} = \frac{5.00\text{ in.}}{11.9\text{ in.}}$$
$$= 0.420 \le 1.0$$

$$\frac{c}{h_o} = \frac{5.00\text{ in.}}{9.90\text{ in.}}$$
$$= 0.505 \le 1.0$$

Because $\dfrac{c}{d} \le 1.0$, the plate buckling model adjustment factor:

$$f = 2\left(\frac{c}{d}\right) \qquad \text{(\textit{Manual} Eq. 9-8)}$$
$$= 2(0.420)$$
$$= 0.840$$

Because $\dfrac{c}{h_o} \le 1.0$, the plate buckling coefficient is:

$$k = 2.2\left(\frac{h_o}{c}\right)^{1.65}$$ (*Manual* Eq. 9-10)

$$= 2.2\left(\frac{9.90\text{ in.}}{5.00\text{ in.}}\right)^{1.65}$$
$$= 6.79$$

For top cope only, the critical buckling stress is:

$$F_{cr} = 26{,}210\left(\frac{t_w}{h_o}\right)^2 f k \le F_y$$ (*Manual* Eq. 9-7)

$$= 26{,}210\left(\frac{0.295\text{ in.}}{9.90\text{ in.}}\right)^2 (0.840)(6.79)$$
$$= 133\text{ ksi} \le F_y$$

Use $F_{cr} = F_y = 50$ ksi.

From AISC *Manual* Equation 9-6:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = \dfrac{\phi F_{cr} S_{net}}{e}$ $= \dfrac{0.90(50\text{ ksi})(8.03\text{ in.}^3)}{5.50\text{ in.}}$ $= 65.7\text{ kips} > 25.0\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{F_{cr} S_{net}}{\Omega e}$ $= \dfrac{50\text{ ksi}(8.03\text{ in.}^3)}{1.67(5.50\text{ in.})}$ $= 43.7\text{ kips} > 16.7\text{ kips}$ **o.k.**

Shear Yielding of Beam Web

$R_n = 0.60F_yA_{gv}$ (*Spec*. Eq. J4-3)
$= 0.60(50\text{ ksi})(0.295\text{ in.})(9.90\text{ in.})$
$= 87.6$ kips

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(87.6\text{ kips})$ $= 87.6\text{ kips} > 25.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{87.6\text{ kips}}{1.50}$ $= 58.4\text{ kips} > 16.7\text{ kips}$ **o.k.**

Shear Rupture of Beam Web

$A_{nv} = t_w[h_o - 2(\frac{13}{16}\text{ in.} + \frac{1}{16}\text{ in.})]$
$= 0.295\text{ in.}(9.90\text{ in.} - 1.75\text{ in.})$
$= 2.40\text{ in.}^2$

$R_n = 0.60F_uA_{nv}$ (*Spec*. Eq. J4-4)
$= 0.60(65\text{ ksi})(2.40\text{ in.}^2)$

$= 93.6$ kips

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(93.6 \text{ kips})$ $= 70.2 \text{ kips} > 25.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{93.6 \text{ kips}}{2.00}$ $= 46.8 \text{ kips} > 16.7 \text{ kips}$ **o.k.**

Beam B:

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_{Bu} = 1.2(18.3 \text{ kips}) + 1.6(55.0 \text{ kips})$ $= 110 \text{ kips}$	$R_{Ba} = 18.3 \text{ kips} + 55.0 \text{ kips}$ $= 73.3 \text{ kips}$

Bolt Shear and Bolt Bearing, Shear Yielding, Shear Rupture and Block Shear Rupture of Angles

From AISC *Manual* Table 10-1, for five rows of bolts and ¼-in. angle thickness:

LRFD	ASD
$\phi R_n = 125 \text{ kips} > 110 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 83.3 \text{ kips} > 73.3 \text{ kips}$ **o.k.**

Bolt Bearing and Block Shear Rupture of Beam Web

From AISC *Manual* Table 10-1, for five rows of bolts and $L_{ev} = 1\frac{1}{4}$ in. and $L_{eh} = 1\frac{1}{2}$ in.:

LRFD	ASD
$\phi R_n = 312 \text{ kips/in.}(0.380 \text{ in.})$ $= 119 \text{ kips} > 110 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 208 \text{ kips/in.}(0.380 \text{ in.})$ $= 79.0 \text{ kips} > 73.3 \text{ kips}$ **o.k.**

Coped Beam Strength (AISC Manual Part 9)

Flexural Yielding and Local Web Buckling

Verify parameters.

$$c \le 2d$$
$$5.00 \text{ in.} \le 2(20.8 \text{ in.})$$
$$\le 41.6 \text{ in.} \quad \textbf{o.k.}$$

$$d_c \le \frac{d}{2}$$
$$2.00 \text{ in.} \le \frac{20.8 \text{ in.}}{2}$$
$$\le 10.4 \text{ in.} \quad \textbf{o.k.}$$

$$\frac{c}{d} = \frac{5.00 \text{ in.}}{20.8 \text{ in.}}$$
$$= 0.240 \leq 1.0$$

$$\frac{c}{h_o} = \frac{5.00 \text{ in.}}{18.8 \text{ in.}}$$
$$= 0.266 \leq 1.0$$

Because $\frac{c}{d} \leq 1.0,$ the plate buckling model adjustment factor is:

$$f = \frac{2c}{d} \quad \text{(\textit{Manual} Eq. 9-8)}$$
$$= 2(0.240)$$
$$= 0.480$$

Because $\frac{c}{h_o} \leq 1.0$, the plate buckling coefficient is:

$$k = 2.2\left(\frac{h_o}{c}\right)^{1.65} \quad \text{(\textit{Manual} Eq. 9-10)}$$
$$= 2.2\left(\frac{18.8 \text{ in.}}{5.00 \text{ in.}}\right)^{1.65}$$
$$= 19.6$$

$$F_{cr} = 26{,}210\left(\frac{t_w}{h_o}\right)^2 f\,k \leq F_y \quad \text{(\textit{Manual} Eq. 9-7)}$$
$$= 26{,}210\left(\frac{0.380 \text{ in.}}{18.8 \text{ in.}}\right)^2 (0.480)(19.6)$$
$$= 101 \text{ ksi} \leq F_y$$

Use $F_{cr} = F_y = 50$ ksi.

From AISC *Manual* Equation 9-6:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = \frac{\phi F_{cr} S_{net}}{e}$ $= \frac{0.90(50 \text{ ksi})(32.5 \text{ in.}^3)}{5.50 \text{ in.}}$ $= 266 \text{ kips} > 110 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\frac{R_n}{\Omega} = \frac{F_{cr} S_{net}}{\Omega e}$ $= \frac{50 \text{ ksi}(32.5 \text{ in.}^3)}{1.67(5.50 \text{ in.})}$ $= 177 \text{ kips} > 73.3 \text{ kips}$ **o.k.**

Shear Yielding of Beam Web

$R_n = 0.60F_yA_{gv}$ (*Spec*. Eq. J4-3)
$= 0.60(50\text{ ksi})(0.380\text{ in.})(18.8\text{ in.})$
$= 214\text{ kips}$

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(214\text{ kips})$ $= 214\text{ kips} > 110\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{214\text{ kips}}{1.50}$ $= 143\text{ kips} > 73.3\text{ kips}$ **o.k.**

Shear Rupture of Beam Web

$A_{nv} = t_w[h_o - (5)(^{13}/_{16}\text{ in.} + {}^{1}/_{16}\text{ in.})]$
$= 0.380\text{ in.}(18.8\text{ in.} - 4.38\text{ in.})$
$= 5.48\text{ in.}^2$

$R_n = 0.60F_uA_{nv}$ (*Spec*. Eq. J4-4)
$= 0.60(65\text{ ksi})(5.48\text{ in.}^2)$
$= 214\text{ kips}$

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(214\text{ kips})$ $= 161\text{ kips} > 110\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{214\text{ kips}}{2.00}$ $= 107\text{ kips} > 73.3\text{ kips}$ **o.k.**

Supporting Girder

Supporting Girder Web

The required bearing strength per bolt is greatest for the bolts that are loaded by both connections. Thus, for the design of these four critical bolts, the required strength is determined as follows:

LRFD	ASD
From Beam A, each bolt must support one-fourth of 25.0 kips or 6.25 kips/bolt. From Beam B, each bolt must support one-tenth of 110 kips or 11.0 kips/bolt. Thus, $R_u = 6.25\text{ kips/bolt} + 11.0\text{ kips/bolt}$ $= 17.3\text{ kips/bolt}$ From AISC *Manual* Table 7-4, the allowable bearing strength per bolt is:	From Beam A, each bolt must support one-fourth of 16.7 kips or 4.18 kips/bolt. From Beam B, each bolt must support one-tenth of 73.3 kips or 7.33 kips/bolt. Thus, $R_a = 4.18\text{ kips/bolt} + 7.33\text{ kips/bolt}$ $= 11.5\text{ kips/bolt}$ From AISC *Manual* Table 7-4, the allowable bearing strength per bolt is:

$\phi r_n = 87.8$ kips/in.(0.520 in.) $= 45.7$ kips/bolt > 17.3 kips/bolt **o.k.**	$\dfrac{r_n}{\Omega} = 58.5$ kips/in.(0.520 in.) $= 30.4$ kips/bolt > 11.5 kips/bolt **o.k.**

The tabulated values may be verified by hand calculations, as follows:

From AISC *Specification* Equation J3-6a:

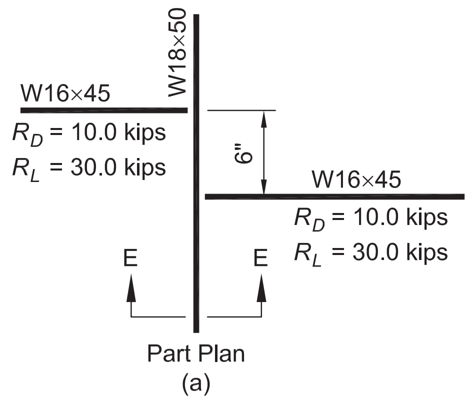
LRFD	ASD
$\phi = 0.75$ $\phi r_n = \phi 1.2 l_c t F_u \le \phi 2.4 d t F_u$ $l_c = 3.00\text{ in.} - {}^{13}\!/_{16}\text{ in.}$ $= 2.19$ in.	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega} \le \dfrac{2.4 d t F_u}{\Omega}$ $l_c = 3.00\text{ in.} - {}^{13}\!/_{16}\text{ in.}$ $= 2.19$ in.
$\phi 1.2 l_c t F_u = 0.75(1.2)(2.19\text{ in.})(0.520\text{ in.})(65\text{ ksi})$ $= 66.6$ kips $\phi(2.4 d t F_u) = 0.75(2.4)(0.750\text{ in.})(0.520\text{ in.})(65\text{ ksi})$ $= 45.6\text{ kips} < 66.6\text{ kips}$ $\phi r_n = 45.6$ kips/bolt > 17.3 kips/bolt **o.k.**	$\dfrac{1.2 l_c t F_u}{\Omega} = \dfrac{1.2(2.19\text{ in.})(0.520\text{ in.})(65\text{ ksi})}{2.00}$ $= 44.4$ kips $\dfrac{2.4 d t F_u}{\Omega} = \dfrac{2.4(0.750\text{ in.})(0.520\text{ in.})(65.0\text{ ksi})}{2.00}$ $= 30.4\text{ kips} < 44.4\text{ kips}$ $\dfrac{r_n}{\Omega} = 30.4$ kips/bolt > 11.5 kips/bolt **o.k.**

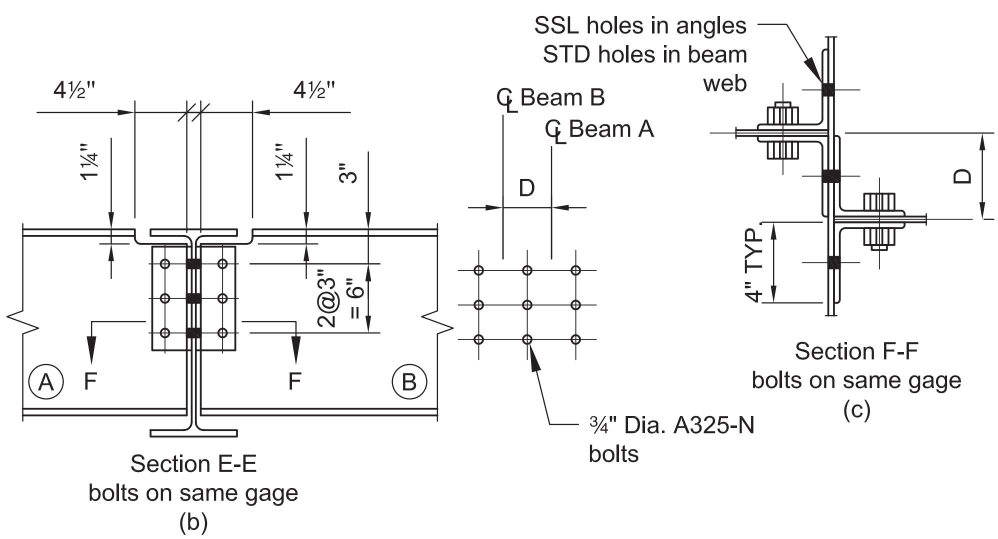
Return to Table of Contents

EXAMPLE II.A-9 OFFSET ALL-BOLTED DOUBLE-ANGLE CONNECTIONS (BEAMS-TO-GIRDER WEB)

Given:

Two all-bolted double-angle connections are made back-to-back with offset beams. Design the connections to accommodate an offset of 6 in. Use an ASTM A992 beam, and ASTM A992 beam and ASTM A36 angles.





Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Girder
W18×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Beam
W16×45
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Girder
W18×50
t_w = 0.355 in.
d = 18.0 in.

Beam
W16×45
t_w = 0.345 in.
d = 16.1 in.

Modify the 2L4×3½×¼ SLBB connection designed in Example II.A-4 to work in the configuration shown in the preceding figure. The offset dimension (6 in.) is approximately equal to the gage on the support from the previous example (6¼ in.) and, therefore, is not recalculated.

Thus, the bearing strength of the middle vertical row of bolts (through both connections), which carries a portion of the reaction for both connections, must be verified for this new configuration.

For each beam,

R_D = 10 kips
R_L = 30 kips

From Chapter 2 of ASCE 7, the required strength is:

LRFD	ASD
R_u = 1.2(10 kips) + 1.6(30 kips) = 60.0 kips	R_a = 10 kips + 30 kips = 40.0 kips

Bolt Shear

LRFD	ASD
$r_u = \dfrac{60.0\text{ kips}}{6\text{ bolts}}$ = 10.0 kips/bolt From AISC *Manual* Table 7-1, the available shear strength of a single bolt in double shear is: 17.9 kips/bolt > 10.0 kips/bolt **o.k.**	$r_a = \dfrac{40.0\text{ kips}}{6\text{ bolts}}$ = 6.67 kips/bolt From AISC *Manual* Table 7-1, the available shear strength of a single bolt in single shear is: 11.9 kips/bolt > 6.67 kips/bolt **o.k.**

Supporting Girder Web

At the middle vertical row of bolts, the required bearing strength for one bolt is the sum of the required shear strength per bolt for each connection. The available bearing strength per bolt is determined from AISC *Manual* Table 7-4.

LRFD	ASD
$r_u = 2(10.0 \text{ kips/bolt})$ $= 20.0 \text{ kips/bolt}$ $\phi r_n = 87.8 \text{ kips/in}(0.355 \text{ in.})$ $= 31.2 \text{ kips/bolt} > 20.0 \text{ kips/bolt}$ **o.k.**	$r_a = 2(6.67 \text{ kips/bolt})$ $= 13.3 \text{ kips/bolt}$ $\dfrac{r_n}{\Omega} = 58.5 \text{ kips/in.}(0.355 \text{ in.})$ $= 20.8 \text{ kips/bolt} > 13.3 \text{ kips/bolt}$ **o.k.**

Note: If the bolts are not spaced equally from the supported beam web, the force in each column of bolts should be determined by using a simple beam analogy between the bolts, and applying the laws of statics.

EXAMPLE II.A-10 SKEWED DOUBLE BENT-PLATE CONNECTION (BEAM-TO-GIRDER WEB)

Given:

Design the skewed double bent-plate connection between an ASTM A992 W16×77 beam and ASTM A992 W27×94 girder-web to support the following beam end reactions:

R_D = 13.3 kips
R_L = 40.0 kips

Use ⅞-in.-diameter ASTM A325-N or F1852-N bolts in standard holes through the support and ASTM A36 plates. Use 70-ksi electrode welds to the supported beam.

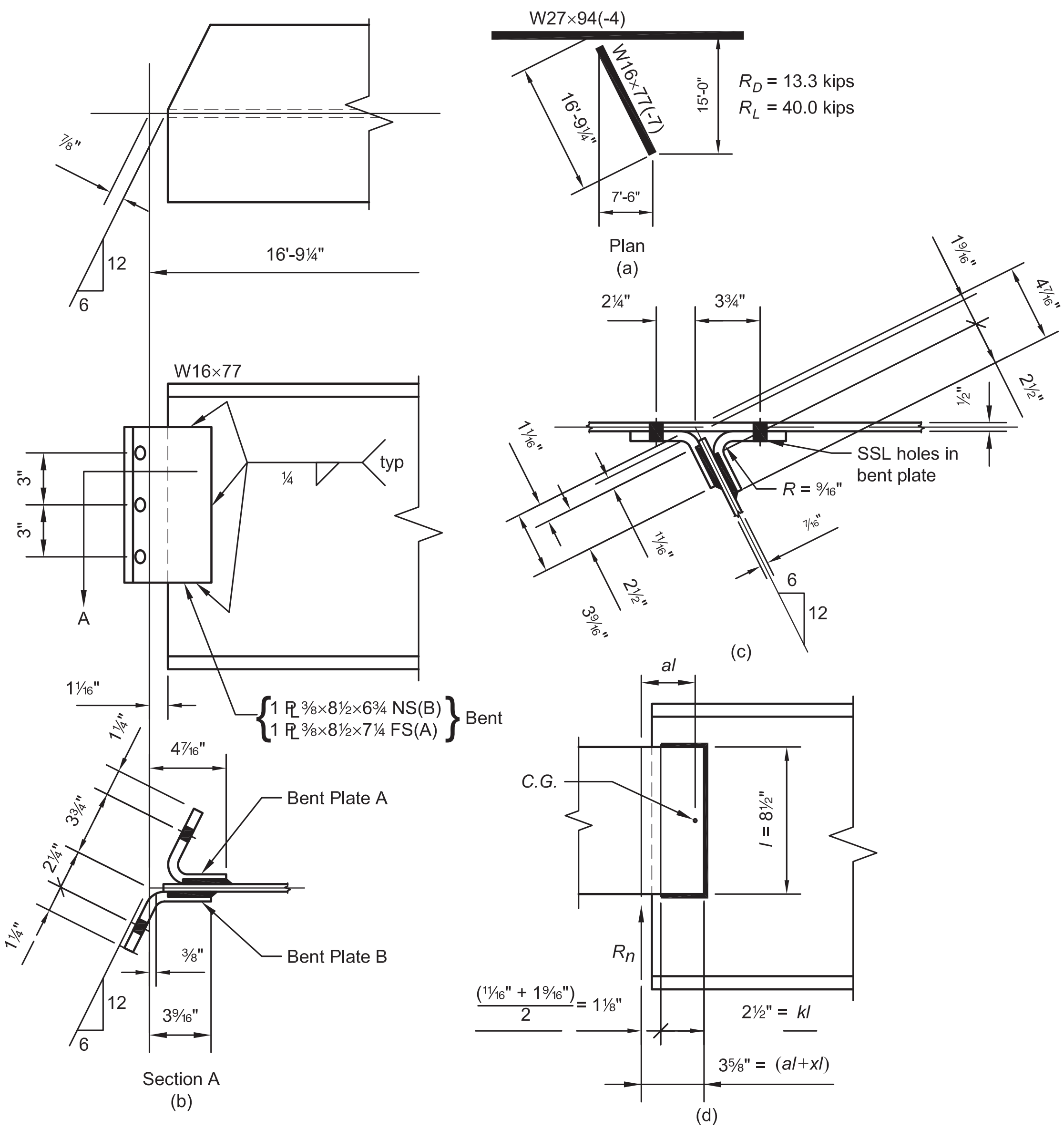


Fig. II.A-10. Skewed Double Bent-Plate Connection (Beam-to-Girder Web)

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W16×77
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Girder
W27×94
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×77
t_w = 0.455 in.
d = 16.5 in.

Girder
W27×94
t_w = 0.490 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2 (13.3 kips) + 1.6 (40.0 kips) = 80.0 kips	R_a = 13.3 kips + 40.0 kips = 53.3 kips

Using figure (c) of the connection, assign load to each vertical row of bolts by assuming a simple beam analogy between bolts and applying the laws of statics.

LRFD	ASD
Required strength for bent plate A: $R_u = \dfrac{80.0 \text{ kips } (2\frac{1}{4} \text{ in.})}{6.00 \text{ in.}}$ = 30.0 kips Required strength for bent plate B: $R_u = 80.0 \text{ kips} - 30.0 \text{ kips}$ = 50.0 kips	Required strength for bent plate A: $R_a = \dfrac{53.3 \text{ kips } (2\frac{1}{4} \text{ in.})}{6.00 \text{ in.}}$ = 20.0 kips Required strength for bent plate B: $R_a = 53.3 \text{ kips} - 20.0 \text{ kips}$ = 33.3 kips

Assume that the welds across the top and bottom of the plates will be 2½ in. long, and that the load acts at the intersection of the beam centerline and the support face.

While the welds do not coincide on opposite faces of the beam web and the weld groups are offset, the locations of the weld groups will be averaged and considered identical. See figure (d).

Weld Design

Assume a plate length of 8½ in.

$$k = \frac{kl}{l}$$
$$= \frac{2\frac{1}{2}\text{ in.}}{8\frac{1}{2}\text{ in.}}$$
$$= 0.294$$

$$xl = \frac{2\frac{1}{2}\text{ in.}(1\frac{1}{4}\text{ in.})(2)}{2\frac{1}{2}\text{ in.}(2) + 8\frac{1}{2}\text{ in.}}$$
$$= 0.463\text{ in.}$$

$$a = \frac{(al + xl) - xl}{l}$$
$$= \frac{3\frac{5}{8}\text{ in} - 0.463\text{ in.}}{8.50\text{ in.}}$$
$$= 0.373$$

Interpolating from AISC *Manual* Table 8-8, with $\theta = 0°$, $a = 0.373$, and $k = 0.294$,

$$C = 2.52$$

The required weld size for two such welds using AISC *Manual* Equation 8-13 is:

LRFD	ASD
$\phi = 0.75$ $D_{req} = \frac{R_u}{\phi C C_1 l}$ $= \frac{50.0\text{ kips}}{0.75(2.52)(1.0)(8\frac{1}{2}\text{ in.})}$ $= 3.11 \rightarrow$ 4 sixteenths	$\Omega = 2.00$ $D_{req} = \frac{\Omega R_a}{C C_1 l}$ $= \frac{2.00(33.3\text{ kips})}{2.52(1.0)(8\frac{1}{2}\text{ in.})}$ $= 3.11 \rightarrow$ 4 sixteenths

Use ¼-in. fillet welds and at least 5⁄16-in.-thick bent plates to allow for the welds.

Beam Web Thickness

According to Part 9 of the AISC *Manual*, with F_{EXX} = 70 ksi on both sides of the connection, the minimum thickness required to match the available shear rupture strength of the connection element to the available shear rupture strength of the base metal is:

$$t_{min} = \frac{6.19D}{F_u} \qquad \text{(\textit{Manual} Eq. 9-3)}$$
$$= \frac{6.19(3.11\text{ sixteenths})}{65\text{ ksi}}$$
$$= 0.296\text{ in.} < 0.455\text{ in.} \qquad \textbf{o.k.}$$

Bolt Shear

LRFD	ASD
Maximum shear to one bent plate = 50.0 kips Try 3 rows of ⅞-in.-diameter ASTM A325-N bolts. From AISC *Manual* Table 7-1: $\phi R_n = n(\phi r_n)$ $= 3 \text{ bolts}(24.3 \text{ kips/bolt})$ $= 72.9 \text{ kips} > 50.0 \text{ kips}$ **o.k.**	Maximum shear to one bent plate = 33.3 kips Try 3 rows of ⅞-in.-diameter ASTM A325-N bolts. From AISC *Manual* Table 7-1: $\dfrac{R_n}{\Omega} = n\left(\dfrac{r_n}{\Omega}\right)$ $= 3 \text{ bolts}(16.2 \text{ kips/bolt})$ $= 48.6 \text{ kips} > 33.3 \text{ kips}$ **o.k.**

Bearing on Support

From AISC *Manual* Table 7-4 with 3-in. spacing in standard holes:

LRFD	ASD
$\phi r_n = 102 \text{ kips/in.}(0.490 \text{ in.})(3 \text{ bolts})$ $= 150 \text{ kips} > 50.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 68.3 \text{ kips/in.}(0.490 \text{ in.})(3 \text{ bolts})$ $= 100 \text{ kips} > 33.3 \text{ kips}$ **o.k.**

Bent Plate Design

Try a 5⁄16 in plate.

LRFD	ASD
Bearing on plate from AISC *Manual* Tables 7-4 and 7-5: $\phi r_{ni} = 91.4 \text{ kips/in.}$ $\phi r_{no} = 40.8 \text{ kips/in.}$ $\phi R_n = \begin{bmatrix}(91.4 \text{ kips/in.})(2 \text{ bolts}) \\ +(40.8 \text{ kips/in.})(1 \text{ bolt})\end{bmatrix}(5/16 \text{ in.})$ $= 69.9 \text{ kips} > 50.0 \text{ kips}$ **o.k.** Shear yielding of plate using AISC *Specification* Equation J4-3: $\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(8\tfrac{1}{2} \text{ in.})(5/16 \text{ in.})$ $= 57.4 \text{ kips} > 50.0 \text{ kips}$ **o.k.**	Bearing on plate from AISC *Manual* Tables 7-4 and 7-5: $\dfrac{r_{ni}}{\Omega} = 60.9 \text{ kips/in.}$ $\dfrac{r_{no}}{\Omega} = 27.2 \text{ kips/in.}$ $\dfrac{R_n}{\Omega} = \begin{bmatrix}(60.9 \text{ kips/in.})(2 \text{ bolts}) \\ +(27.2 \text{ kips/in.})(1 \text{ bolt})\end{bmatrix}(5/16 \text{ in.})$ $= 46.6 \text{ kips} > 33.3 \text{ kips}$ **o.k.** Shear yielding of plate using AISC *Specification* Equation J4-3: $\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})(8\tfrac{1}{2} \text{ in.})(5/16 \text{ in.})}{1.50}$ $= 38.3 \text{ kips} > 33.3 \text{ kips}$ **o.k.**

<table>
<tr><th>LRFD</th><th>ASD</th></tr>
<tr><td>
Shear rupture of plate using AISC Specification Equation J4-4:

$A_{nv} = [8\frac{1}{2} \text{ in.} - 3(1.00 \text{ in.})](\frac{5}{16} \text{ in.})$
$= 1.72 \text{ in.}^2$

$\phi = 0.75$

$\phi R_n = \phi 0.60 F_u A_{nv}$
$= 0.75(0.60)(58 \text{ ksi})(1.72 \text{ in.}^2)$
$= 44.9 \text{ kips} < 50.0 \text{ kips}$ n.g.

Increase the plate thickness to ⅜ in.

$A_{nv} = [8\frac{1}{2} \text{ in.} - 3(1.00 \text{ in.})](\frac{3}{8} \text{ in.})$
$= 2.06 \text{ in.}^2$

$\phi = 0.75$

$\phi R_n = 0.75(0.60)(58 \text{ ksi})(2.06 \text{ in.}^2)$
$= 53.8 \text{ kips} > 50.0 \text{ kips}$ o.k.

Block shear rupture of plate using AISC Specification Equation J4-5 with $n = 3$, $L_{ev} = L_{eh} = 1\frac{1}{4}$ in., $U_{bs} = 1$:

$\phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv})$

Tension rupture component from AISC Manual Table 9-3a:

$\phi U_{bs} F_u A_{nt} = (1.0)(32.6 \text{ kips/in.})(\frac{3}{8} \text{ in.})$

Shear yielding component from AISC Manual Table 9-3b:

$\phi 0.60 F_y A_{gv} = 117 \text{ kips/in.}(\frac{3}{8} \text{ in.})$

Shear rupture component from AISC Manual Table 9-3c:

$\phi 0.60 F_u A_{nv} = 124 \text{ kips/in.}(\frac{3}{8} \text{ in.})$
</td><td>
Shear rupture of plate using AISC Specification Equation J4-4:

$A_{nv} = [8\frac{1}{2} \text{ in.} - 3(1.00 \text{ in.})](\frac{5}{16} \text{ in.})$
$= 1.72 \text{ in.}^2$

$\Omega = 2.00$

$\frac{R_n}{\Omega} = \frac{0.60 F_u A_{nv}}{\Omega}$
$= \frac{0.60(58 \text{ ksi})(1.72 \text{ in.}^2)}{2.00}$
$= 29.9 \text{ kips} < 33.3 \text{ kips}$ n.g.

Increase the plate thickness to ⅜ in.

$A_{nv} = [8\frac{1}{2} \text{ in.} - 3(1.00 \text{ in.})](\frac{3}{8} \text{ in.})$
$= 2.06 \text{ in.}^2$

$\Omega = 2.00$

$\frac{R_n}{\Omega} = \frac{0.60(58 \text{ ksi})(2.06 \text{ in.}^2)}{2.00}$
$= 35.8 \text{ kips} > 33.3 \text{ kips}$ o.k.

Block shear rupture of plate using AISC Specification Equation J4-5 with $n = 3$, $L_{ev} = L_{eh} = 1\frac{1}{4}$ in., $U_{bs} = 1$:

$\frac{R_n}{\Omega} = \frac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\frac{0.60 F_y A_{gv}}{\Omega},\ \frac{0.60 F_u A_{nv}}{\Omega}\right)$

Tension rupture component from AISC Manual Table 9-3a:

$\frac{U_{bs} F_u A_{nt}}{\Omega} = (1.0)(21.8 \text{ kips/in.})(\frac{3}{8} \text{ in.})$

Shear yielding component from AISC Manual Table 9-3b:

$\frac{0.60 F_y A_{gv}}{\Omega} = 78.3 \text{ kips/in.}(\frac{3}{8} \text{ in.})$

Shear rupture component from AISC Manual Table 9-3c:

$\frac{0.60 F_u A_{nv}}{\Omega} = 82.6 \text{ kips/in.}(\frac{3}{8} \text{ in.})$
</td></tr>
</table>

LRFD	ASD
$\phi R_n = (32.6\text{ kips/in.} + 117\text{ kips/in.})(\tfrac{3}{8}\text{ in.})$ $= 56.1\text{ kips} > 50.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (21.8\text{ kips/in.} + 78.3\text{ kips/in.})(\tfrac{3}{8}\text{ in.})$ $= 37.5\text{ kips} > 33.3\text{ kips}$ **o.k.**

Thus, the configuration shown in Figure II.A-10 can be supported using ⅜-in. bent plates, and ¼-in. fillet welds.

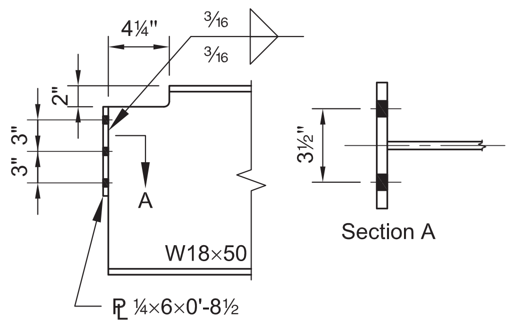
EXAMPLE II.A-11 SHEAR END-PLATE CONNECTION (BEAM TO GIRDER WEB)

Given:

Design a shear end-plate connection to connect an ASTM A992 W18×50 beam to an ASTM A992 W21×62 girder web, to support the following beam end reactions:

R_D = 10 kips
R_L = 30 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes, 70-ksi electrodes and ASTM A36 plates.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Girder
W21×62
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1 and 9-2 and AISC *Manual* Figure 9-2, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
t_w = 0.355 in.
S_{net} = 23.4 in.^{3}
c = 4¼ in.
d_c = 2 in.
e = 4½ in.

h_o = 16.0 in.

Girder
W21×62
t_w = 0.400 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2 (10 kips) + 1.6 (30 kips) = 60.0 kips	R_a = 10 kips + 30 kips = 40.0 kips

Bolt Shear and Bolt Bearing, Shear Yielding, Shear Rupture, and Block Shear Rupture of End-Plate

From AISC *Manual* Table 10-4, for 3 rows of bolts and ¼-in. plate thickness:

LRFD	ASD
ϕR_n = 76.4 kips > 60.0 kips **o.k.**	$\frac{R_n}{\Omega}$ = 50.9 kips > 40.0 kips **o.k.**

Weld Shear and Beam Web Shear Rupture

Try 3⁄16-in. weld. From AISC *Manual* Table 10-4, the minimum beam web thickness is,

$t_{w\,min}$ = 0.286 in. < 0.355 in. **o.k.**

From AISC *Manual* Table 10-4:

LRFD	ASD
ϕR_n = 67.9 kips > 60.0 kips **o.k.**	$\frac{R_n}{\Omega}$ = 45.2 kips > 40.0 kips **o.k.**

Bolt Bearing on Girder Web

From AISC *Manual* Table 10-4:

LRFD	ASD
ϕR_n = 526 kip/in.(0.400 in.) = 210 kips > 60.0 kips **o.k.**	$\frac{R_n}{\Omega}$ = 351 kip/in.(0.400 in.) = 140 kips > 40.0 kips **o.k.**

Coped Beam Strength

As was shown in Example II.A-4, the coped section does not control the design. **o.k.**

Beam Web Shear

As was shown in Example II.A-4, beam web shear does not control the design. **o.k.**

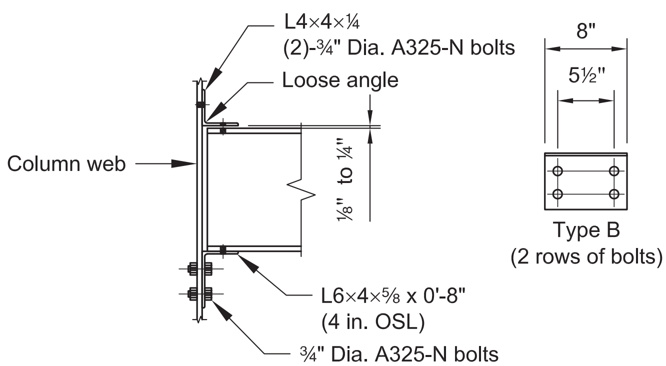
EXAMPLE II.A-12 ALL-BOLTED UNSTIFFENED SEATED CONNECTION (BEAM-TO-COLUMN WEB)

Given:

Design an all-bolted unstiffened seated connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column web to support the following end reactions:

R_D = 9.0 kips
R_L = 27.5 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and ASTM A36 angles.



Note: For calculation purposes, assume setback is equal to ¾ in. to account for possible beam underrun.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
W16×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×90
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×50
t_w = 0.380 in.

d = 16.3 in.
b_f = 7.07 in.
t_f = 0.630 in.
k_{des} = 1.03 in.

Column
W14×90
t_w = 0.440 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2\,(9.0\text{ kips}) + 1.6\,(27.5\text{ kips})$ $= 54.8\text{ kips}$	$R_a = 9.0\text{ kips} + 27.5\text{ kips}$ $= 36.5\text{ kips}$

Web Local Yielding Bearing Length (AISC Specification Section J10.2):

$l_{b\,min}$ is the length of bearing required for the limit states of web local yielding and web local crippling on the beam, but not less than k_{des}.

From AISC *Manual* Table 9-4:

LRFD	ASD
$l_{b\,min} = \dfrac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ (from *Manual* Eq. 9-45a) $= \dfrac{54.8\text{ kips} - 48.9\text{ kips}}{19.0\text{ kips/in.}} \geq 1.03\text{ in.}$ $= 0.311\text{ in.} < 1.03\text{ in.}$ Use $l_{bmin} = 1.03$ in.	$l_{b\,min} = \dfrac{R_a - R_1/\Omega}{R_2/\Omega} \geq k_{des}$ (from *Manual* Eq. 9-45b) $= \dfrac{36.5\text{ kips} - 32.6\text{ kips}}{12.7\text{ kips/in.}} \geq 1.03\text{ in.}$ $= 0.307\text{ in.} < 1.03\text{ in.}$ Use $l_{bmin} = 1.03$ in.

Web Local Crippling Bearing Length (AISC Specification Section J10.3):

$$\left(\frac{l_b}{d}\right)_{\max} = \frac{3.25\text{ in.}}{16.3\text{ in.}}$$
$$= 0.199 < 0.2$$

From AISC *Manual* Table 9-4, when $\dfrac{l_b}{d} \leq 0.2$,

LRFD	ASD
$l_{b\,min} = \dfrac{R_u - \phi R_3}{\phi R_4}$ (from *Manual* Eq. 9-47a) $= \dfrac{54.8\text{ kips} - 67.2\text{ kips}}{5.79\text{ kips/in.}}$ which results in a negative quantity. Therefore, $l_{b\,min} = k_{des} = 1.03$ in.	$l_{b\,min} = \dfrac{R_a - R_3/\Omega}{R_4/\Omega}$ (from *Manual* Eq. 9-47b) $= \dfrac{36.5\text{ kips} - 44.8\text{ kips}}{3.86\text{ kips/in.}}$ which results in a negative quantity. Therefore, $l_{b\,min} = k_{des} = 1.03$ in.

Shear Yielding and Flexural Yielding of Angle and Local Yielding and Crippling of Beam Web

Try an 8-in. angle length with a ⅝ in. thickness, a 3½-in. minimum outstanding leg and $l_{b\,req}$ = 1.03 in.

Conservatively, use l_b = 1 1/16 in.

From AISC *Manual* Table 10-5:

LRFD	ASD
$\phi R_n = 90.0 \text{ kips} > 54.8 \text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = 59.9 \text{ kips} > 36.5 \text{ kips}$ **o.k.**

Try L6×4×⅝ (4-in. OSL), 8-in. long with 5½-in. bolt gage, connection type B (four bolts).

From AISC *Manual* Table 10-5, for ¾-in. diameter ASTM A325-N bolts:

LRFD	ASD
$\phi R_n = 71.6 \text{ kips} > 54.8 \text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = 47.7 \text{ kips} > 36.5 \text{ kips}$ **o.k.**

Bolt Bearing on the Angle

LRFD	ASD
Required bearing strength: $r_u = \frac{54.8 \text{ kips}}{4 \text{ bolts}}$ $= 13.7 \text{ kips/bolt}$ By inspection, tear-out does not control; therefore, only the limit on AISC *Specification* Equation J3-6a need be checked. From AISC *Specification* Equation J3-6a: $\phi R_n = \phi 2.4 dtF_u$ $= 0.75(2.4)(\text{¾ in.})(\text{⅝ in.})(58 \text{ ksi})$ $= 48.9 \text{ kips} > 13.7 \text{ kips}$ **o.k.**	Required bearing strength: $r_a = \frac{36.5 \text{ kips}}{4 \text{ bolts}}$ $= 9.13 \text{ kips/bolt}$ By inspection, tear-out does not control; therefore, only the limit on AISC *Specification* Equation J3-6a need be checked. From AISC *Specification* Equation J3-6a: $\frac{R_n}{\Omega} = \frac{2.4 dtF_u}{\Omega}$ $= \frac{2.4(\text{¾ in.})(\text{⅝ in.})(58 \text{ ksi})}{2.00}$ $= 32.6 \text{ kips} > 9.13 \text{ kips}$ **o.k.**

Bolt Bearing on the Column

LRFD	ASD
$\phi R_n = \phi 2.4 dtF_u$ $= 0.75(2.4)(\text{¾ in.})(0.440 \text{ in.})(65 \text{ ksi})$ $= 38.6 \text{ kips} > 13.7 \text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = \frac{2.4 dtF_u}{\Omega}$ $= \frac{2.4(\text{¾ in.})(0.440 \text{ in.})(65 \text{ ksi})}{2.00}$ $= 25.7 \text{ kips} > 9.13 \text{ kips}$ **o.k.**

Top Angle and Bolts

Use an L4×4×¼ with two ¾-in.-diameter ASTM A325-N or F1852-N bolts through each leg.

Return to Table of Contents

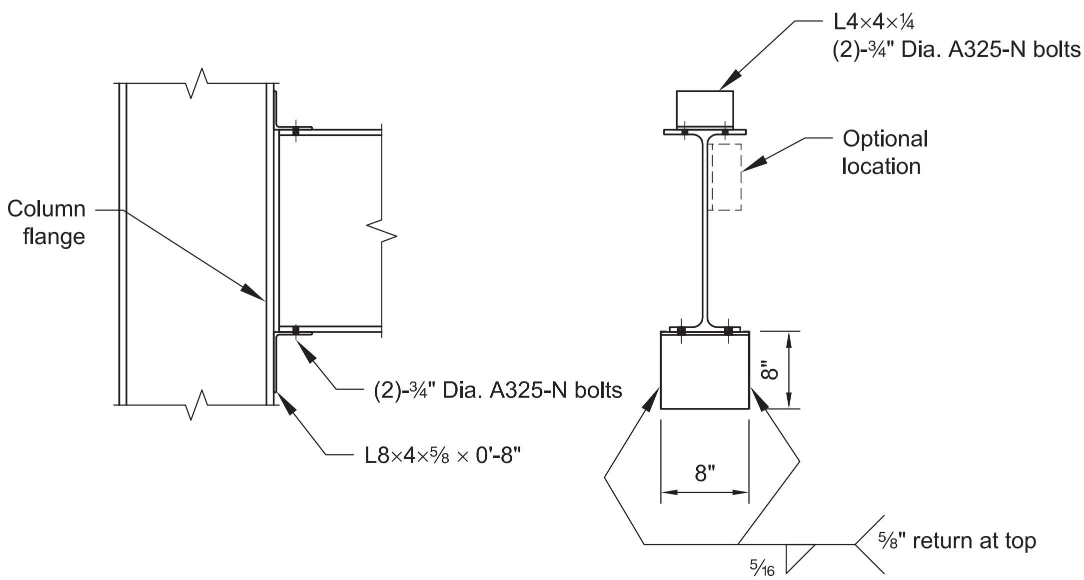
EXAMPLE II.A-13 BOLTED/WELDED UNSTIFFENED SEATED CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design an unstiffened seated connection between an ASTM A992 W21×62 beam and an ASTM A992 W14×61 column flange to support the following beam end reactions:

R_D = 9.0 kips
R_L = 27.5 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes to connect the supported beam to the seat and top angles. Use 70-ksi electrode welds to connect the seat and top angles to the column flange and ASTM A36 angles.



Note: For calculation purposes, assume setback is equal to ¾ in. to account for possible beam underrun.

Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
W21×62
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×61
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W21×62
$t_w = 0.400$ in.
$d = 21.0$ in.
$b_f = 8.24$ in.
$t_f = 0.615$ in.
$k_{des} = 1.12$ in.

Column
W14×61
$t_f = 0.645$ in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2\text{ (9.0 kips)} + 1.6\text{ (27.5 kips)}$ $= 54.8\text{ kips}$	$R_a = 9.0\text{ kips} + 27.5\text{ kips}$ $= 36.5\text{ kips}$

Web Local Yielding Bearing Length (AISC Specification Section J10.2):

$l_{b\,min}$ is the length of bearing required for the limit states of web local yielding and web local crippling of the beam, but not less than k_{des}.

From AISC *Manual* Table 9-4:

LRFD	ASD
$l_{b\,min} = \dfrac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ (from *Manual* Eq. 9-45a) $= \dfrac{54.8\text{ kips} - 56.0\text{ kips}}{20.0\text{ kips/in.}} \geq 1.12\text{ in.}$ which results in a negative quantity. Therefore, $l_{b\,min} = k_{des} = 1.12$ in.	$l_{b\,min} = \dfrac{R_a - R_1 / \Omega}{R_2 / \Omega} \geq k_{des}$ (from *Manual* Eq. 9-45b) $= \dfrac{36.5\text{ kips} - 37.3\text{ kips}}{13.3\text{ kips/in.}} \geq 1.12\text{ in.}$ which results in a negative quantity. Therefore, $l_{b\,min} = k_{des} = 1.12$ in.

Web Local Crippling Bearing Length (AISC Specification Section J10.3):

$$\left(\frac{l_b}{d}\right)_{max} = \frac{3\tfrac{1}{4}\text{ in.}}{21.0\text{ in.}}$$
$$= 0.155 < 0.2$$

From AISC *Manual* Table 9-4, when $\dfrac{l_b}{d} \le 0.2$,

LRFD	ASD
$l_{b\,min} = \dfrac{R_u - \phi R_3}{\phi R_4}$ (from *Manual* Eq. 9-47a) $= \dfrac{54.8 \text{ kips} - 71.7 \text{ kips}}{5.37 \text{ kips/in.}}$ which results in a negative quantity. Therefore, $l_{b\,min} = k_{des} = 1.12$ in.	$l_{b\,min} = \dfrac{R_a - R_3/\Omega}{R_4/\Omega}$ (from *Manual* Eq. 9-47b) $= \dfrac{36.5 \text{ kips} - 47.8 \text{ kips}}{3.58 \text{ kips/in.}}$ which results in a negative quantity. Therefore, $l_{b\,min} = k_{des} = 1.12$ in.

Shear Yielding and Flexural Yielding of Angle and Local Yielding and Crippling of Beam Web

Try an 8-in. angle length with a ⅝-in. thickness and a 3½-in. minimum outstanding leg.

Conservatively, use l_b = 1⅛ in.

From AISC *Manual* Table 10-6:

LRFD	ASD
ϕR_n = 81.0 kips > 54.8 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 53.9 kips > 36.5 kips **o.k.**

Try an L8×4×⅝ (4 in. OSL), 8 in. long with 5⁄16-in. fillet welds.

From AISC *Manual* Table 10-6:

LRFD	ASD
ϕR_n = 66.7 kips > 54.8 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 44.5 kips > 36.5 kips **o.k.**

Use two ¾-in.-diameter ASTM A325-N bolts to connect the beam to the seat angle.

The strength of the bolts, welds and angles must be verified if horizontal forces are added to the connection.

Top Angle, Bolts and Welds

Use an L4×4×¼ with two ¾-in.-diameter ASTM A325-N or F1852-N bolts through the supported beam leg of the angle. Use a 3⁄16-in. fillet weld along the toe of the angle to the column flange. See the discussion in AISC *Manual* Part 10.

Return to Table of Contents

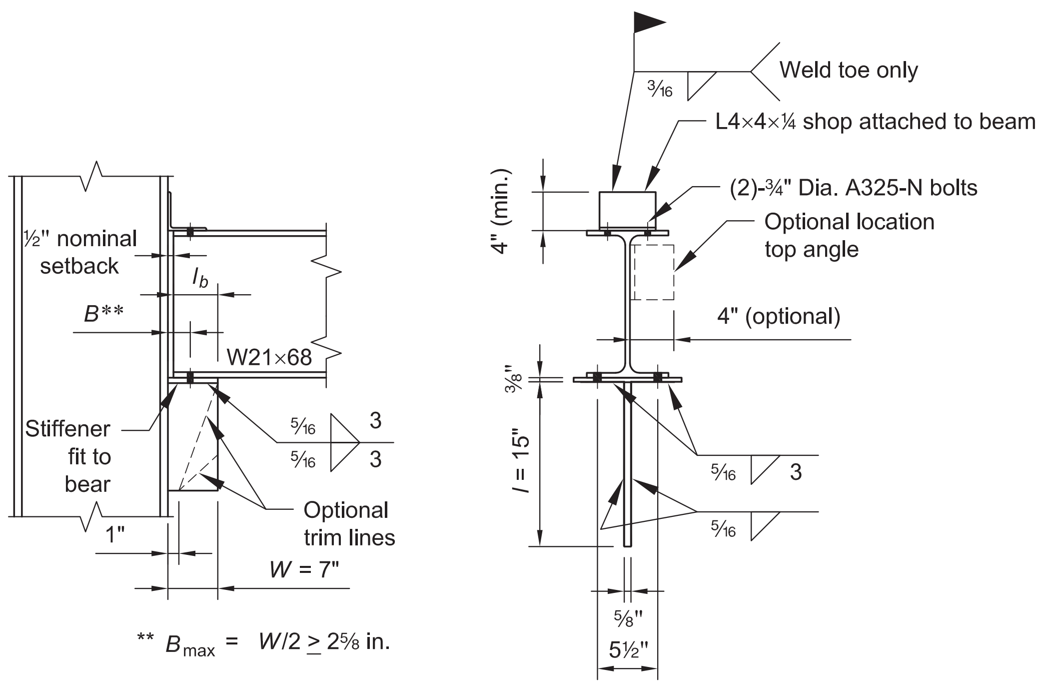
EXAMPLE II.A-14 STIFFENED SEATED CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a stiffened seated connection between an ASTM A992 W21×68 beam and an ASTM A992 W14×90 column flange, to support the following end reactions:

R_D = 21 kips
R_L = 62.5 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes to connect the supported beam to the seat plate and top angle. Use 70-ksi electrode welds to connect the stiffener and top angle to the column flange and ASTM A36 plates and angles.



Note: For calculation purposes, assume setback is equal to ¾ in. to account for possible beam underrun.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W21×68
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×90
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles and plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W21×68
t_w = 0.430 in.
d = 21.1 in.
b_f = 8.27 in.
t_f = 0.685 in.
k_{des} = 1.19 in.

Column
W14×90
t_f = 0.710 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2\text{ (21 kips)} + 1.6\text{ (62.5 kips)}$ $= 125\text{ kips}$	$R_a = 21\text{ kips} + 62.5\text{ kips}$ $= 83.5\text{ kips}$

Required Stiffener Width, W

For web local crippling, assume $l_b/d > 0.2$.

From AISC *Manual* Table 9-4, W_{min} for local crippling is:

LRFD	ASD
$W_{min} = \dfrac{R_u - \phi R_5}{\phi R_6} + \text{ setback}$ $= \dfrac{125\text{ kips} - 75.9\text{ kips}}{7.95\text{ kips / in.}} + \text{¾ in.}$ $= 6.93\text{ in.}$	$W_{min} = \dfrac{R_a - R_5 / \Omega}{R_6 / \Omega} + \text{ setback}$ $= \dfrac{83.5\text{ kips} - 50.6\text{ kips}}{5.30\text{ kips / in.}} + \text{¾ in.}$ $= 6.96\text{ in.}$

From AISC *Manual* Table 9-4, W_{min} for web local yielding is:

LRFD	ASD
$W_{min} = \dfrac{R_u - \phi R_1}{\phi R_2} + \text{ setback}$ $= \dfrac{125\text{ kips} - 64.0\text{ kips}}{21.5\text{ kips / in.}} + \text{¾ in.}$ $= 3.59\text{ in.} < 6.93\text{ in.}$	$W_{min} = \dfrac{R_a - R_1 / \Omega}{R_2 / \Omega} + \text{ setback}$ $= \dfrac{83.5\text{ kips} - 42.6\text{ kips}}{14.3\text{ kips / in.}} + \text{¾ in.}$ $= 3.61\text{ in.} < 6.96\text{ in.}$

Use W = 7 in.

Check assumption:

$$\frac{l_b}{d} = \frac{7.00\text{ in.} - \text{¾ in.}}{21.1\text{ in.}}$$

= 0.296 > 0.2 **o.k.**

Stiffener Length, L, and Stiffener to Column Flange Weld Size

Try a stiffener with L = 15 in. and 5⁄16-in. fillet welds.

From AISC *Manual* Table 10-8:

LRFD	ASD
ϕR_n = 139 kips > 125 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 93.0 kips > 83.5 kips **o.k.**

Seat Plate Welds (AISC Manual Part 10)

Use 5⁄16-in. fillet welds on each side of the stiffener. Minimum length of seat plate to column flange weld is 0.2(L) = 3 in. The weld between the seat plate and stiffener plate is required to have a strength equal to or greater than the weld between the seat plate and the column flange, use 5⁄16-in. fillet welds on each side of the stiffener to the seat plate; length of weld = 6 in.

Seat Plate Dimensions (AISC Manual Part 10)

A width of 9 in. is adequate to accommodate two ¾-in.-diameter ASTM A325-N bolts on a 5½ in. gage connecting the beam flange to the seat plate.

Use a PL⅜ in.×7 in.×9 in. for the seat.

Stiffener Plate Thickness (AISC Manual Part 10)

Determine the minimum plate thickness to develop the stiffener to the seat plate weld.

$$\begin{aligned} t_{min} &= 2w \\ &= 2(5/16 \text{ in.}) \\ &= 5/8 \text{ in.} \end{aligned}$$

Determine the minimum plate thickness for a stiffener with F_y = 36 ksi and a beam with F_y = 50 ksi.

$$\begin{aligned} t_{min} &= \frac{50 \text{ ksi}}{36 \text{ ksi}} t_w \\ &= \frac{50 \text{ ksi}}{36 \text{ ksi}} (0.430 \text{ in.}) \\ &= 0.597 \text{ in.} < 5/8 \text{ in.} \end{aligned}$$

Use a PL ⅝ in.×7 in.×1 ft 3 in.

Top Angle, Bolts and Welds

Use an L4×4×¼ with two ¾-in.-diameter A325-N or F1852-N bolts through the supported beam leg of the angle. Use a 3⁄16-in. fillet weld along the toe of the angle to the column flange.

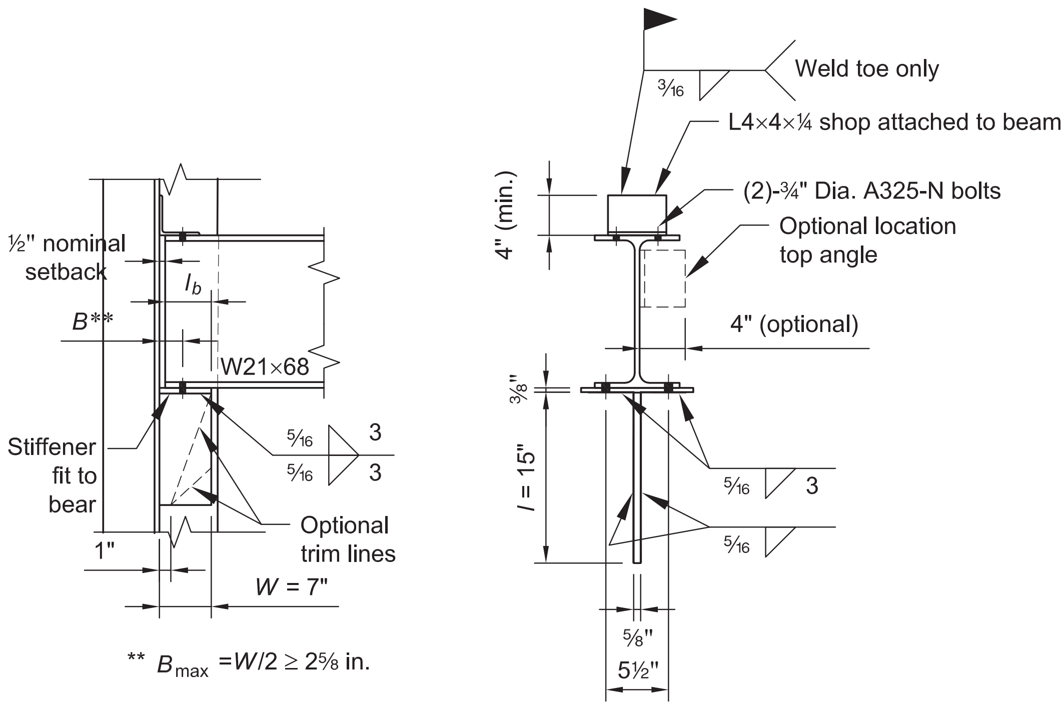
EXAMPLE II.A-15 STIFFENED SEATED CONNECTION (BEAM-TO-COLUMN WEB)

Given:

Design a stiffened seated connection between an ASTM A992 W21×68 beam and an ASTM A992 W14×90 column web to support the following beam end reactions:

R_D = 21 kips
R_L = 62.5 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes to connect the supported beam to the seat plate and top angle. Use 70-ksi electrode welds to connect the stiffener and top angle to the column web. Use ASTM A36 angles and plates.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W21×68
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×90
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles and Plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W21×68
t_w = 0.430 in.
d = 21.1 in.
b_f = 8.27 in.
t_f = 0.685 in.
k_{des} = 1.19 in.

Column
W14×90
t_w = 0.440 in.
T = 10 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2 (21 kips) + 1.6 (62.5 kips) = 125 kips	R_a = 21 kips + 62.5 kips = 83.5 kips

Required Stiffener Width, W

As calculated in Example II.A-14, use W = 7 in.

Stiffener Length, L, and Stiffener to Column Web Weld Size

As calculated in Example II.A-14, use L = 15 in. and 5⁄16-in. fillet welds.

Seat Plate Welds (AISC Manual Part 10)

As calculated in Example II.A-14, use 3 in. of 5⁄16-in. weld on both sides of the seat plate for the seat plate to column web welds and for the seat plate to stiffener welds.

Seat Plate Dimensions (AISC Manual Part 10)

For a column web support, the maximum distance from the face of the support to the line of the bolts between the beam flange and seat plate is 3½ in. The PL⅜ in.×7 in.×9 in. selected in Example II.A-14 will accommodate these bolts.

Stiffener Plate Thickness (AISC Manual Part 10)

As calculated in Example II.A-14, use a PL⅝ in.×7 in.×1 ft 3 in.

Top Angle, Bolts and Welds

Use an L4×4×¼ with two ¾-in.-diameter ASTM A325-N bolts through the supported beam leg of the angle. Use a 3⁄16-in. fillet weld along the toe of the angle to the column web.

Column Web

If only one side of the column web has a stiffened seated connection, then,

$$t_{w\,min} = \frac{3.09D}{F_u} \qquad \text{(}\textit{Manual}\text{ Eq. 9-2)}$$

$$= \frac{3.09(5\text{ sixteenths})}{65\text{ ksi}}$$

$$= 0.238\text{ in.}$$

If both sides of the column web have a stiffened seated connection, then,

$$t_{w\,min} = \frac{6.19D}{F_u} \qquad \text{(}\textit{Manual}\text{ Eq. 9-3)}$$

$$= \frac{6.19(5\text{ sixteenths})}{65\text{ ksi}}$$

$$= 0.476\text{ in.}$$

Column t_w = 0.440 in., which is sufficient for the one-sided stiffened seated connection shown.

Note: Additional detailing considerations for stiffened seated connections are given in Part 10 of the AISC *Manual*.

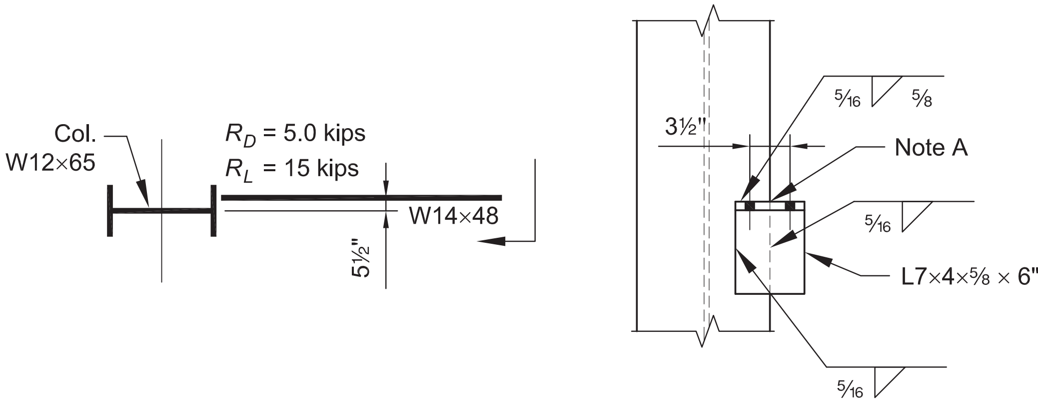
EXAMPLE II.A-16 OFFSET UNSTIFFENED SEATED CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Determine the seat angle and weld size required for the unstiffened seated connection between an ASTM A992 W14×48 beam and an ASTM A992 W12×65 column flange connection with an offset of 5½ in., to support the following beam end reactions:

R_D = 5.0 kips
R_L = 15 kips

Use 70-ksi electrode welds to connect the seat angle to the column flange and an ASTM A36 angle.



Note A: End return is omitted because the AWS Code does not permit weld returns to be carried around the corner formed by the column flange toe and seat angle heel.

Note B: Beam and top angle not shown for clarity.

Note C: The nominal setback of the beam from the face of the flange is ½ in. A setback of ¾ in. is used in the calculations to accommodate potential beam underrun.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W14×48
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W12×65
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle

ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W14×48
t_w = 0.340 in.
d = 13.8 in.
b_f = 8.03 in.
t_f = 0.595 in.
k_{des} = 1.19 in.

Column
W12×65
b_f = 12.0 in.
t_f = 0.605 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2 (5.0 kips) + 1.6 (15 kips) = 30.0 kips	R_a = 5.0 kips + 15 kips = 20.0 kips

Web Local Yielding Bearing Length (AISC Specification Section J10.2):

$l_{b\ min}$ is the length of bearing required for the limit states of web local yielding and web local crippling, but not less than k_{des}.

From AISC *Manual* Table 9-4:

LRFD	ASD
$l_{b\ min} = \dfrac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ (from *Manual* Eq. 9-45a) $= \dfrac{30.0 \text{ kips} - 50.6 \text{ kips}}{17.0 \text{ kips / in.}} > 1.19 \text{ in.}$ which results in a negative quantity. Therefore, $l_{b\ min} = k_{des}$ = 1.19 in.	$l_{b\ min} = \dfrac{R_a - (R_1 / \Omega)}{(R_2 / \Omega)} \geq k_{des}$ (from *Manual* Eq. 9-45b) $= \dfrac{20.0 \text{ kips} - 33.7 \text{ kips}}{11.3 \text{ kips / in.}} > 1.19 \text{ in.}$ which results in a negative quantity. Therefore, $l_{b\ min} = k_{des}$ = 1.19 in.

Web Local Crippling Bearing Length (AISC Specification Section J10.3):

From AISC *Manual* Table 9-4, when $\dfrac{l_b}{d} \leq 0.2$,

LRFD	ASD
$l_{b\ min} = \dfrac{R_u - \phi R_3}{\phi R_4}$ (from *Manual* Eq. 9-47a) $= \dfrac{30.0 \text{ kips} - 55.2 \text{ kips}}{5.19 \text{ kips/in.}}$	$l_{b\ min} = \dfrac{R_a - (R_3 / \Omega)}{(R_4 / \Omega)}$ (from *Manual* Eq. 9-47b) $= \dfrac{20.0 \text{ kips} - 36.8 \text{ kips}}{3.46 \text{ kips/in.}}$ which results in a negative quantity.

which results in a negative quantity. Therefore, $l_{b,req} = k_{des}$ = 1.19 in.	Therefore, $l_{b,req} = k_{des}$ = 1.19 in.

Seat Angle and Welds

The required strength for the righthand weld can be determined by summing moments about the lefthand weld.

LRFD	ASD
$R_{uR} = \dfrac{30.0\text{ kips}(3.00\text{ in.})}{3.50\text{ in.}}$ $= 25.7$ kips	$R_{aR} = \dfrac{20.0\text{ kips}(3.00\text{ in.})}{3.50\text{ in.}}$ $= 17.1$ kips

Conservatively design the seat for twice the force in the more highly loaded weld. Therefore design the seat for the following:

LRFD	ASD
R_u = 2(25.7 kips) = 51.4 kips	R_a = 2(17.1 kips) = 34.2 kips

Try a 6-in. angle length with a ⅝-in. thickness.

From AISC *Manual* Table 10-6, with $l_{b,req}$ = 1³⁄₁₆ in.:

LRFD	ASD
ϕR_n = 55.2 kips > 51.4 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 36.7 kips > 34.2 kips **o.k.**

For an L7×4 (OSL) angle with ⁵⁄₁₆-in. fillet welds, the weld strength from AISC *Manual* Table 10-6 is:

LRFD	ASD
ϕR_n = 53.4 kips > 51.4 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 35.6 kips > 34.2 kips **o.k.**

Use L7×4×⅝×6 in. for the seat angle. Use two ¾-in.-diameter ASTM A325-N or F1852-N bolts to connect the beam to the seat angle. Weld the angle to the column with ⁵⁄₁₆-in. fillet welds.

Top Angle, Bolts and Welds

Use an L4×4×¼ with two ¾-in.-diameter ASTM A325-N or F1852-N bolts through the outstanding leg of the angle.

Use a ³⁄₁₆-in. fillet weld along the toe of the angle to the column flange (maximum size permitted by AISC *Specification* Section J2.2b).

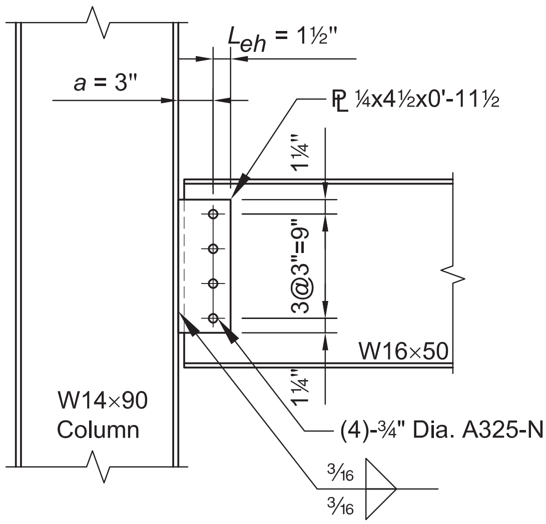
EXAMPLE II.A-17 SINGLE-PLATE CONNECTION (CONVENTIONAL—BEAM-TO-COLUMN FLANGE)

Given:

Design a single-plate connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column flange to support the following beam end reactions:

R_D = 8.0 kips
R_L = 25 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes, 70-ksi electrode welds and an ASTM A36 plate.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W16×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×90
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×50
t_w = 0.380 in.
d = 16.3 in.
t_f = 0.630 in.

Column
W14×90
t_f = 0.710 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(8.0 kips) + 1.6(25 kips) = 49.6 kips	R_a = 8.0 kips + 25 kips = 33.0 kips

Bolt Shear, Weld Shear, and Bolt Bearing, Shear Yielding, Shear Rupture, and Block Shear Rupture of the Plate

Try four rows of bolts, ¼-in. plate thickness, and 3⁄16-in. fillet weld size.

From AISC *Manual* Table 10-10a:

LRFD	ASD
ϕR_n = 52.2 kips > 49.6 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 34.8 kips > 33.0 kips **o.k.**

Bolt Bearing for Beam Web

Block shear rupture, shear yielding and shear rupture will not control for an uncoped section.

From AISC *Manual* Table 10-1, for an uncoped section, the beam web available strength is:

LRFD	ASD
ϕR_n = 351 kips/in.(0.380 in.) = 133 kips > 49.6 kips **o.k.**	$\dfrac{R_n}{\Omega} = 234\text{ kips/in.}\left(0.380\text{ in.}\right)$ = 88.9 kips > 33.0 kips **o.k.**

Note: To provide for stability during erection, it is recommended that the minimum plate length be one-half the T-dimension of the beam to be supported. AISC *Manual* Table 10-1 may be used as a reference to determine the recommended maximum and minimum connection lengths for a supported beam.

Return to Table of Contents

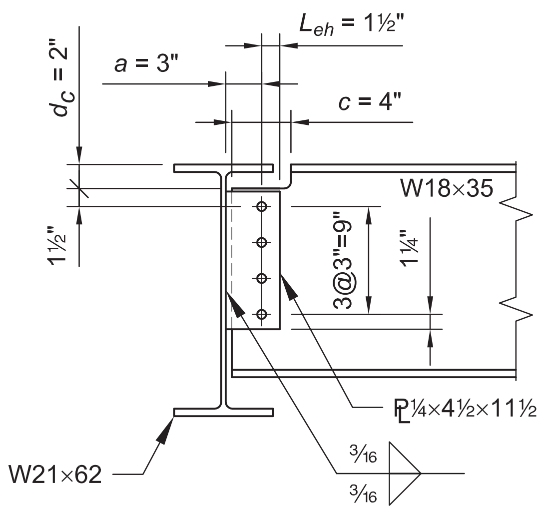
EXAMPLE II.A-18 SINGLE-PLATE CONNECTION (BEAM-TO-GIRDER WEB)

Given:

Design a single-plate connection between an ASTM A992 W18×35 beam and an ASTM A992 W21×62 girder web to support the following beam end reactions:

R_D = 6.5 kips
R_L = 20 kips

The top flange is coped 2 in. deep by 4 in. long, L_{ev} = 1½ in. Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes, 70-ksi electrode welds and an ASTM A36 plate.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×35
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Girder
W21×62
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1 and Figure 9-2, the geometric properties are as follows:

Beam
W18×35
t_w = 0.300 in.
d = 17.7 in.
t_f = 0.425 in.
c = 4.00 in.
d_c = 2.00 in.
h_o = 15.7 in.

Girder
W21×62
t_w = 0.400 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(6.5 kips) + 1.6(20 kips) = 39.8 kips	R_a = 6.5 kips + 20 kips = 26.5 kips

Bolt Shear, Weld Shear, and Bolt Bearing, Shear Yielding, Shear Rupture, and Block Shear Rupture of the Plate

Try four rows of bolts, ¼-in. plate thickness, and 3⁄16-in. fillet weld size.

From AISC *Manual* Table 10-10a:

LRFD	ASD
ϕR_n = 52.2 kips > 39.8 kips **o.k.**	$\frac{R_n}{\Omega}$ = 34.8 kips > 26.5 kips **o.k.**

Bolt Bearing and Block Shear Rupture for Beam Web

From AISC *Manual* Table 10-1, for a coped section with n = 4, L_{ev} = 1½ in., and L_{eh} > 1¾ in.:

LRFD	ASD
ϕR_n = 269 kips/in.(0.300 in.) = 80.7 kips > 39.8 kips **o.k.**	$\frac{R_n}{\Omega} = 180 \text{ kips/in.}(0.300 \text{ in.})$ = 54.0 kips > 26.5 kips **o.k.**

Shear Rupture of the Girder Web at the Weld

$$t_{min} = \frac{3.09D}{F_u} \qquad (\textit{Manual} \text{ Eq. 9-2})$$

$$= \frac{3.09(3\,\text{sixteenths})}{65 \text{ ksi}}$$

$$= 0.143 \text{ in.} < 0.400 \text{ in.} \qquad \textbf{o.k.}$$

Note: For coped beam sections, the limit states of flexural yielding and local buckling should be checked independently per AISC *Manual* Part 9. The supported beam web should also be checked for shear yielding and shear rupture per AISC *Specification* Section J4.2. However, for the shallow cope in this example, these limit states do not govern. For an illustration of these checks, see Example II.A-4.

Return to Table of Contents

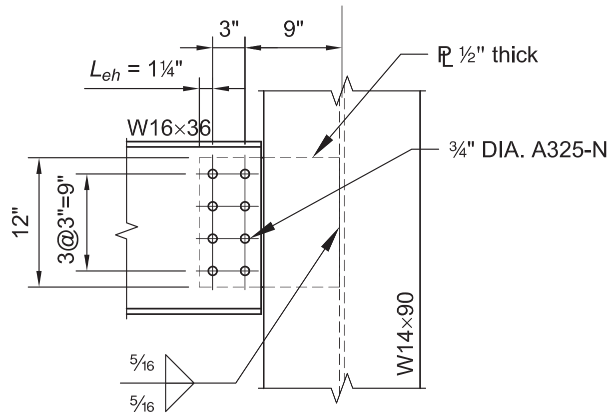
EXAMPLE II.A-19 EXTENDED SINGLE-PLATE CONNECTION (BEAM-TO-COLUMN WEB)

Given:

Design the connection between an ASTM A992 W16×36 beam and the web of an ASTM A992 W14×90 column, to support the following beam end reactions:

R_D = 6.0 kips
R_L = 18 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and an ASTM A36 plate. The beam is braced by the floor diaphragm. The plate is assumed to be thermally cut.



Note: All dimensional limitations are satisfied.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W16×36
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×90
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam

W16×36
t_w = 0.295 in.
d = 15.9 in.

Column
W14×90
t_w = 0.440 in.
b_f = 14.5 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(6.0 kips) + 1.6(18 kips) = 36.0 kips	R_a = 6.0 kips + 18 kips = 24.0 kips

Determine the distance from the support to the first line of bolts and the distance to the center of gravity of the bolt group.

a = 9.00 in.

e = 9.00 in. +1.50 in.
= 10.5 in.

Bearing Strength of One Bolt on the Beam Web

Tear out does not control by inspection.

From AISC *Manual* Table 7-4, determine the bearing strength (right side of AISC *Specification* Equation J3-6a):

LRFD	ASD
$\phi r_n = 87.8 \text{ kips/in.}(0.295 \text{ in.})$ $= 25.9 \text{ kips}$	$\dfrac{r_n}{\Omega} = 58.5 \text{ kips/in.}(0.295 \text{ in.})$ $= 17.3 \text{ kips}$

Shear Strength of One Bolt

From AISC *Manual* Table 7-1:

LRFD	ASD
$\phi r_n = 17.9 \text{ kips}$	$\dfrac{r_n}{\Omega} = 11.9 \text{ kips}$

Therefore, shear controls over bearing.

Strength of the Bolt Group

By interpolating AISC *Manual* Table 7-7, with e = 10.5 in.:

C = 2.33

LRFD	ASD
$\phi R_n = C\phi r_n$ $= 2.33(17.9 \text{ kips})$ $= 41.7 \text{ kips} > 36.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{Cr_n}{\Omega}$ $= 2.33(11.9 \text{ kips})$ $= 27.7 \text{ kips} > 24.0 \text{ kips}$ **o.k.**

Maximum Plate Thickness

Determine the maximum plate thickness, t_{max}, that will result in the plate yielding before the bolts shear.

F_{nv} = 54 ksi from AISC *Specification* Table J3.2

C' = 26.0 in. from AISC *Manual* Table 7-7 for the moment-only case

$$M_{max} = \frac{F_{nv}}{0.90}\left(A_b C'\right) \qquad \text{(\textit{Manual} Eq. 10-3)}$$
$$= \frac{54 \text{ ksi}}{0.90}\left(0.442 \text{ in.}^2\right)\left(26.0 \text{ in.}\right)$$
$$= 690 \text{ kip-in.}$$

$$t_{max} = \frac{6M_{max}}{F_y d^2} \qquad \text{(\textit{Manual} Eq. 10-2)}$$
$$= \frac{6\left(690 \text{ kip-in.}\right)}{36 \text{ ksi}\left(12.0 \text{ in.}\right)^2}$$
$$= 0.799 \text{ in.}$$

Try a plate thickness of ½ in.

Bolt Bearing on Plate

$$l_c = 1.50 \text{ in.} - \frac{{}^{13}\!/_{16} \text{ in.}}{2}$$
$$= 1.09 \text{ in.}$$

$$R_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad \text{(\textit{Spec}. Eq. J3-6a)}$$
1.2(1.09 in.)(½ in.)(58 ksi) ≤ 2.4(¾ in.)(½ in.)(58 ksi)
37.9 kips/bolt ≤ 52.2 kips/bolt

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(37.9 \text{ kips/bolt})$ $= 28.4 \text{ kips/bolt} > 17.9 \text{ kip/bolt}$	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{37.9 \text{ kips/bolt}}{2.00}$ $= 19.0 \text{ kips/bolt} > 11.9 \text{ kips/bolt}$

Therefore, bolt shear controls.

Shear Yielding of Plate

Using AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(12.0\text{ in.})(\tfrac{1}{2}\text{ in.})$ $= 130\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(12.0\text{ in.})(\tfrac{1}{2}\text{ in.})}{1.50}$ $= 86.4\text{ kips} > 24.0\text{ kips}$ **o.k.**

Shear Rupture of Plate

$$A_{nv} = t_p\left[d - n\left(d_b + \tfrac{1}{8}\text{ in.}\right)\right]$$
$$= \tfrac{1}{2}\text{ in.}\left[12.0\text{ in.} - 4\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]$$
$$= 4.25\text{ in.}^2$$

Using AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58\text{ ksi})(4.25\text{ in.}^2)$ $= 111\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58\text{ ksi})(4.25\text{ in.}^2)}{2.00}$ $= 74.0\text{ kips} > 24.0\text{ kips}$ **o.k.**

Block Shear Rupture of Plate

$n = 4$, $L_{ev} = 1\tfrac{1}{2}$ in., $L_{eh} = 4\tfrac{1}{4}$ in.

Using AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv}, \phi 0.60 F_u A_{nv})$	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega}, \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$
Tension rupture component:	Tension rupture component:
$U_{bs} = 0.5$ from AISC *Specification* Section J4.3	$U_{bs} = 0.5$ from AISC *Specification* Section J4.3
$A_{nt} = \tfrac{1}{2}\text{ in.}\left[4\tfrac{1}{4}\text{ in.} - 1.5\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]$ $= 1.47\text{ in.}^2$	$A_{nt} = \tfrac{1}{2}\text{ in.}\left[4\tfrac{1}{4}\text{ in.} - 1.5\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]$ $= 1.47\text{ in.}^2$
$\phi U_{bs} F_u A_{nt} = 0.75(0.5)(58\text{ ksi})(1.47\text{ in.}^2)$ $= 32.0\text{ kips}$	$\dfrac{U_{bs} F_u A_{nt}}{\Omega} = \dfrac{0.5(58\text{ ksi})(1.47\text{ in.}^2)}{2.00}$ $= 21.3\text{ kips}$

LRFD	ASD
Shear yielding component from AISC *Manual* Table 9-3b: $\phi 0.60 F_y A_{gv} = 170 \text{ kips/in.}(\frac{1}{2} \text{ in.})$ $= 85.0 \text{ kips}$	Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60 F_y A_{gv}}{\Omega} = 113 \text{ kips/in.}(\frac{1}{2} \text{ in.})$ $= 56.5 \text{ kips}$
Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv} = 194 \text{ kips/in.}(\frac{1}{2} \text{ in.})$ $= 97.0 \text{ kips}$ $\phi R_n = 32.0 \text{ kips} + 85.0 \text{ kips}$ $= 117 \text{ kips} > 36.0 \text{ kips}$ **o.k.**	Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60 F_u A_{nv}}{\Omega} = 129 \text{ kips/in.}(\frac{1}{2} \text{ in.})$ $= 64.5 \text{ kips}$ $\dfrac{R_n}{\Omega} = 21.3 \text{ kips} + 56.5 \text{ kips}$ $= 77.8 \text{ kips} > 24.0 \text{ kips}$ **o.k.**

Shear Yielding, Shear Buckling and Flexural Yielding of Plate

Check local buckling of plate

This check is analogous to the local buckling check for doubly coped beams as illustrated in AISC *Manual* Part 9, where c = 9 in. and h_o = 12 in.

$$\lambda = \frac{h_o \sqrt{F_y}}{10 t_w \sqrt{475 + 280 \left(\dfrac{h_o}{c} \right)^2}} \qquad \text{(\textit{Manual} Eq. 9-18)}$$

$$= \frac{(12.0 \text{ in.}) \sqrt{36 \text{ ksi}}}{10(\frac{1}{2} \text{ in.}) \sqrt{475 + 280 \left(\dfrac{12.0 \text{ in.}}{9.00 \text{ in.}} \right)^2}}$$

$$= 0.462$$

$\lambda \leq 0.7$, therefore, $Q = 1.0$

$QF_y = F_y$

Therefore, plate buckling is not a controlling limit state.

From AISC *Manual* Equation 10-4:

LRFD	ASD
$\left(\dfrac{V_u}{\phi_v V_n} \right)^2 + \left(\dfrac{M_u}{\phi_b M_n} \right)^2 \leq 1.0$ From preceding calculations: $V_u = 36.0 \text{ kips}$	$\left(\dfrac{V_a}{V_n / \Omega_v} \right)^2 + \left(\dfrac{M_a}{M_n / \Omega_b} \right)^2 \leq 1.0$ From preceding calculations: $V_a = 24.0 \text{ kips}$

LRFD	ASD
$\phi_v V_n = 130 \text{ kips}$	$\dfrac{V_n}{\Omega_v} = 86.4 \text{ kips}$
$M_u = V_u e$ $= 36.0 \text{ kips}(9.00 \text{ in.})$ $= 324 \text{ kip-in.}$	$M_a = V_a e$ $= 24.0 \text{ kips}(9.00 \text{ in.})$ $= 216 \text{ kip-in.}$
$\phi_b = 0.90$	$\Omega = 1.67$
$\phi_b M_n = \phi_b Q F_y Z_{pl}$ $= 0.90(1.0)(36 \text{ ksi})\left[\dfrac{\frac{1}{2} \text{ in.}(12.0 \text{ in.})^2}{4}\right]$ $= 583 \text{ kip-in.}$	$\dfrac{M_n}{\Omega_b} = \dfrac{QF_y Z_{pl}}{\Omega_b}$ $= 1.0(36 \text{ ksi})\left[\dfrac{\frac{1}{2} \text{ in.}(12.0 \text{ in.})^2 / 4}{1.67}\right]$ $= 388 \text{ kip-in.}$
$\left(\dfrac{36.0 \text{ kips}}{130 \text{ kips}}\right)^2 + \left(\dfrac{324 \text{ kip-in.}}{583 \text{ kip-in.}}\right)^2 = 0.386 \le 1.0$ **o.k.**	$\left(\dfrac{24.0 \text{ kips}}{86.4 \text{ kips}}\right)^2 + \left(\dfrac{216 \text{ kip-in.}}{388 \text{ kip-in.}}\right)^2 = 0.387 \le 1.0$ **o.k.**

Flexural Rupture of Plate

$Z_{net} = 12.8 \text{ in.}^3$ from AISC *Manual* Table 15-3

From AISC *Manual* Equation 9-4:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi M_n = \phi F_u Z_{net}$ $= 0.75(58 \text{ ksi})(12.8 \text{ in.}^3)$ $= 557 \text{ kip-in.} > 324 \text{ kip-in.}$ **o.k.**	$\dfrac{M_n}{\Omega} = \dfrac{F_u Z_{net}}{\Omega}$ $= \dfrac{58 \text{ ksi}(12.8 \text{ in.}^3)}{2.00}$ $= 371 \text{ kip-in.} > 216 \text{ kip-in.}$ **o.k.**

Weld Between Plate and Column Web (AISC Manual Part 10)

$$w = \tfrac{5}{8} t_p$$
$$= \tfrac{5}{8}(\tfrac{1}{2} \text{ in.})$$
$= 0.313$ in., therefore, use a $\frac{5}{16}$-in. fillet weld on both sides of the plate.

Strength of Column Web at Weld

$$t_{min} = \frac{3.09D}{F_u} \quad \text{(}\textit{Manual}\text{ Eq. 9-2)}$$
$$= \frac{3.09(5 \text{ sixteenths})}{65 \text{ ksi}}$$
$= 0.238 \text{ in.} < 0.440 \text{ in.}$ **o.k.**

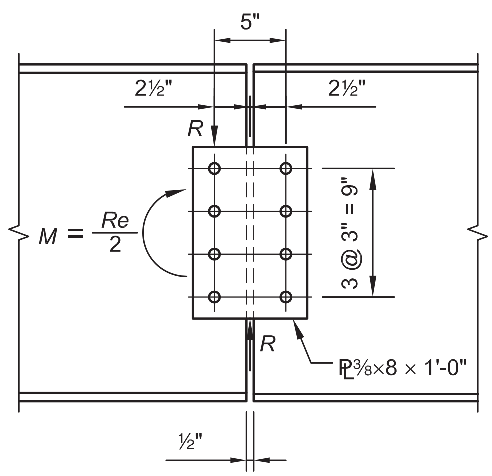
EXAMPLE II.A-20 ALL-BOLTED SINGLE-PLATE SHEAR SPLICE

Given:

Design an all-bolted single-plate shear splice between an ASTM A992 W24×55 beam and an ASTM A992 W24×68 beam.

R_D = 10 kips
R_L = 30 kips

Use ⅞-in.-diameter ASTM A325-N or F1852-N bolts in standard holes with 5 in. between vertical bolt rows and an ASTM A36 plate.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W24×55
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Beam
W24×68
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W24×55

t_w = 0.395 in.

Beam
W24×68
t_w = 0.415 in.

Bolt Group Design

Note: When the splice is symmetrical, the eccentricity of the shear to the center of gravity of either bolt group is equal to half the distance between the centroids of the bolt groups. Therefore, each bolt group can be designed for the shear, R_u or R_a, and one-half the eccentric moment, $R_u e$ or $R_a e$.

Using a symmetrical splice, each bolt group will carry one-half the eccentric moment. Thus, the eccentricity on each bolt group, $e/2$ = 2½ in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(10\text{ kips}) + 1.6(30\text{ kips})$ $= 60.0\text{ kips}$	$R_a = 10\text{ kips} + 30\text{ kips}$ $= 40.0\text{ kips}$

Bolt Shear

From AISC *Manual* Table 7-1:

LRFD	ASD
$\phi r_n = 24.3\text{ kips/bolt}$	$\dfrac{r_n}{\Omega} = 16.2\text{ kips/bolt}$

Bolt Bearing on ⅜-in. Plate

Note: The available bearing strength based on edge distance will conservatively be used for all of the bolts.

$$l_c = 1.50\text{ in.} - \frac{15/16\text{ in.}}{2}$$
$$= 1.03\text{ in.}$$

$$r_n = 1.2 l_c t F_u \le 2.4 d t F_u \qquad \text{(Spec. Eq. J3-6a)}$$
$$= 1.2(1.03\text{ in.})(3/8\text{ in.})(58\text{ ksi}) \le 2.4(7/8\text{ in.})(3/8\text{ in.})(58\text{ ksi})$$
$$= 26.9\text{ kips/bolt} \le 45.7\text{ kips/bolt}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(26.9\text{ kips})$ $= 20.2\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{26.9\text{ kips}}{2.00}$ $= 13.5\text{ kips/bolt}$

Note: By inspection, bearing on the webs of the W24 beams will not govern.

Since bearing is more critical,

LRFD	ASD
$C_{min} = \dfrac{R_u}{\phi r_n}$ $= \dfrac{60.0 \text{ kips}}{20.2 \text{ kips/bolt}}$ $= 2.97$ By interpolating AISC *Manual* Table 7-6, with $n = 4$, $\theta = 0°$ and $e_x = 2\tfrac{1}{2}$ in.: $C = 3.07 > 2.97$ **o.k.**	$C_{min} = \dfrac{R_a}{r_n / \Omega}$ $= \dfrac{40.0 \text{ kips}}{13.5 \text{ kips/bolt}}$ $= 2.96$ By interpolating AISC *Manual* Table 7-6, with $n = 4$, $\theta = 0°$ and $e_x = 2\tfrac{1}{2}$ in.: $C = 3.07 > 2.96$ **o.k.**

Flexural Yielding of Plate

Try PL⅜ in. × 8 in. × 1'-0".

The required flexural strength is:

LRFD	ASD
$M_u = \dfrac{R_u e}{2}$ $= \dfrac{60.0 \text{ kips}(5.00 \text{ in.})}{2}$ $= 150 \text{ kip-in.}$ $\phi = 0.90$ $\phi M_n = \phi F_y Z_x$ $= 0.90(36 \text{ ksi})\left[\dfrac{\tfrac{3}{8} \text{ in.}(12.0 \text{ in.})^2}{4}\right]$ $= 437 \text{ kip-in.} > 150 \text{ kip-in.}$ **o.k.**	$M_a = \dfrac{R_a e}{2}$ $= \dfrac{40.0 \text{ kips}(5.00 \text{ in.})}{2}$ $= 100 \text{ kip-in.}$ $\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{F_y Z_x}{\Omega}$ $= \dfrac{36 \text{ ksi}}{1.67}\left[\dfrac{\tfrac{3}{8} \text{ in.}(12.0 \text{ in.})^2}{4}\right]$ $= 291 \text{ kip-in.} > 100 \text{ kip-in.}$ **o.k.**

Flexural Rupture of Plate

$Z_{net} = 9.00 \text{ in.}^3$ from AISC *Manual* Table 15-3

From AISC *Manual* Equation 9-4:

LRFD	ASD
$\phi = 0.75$ $\phi M_n = \phi F_u Z_{net}$ $= 0.75(58 \text{ ksi})(9.00 \text{ in}^3)$ $= 392 \text{ kip-in.} > 150 \text{ kip-in.}$ **o.k.**	$\Omega = 2.00$ $\dfrac{M_n}{\Omega} = \dfrac{F_u Z_{net}}{\Omega}$ $= \dfrac{58 \text{ ksi}(9.00 \text{ in.}^3)}{2.00}$ $= 261 \text{ kip-in.} > 100 \text{ kip-in.}$ **o.k.**

Shear Yielding of Plate

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(12.0 \text{ in.})(3/8 \text{ in.})$ $= 97.2 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})(12.0 \text{ in.})(3/8 \text{ in.})}{1.50}$ $= 64.8 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Shear Rupture of Plate

$A_{nv} = 3/8 \text{ in.}[12.0 \text{ in.} - 4(15/16 \text{ in.} + 1/16 \text{ in.})]$
$= 3.00 \text{ in.}^2$

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58 \text{ ksi})(3.00 \text{ in.}^2)$ $= 78.3 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58 \text{ ksi})(3.00 \text{ in.}^2)}{2.00}$ $= 52.2 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Block Shear Rupture of Plate

$L_{eh} = L_{ev} = 1\tfrac{1}{2} \text{ in.}$

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv})$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt} = 1.0(43.5 \text{ kips/in.})(3/8 \text{ in.})$	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = 1.0(29.0 \text{ kips/in.})(3/8 \text{ in.})$

LRFD	ASD
Shear yielding component from AISC *Manual* Table 9-3b: $\phi 0.60 F_y A_{gv} = 170$ kips/in.(⅜ in.)	Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60 F_y A_{gv}}{\Omega} = 113$ kips/in.(⅜ in.)
Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv} = 183$ kips/in.(⅜ in.) $\phi R_n = (43.5 \text{ kips/in.} + 170 \text{ kips/in.})(⅜ \text{ in.})$ $= 80.1 \text{ kips} > 60.0 \text{ kips}$ **o.k.**	Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60 F_u A_{nv}}{\Omega} = 122$ kips/in.(⅜ in.) $\dfrac{R_n}{\Omega} = (29.0 \text{ kips/in.} + 113 \text{ kips/in.})(⅜ \text{ in.})$ $= 53.3 \text{ kips} > 40.0 \text{ kips}$ **o.k.**

Use PL⅜ in. × 8 in. × 1 ft 0 in.

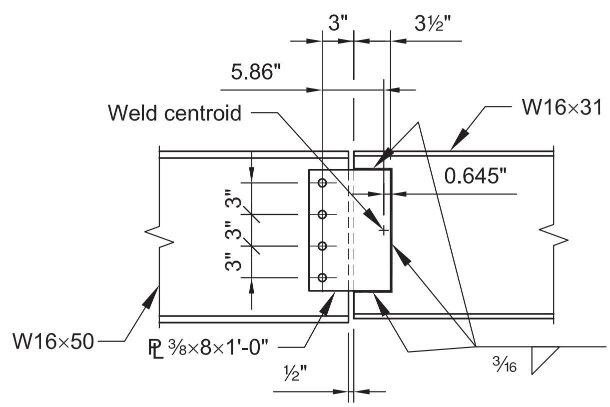
EXAMPLE II.A-21 BOLTED/WELDED SINGLE-PLATE SHEAR SPLICE

Given:

Design a single-plate shear splice between an ASTM A992 W16×31 beam and an ASTM A992 W16×50 beam to support the following beam end reactions:

R_D = 8.0 kips
R_L = 24.0 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts through the web of the W16×50 and 70-ksi electrode welds to the web of the W16×31. Use an ASTM A36 plate.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W16×31
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Beam
W16×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×31
t_w = 0.275 in.

Beam
W16×50
t_w = 0.380 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(8.0 kips) + 1.6(24 kips) = 48.0 kips	R_a = 8.0 kips + 24 kips = 32.0 kips

Weld Design

Since the splice is unsymmetrical and the weld group is more rigid, it will be designed for the full moment from the eccentric shear.

Assume PL⅜ in. × 8 in. × 1 ft 0 in. This plate size meets the dimensional and other limitations of a single-plate connection with a conventional configuration from AISC *Manual* Part 10.

Use AISC *Manual* Table 8-8 to determine the weld size.

$$k = \frac{kl}{l}$$
$$= \frac{3\tfrac{1}{2}\text{ in.}}{12.0\text{ in.}}$$
$$= 0.292$$

$$xl = \frac{(kl)^2}{2(kl)+l}$$
$$= \frac{(3\tfrac{1}{2}\text{ in})^2}{2(3\tfrac{1}{2}\text{ in.})+12.0\text{ in.}}$$
$$= 0.645\text{ in.}$$

$$al = 6.50\text{ in.} - 0.645\text{ in.}$$
$$= 5.86\text{ in.}$$

$$a = \frac{al}{l}$$
$$= \frac{5.86\text{ in.}}{12.0\text{ in.}}$$
$$= 0.488$$

By interpolating AISC *Manual* Table 8-8, with $\theta = 0°$,

$$C = 2.15$$

The required weld size is:

LRFD	ASD
$D_{req} = \dfrac{P_u}{\phi C C_1 l}$	$D_{req} = \dfrac{P_a\,\Omega}{C C_1 l}$

$= \dfrac{48.0\text{ kips}}{0.75(2.15)(1.0)(12.0\text{ in.})}$ $= 2.48 \rightarrow 3\text{ sixteenths}$	$= \dfrac{32.0\text{ kips}(2.00)}{2.15(1.0)(12.0\text{ in.})}$ $= 2.48 \rightarrow 3\text{ sixteenths}$

The minimum weld size from AISC *Specification* Table J2.4 is 3⁄16 in.

Use a 3⁄16-in. fillet weld.

Shear Rupture of W16×31 Beam Web at Weld

For fillet welds with F_{EXX} = 70 ksi on one side of the connection, the minimum thickness required to match the available shear rupture strength of the connection element to the available shear rupture strength of the base metal is:

$$t_{min} = \frac{3.09D}{F_u} \qquad \text{(Manual Eq. 9-2)}$$

$$= \frac{3.09(2.48\text{ sixteenths})}{65\text{ ksi}}$$

$$= 0.118 < 0.275\text{ in.} \qquad \textbf{o.k.}$$

Bolt Group Design

Since the weld group was designed for the full eccentric moment, the bolt group will be designed for shear only.

LRFD	ASD
Bolt shear strength from AISC *Manual* Table 7-1: ϕr_n = 17.9 kips/bolt For bearing on the 3⁄8-in.-thick single plate, conservatively use the design values provided for L_e = 1¼ in. Note: By inspection, bearing on the web of the W16×50 beam will not govern. From AISC *Manual* Table 7-5: $\phi r_n = 44.0\text{ kips/in./bolt}(3/8\text{ in.})$ $= 16.5\text{ kips/bolt}$ Since bolt bearing is more critical than bolt shear, $n_{min} = \dfrac{R_u}{\phi r_n}$ $= \dfrac{48.0\text{ kips}}{16.5\text{ kips/bolt}}$ $= 2.91\text{ bolts} < 4\text{ bolts}$ **o.k.**	Bolt shear strength from AISC *Manual* Table 7-1: $\dfrac{r_n}{\Omega}$ = 11.9 kips/bolt For bearing on the 3⁄8-in.-thick single plate, conservatively use the design values provided for L_e = 1¼ in. Note: By inspection, bearing on the web of the W16×50 beam will not govern. From AISC *Manual* Table 7-5: $\dfrac{r_n}{\Omega} = 29.4\text{ kips/in./bolt}(3/8\text{ in.})$ $= 11.0\text{ kips/bolt}$ Since bolt bearing is more critical than bolt shear, $n_{min} = \dfrac{R_u}{r_n / \Omega}$ $= \dfrac{32.0\text{ kips}}{11.0\text{ kips/bolt}}$ $= 2.91\text{ bolts} < 4\text{ bolts}$ **o.k.**

Flexural Yielding of Plate

As before, try a PL⅜ in. × 8 in. × 1 ft 0 in.

The required flexural strength is:

LRFD	ASD
$M_u = R_u e$ $= 48.0\text{ kips}(5.86\text{ in.})$ $= 281\text{ kip-in.}$ $\phi = 0.90$ $\phi M_n = \phi F_y Z_x$ $= 0.9(36\text{ ksi})\left[\dfrac{\frac{3}{8}\text{ in.}(12.0\text{ in.})^2}{4}\right]$ $= 437\text{ kip-in.} > 281\text{ kip-in.}$ **o.k.**	$M_a = R_a e$ $= 32.0\text{ kips}(5.86\text{ in.})$ $= 188\text{ kip-in.}$ $\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{F_y Z_x}{\Omega}$ $= \dfrac{36\text{ ksi}}{1.67}\left[\dfrac{\frac{3}{8}\text{ in.}(12.0\text{ in.})^2}{4}\right]$ $= 291\text{ kip-in.} > 188\text{ kip-in.}$ **o.k.**

Shear Yielding of Plate

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(12.0\text{ in.})(\frac{3}{8}\text{ in.})$ $= 97.2\text{ kips} > 48.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(12.0\text{ in.})(\frac{3}{8}\text{ in.})}{1.50}$ $= 64.8\text{ kips} > 32.0\text{ kips}$ **o.k.**

Shear Rupture of Plate

$$A_{nv} = \tfrac{3}{8}\text{ in.}\left[12.0\text{ in.} - 4\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]$$
$$= 3.19\text{ in.}^2$$

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58\text{ ksi})(3.19\text{ in.}^2)$ $= 83.3\text{ kips} > 48.0\text{ kip}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58\text{ ksi})(3.19\text{ in.}^2)}{2.00}$ $= 55.5\text{ kips} > 32.0\text{ kips}$ **o.k.**

Block Shear Rupture of Plate

$L_{eh} = L_{ev} = 1\frac{1}{2}$ in.

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv}, \ \phi 0.60 F_u A_{nv})$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt} = 1.0(46.2 \text{ kips/in.})(\frac{3}{8} \text{ in.})$ Shear yielding component from AISC *Manual* Table 9-3b: $\phi 0.60 F_y A_{gv} = 170 \text{ kips/in.}(\frac{3}{8} \text{ in.})$ Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv} = 194 \text{ kips/in.}(\frac{3}{8} \text{ in.})$ $\phi R_n = (46.2 \text{ kips/in.} + 170 \text{ kips/in.})(\frac{3}{8} \text{ in.})$ $= 81.1 \text{ kips} > 48.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega}, \ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = 1.0(30.8 \text{ kips/in.})(\frac{3}{8} \text{ in.})$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60 F_y A_{gv}}{\Omega} = 113 \text{ kips/in.}(\frac{3}{8} \text{ in.})$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60 F_u A_{nv}}{\Omega} = 129 \text{ kips/in.}(\frac{3}{8} \text{ in.})$ $\dfrac{R_n}{\Omega} = (30.8 \text{ kips/in.} + 113 \text{ kips/in.})(\frac{3}{8} \text{ in.})$ $= 53.9 \text{ kips} > 32.0 \text{ kips}$ **o.k.**

Use PL⅜ in. × 8 in. × 1 ft 0 in.

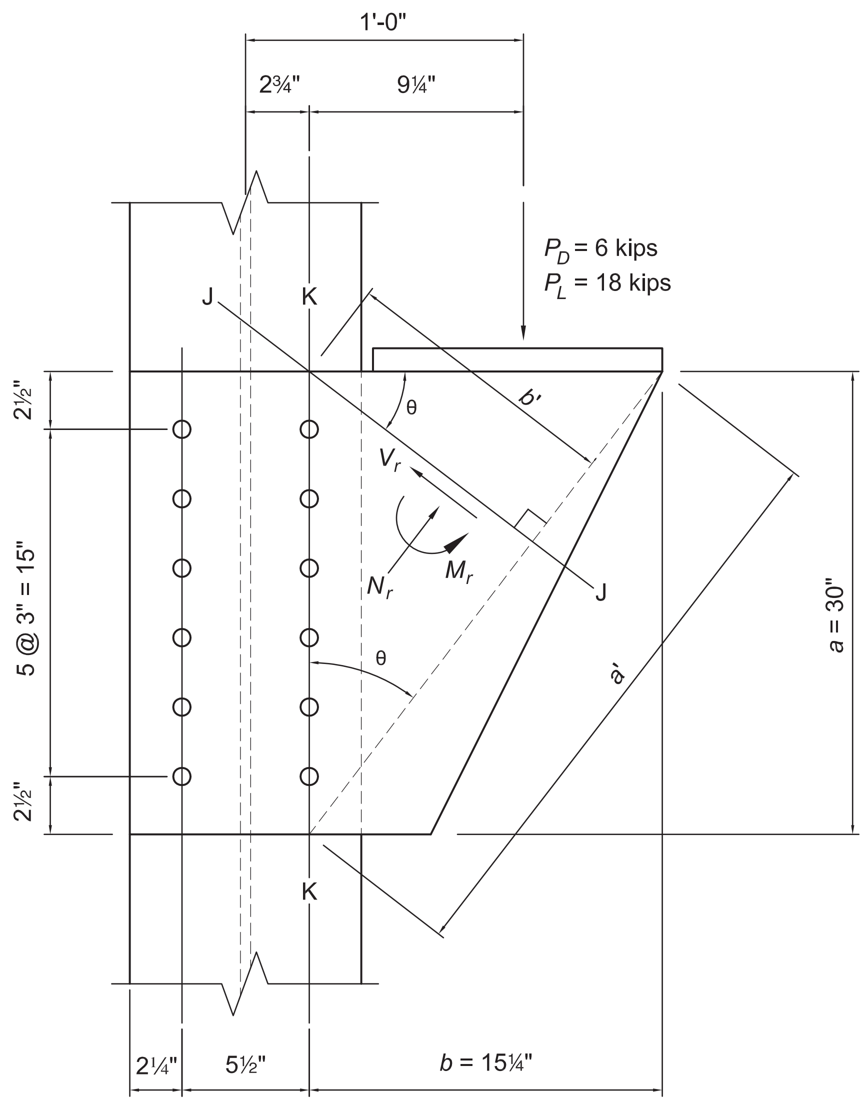
EXAMPLE II.A-22 BOLTED BRACKET PLATE DESIGN

Given:

Design a bracket plate to support the following loads:

$P_D = 6$ kips
$P_L = 18$ kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and an ASTM A36 plate. Assume the column has sufficient available strength for the connection.



Solution:

For discussion of the design of a bracket plate, see AISC *Manual* Part 15.

From AISC *Manual* Table 2-5, the material properties are as follows:

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(6 kips) + 1.6(18 kips) = 36.0 kips	R_a = 6 kips + 18 kips = 24.0 kips

Bolt Design

LRFD	ASD
Bolt shear from AISC *Manual* Table 7-1: $\phi r_n = 17.9$ kips	Bolt shear from AISC *Manual* Table 7-1: $\dfrac{r_n}{\Omega} = 11.9$ kips
For bearing on the bracket plate: Try PL⅜ in. × 20 in., $L_e \geq 2$ in. From AISC *Manual* Table 7-5: ϕr_n = 78.3 kips/bolt/in.(⅜ in.) = 29.4 kips/bolt Bolt shear controls. By interpolating AISC *Manual* Table 7-8 with θ = 0°, a 5½ in. gage with s = 3 in., e_x = 12.0 in., n = 6 and C = 4.53: $C_{min} = \dfrac{R_u}{\phi r_n}$ $= \dfrac{36.0 \text{ kips}}{17.9 \text{ kips/bolt}}$ $= 2.01$ C = 4.53 > 2.01 **o.k.**	For bearing on the bracket plate: Try PL⅜ in. × 20 in., $L_e \geq 2$ in. From AISC *Manual* Table 7-5: r_n/Ω = 52.2 kips/bolt/in.(⅜ in.) = 19.6 kips/bolt Bolt shear controls. By interpolating AISC *Manual* Table 7-8 with θ = 0°, a 5½ in. gage with s = 3 in., e_x = 12.0 in., n = 6, and C = 4.53: $C_{min} = \dfrac{\Omega R_a}{r_n}$ $= \dfrac{24.0 \text{ kips}}{11.9 \text{ kips/bolt}}$ $= 2.02$ C = 4.53 > 2.02 **o.k.**

Flexural Yielding of Bracket Plate on Line K

The required strength is:

LRFD	ASD
$M_u = P_u e$ (*Manual* Eq. 15-1a) $= (36.0 \text{ kips})(12.0 \text{ in.} - 2\tfrac{3}{4} \text{ in.})$ $= 333 \text{ kip-in.}$	$M_a = P_a e$ (*Manual* Eq. 15-1b) $= (24.0 \text{ kips})(12.0 \text{ in.} - 2\tfrac{3}{4} \text{ in.})$ $= 222 \text{ kip-in.}$

From AISC *Manual* Equation 15-2:

LRFD	ASD
$\phi = 0.90$ $\phi M_n = \phi F_y Z$ $= 0.90(36 \text{ ksi})\left(\dfrac{\tfrac{3}{8} \text{ in.}(20.0 \text{ in.})^2}{4}\right)$ $= 1{,}220 \text{ kip-in.} > 333 \text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{F_y Z}{\Omega}$ $= \dfrac{36 \text{ ksi}\left(\dfrac{\tfrac{3}{8} \text{ in.}(20.0 \text{ in.})^2}{4}\right)}{1.67}$ $= 808 \text{ kip-in.} > 222 \text{ kip-in.}$ **o.k.**

Flexural Rupture of Bracket Plate on Line K

From Table 15-3, for a ⅜-in.-thick bracket plate, with ¾-in. bolts and six bolts in a row, $Z_{net} = 21.5 \text{ in.}^3$.

From AISC *Manual* Equation 15-3:

LRFD	ASD
$\phi = 0.75$ $\phi M_n = \phi F_u Z_{net}$ $= 0.75(58 \text{ ksi})(21.5 \text{ in.}^3)$ $= 935 \text{ kip-in.} > 333 \text{ kip-in.}$ **o.k.**	$\Omega = 2.00$ $\dfrac{M_n}{\Omega} = \dfrac{F_u Z_{net}}{\Omega}$ $= \dfrac{58 \text{ ksi}(21.5 \text{ in.}^3)}{2.00}$ $= 624 \text{ kip-in.} > 222 \text{ kip-in.}$ **o.k.**

Shear Yielding of Bracket Plate on Line J

$$\tan\theta = \frac{b}{a} = \frac{15\tfrac{1}{4} \text{ in.}}{20.0 \text{ in.}}$$

$$\theta = 37.3°$$

$$b' = a \sin\theta = 20.0 \text{ in.} (\sin 37.3°) = 12.1 \text{ in.}$$

LRFD	ASD
$V_r = V_u = P_u \sin\theta$ (*Manual* Eq. 15-6a) $= 36.0 \text{ kips}(\sin 37.3°)$ $= 21.8 \text{ kips}$ $V_n = 0.6F_y tb'$ (*Manual* Eq. 15-7) $= 0.6(36 \text{ ksi})(\tfrac{3}{8} \text{ in.})(12.1 \text{ in.})$ $= 98.0 \text{ kips}$	$V_r = V_a = P_a \sin\theta$ (*Manual* Eq. 15-6b) $= 24.0 \text{ kips}(\sin 37.3°)$ $= 14.5 \text{ kips}$ $V_n = 0.6F_y tb'$ (*Manual* Eq. 15-7) $= 0.6(36 \text{ ksi})(\tfrac{3}{8} \text{ in.})(12.1 \text{ in.})$ $= 98.0 \text{ kips}$

LRFD	ASD
$\phi = 1.00$ $\phi V_n = 1.00(98.0 \text{ kips})$ $= 98.0 \text{ kips} > 21.8 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{V_n}{\Omega} = \dfrac{98.0 \text{ kips}}{1.50}$ $= 65.3 \text{ kips} > 14.5 \text{ kips}$ **o.k.**

Local Yielding and Local Buckling of Bracket Plate on Line J

For local yielding:

$$F_{cr} = F_y = 36 \text{ ksi} \qquad (\textit{Manual} \text{ Eq. 15-13})$$

For local buckling:

$$F_{cr} = QF_y \qquad (\textit{Manual} \text{ Eq. 15-14})$$

where

$$a' = \frac{a}{\cos\theta} = \frac{20.0 \text{ in.}}{\cos 37.3^\circ} = 25.1 \text{ in.} \qquad (\textit{Manual} \text{ Eq. 15-18})$$

$$\lambda = \frac{\left(\dfrac{b'}{t}\right)\sqrt{F_y}}{5\sqrt{475 + 1{,}120\left(\dfrac{b'}{a'}\right)^2}} \qquad (\textit{Manual} \text{ Eq. 15-17})$$

$$= \frac{\left(\dfrac{12.1 \text{ in.}}{3/8 \text{ in.}}\right)\sqrt{36 \text{ ksi}}}{5\sqrt{475 + 1{,}120\left(\dfrac{12.1 \text{ in.}}{25.1 \text{ in.}}\right)^2}} = 1.43$$

Because $1.41 < \lambda$

$$Q = \frac{1.30}{\lambda^2} = \frac{1.30}{(1.43)^2} = 0.636 \qquad (\textit{Manual} \text{ Eq. 15-16})$$

$$F_{cr} = QF_y = 0.636(36 \text{ ksi}) = 22.9 \text{ ksi} \qquad (\textit{Manual} \text{ Eq. 15-14})$$

Local buckling controls over local yielding.

Interaction of Normal and Flexural Strengths
Check that *Manual* Equation 15-10 is satisfied:

LRFD	ASD
$N_r = N_u = P_u \cos\theta$ (*Manual* Eq. 15-9a) $= 36.0 \text{ kips}(\cos 37.3°)$ $= 28.6 \text{ kips}$ $N_n = F_{cr}tb'$ (*Manual* Eq. 15-11) $= 22.9 \text{ ksi}(3/8 \text{ in.})(12.1 \text{ in.})$ $= 104 \text{ kips}$ $\phi = 0.90$ $N_c = \phi N_n = 0.90(104 \text{ kips})$ $= 93.6 \text{ kips}$ $M_r = M_u = P_u e - N_u(b'/2)$ (*Manual* Eq. 15-8a) $= 36.0 \text{ kips}(9\tfrac{1}{4} \text{ in.}) - 28.6 \text{ kips}(12.1 \text{ in.}/2)$ $= 160 \text{ kip-in.}$ $M_n = \dfrac{F_{cr}t(b')^2}{4}$ (*Manual* Eq. 15-12) $= \dfrac{22.9 \text{ ksi}(3/8 \text{ in.})(12.1 \text{ in.})^2}{4}$ $= 314 \text{ kip-in.}$ $M_c = \phi M_n = 0.90(314 \text{ kip-in.})$ $= 283 \text{ kip-in.}$ $\dfrac{N_r}{N_c} + \dfrac{M_r}{M_c} \le 1.0$ (*Manual* Eq. 15-10) $= \dfrac{28.6 \text{ kips}}{93.6 \text{ kips}} + \dfrac{160 \text{ kip-in.}}{283 \text{ kip-in.}} \le 1.0$ $= 0.871 \le 1.0$ **o.k.**	$N_r = N_a = P_a \cos\theta$ (*Manual* Eq. 15-9b) $= 24.0 \text{ kips}(\cos 37.3°)$ $= 19.1 \text{ kips}$ $N_n = F_{cr}tb'$ (*Manual* Eq. 15-11) $= 22.9 \text{ ksi}(3/8 \text{ in.})(12.1 \text{ in.})$ $= 104 \text{ kips}$ $\Omega = 1.67$ $N_c = \dfrac{N_n}{\Omega} = \dfrac{104 \text{ kips}}{1.67}$ $= 62.3 \text{ kips}$ $M_r = M_a = P_a e - N_a(b'/2)$ (*Manual* Eq. 15-8b) $= 24.0 \text{ kips}(9\tfrac{1}{4} \text{ in.}) - 19.1 \text{ kips}(12.1 \text{ in.}/2)$ $= 106 \text{ kip-in.}$ $M_n = \dfrac{F_{cr}t(b')^2}{4}$ (*Manual* Eq. 15-12) $= \dfrac{22.9 \text{ ksi}(3/8 \text{ in.})(12.1 \text{ in.})^2}{4}$ $= 314 \text{ kip-in.}$ $M_c = \dfrac{M_n}{\Omega} = \dfrac{314 \text{ kip-in.}}{1.67}$ $= 188 \text{ kip-in.}$ $\dfrac{N_r}{N_c} + \dfrac{M_r}{M_c} \le 1.0$ (*Manual* Eq. 15-10) $= \dfrac{19.1 \text{ kips}}{62.3 \text{ kips}} + \dfrac{106 \text{ kip-in.}}{188 \text{ kip-in.}} \le 1.0$ $= 0.870 \le 1.0$ **o.k.**

Shear Yielding of Bracket Plate on Line K (using AISC Specification Equation J4-3)

$$R_n = 0.60F_yA_{gv}$$ (*Spec*. Eq. J4-3)

$$= 0.6(36 \text{ ksi})(20.0 \text{ in.})(3/8 \text{ in.})$$
$$= 162 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(162 \text{ kips})$ $= 162 \text{ kips} > 36.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{162 \text{ kips}}{1.50}$ $= 108 \text{ kips} > 24.0 \text{ kips}$ **o.k.**

Shear Rupture of Bracket Plate on Line K (using AISC Specification Equation J4-4)

$$A_{nv} = \left[20.0 \text{ in.} - 6\left(13/16 \text{ in.} + 1/16 \text{ in.}\right)\right]\left(3/8 \text{ in.}\right)$$
$$= 5.53 \text{ in.}^2$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58 \text{ ksi})(5.53 \text{ in.}^2)$ $= 144 \text{ kips} > 36.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58 \text{ ksi})(5.53 \text{ in.}^2)}{2.00}$ $= 96.2 \text{ kips} > 24.0 \text{ kips}$ **o.k.**

Return to Table of Contents

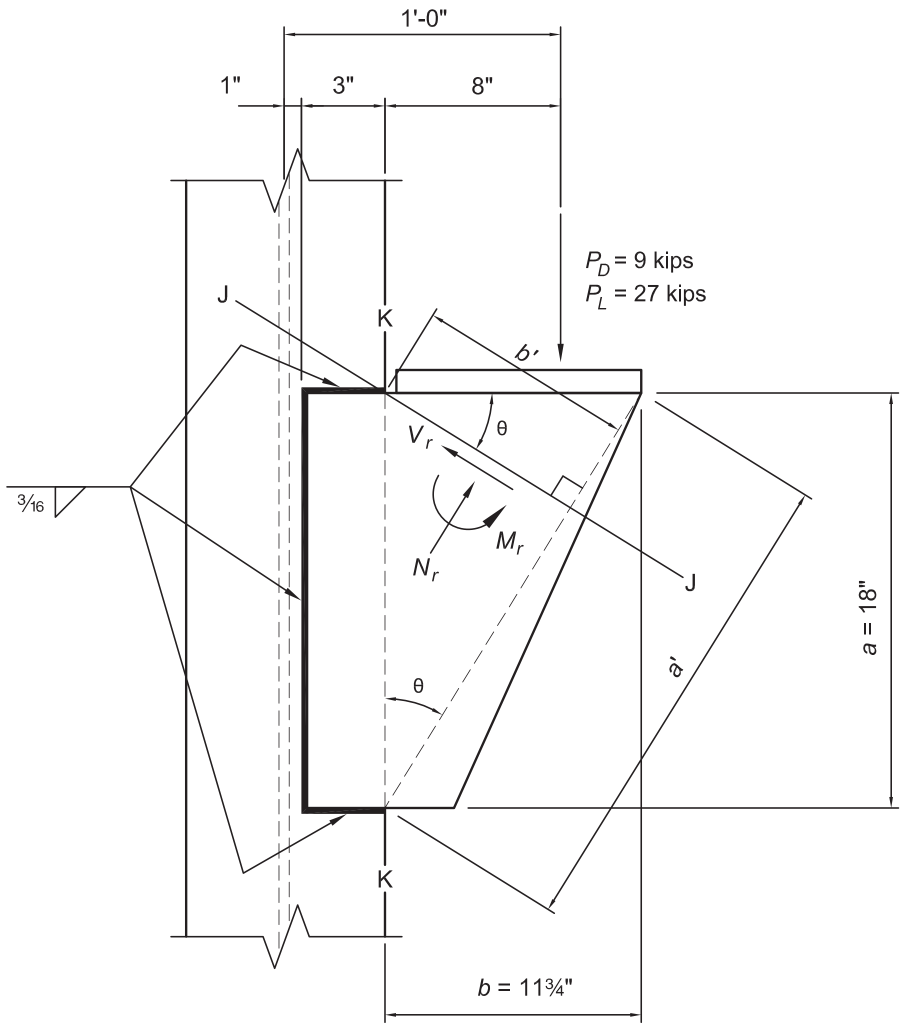
EXAMPLE II.A-23 WELDED BRACKET PLATE DESIGN

Given:

Design a welded bracket plate, using 70-ksi electrodes, to support the following loads:

P_D = 9 kips
P_L = 27 kips

Assume the column has sufficient available strength for the connection. Use an ASTM A36 plate.



Solution:

From AISC *Manual* Table 2-5, the material properties are as follows:

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(9 \text{ kips}) + 1.6(27 \text{ kips})$ $= 54.0 \text{ kips}$	$R_a = 9 \text{ kips} + 27 \text{ kips}$ $= 36.0 \text{ kips}$

Try PL½ in. × 18 in.

Try a C-shaped weld with kl = 3 in. and l = 18 in.

$$k = \frac{kl}{l} = \frac{3.00 \text{ in.}}{18.0 \text{ in.}} = 0.167$$

$$xl = \frac{(kl)^2}{2(kl)+l} = \frac{(3.00 \text{ in.})^2}{2(3.00 \text{ in.})+18.00 \text{ in.}} = 0.375 \text{ in.}$$

$$al = 11.0 \text{ in.} - 0.375 \text{ in.} = 10.6 \text{ in.}$$

$$al = \frac{al}{l} = \frac{10.6 \text{ in.}}{18.0 \text{ in.}} = 0.589$$

Interpolate AISC *Manual* Table 8-8 using θ = 0°, k = 0.167, and a = 0.589.

C = 1.49

From AISC *Manual* Table 8-3:

C_1 = 1.0 for E70 electrode

From AISC *Manual* Equation 8-13:

LRFD	ASD
$\phi = 0.75$ $D_{min} = \frac{P_u}{\phi C C_1 l} = \frac{54.0 \text{ kips}}{0.75(1.49)(1.0)(18.0 \text{ in.})} = 2.68 \rightarrow 3 \text{ sixteenths}$	$\Omega = 2.00$ $D_{min} = \frac{\Omega P_a}{C C_1 l} = \frac{2.00(36.0 \text{ kips})}{1.49(1.0)(18.0 \text{ in.})} = 2.68 \rightarrow 3 \text{ sixteenths}$

From AISC *Specification* Section J2.2(b):

w_{max} = ½ in. – 1⁄16 in.
= 7⁄16 in. ≥ 3⁄16 in. **o.k.**

From AISC *Specification* Table J2.4:

w_{min} = 3⁄16 in.

Use a 3⁄16-in. fillet weld.

Flexural Yielding of Bracket Plate

Conservatively taking the required moment strength of the plate as equal to the moment strength of the weld group,

LRFD	ASD
$M_u = P_u(al)$ $= 54.0 \text{ kips}(10.6 \text{ in.})$ $= 572 \text{ kip-in.}$	$M_a = P_a(al)$ $= 36.0 \text{ kips}(10.6 \text{ in.})$ $= 382 \text{ kip-in.}$

$$M_n = F_y Z \qquad (\textit{Manual} \text{ Eq. 15-2})$$

$$= (36 \text{ ksi})\frac{½ \text{ in.}(18.0 \text{ in.})^2}{4}$$

$$= 1{,}460 \text{ kip-in.}$$

LRFD	ASD
$\phi = 0.90$ $\phi M_n = 0.90(1{,}460 \text{ kip-in.})$ $= 1{,}310 \text{ kip-in.}$ $1{,}310 \text{ kip-in.} > 572 \text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{1{,}460 \text{ kip-in.}}{1.67}$ $= 874 \text{ kip-in.}$ $874 \text{ kip-in.} > 382 \text{ kip-in.}$ **o.k.**

Shear Yielding of Bracket Plate on Line J

$$\tan\theta = \frac{b}{a}$$

$$= \frac{11¾ \text{ in.}}{18.0 \text{ in.}}$$

$$\theta = 33.1°$$

$$b' = a \sin\theta$$
$$= 18.0 \text{ in.} (\sin 33.1°)$$
$$= 9.83 \text{ in.}$$

LRFD	ASD
$V_r = V_u = P_u \sin\theta$ (*Manual* Eq. 15-6a) $= 54.0 \text{ kips}(\sin 33.1°)$ $= 29.5 \text{ kips}$ $V_n = 0.6F_y tb'$ (*Manual* Eq. 15-7) $= 0.6(36 \text{ ksi})(½ \text{ in.})(9.83 \text{ in.})$ $= 106 \text{ kips}$	$V_r = V_a = P_a \sin\theta$ (*Manual* Eq. 15-6b) $= 36.0 \text{ kips}(\sin 33.1°)$ $= 19.7 \text{ kips}$ $V_n = 0.6F_y tb'$ (*Manual* Eq. 15-7) $= 0.6(36 \text{ ksi})(½ \text{ in.})(9.83 \text{ in.})$ $= 106 \text{ kips}$

LRFD	ASD
$\phi = 1.00$ $\phi V_n = 1.00(106\text{ kips})$ $= 106\text{ kips} > 29.5\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{V_n}{\Omega} = \dfrac{106\text{ kips}}{1.50}$ $= 70.7\text{ kips} > 19.7\text{ kips}$ **o.k.**

Local Yielding and Local Buckling of Bracket Plate on Line J

For local yielding:

$$F_{cr} = F_y = 36\text{ ksi} \quad (\textit{Manual}\text{ Eq. 15-13})$$

For local buckling:

$$F_{cr} = QF_y \quad (\textit{Manual}\text{ Eq. 15-14})$$

where

$$a' = \frac{a}{\cos\theta} \quad (\textit{Manual}\text{ Eq. 15-18})$$
$$= \frac{18.0\text{ in.}}{\cos 33.1^\circ}$$
$$= 21.5\text{ in.}$$

$$\lambda = \frac{\left(\dfrac{b'}{t}\right)\sqrt{F_y}}{5\sqrt{475 + 1{,}120\left(\dfrac{b'}{a'}\right)^2}} \quad (\textit{Manual}\text{ Eq. 15-17})$$
$$= \frac{\left(\dfrac{9.83\text{ in.}}{\frac{1}{2}\text{ in.}}\right)\sqrt{36\text{ ksi}}}{5\sqrt{475 + 1{,}120\left(\dfrac{9.83\text{ in.}}{21.5\text{ in.}}\right)^2}}$$
$$= 0.886$$

Because $0.70 < \lambda \leq 1.41$,

$$Q = 1.34 - 0.486\lambda \quad (\textit{Manual}\text{ Eq. 15-15})$$
$$= 1.34 - 0.486(0.886)$$
$$= 0.909$$

$$F_{cr} = QF_y \quad (\textit{Manual}\text{ Eq. 15-14})$$
$$= 0.909(36\text{ ksi})$$
$$= 32.7\text{ ksi}$$

Local buckling controls over local yielding. Therefore, the required and available normal and flexural strengths are determined as follows:

LRFD	ASD
$N_r = N_u = P_u \cos\theta$ (*Manual* Eq. 15-9a) $= 54.0 \text{ kips}(\cos 33.1°)$ $= 45.2 \text{ kips}$ $N_n = F_{cr}tb'$ (*Manual* Eq. 15-11) $= 32.7 \text{ ksi}(½ \text{ in.})(9.83 \text{ in.})$ $= 161 \text{ kips}$ $\phi = 0.90$ $N_c = \phi N_n = 0.90(161 \text{ kips})$ $= 145 \text{ kips}$	$N_r = N_a = P_a \cos\theta$ (*Manual* Eq. 15-9b) $= 36.0 \text{ kips}(\cos 33.1°)$ $= 30.2 \text{ kips}$ $N_n = F_{cr}tb'$ (*Manual* Eq. 15-11) $= 32.7 \text{ ksi}(½ \text{ in.})(9.83 \text{ in.})$ $= 161 \text{ kips}$ $\Omega = 1.67$ $N_c = \dfrac{N_n}{\Omega} = \dfrac{161 \text{ kips}}{1.67}$ $= 96.4 \text{ kips}$
$M_r = M_u = P_u e - N_u(b'/2)$ (*Manual* Eq. 15-8a) $= 54.0 \text{ kips}(8.00 \text{ in.}) - 45.2 \text{ kips}(9.83 \text{ in.}/2)$ $= 210 \text{ kip-in.}$ $M_n = \dfrac{F_{cr}t(b')^2}{4}$ (*Manual* Eq. 15-12) $= \dfrac{32.7 \text{ ksi}(½ \text{ in.})(9.83 \text{ in.})^2}{4}$ $= 395 \text{ kip-in.}$ $M_c = \phi M_n = 0.90(395 \text{ kip-in.})$ $= 356 \text{ kip-in.}$ $\dfrac{N_r}{N_c} + \dfrac{M_r}{M_c} \le 1.0$ (*Manual* Eq. 15-10) $= \dfrac{45.2 \text{ kips}}{145 \text{ kips}} + \dfrac{210 \text{ kip-in.}}{356 \text{ kip-in.}} \le 1.0$ $= 0.902 \le 1.0$ **o.k.**	$M_r = M_a = P_a e - N_a(b'/2)$ (*Manual* Eq. 15-8b) $= 36.0 \text{ kips}(8.00 \text{ in.}) - 30.1 \text{ kips}(9.83 \text{ in.}/2)$ $= 140 \text{ kip-in.}$ $M_n = \dfrac{F_{cr}t(b')^2}{4}$ (*Manual* Eq. 15-12) $= \dfrac{32.7 \text{ ksi}(½ \text{ in.})(9.83 \text{ in.})^2}{4}$ $= 395 \text{ kip-in.}$ $M_c = \dfrac{M_n}{\Omega} = \dfrac{395 \text{ kip-in.}}{1.67}$ $= 237 \text{ kip-in.}$ $\dfrac{N_r}{N_c} + \dfrac{M_r}{M_c} \le 1.0$ (*Manual* Eq. 15-10) $= \dfrac{30.2 \text{ kips}}{96.4 \text{ kips}} + \dfrac{140 \text{ kip-in.}}{237 \text{ kip-in.}} \le 1.0$ $= 0.902 \le 1.0$ **o.k.**

Shear Yielding of Bracket Plate on Line K

$$R_n = 0.60F_y A_{gv}$$ (*Spec*. Eq. J4-3)

$$= 0.60(36 \text{ ksi})(18.0 \text{ in.})(½ \text{ in.})$$
$$= 194 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(194 \text{ kips})$ $= 194 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{194 \text{ kips}}{1.50}$ $= 129 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

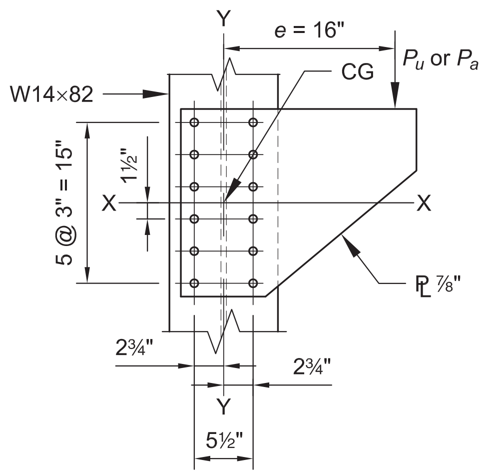
EXAMPLE II.A-24 ECCENTRICALLY LOADED BOLT GROUP (IC METHOD)

Given:

Determine the largest eccentric force, acting vertically and at a 15° angle, which can be supported by the available shear strength of the bolts using the instantaneous center of rotation method. Use ⅞-in.-diameter ASTM A325-N or F1852-N bolts in standard holes. Assume that bolt shear controls over bearing. Use AISC *Manual* Table 7-8.

Solution A (θ = 0°):

Assume the load is vertical (θ = 0°) as shown:



From AISC *Manual* Table 7-8, with θ = 0°, s = 3.00 in., e_x = 16.0 in. and n = 6:

$C = 3.55$

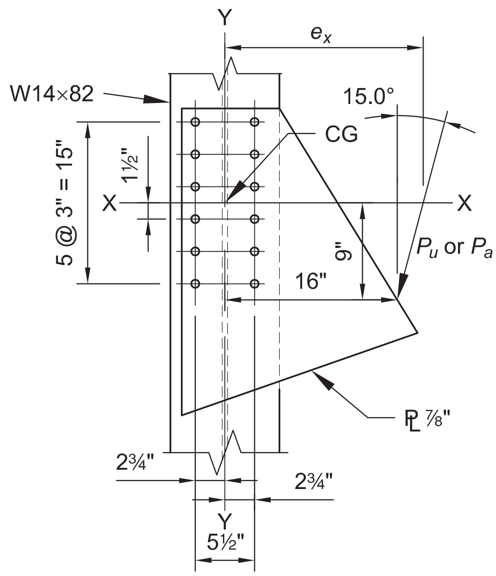
From AISC *Manual* Table 7-1:

LRFD	ASD
$\phi r_n = 24.3$ kips	$\dfrac{r_n}{\Omega} = 16.2$ kips
$\phi R_n = C\phi r_n$ $= 3.55(24.3\text{ kips})$ $= 86.3$ kips	$\dfrac{R_n}{\Omega} = C\dfrac{r_n}{\Omega}$ $= 3.55(16.2\text{ kips})$ $= 57.5$ kips
Thus, P_u must be less than or equal to 86.3 kips.	Thus, P_a must be less than or equal to 57.5 kips.

Note: The eccentricity of the load significantly reduces the shear strength of the bolt group.

Solution B (θ = 15°):

Assume the load acts at an angle of 15° with respect to vertical (θ = 15°) as shown:



$e_x = 16.0\text{ in.} + 9.00\text{ in.}(\tan 15°)$
$= 18.4\text{ in.}$

By interpolating AISC *Manual* Table 7-8, with θ = 15°, s = 3.00 in., e_x = 18.4 in., and n = 6:

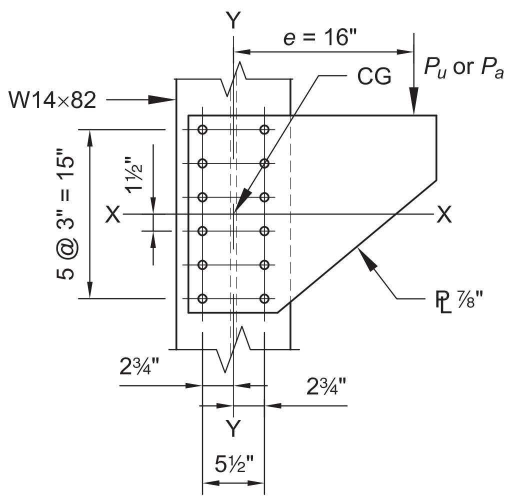
$C = 3.21$

LRFD	ASD
$\phi R_n = C\phi r_n$ $= 3.21(24.3\text{ kips})$ $= 78.0\text{ kips}$ Thus, P_u must be less than or equal to 78.0 kips.	$\dfrac{R_n}{\Omega} = C\dfrac{r_n}{\Omega}$ $= 3.21(16.2\text{ kips})$ $= 52.0\text{ kips}$ Thus, P_a must be less than or equal to 52.0 kips.

EXAMPLE II.A-25 ECCENTRICALLY LOADED BOLT GROUP (ELASTIC METHOD)

Given:

Determine the largest eccentric force that can be supported by the available shear strength of the bolts using the elastic method for θ = 0°. Compare the result with that of the previous example. Use ⅞-in.-diameter ASTM A325-N or F1852-N bolts in standard holes. Assume that bolt shear controls over bearing.



Solution:

LRFD	ASD
Direct shear force per bolt: $r_{pxu} = 0$ $r_{pyu} = \dfrac{P_u}{n}$ (*Manual* Eq. 7-2a) $= \dfrac{P_u}{12}$ Additional shear force due to eccentricity: Polar moment of inertia: $I_x \approx \Sigma y^2$ $= 4(7.50 \text{ in.})^2 + 4(4.50 \text{ in.})^2 + 4(1.50 \text{ in.})^2$ $= 315 \dfrac{\text{in.}^4}{\text{in.}^2}$ $I_y \approx \Sigma x^2$ $= 12(2.75 \text{ in.})^2$ $= 90.8 \dfrac{\text{in.}^4}{\text{in.}^2}$	Direct shear force per bolt: $r_{pxa} = 0$ $r_{pya} = \dfrac{P_a}{n}$ (*Manual* Eq. 7-2b) $= \dfrac{P_a}{12}$ Additional shear force due to eccentricity: Polar moment of inertia: $I_x \approx \Sigma y^2$ $= 4(7.50 \text{ in.})^2 + 4(4.50 \text{ in.})^2 + 4(1.50 \text{ in.})^2$ $= 315 \dfrac{\text{in.}^4}{\text{in.}^2}$ $I_y \approx \Sigma x^2$ $= 12(2.75 \text{ in.})^2$ $= 90.8 \dfrac{\text{in.}^4}{\text{in.}^2}$

LRFD	ASD
$I_p \approx I_x + I_y$ $= 315\dfrac{\text{in.}^4}{\text{in.}^2} + 90.8\dfrac{\text{in.}^4}{\text{in.}^2}$ $= 406\dfrac{\text{in.}^4}{\text{in.}^2}$	$I_p \approx I_x + I_y$ $= 315\dfrac{\text{in.}^4}{\text{in.}^2} + 90.8\dfrac{\text{in.}^4}{\text{in.}^2}$ $= 406\dfrac{\text{in.}^4}{\text{in.}^2}$
$r_{mxu} = \dfrac{P_u e c_y}{I_p}$ (*Manual* Eq. 7-6a) $= \dfrac{P_u(16.0\text{ in.})(7.50\text{ in.})}{406\text{ in.}^4/\text{in.}^2}$ $= 0.296P_u$	$r_{mxa} = \dfrac{P_a e c_y}{I_p}$ (*Manual* Eq. 7-6b) $= \dfrac{P_a(16.0\text{ in.})(7.50\text{ in.})}{406\text{ in.}^4/\text{in.}^2}$ $= 0.296P_a$
$r_{myu} = \dfrac{P_u e c_x}{I_p}$ (*Manual* Eq. 7-7a) $= \dfrac{P_u(16.0\text{ in.})(2.75\text{ in.})}{406\text{ in.}^4/\text{in.}^2}$ $= 0.108P_u$	$r_{mya} = \dfrac{P_a e c_x}{I_p}$ (*Manual* Eq. 7-7b) $= \dfrac{P_a(16.0\text{ in.})(2.75\text{ in.})}{406\text{ in.}^4/\text{in.}^2}$ $= 0.108P_a$
Resultant shear force:	Resultant shear force:
$r_u = \sqrt{(r_{pxu} + r_{mxu})^2 + (r_{pyu} + r_{myu})^2}$ (*Manual* Eq. 7-8a) $= \sqrt{(0 + 0.296P_u)^2 + \left(\dfrac{P_u}{12} + 0.108P_u\right)^2}$ $= 0.353P_u$	$r_a = \sqrt{(r_{pxa} + r_{mxa})^2 + (r_{pya} + r_{mya})^2}$ (*Manual* Eq. 7-8b) $= \sqrt{(0 + 0.296P_a)^2 + \left(\dfrac{P_a}{12} + 0.108P_a\right)^2}$ $= 0.353P_a$
Since r_u must be less than or equal to the available strength,	Since r_a must be less than or equal to the available strength,
$P_u \le \dfrac{\phi r_n}{0.353}$ $= \dfrac{24.3\text{ kips}}{0.353}$ $= 68.8\text{ kips}$	$P_a \le \dfrac{r_n / \Omega}{0.353}$ $= \dfrac{16.2\text{ kips}}{0.353}$ $= 45.9\text{ kips}$

Note: The elastic method, shown here, is more conservative than the instantaneous center of rotation method, shown in Example II.A-24.

Return to Table of Contents

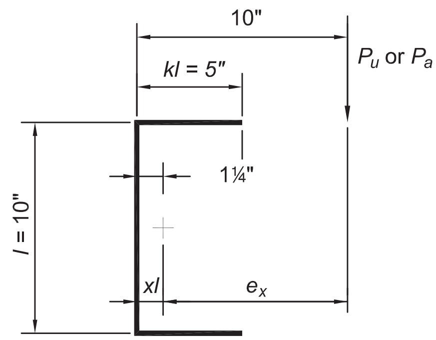
EXAMPLE II.A-26 ECCENTRICALLY LOADED WELD GROUP (IC METHOD)

Given:

Determine the largest eccentric force, acting vertically and at a 75° angle, that can be supported by the available shear strength of the weld group, using the instantaneous center of rotation method. Use a ⅜-in. fillet weld and 70-ksi electrodes. Use AISC *Manual* Table 8-8.

Solution A (θ = 0°):

Assume that the load is vertical (θ = 0°) as shown:



$$l = 10.0 \text{ in.}$$

$$kl = 5.00 \text{ in.}$$

$$\begin{aligned} k &= \frac{kl}{l} \\ &= \frac{5.00 \text{ in.}}{10.0 \text{ in.}} \\ &= 0.500 \end{aligned}$$

$$\begin{aligned} xl &= \frac{(kl)^2}{2(kl)+l} \\ &= \frac{(5.00 \text{ in.})^2}{2(5.00 \text{ in.})+10.0 \text{ in.}} \\ &= 1.25 \text{ in.} \end{aligned}$$

$$xl + al = 10.0 \text{ in.}$$

$$1.25 \text{ in.} + a(10.0 \text{ in.}) = 10.0 \text{ in.}$$

$$a = 0.875$$

By interpolating AISC *Manual* Table 8-8, with θ = 0°, a = 0.875 and k = 0.500:

$$C = 1.88$$

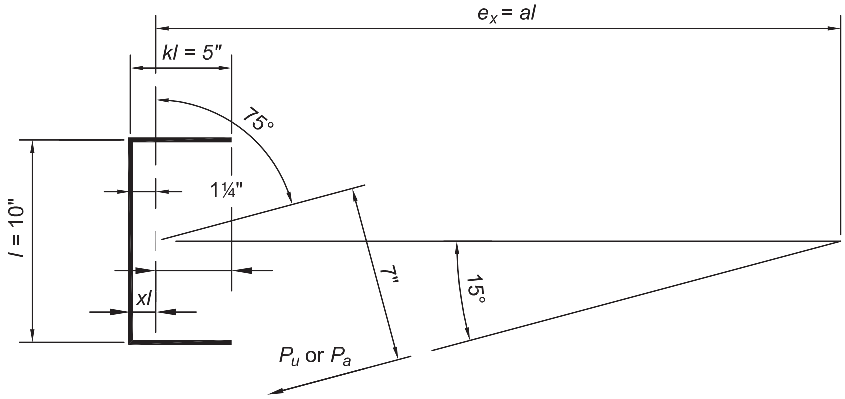
From AISC *Manual* Equation 8-13:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi C C_1 D l$ $= 0.75(1.88)(1.0)(6 \text{ sixteenths})(10.0 \text{ in.})$ $= 84.6 \text{ kips}$ Thus, P_u must be less than or equal to 84.6 kips.	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{C C_1 D l}{\Omega}$ $= \dfrac{1.88(1.0)(6 \text{ sixteenths})(10.0 \text{ in.})}{2.00}$ $= 56.4 \text{ kips}$ Thus, P_a must be less than or equal to 56.4 kips.

Note: The eccentricity of the load significantly reduces the shear strength of this weld group as compared to the concentrically loaded case.

Solution B (θ = 75°):

Assume that the load acts at an angle of 75° with respect to vertical (θ = 75°) as shown:



As determined in Solution A,

$k = 0.500$ and $xl = 1.25$ in.

$$e_x = al$$
$$= \frac{7.00 \text{ in.}}{\sin 15°}$$
$$= 27.0 \text{ in.}$$

$$a = \frac{e_x}{l}$$
$$= \frac{27.0 \text{ in.}}{10.0 \text{ in.}}$$
$$= 2.70$$

By interpolating AISC *Manual* Table 8-8, with $\theta = 75^o$, $a = 2.70$ and $k = 0.500$:

$$C = 1.99$$

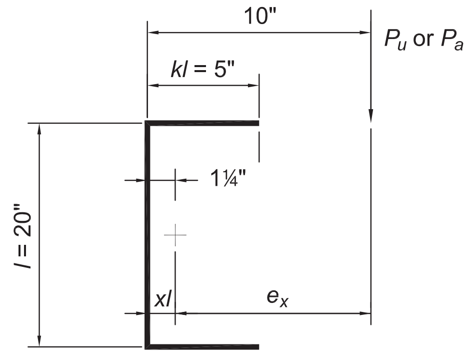
From AISC *Manual* Equation 8-13:

LRFD	ASD
$\phi R_n = \phi C C_1 D l$ $= 0.75(1.99)(1.0)(6 \text{ sixteenths})(10.0 \text{ in.})$ $= 89.6 \text{ kips}$ Thus, P_u must be less than or equal to 89.6 kips.	$\dfrac{R_n}{\Omega} = \dfrac{C C_1 D l}{\Omega}$ $= \dfrac{1.99(1.0)(6 \text{ sixteenths})(10.0 \text{ in.})}{2.00}$ $= 59.7 \text{ kips}$ Thus, P_a must be less than or equal to 59.7 kips.

EXAMPLE II.A-27 ECCENTRICALLY LOADED WELD GROUP (ELASTIC METHOD)

Given:

Determine the largest eccentric force that can be supported by the available shear strength of the welds in the connection, using the elastic method. Compare the result with that of the previous example. Use ⅜-in. fillet welds and 70-ksi electrodes.



Solution:

Direct Shear Force per Inch of Weld

LRFD	ASD
$r_{pux} = 0$	$r_{pax} = 0$
$r_{puy} = \dfrac{P_u}{l}$ (*Manual* Eq. 8-5a)	$r_{pay} = \dfrac{P_a}{l}$ (*Manual* Eq. 8-5b)
$= \dfrac{P_u}{20.0\text{ in.}}$	$= \dfrac{P_a}{20.0\text{ in.}}$
$= 0.0500\dfrac{P_u}{\text{in.}}$	$= 0.0500\dfrac{P_a}{\text{in.}}$

Additional Shear Force due to Eccentricity

Determine the polar moment of inertia referring to the AISC *Manual* Figure 8-6:

$$I_x = \frac{l^3}{12} + 2(kl)(y^2)$$
$$= \frac{(10.0\text{ in.})^3}{12} + 2(5.00\text{ in.})(5.00\text{ in.})^2$$
$$= 333\text{ in.}^4\text{/in.}$$

$$I_y = \frac{2(kl)^3}{12} + 2(kl)\left(\frac{kl}{2} - xl\right)^2 + l(xl)^2$$
$$= \frac{2(5.00\text{ in.})^3}{12} + 2(5.00\text{ in.})(2.50\text{ in.} - 1\tfrac{1}{4}\text{ in.})^2 + (10.0\text{ in.})(1\tfrac{1}{4}\text{ in.})^2$$
$$= 52.1\text{ in.}^4\text{/in.}$$

$$I_p = I_x + I_y$$
$$= 333 \text{ in.}^4\text{/in.} + 52.1 \text{ in.}^4\text{/in.}$$
$$= 385 \text{ in.}^4\text{/in.}$$

LRFD	ASD
$r_{mux} = \dfrac{P_u e c_y}{I_p}$ (*Manual* Eq. 8-9a) $= \dfrac{P_u(8.75\text{ in.})(5.00\text{ in.})}{385\text{ in.}^4\text{/in.}}$ $= \dfrac{0.114P_u}{\text{in.}}$	$r_{max} = \dfrac{P_a e c_y}{I_p}$ (*Manual* Eq. 8-9b) $= \dfrac{P_a(8.75\text{ in.})(5.00\text{ in.})}{385\text{ in.}^4\text{/in.}}$ $= \dfrac{0.114P_a}{\text{in.}}$
$r_{muy} = \dfrac{P_u e c_x}{I_p}$ (*Manual* Eq. 8-10a) $= \dfrac{P_u(8.75\text{ in.})(3.75\text{ in.})}{385\text{ in.}^4\text{/in.}}$ $= \dfrac{0.0852P_u}{\text{in.}}$	$r_{may} = \dfrac{P_a e c_x}{I_p}$ (*Manual* Eq. 8-10b) $= \dfrac{P_a(8.75\text{ in.})(3.75\text{ in.})}{385\text{ in.}^4\text{/in.}}$ $= \dfrac{0.0852P_a}{\text{in.}}$
Resultant shear force: $r_u = \sqrt{(r_{pux} + r_{mux})^2 + (r_{puy} + r_{muy})^2}$ (*Manual* Eq. 8-11a) $= \sqrt{\left(0 + \dfrac{0.114P_u}{\text{in.}}\right)^2 + \left(\dfrac{0.0500P_u}{\text{in.}} + \dfrac{0.0852P_u}{\text{in.}}\right)^2}$ $= \dfrac{0.177P_u}{\text{in.}}$	Resultant shear force: $r_a = \sqrt{(r_{pax} + r_{max})^2 + (r_{pay} + r_{may})^2}$ (*Manual* Eq. 8-11b) $= \sqrt{\left(0 + \dfrac{0.114P_a}{\text{in.}}\right)^2 + \left(\dfrac{0.0500P_a}{\text{in.}} + \dfrac{0.0852P_a}{\text{in.}}\right)^2}$ $= \dfrac{0.177P_a}{\text{in.}}$
Since r_u must be less than or equal to the available strength, from AISC *Manual* Equation 8-2a, $r_u = 0.177P_u \leq \phi r_n$ $P_u \leq \dfrac{\phi r_n}{0.177}$ $\leq \dfrac{1.392\text{ kips/in.}}{\text{sixteenth}}(6\text{ sixteenths})\left(\dfrac{\text{in.}}{0.177}\right)$ ≤ 47.2 kips	Since r_a must be less than or equal to the available strength, from AISC *Manual* Equation 8-2b, $r_a = 0.177P_a \leq r_n/\Omega$ $P_a \leq \dfrac{r_n/\Omega}{0.177}$ $\leq \dfrac{0.928\text{ kip/in.}}{\text{sixteeth}}(6\text{ sixteenths})\left(\dfrac{\text{in.}}{0.177}\right)$ ≤ 31.5 kips

Note: The strength of the weld group predicted by the elastic method, as shown here, is significantly less than that predicted by the instantaneous center of rotation method in Example II.A-26.

Return to Table of Contents

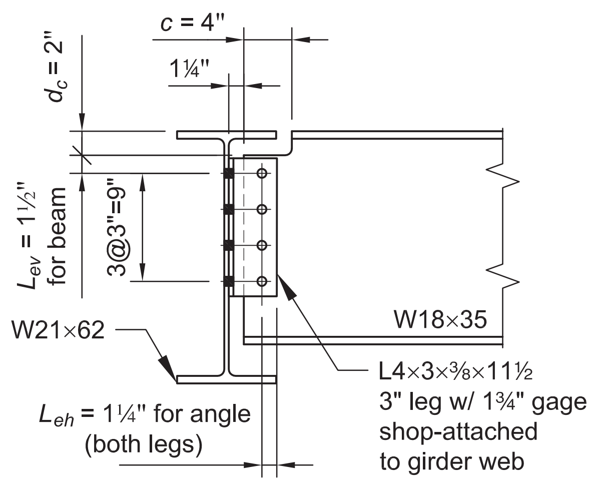
EXAMPLE II.A-28 ALL-BOLTED SINGLE-ANGLE CONNECTION (BEAM-TO-GIRDER WEB)

Given:

Design an all-bolted single-angle connection (Case I in Table 10-11) between an ASTM A992 W18×35 beam and an ASTM A992 W21×62 girder web, to support the following beam end reactions:

R_D = 6.5 kips
R_L = 20 kips

The top flange is coped 2 in. deep by 4 in. long, L_{ev} = 1½ in. and L_{eh} = 1½ in. (assumed to be 1¼ in. for calculation purposes to account for possible underrun in beam length). Use ¾-in.-diameter A325-N or F1852-N bolts in standard holes and an ASTM A36 angle.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
W18×35
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Girder
W21×62
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1 and Figure 9-2, the geometric properties are as follows:

Beam
W18×35
t_w = 0.300 in.
d = 17.7 in.
t_f = 0.425 in.
c = 4.00 in.
d_c = 2.00 in.
e = 5.25 in.
h_0 = 15.7 in.

Girder
W21×62
t_w = 0.400 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(6.5 kips) + 1.6(20 kips) = 39.8 kips	R_a = 6.5 kips + 20 kips = 26.5 kips

Bolt Design

Check eccentricity of connection.

For the 4-in. angle leg attached to the supported beam (W18x35):

e = 2.75 ≤ 3.00 in., therefore, eccentricity does not need to be considered for this leg.

For the 3-in. angle leg attached to the supporting girder (W21x62):

$$e = 1.75\text{ in.} + \frac{0.300\text{ in.}}{2}$$

= 1.90 in. ≤ 2.50 in., therefore, AISC *Manual* Table 10-11 may conservatively be used for bolt shear.

From AISC *Manual* Table 7-1, the single bolt shear strength is:

LRFD	ASD
$\phi r_n = 17.9\text{ kips}$	$\dfrac{r_n}{\Omega} = 11.9\text{ kips}$

From AISC *Manual* Table 7-5, the single bolt bearing strength on a ⅜-in.-thick angle is:

LRFD	ASD
$\phi r_n = 44.0\text{ kips/in.}(⅜\text{ in.})$ $= 16.5\text{ kips}$ Bolt bearing is more critical than bolt shear in this example; thus, ϕr_n = 16.5 kips. $C_{min} = \dfrac{R_u}{\phi r_n}$	$\dfrac{r_n}{\Omega} = 29.4\text{ kips/in.}(⅜\text{ in.})$ $= 11.0\text{ kips}$ Bolt bearing is more critical than bolt shear in this example; thus, r_n/Ω = 11.0 kips. $C_{min} = \dfrac{R_a}{r_n / \Omega}$

LRFD	ASD
$= \dfrac{39.8\text{ kips}}{16.5\text{ kips/bolt}}$ $= 2.41$ Try a four-bolt connection. From AISC *Manual* Table 10-11: $C = 3.07 > 2.41$ **o.k.**	$= \dfrac{26.5\text{ kips}}{11.0\text{ kips/bolt}}$ $= 2.41$ Try a four-bolt connection. From AISC *Manual* Table 10-11: $C = 3.07 > 2.41$ **o.k.**

The 3-in. leg will be shop bolted to the girder web and the 4-in. leg will be field bolted to the beam web.

Shear Yielding of Angle

$$\begin{aligned} R_n &= 0.60F_yA_{gv} \\ &= 0.60(36\text{ ksi})(11\tfrac{1}{2}\text{ in.})(\tfrac{3}{8}\text{ in.}) \\ &= 93.2\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-3)

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(93.2\text{ kips})$ $= 93.2\text{ kips} > 39.8\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{93.2\text{ kips}}{1.50}$ $= 62.1\text{ kips} > 26.5$ **o.k.**

Shear Rupture of Angle

$$\begin{aligned} A_{nv} &= \tfrac{3}{8}\text{ in.}[11\tfrac{1}{2}\text{ in.} - 4(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})] \\ &= 3.00\text{ in.}^2 \end{aligned}$$

$$\begin{aligned} R_n &= 0.60F_uA_{nv} \\ &= 0.60(58\text{ ksi})(3.00\text{ in.}^2) \\ &= 104\text{ kips} \end{aligned}$$ (*Spec*. Eq. J4-4)

From AISC *Specification* Section J4.2:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(104\text{ kisp})$ $= 78.0\text{ kips} > 39.8\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{104\text{ kips}}{2.00}$ $= 52.0\text{ kips} > 26.5\text{ kips}$ **o.k.**

Block Shear Rupture of Angle

$n = 4$
$L_{ev} = L_{eh} = 1\tfrac{1}{4}$ in.

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv})$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt} = 1.0(35.3\text{ kips/in.})(\tfrac{3}{8}\text{ in.})$ Shear yielding component from AISC *Manual* Table 9-3b: $\phi 0.60 F_y A_{gv} = 166\text{ kips/in.}(\tfrac{3}{8}\text{ in.})$ Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv} = 188\text{ kips/in.}(\tfrac{3}{8}\text{ in.})$ $\phi R_n = (35.3\text{ kips/in.} + 166\text{ kips/in.})(\tfrac{3}{8}\text{ in.})$ $= 75.5\text{ kips} > 39.8\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = 1.0(23.6\text{ kips/in.})(\tfrac{3}{8}\text{ in.})$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60 F_y A_{gv}}{\Omega} = 111\text{ kips/in.}(\tfrac{3}{8}\text{ in.})$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60 F_u A_{nv}}{\Omega} = 125\text{ kips/in.}(\tfrac{3}{8}\text{ in.})$ $\dfrac{R_n}{\Omega} = (23.6\text{ kips/in.} + 111\text{ kips/in.})(\tfrac{3}{8}\text{ in.})$ $= 50.5\text{ kips} > 26.5\text{ kips}$ **o.k.**

Flexural Yielding of Support-Leg of Angle

The required strength is:

LRFD	ASD
$M_u = R_u e$ $= 39.8\text{ kips}\left(1\tfrac{3}{4}\text{ in.} + \dfrac{0.300\text{ in.}}{2}\right)$ $= 75.6\text{ kip-in.}$ $\phi = 0.90$ $\phi M_n = \phi F_y Z_x$ $= 0.90(36\text{ ksi})\left[\dfrac{\tfrac{3}{8}\text{ in.}(11\tfrac{1}{2}\text{ in.})^2}{4}\right]$ $= 402\text{ kip-in.} > 75.6\text{ kip-in.}$ **o.k.**	$M_a = R_a e$ $= 26.5\text{ kips}\left(1\tfrac{3}{4}\text{ in.} + \dfrac{0.300\text{ in.}}{2}\right)$ $= 50.4\text{ kip-in.}$ $\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{F_y Z_x}{\Omega}$ $= \dfrac{36\text{ ksi}}{1.67}\left[\dfrac{\tfrac{3}{8}\text{ in.}(11\tfrac{1}{2}\text{ in.})^2}{4}\right]$ $= 267\text{ kip-in.} > 50.4\text{ kip-in.}$ **o.k.**

Flexural Rupture of Support-Leg of Angle

$$Z_{net} = \tfrac{3}{8}\text{ in.}\left[\frac{(11\tfrac{1}{2}\text{ in.})^2}{4} - 2(0.875\text{ in.})(4.50\text{ in.}) - 2(0.875\text{ in.})(1.50\text{ in.})\right]$$
$$= 8.46\text{ in.}^3$$

From AISC *Manual* Equation 9-4:

LRFD	ASD
$\phi_b = 0.75$ $\phi_b M_n = \phi_b F_u Z_{net}$ $= 0.75(58\text{ ksi})(8.46\text{ in.}^3)$ $= 368\text{ kip-in.} > 75.6\text{ kip-in.}$ **o.k.**	$\Omega_b = 2.00$ $\dfrac{M_n}{\Omega_b} = \dfrac{F_u Z_{net}}{\Omega_b}$ $= \dfrac{58\text{ ksi}(8.46\text{ in.}^3)}{2.00}$ $= 245\text{ kip-in.} > 50.4\text{ kip-in.}$ **o.k.**

Bolt Bearing and Block Shear Rupture of Beam Web

$n = 4$
$L_{ev} = L_{eh} = 1\tfrac{1}{2}$ in.

(L_{eh} assumed to be 1¼ in. for calculation purposes to provide for possible underrun in beam length.)

From AISC *Manual* Table 10-1:

LRFD	ASD
$\phi R_n = 257\text{ kips/in.}(0.300\text{ in.})$ $= 77.1\text{ kips} > 39.8\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 171\text{ kips/in.}(0.300\text{ in.})$ $= 51.3\text{ kips} > 26.5\text{ kips}$ **o.k.**

Note: For coped beam sections, the limit states of flexural yielding and local buckling should be checked independently per AISC *Manual* Part 9. The supported beam web should also be checked for shear yielding and shear rupture per AISC *Specification* Section J4.2. However, for the shallow cope in this example, these limit states do not govern. For an illustration of these checks, see Example II.A-4.

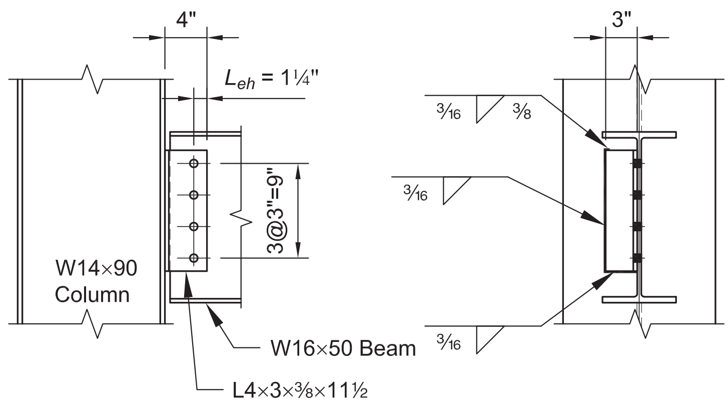
EXAMPLE II.A-29 BOLTED/WELDED SINGLE-ANGLE CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a single-angle connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column flange to support the following beam end reactions:

R_D = 9.0 kips
R_L = 27 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts to connect the supported beam to an ASTM A36 single angle. Use 70-ksi electrode welds to connect the single angle to the column flange.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
W16×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×90
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angle
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W16×50
t_w = 0.380 in.
d = 16.3in.
t_f = 0.630 in.

Column
W14×90
t_f = 0.710

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(9.0 kips) + 1.6(27 kips) = 54.0 kips	R_a = 9.0 kips + 27 kips = 36.0 kips

Single Angle, Bolts and Welds

Check eccentricity of the connection.

For the 4-in. angle leg attached to the supported beam:

e = 2.75 in. ≤ 3.00 in., therefore, eccentricity does not need to be considered for this leg.

For the 3-in. angle leg attached to the supporting column flange:

Since the half-web dimension of the W16x50 supported beam is less than ¼ in., AISC *Manual* Table 10-12 may conservatively be used.

Try a four-bolt single-angle (L4×3×⅜).

From AISC *Manual* Table 10-12:

LRFD	ASD
Bolt and angle available strength: ϕR_n = 71.4 kips > 54.0 kips **o.k.** Weld available strength: With a 3⁄16-in fillet weld size: ϕR_n = 56.6 kips > 54.0 kips **o.k.**	Bolt and angle available strength: $\dfrac{R_n}{\Omega}$ = 47.6 kips > 36.0 kips **o.k.** Weld available strength: With a 3⁄16-in. fillet weld size: $\dfrac{R_n}{\Omega}$ = 37.8 kips > 36.0 kips **o.k.**

Support Thickness

The minimum support thickness for the 3⁄16-in. fillet welds is:

$$t_{min} = \frac{3.09D}{F_u} \qquad \text{(}\textit{Manual}\text{ Eq. 9-2)}$$

$$= \frac{3.09(3 \text{ sixteenths})}{65 \text{ ksi}}$$
$= 0.143$ in. < 0.710 in. **o.k.**

Note: The minimum thickness values listed in Table 10-12 are for conditions with angles on both sides of the web.

Use a four-bolt single-angle L4×3×⅜. The 3-in. leg will be shop welded to the column flange and the 4-in. leg will be field bolted to the beam web.

Supported Beam Web

From AISC *Manual* Table 7-4, with s = 3.00 in., ¾-in.-diameter bolts and standard holes, the bearing strength of the beam web is:

LRFD	ASD
$\phi R_n = \phi r_n t_w n$ $= 87.8$ kips/in.(0.380 in.)(4 bolts) $= 133$ kips > 54.0 kips **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{r_n t_w n}{\Omega}$ $= 58.5$ kips/in.(0.380 in.)(4 bolts) $= 88.9$ kips > 36.0 kips **o.k.**

Return to Table of Contents

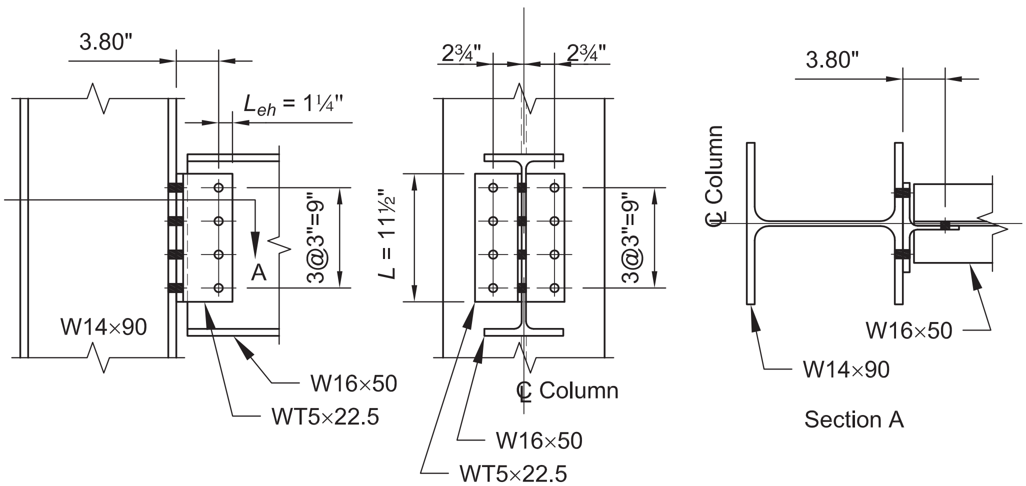
EXAMPLE II.A-30 ALL-BOLTED TEE CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design an all-bolted tee connection between an ASTM A992 W16×50 beam and an ASTM A992 W14×90 column flange to support the following beam end reactions:

R_D = 9.0 kips
R_L = 27 kips

Use ¾-in.-diameter ASTM A325 bolts in standard holes. Try an ASTM A992 WT5×22.5 with a four-bolt connection.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
W16×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×90
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Tee
WT5×22.5
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Tables 1-1 and 1-8, the geometric properties are as follows:

Beam
W16×50
t_w = 0.380 in.
d = 16.3 in.
t_f = 0.630 in.

Column
W14×90
t_f = 0.710 in.

Tee
WT5×22.5
d = 5.05 in.
b_f = 8.02 in.
t_f = 0.620 in.
t_s = 0.350 in.
k_1 = 13⁄16 in. (see W10×45 AISC *Manual* Table 1-1)
k_{des} = 1.12 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(9.0 \text{ kips}) + 1.6(27 \text{ kips})$ $= 54.0 \text{ kips}$	$R_a = 9.0 \text{ kips} + 27 \text{ kips}$ $= 36.0 \text{ kips}$

Limitation on Tee Stem Thickness

See rotational ductility discussion at the beginning of the AISC *Manual* Part 9.

$$t_{s\,max} = \frac{d}{2} + \tfrac{1}{16} \text{ in.} \qquad (\textit{Manual} \text{ Eq. 9-38})$$

$$= \frac{\tfrac{3}{4} \text{ in.}}{2} + \tfrac{1}{16} \text{ in.}$$

$$= 0.438 \text{ in.} > 0.350 \text{ in.} \qquad \textbf{o.k.}$$

Limitation on Bolt Diameter for Bolts through Tee Flange

Note: The bolts are not located symmetrically with respect to the centerline of the tee.

b = flexible width in connection element

$$= 2.75 \text{ in.} - \frac{t_s}{2} - \frac{t_w}{2} - k_1$$

$$= 2.75 \text{ in.} - \frac{0.350 \text{ in.}}{2} - \frac{0.380 \text{ in.}}{2} - \tfrac{13}{16} \text{ in.}$$

$$= 1.57 \text{ in.}$$

$$d_{min} = 0.163 t_f \sqrt{\frac{F_y}{b}\left(\frac{b^2}{L^2} + 2\right)} \leq 0.69\sqrt{t_s} \qquad (\textit{Manual} \text{ Eq. 9-37})$$

$$= 0.163(0.620 \text{ in.})\sqrt{\frac{50 \text{ ksi}}{1.57 \text{ in.}}\left(\frac{(1.57 \text{ in.})^2}{(11\tfrac{1}{2} \text{ in.})^2} + 2\right)} \leq 0.69\sqrt{0.350 \text{ in.}}$$

$$= 0.810 \text{ in.} \leq 0.408 \text{ in.}$$

Use d_{min} = 0.408 in.

d = ¾ in. > d_{min} = 0.408 in. **o.k.**

Since the connection is rigid at the support, the bolts through the tee stem must be designed for shear, but do not need to be designed for an eccentric moment.

Shear and Bearing for Bolts through Beam Web

LRFD	ASD
Since bolt shear is more critical than bolt bearing in this example, ϕr_n = 17.9 kips from AISC *Manual* Table 7-1. Thus, $\phi R_n = n\phi r_n$ $= 4 \text{ bolts}(17.9 \text{ kips})$ $= 71.6 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	Since bolt shear is more critical than bolt bearing in this example, r_n/Ω = 11.9 kips from AISC *Manual* Table 7-1. Thus, $\dfrac{R_n}{\Omega} = \dfrac{nr_n}{\Omega}$ $= 4 \text{ bolts}(11.9 \text{ kips})$ $= 47.6 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Flexural Yielding of Stem

The flexural yielding strength is checked at the junction of the stem and the fillet.

LRFD	ASD
$M_u = P_u e$ $= (54.0 \text{ kips})(3.80 \text{ in.} - 1.12 \text{ in.})$ $= 145 \text{ kip-in.}$ $\phi = 0.90$ $\phi M_n = \phi F_y Z_x$ $= 0.90(50 \text{ ksi})\left(\dfrac{0.350 \text{ in.}(11\tfrac{1}{2} \text{ in.})^2}{4}\right)$ $= 521 \text{ kip-in.} > 145 \text{ kip-in.}$ **o.k.**	$M_a = P_a e$ $= 36.0 \text{ kips}(3.80 \text{ in.} - 1.12 \text{ in.})$ $= 96.5 \text{ kip-in.}$ $\Omega = 1.67$ $\dfrac{M_u}{\Omega} = \dfrac{F_y Z_x}{\Omega}$ $= \dfrac{50 \text{ ksi}\left(\dfrac{0.350 \text{ in.}(11\tfrac{1}{2} \text{ in.})^2}{4}\right)}{1.67}$ $= 346 \text{ kip-in.} > 96.5 \text{ kip-in.}$ **o.k.**

Shear Yielding of Stem

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00\left[0.60(50 \text{ ksi})(11\tfrac{1}{2} \text{ in.})(0.350 \text{ in.})\right]$ $= 121 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(50 \text{ ksi})(11\tfrac{1}{2} \text{ in.})(0.350 \text{ in.})}{1.50}$ $= 80.5 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Shear Rupture of Stem

$$A_{nv} = \left[11\tfrac{1}{2}\text{ in.} - 4\left(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}\right)\right]0.350\text{ in.}$$
$$= 2.80\text{ in.}^2$$

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(65\text{ ksi})(2.80\text{ in.}^2)$ $= 81.9\text{ kips} > 54.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(65\text{ ksi})(2.80\text{ in.}^2)}{2.00}$ $= 54.6\text{ kips} > 36.0\text{ kips}$ **o.k.**

Block Shear Rupture of Stem

$$L_{eh} = L_{ev} = 1\tfrac{1}{4}\text{ in.}$$

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv})$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt} = 39.6\text{ kips/in.}\ (0.350\text{ in.})$ Shear yielding component from AISC *Manual* Table 9-3b: $\phi 0.60 F_y A_{gv} = 231\text{ kips/in.}\ (0.350\text{ in.})$ Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv} = 210\text{ kips/in.}\ (0.350\text{ in.})$ $\phi R_n = (39.6\text{ kips/in.} + 210\text{ kips/in.})(0.350\text{in.})$ $= 87.4\text{ kips} > 54.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = 26.4\text{ kips/in.}\ (0.350\text{ in.})$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60 F_y A_{gv}}{\Omega} = 154\text{ kips/in.}\ (0.350\text{ in.})$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60 F_u A_{nv}}{\Omega} = 140\text{ kips/in.}\ (0.350\text{ in.})$ $\dfrac{R_n}{\Omega} = (26.4\text{ kips/in.} + 140\text{ kips/in.})(0.350\text{ in.})$ $= 58.2\text{ kips} > 36.0\text{ kips}$ **o.k.**

Since the connection is rigid at the support, the bolts attaching the tee flange to the support must be designed for the shear and the eccentric moment.

Bolt Group at Column

Check bolts for shear and bearing combined with tension due to eccentricity.

The following calculation follows the Case II approach in the Section "Eccentricity Normal to the Plane of the Faying Surface" in Part 7 of the AISC *Manual*.

LRFD	ASD
Tensile force per bolt, r_{ut}: $r_{ut} = \dfrac{P_u e}{n' d_m}$ (*Manual* Eq. 7-14a) $= \dfrac{54.0 \text{ kips}(3.80 \text{ in.})}{4 \text{ bolts}(6.00 \text{ in.})}$ $= 8.55$ kips/bolt Design strength of bolts for tension-shear interaction: When threads are not excluded from the shear planes of ASTM A325 bolts, from AISC Manual Table 7-1, $\phi r_n = 17.9$ kips/bolt. $r_{uv} = \dfrac{P_u}{n}$ (*Spec.* Eq. 7-13a) $= \dfrac{54.0 \text{ kips}}{8 \text{ bolts}}$ $= 6.75$ kips/bolt < 17.9 kips/bolt **o.k.** $f_{rv} = \dfrac{6.75 \text{ kips/bolt}}{0.442 \text{ in.}^2}$ $= 15.3$ ksi From AISC *Specification* Table J3.2: $F_{nt} = 90$ ksi $F_{nv} = 54$ ksi $\phi = 0.75$ $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec.* Eq. J3-3a) $= 1.3(90 \text{ ksi}) - \left(\dfrac{90 \text{ ksi}}{0.75(54 \text{ ksi})}\right)(15.3 \text{ ksi})$ $= 83.0 \text{ ksi} \le 90 \text{ ksi}$ From AISC *Specification* Equation J3-2: $\phi R_n = \phi F'_{nt} A_b$	Tensile force per bolt, r_{at}: $r_{at} = \dfrac{P_a e}{n' d_m}$ (*Manual* Eq. 7-14b) $= \dfrac{36.0 \text{ kips}(3.80 \text{ in.})}{4 \text{ bolts}(6.00 \text{ in.})}$ $= 5.70$ kips/bolt Allowable strength of bolts for tension-shear interaction: When threads are not excluded from the shear planes of ASTM A325 bolts, from AISC Manual Table 7-1, $r_n/\Omega = 11.9$ kips/bolt. $r_{av} = \dfrac{P_a}{n}$ (*Spec.* Eq. 7-13b) $= \dfrac{36.0 \text{ kips}}{8 \text{ bolts}}$ $= 4.50$ kips/bolt < 11.9 kips/bolt **o.k.** $f_{rv} = \dfrac{4.50 \text{ kips/bolt}}{0.442 \text{ in.}^2}$ $= 10.2$ ksi From AISC *Specification* Table J3.2: $F_{nt} = 90$ ksi $F_{nv} = 54$ ksi $\Omega = 2.00$ $F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec.* Eq. J3-3b) $= 1.3(90 \text{ ksi}) - \left(\dfrac{2.00(90 \text{ ksi})}{54 \text{ ksi}}\right)(10.2 \text{ ksi})$ $= 83.0 \text{ ksi} < 90 \text{ ksi}$ From AISC *Specification* Equation J3-2:

$= 0.75(83.0 \text{ ksi})(0.442 \text{ in.}^2)$ $= 27.5 \text{ kips/bolt} > 8.55 \text{ kips/bolt}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F'_{nt} A_b}{\Omega}$ $= 83.0 \text{ ksi}(0.442 \text{ in.}^2)/2.00$ $= 18.3 \text{ kips/bolt} > 5.70 \text{ kips/bolt}$ **o.k.**

With L_e = 1¼ in. and s = 3 in., the bearing strength of the tee flange exceeds the single shear strength of the bolts. Therefore, bearing strength is o.k.

Prying Action (AISC Manual Part 9)

By inspection, prying action in the tee will control over prying action in the column.

Note: The bolts are not located symmetrically with respect to the centerline of the tee.

$$b = 2\tfrac{3}{4} \text{ in.} + \frac{0.380 \text{ in.}}{2}$$
$$= 2.94 \text{ in.}$$

$$a = \frac{8.02 \text{ in.}}{2} - 2\tfrac{3}{4} \text{ in.} - \frac{0.380 \text{ in.}}{2} - \frac{0.350 \text{ in.}}{2}$$
$$= 0.895 \text{ in.}$$

$$b' = b - \frac{d_b}{2} \qquad \text{(\textit{Manual} Eq. 9-21)}$$
$$= 2.94 \text{ in.} - \frac{\tfrac{3}{4} \text{ in.}}{2}$$
$$= 2.57 \text{ in.}$$

Since a = 0.895 in. is less than 1.25b = 3.68 in., use a = 0.895 in. for calculation purposes.

$$a' = a + \frac{d_b}{2} \qquad \text{(\textit{Manual} Eq. 9-27)}$$
$$= 0.895 \text{ in.} + \frac{\tfrac{3}{4} \text{ in.}}{2}$$
$$= 1.27 \text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad \text{(\textit{Manual} Eq. 9-26)}$$
$$= \frac{2.57 \text{ in.}}{1.27 \text{ in.}}$$
$$= 2.02$$

LRFD	ASD
$T_u = r_{ut} = 8.55 \text{ kips/bolt}$	$T_a = r_{at} = 5.70 \text{ kips/bolt}$
$B_u = \phi r_n = 27.5 \text{ kips/bolt}$	$B_a = r_n / \Omega = 18.3 \text{ kips/bolt}$
$\beta = \dfrac{1}{\rho}\left(\dfrac{B_u}{T_u} - 1\right)$ (*Manual* Eq. 9-25)	$\beta = \dfrac{1}{\rho}\left(\dfrac{B_a}{T_a} - 1\right)$ (*Manual* Eq. 9-25)

$= \dfrac{1}{2.02}\left(\dfrac{27.5 \text{ kips/bolt}}{8.55 \text{ kips/bolt}} - 1\right)$ $= 1.10$	$= \dfrac{1}{2.02}\left(\dfrac{18.3 \text{ kips/bolt}}{5.70 \text{ kips/bolt}} - 1\right)$ $= 1.09$

Since $\beta \geq 1$, set $\alpha' = 1.0$

$$p = 1\tfrac{1}{4} \text{ in.} + \frac{3.00 \text{ in.}}{2}$$
$$= 2.75 \text{ in.}$$
$$\leq s = 3.00 \text{ in.}$$

$$\delta = 1 - \frac{d'}{p}$$ (*Manual* Eq. 9-24)

$$= 1 - \frac{\tfrac{13}{16} \text{ in.}}{2.75 \text{ in.}}$$
$$= 0.705$$

LRFD	ASD
$\phi = 0.90$ $t_{min} = \sqrt{\dfrac{4T_u b'}{\phi p F_u (1+\delta\alpha')}}$ (*Manual* Eq. 9-23a) $= \sqrt{\dfrac{4(8.55 \text{ kips})(2.57 \text{ in.})}{0.90(2.75 \text{ in.})(65 \text{ ksi})\left[1+(0.705)(1.0)\right]}}$ $= 0.566 \text{ in.} < 0.620 \text{ in.}$ **o.k.**	$\Omega = 1.67$ $t_{min} = \sqrt{\dfrac{\Omega 4T_a b'}{p F_u (1+\delta\alpha')}}$ (*Manual* Eq. 9-23b) $= \sqrt{\dfrac{1.67(4)(5.70 \text{ kips})(2.57 \text{ in.})}{2.75 \text{ in.}(65 \text{ ksi})\left[1+(0.705)(1.0)\right]}}$ $= 0.567 \text{ in.} < 0.620 \text{ in.}$ **o.k.**

Similarly, checks of the tee flange for shear yielding, shear rupture, and block shear rupture will show that the tee flange is o.k.

Bolt Bearing on Beam Web

From AISC *Manual* Table 10-1, for four rows of ¾-in.-diameter bolts and an uncoped beam:

LRFD	ASD
$\phi R_n = 351 \text{ kips/in.}(0.380 \text{ in.})$ $= 133 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 234 \text{ kips/in.}(0.380 \text{ in.})$ $= 88.9 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Bolt Bearing on Column Flange

From AISC *Manual* Table 10-1, for four rows of ¾-in.-diameter bolts:

LRFD	ASD
$\phi R_n = 702 \text{ kips/in.}(0.710 \text{ in.})$ $= 498 \text{ kips} > 54.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 468 \text{ kips/in.}(0.710 \text{ in.})$ $= 332 \text{ kips} > 36.0 \text{ kips}$ **o.k.**

Note: Although the edge distance (a = 0.895 in.) for one row of bolts in the tee flange does not meet the minimum value indicated in AISC *Specification* Table J3.4, based on footnote [a], the edge distance provided is acceptable because the provisions of AISC *Specification* Section J3.10 and J4.4 have been met in this case.

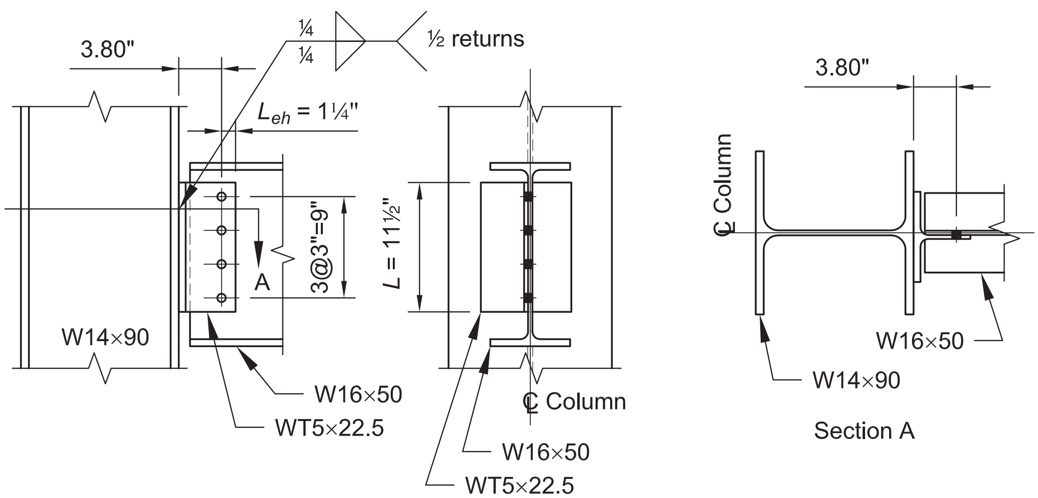
EXAMPLE II.A-31 BOLTED/WELDED TEE CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a tee connection bolted to an ASTM A992 W16×50 supported beam and welded to an ASTM A992 W14×90 supporting column flange, to support the following beam end reactions:

R_D = 6 kips
R_L = 18 kips

Use ¾-in.-diameter Group A bolts in standard holes and 70-ksi electrodes. Try an ASTM A992 WT5×22.5 with four bolts.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Beam
W16×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×90
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Tee
WT5×22.5
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

From AISC *Manual* Tables 1-1 and 1-8, the geometric properties are as follows:

Beam
W16×50
t_w = 0.380 in.
d = 16.3 in.
t_f = 0.630 in.

Column
W14×90
t_f = 0.710 in.

Tee
WT5×22.5
d = 5.05 in.
b_f = 8.02 in.
t_f = 0.620 in.
t_s = 0.350 in.
k_1 = 13⁄16 in. (see W10×45 AISC *Manual* Table 1-1)
k_{des} = 1.12 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(6.0 \text{ kips}) + 1.6(18 \text{ kips})$ $= 36.0 \text{ kips}$	$R_a = 6.0 \text{ kips} + 18 \text{ kips}$ $= 24.0 \text{ kips}$

Limitation on Tee Stem Thickness

See rotational ductility discussion at the beginning of the AISC *Manual* Part 9

$$t_{s\,max} = \frac{d}{2} + \tfrac{1}{16} \text{ in.} \qquad \text{(Manual Eq. 9-38)}$$

$$= \frac{\tfrac{3}{4} \text{ in.}}{2} + \tfrac{1}{16} \text{ in.}$$

$$= 0.438 \text{ in.} > 0.350 \text{ in.} \quad \textbf{o.k.}$$

Weld Design

b = flexible width in connection element

$$= \frac{b_f - 2k_1}{2}$$

$$= \frac{8.02 \text{ in.} - 2(\tfrac{13}{16} \text{ in.})}{2}$$

$$= 3.20 \text{ in.}$$

$$w_{min} = 0.0155\frac{F_y t_f^2}{b}\left(\frac{b^2}{L^2} + 2\right) \le \tfrac{5}{8} t_s \qquad \text{(Manual Eq. 9-36)}$$

$$= 0.0155\left[\frac{50 \text{ ksi}(0.620 \text{ in.})^2}{3.20 \text{ in.}}\right]\left[\left(\frac{(3.20 \text{ in.})^2}{(11\tfrac{1}{2} \text{ in.})^2}\right) + 2\right] \le \tfrac{5}{8}(0.350 \text{ in.})\,.$$

$$= 0.193 \text{ in.} \le 0.219 \text{ in.}$$

The minimum weld size is ¼ in. per AISC *Specification* Table J2.4.

Try ¼-in. fillet welds.

From AISC *Manual* Table 10-2, with n = 4, L = 11½, and weld B = ¼ in.:

LRFD	ASD
ϕR_n = 79.9 kips > 36.0 kips **o.k.**	$\dfrac{R_n}{\Omega}$ = 53.3 kips > 24.0 kips **o.k.**

Use ¼-in. fillet welds.

Supporting Column Flange

From AISC *Manual* Table 10-2, with n = 4, L = 11½, and weld B = ¼ in., the minimum support thickness is 0.190 in.

t_f = 0.710 in. > 0.190 in. **o.k.**

Stem Side of Connection

Since the connection is flexible at the support, the tee stem and bolts must be designed for eccentric shear, where the eccentricity, e_b, is determined as follows:

$$a = d - L_{eh}$$
$$= 5.05 \text{ in.} - 1.25 \text{ in.}$$
$$= 3.80 \text{ in.}$$

$$e_b = a = 3.80 \text{ in.}$$

LRFD	ASD
The tee stem and bolts must be designed for R_u = 36.0 kips and R_ue_b = 36.0 kips(3.80 in.) = 137 kip-in.	The tee stem and bolts must be designed for R_a = 24.0 kips and R_ae_b = 24.0 kips(3.80 in.) = 91.2 kip-in.
Bolt Shear and Bolt Bearing on Tee Stem	*Bolt Shear and Bolt Bearing on Tee Stem*
From AISC *Manual* Table 7-1, the single bolt shear strength is:	From AISC *Manual* Table 7-1, the single bolt shear strength is:
ϕR_n = 17.9 kips	$\dfrac{R_n}{\Omega}$ = 11.9 kips
From AISC *Manual* Table 7-5, the single bolt bearing strength with a 1¼-in. edge distance is:	From AISC *Manual* Table 7-5, the single bolt bearing strength with a 1¼-in. edge distance is:
ϕR_n = 49.4 kips/in.(0.350 in.) = 17.3 kips < 17.9 kips	$\dfrac{R_n}{\Omega}$ = 32.9 kips/in.(0.350 in.) = 11.5 kips < 11.9 kips
Bolt bearing controls.	Bolt bearing controls.
Note: By inspection, bolt bearing on the beam web does not control.	Note: By inspection, bolt bearing on the beam web does not control.
From AISC *Manual* Table 7-6 for θ = 0°, with s = 3 in., e_x = e_b = 3.80 in., and n = 4,	From AISC *Manual* Table 7-6 for θ = 0°, with s = 3 in.,

LRFD	ASD
$C = 2.45$	$e_x = e_b = 3.80$ in., and $n = 4$, $C = 2.45$
Since bolt bearing is more critical than bolt shear in this example, from AISC *Manual* Equation 7-19, $\phi R_n = C\phi r_n$ $= 2.45\text{ (17.3 kips/bolt)}$ $= 42.4\text{ kips} > 36.0\text{ kips}$ **o.k.**	Since bolt bearing is more critical than bolt shear in this example, from AISC *Manual* Equation 7-19, $\dfrac{R_n}{\Omega} = C\dfrac{r_n}{\Omega}$ $= 2.45\text{ (11.5 kips/bolt)}$ $= 28.2\text{ kips} > 24.0\text{ kips}$ **o.k.**

Flexural Yielding of Tee Stem

LRFD	ASD
$\phi = 0.90$ $\phi M_n = \phi F_y Z_x$ $= 0.90(50\text{ ksi})\left[\dfrac{0.350\text{ in.}(11\tfrac{1}{2}\text{ in.})^2}{4}\right]$ $= 521\text{ kip-in.} > 137\text{ kip-in.}$ **o.k.**	$\Omega = 1.67$ $\dfrac{M_n}{\Omega} = \dfrac{F_y Z_x}{\Omega}$ $= \dfrac{50\text{ ksi}}{1.67}\left[\dfrac{0.350\text{ in.}(11\tfrac{1}{2}\text{ in.})^2}{4}\right]$ $= 346\text{ kip-in.} > 91.2\text{ kip-in.}$ **o.k.**

Flexural Rupture of Tee Stem

$$Z_{net} = 0.350\text{ in.}\left[\frac{(11\tfrac{1}{2}\text{ in.})^2}{4} - 2(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(4.50\text{ in.}) - 2(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(1.50\text{ in.})\right]$$
$$= 7.90\text{ in.}^3$$

From AISC *Manual* Equation 9-4:

LRFD	ASD
$\phi = 0.75$ $\phi M_n = \phi F_u Z_{net}$ $= 0.75(65\text{ ksi})(7.90\text{ in.}^3)$ $= 385\text{ kip-in.} > 137\text{ kip-in.}$ **o.k.**	$\Omega = 2.00$ $\dfrac{M_n}{\Omega} = \dfrac{F_u Z_{net}}{\Omega}$ $= \dfrac{65\text{ ksi}(7.90\text{ in.}^3)}{2.00}$ $= 257\text{ kip-in.} > 91.2\text{ kip-in.}$ **o.k.**

Shear Yielding of Tee Stem

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(50\text{ ksi})(11\tfrac{1}{2}\text{ in.})(0.350\text{ in.})$ $= 121\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= 0.60(50\text{ ksi})(11\tfrac{1}{2}\text{ in.})(0.350\text{ in.})/1.50$ $= 80.5\text{ kips} > 24.0\text{ kips}$ **o.k.**

Shear Rupture of Tee Stem

$$A_{nv} = \left[11\tfrac{1}{2}\text{ in.} - 4(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](0.350\text{ in.})$$
$$= 2.80\text{ in.}^2$$

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(65\text{ ksi})(2.80\text{ in.}^2)$ $= 81.9\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(65\text{ ksi})(2.80\text{ in.}^2)}{2.00}$ $= 54.6\text{ kips} > 24.0\text{ kips}$ **o.k.**

Block Shear Rupture of Tee Stem

$$L_{eh} = L_{ev} = 1\tfrac{1}{4}\text{ in.}$$

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv})$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt} = 1.0(39.6\text{ kips/in.})(0.350\text{ in.})$ Shear yielding component from AISC *Manual* Table 9-3b: $\phi 0.60 F_y A_{gv} = 231\text{ kips/in.}(0.350\text{ in.})$ Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv} = 210\text{ kips/in.}(0.350\text{ in.})$ $\phi R_n = (39.6\text{ kips/in} + 210\text{ kips/in.})(0.350\text{in.})$ $= 87.4\text{ kips} > 36.0\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = 1.0(26.4\text{ kips/in.})(0.350\text{ in.})$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60 F_y A_{gv}}{\Omega} = 154\text{ kips/in.}(0.350\text{ in.})$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60 F_u A_{nv}}{\Omega} = 140\text{ kips/in.}(0.350\text{ in.})$ $\dfrac{R_n}{\Omega} = (26.4\text{ kips/in.} + 140\text{ kips/in.})(0.350\text{ in.})$ $= 58.2\text{ kips} > 24.0\text{ kips}$ **o.k.**

Chapter IIB
Fully Restrained (FR) Moment Connections

The design of fully restrained (FR) moment connections is covered in Part 12 of the AISC *Steel Construction Manual*.

Return to Table of Contents

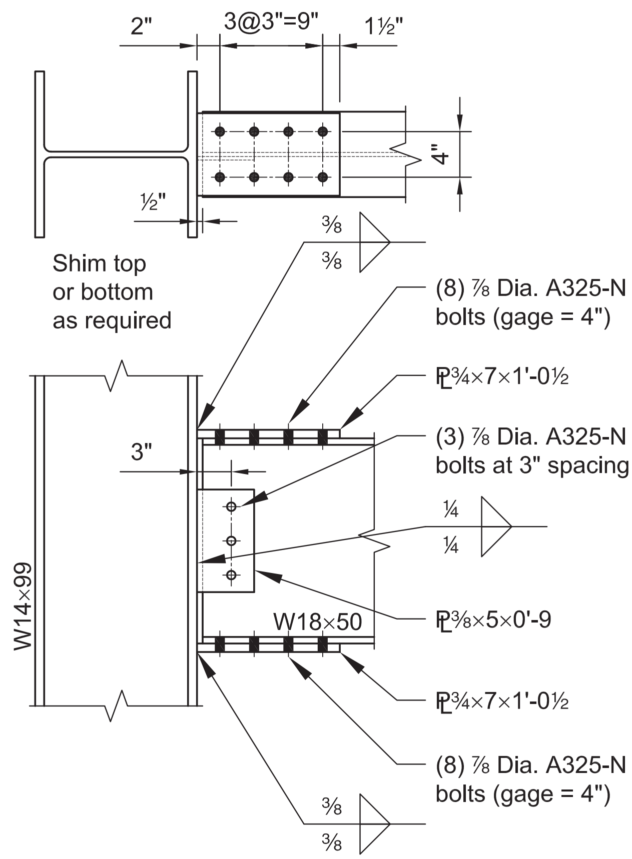
EXAMPLE II.B-1 BOLTED FLANGE-PLATED FR MOMENT CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a bolted flange-plated FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange to transfer the following vertical shear forces and strong-axis moments:

V_D = 7.0 kips M_D = 42 kip-ft
V_L = 21 kips M_L = 126 kip-ft

Use ⅞-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and 70-ksi electrodes. The flange plates are ASTM A36 material. Check the column for stiffening requirements.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
b_f = 7.50 in.
t_f = 0.570 in.
t_w = 0.355 in.
S_x = 88.9 in.^{3}

Column
W14×99
d = 14.2 in.
b_f = 14.6 in.
t_f = 0.780 in.
t_w = 0.485 in.
k_{des} = 1.38 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(7.0 \text{ kips}) + 1.6(21 \text{ kips})$ $= 42.0 \text{ kips}$ $M_u = 1.2(42 \text{ kip-ft}) + 1.6(126 \text{ kip-ft})$ $= 252 \text{ kip-ft}$	$R_a = 7.0 \text{ kips} + 21 \text{ kips}$ $= 28.0 \text{ kips}$ $M_a = 42 \text{ kip-ft} + 126 \text{ kip-ft}$ $= 168 \text{ kip-ft}$

Flexural Strength of Beam (using AISC Specification Section F13.1)

Use two rows of bolts in standard holes.

$$\begin{aligned} A_{fg} &= b_f t_f \\ &= 7.50 \text{ in.}(0.570 \text{ in.}) \\ &= 4.28 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} A_{fn} &= A_{fg} - 2(d_h + \tfrac{1}{16} \text{ in.})t_f \\ &= 4.28 \text{ in.}^2 - 2(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(0.570 \text{ in.}) \\ &= 3.14 \text{ in.}^2 \end{aligned}$$

$$\frac{F_y}{F_u} = \frac{50 \text{ ksi}}{65 \text{ ksi}}$$
$$= 0.769 \leq 0.8, \text{ therefore } Y_t = 1.0.$$

$$F_u A_{fn} = 65 \text{ ksi}\left(3.14 \text{ in.}^2\right)$$
$$= 204 \text{ kips}$$

$$Y_t F_y A_{fg} = 1.0\left(50 \text{ ksi}\right)\left(4.28 \text{ in.}^2\right)$$
$$= 214 \text{ kips} > 204 \text{ kips}$$

Therefore the nominal flexural strength, M_n, at the location of the holes in the tension flange is not greater than:

$$M_n = \frac{F_u A_{fn}}{A_{fg}} S_x \qquad \text{(\textit{Spec}. Eq. F13-1)}$$
$$= \frac{65 \text{ ksi}\left(3.14 \text{ in.}^2\right)}{4.28 \text{ in.}^2}\left(88.9 \text{ in.}^3\right)$$
$$= 4{,}240 \text{ kip-in. or } 353 \text{ kip-ft}$$

LRFD	ASD
$\phi_b = 0.90$ $\phi_b M_n = 0.90\left(353 \text{ kip-ft}\right)$ $= 318 \text{ kip-ft} > 252 \text{ kip-ft}$ **o.k.**	$\Omega_b = 1.67$ $\frac{M_n}{\Omega_b} = \frac{353 \text{ kip-ft}}{1.67}$ $= 211 \text{ kip-ft} > 168 \text{ kip-ft}$ **o.k.**

Note: The available flexural strength of the beam may be less than that determined based on AISC *Specification* Equation F13-1. Other applicable provisions in AISC *Specification* Section F should be checked to possibly determine a lower value for the available flexural strength of the beam.

Single-Plate Web Connection

Try a PL⅜×5×0′-9″, with three ⅞-in.-diameter ASTM A325-N bolts and ¼-in. fillet welds.

LRFD	ASD
Shear strength of bolts from AISC *Manual* Table 7-1: $\phi r_n = 24.3 \text{ kips/bolt}$ Bearing strength of bolts: Bearing on the plate controls over bearing on the beam web. Vertical edge distance = 1.50 in. $l_c = 1.50 \text{ in.} - \frac{15/16 \text{ in.}}{2}$ $= 1.03 \text{ in.}$	Shear strength of bolts from AISC *Manual* Table 7-1: $r_n/\Omega = 16.2 \text{ kips/bolt}$ Bearing strength of bolts: Bearing on the plate controls over bearing on the beam web. Vertical edge distance = 1.50 in. $l_c = 1.50 \text{ in.} - \frac{15/16 \text{ in.}}{2}$ $= 1.03 \text{ in.}$

From AISC *Specification* Equation J-36a:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = \phi 1.2 l_c t F_u \le \phi 2.4 d t F_u$ $0.75(1.2)(1.03\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})$ $\le 0.75(2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})$ $20.2\text{ kips} \le 34.3\text{ kips}$ $\phi r_n = 20.2\text{ kips/bolt}$ From AISC *Manual* Table 7-4 with s = 3 in., $\phi r_n = 91.4\text{ kips/in./bolt}(\tfrac{3}{8}\text{ in.})$ $= 34.3\text{ kips/bolt}$ Bolt bearing strength at the top bolt controls. Determine the coefficient for the eccentrically loaded bolt group from AISC *Manual* Table 7-6. $C_{min} = \dfrac{R_u}{\phi r_n}$ $= \dfrac{42.0\text{ kips}}{20.2\text{ kips}}$ $= 2.08$ Using e = 3.00 in./2 = 1.50 in. and s = 3.00 in., $C = 2.23 > 2.08$ **o.k.** Plate shear yielding, from AISC *Specification* Equation J4-3: $\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(9.00\text{ in.})(\tfrac{3}{8}\text{ in.})$ $= 72.9\text{ kips} > 42.0\text{ kips}$ **o.k.** Plate shear rupture, from AISC *Specification* Equation J4-4: Total length of bolt holes:	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega} \le \dfrac{2.4 d t F_u}{\Omega}$ $\dfrac{1.2(1.03\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $\le \dfrac{2.4(\tfrac{7}{8}\text{ in.})(\tfrac{3}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $13.4\text{ kips} \le 22.8\text{ kips}$ $r_n/\Omega = 13.4\text{ kips/bolt}$ From AISC *Manual* Table 7-4 with s = 3 in., $r_n/\Omega = 60.9\text{ kips/in./bolt}(\tfrac{3}{8}\text{ in.})$ $= 22.8\text{ kips/bolt}$ Bolt bearing strength at the top bolt controls. Determine the coefficient for the eccentrically loaded bolt group from AISC *Manual* Table 7-6. $C_{min} = \dfrac{R_a}{r_n/\Omega}$ $= \dfrac{28.0\text{ kips}}{13.4\text{ kips}}$ $= 2.09$ Using e = 3.00 in./2 = 1.50 in. and s = 3.00 in., $C = 2.23 > 2.09$ **o.k.** Plate shear yielding, from AISC *Specification* Equation J4-3: $\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(9.00\text{ in.})(\tfrac{3}{8}\text{ in.})}{1.50}$ $= 48.6\text{ kips} > 28.0\text{ kips}$ **o.k.** Plate shear rupture, from AISC *Specification* Equation J4-4: Total length of bolt holes:

LRFD	ASD
3 bolts(15⁄16 in. + 1⁄16 in.) = 3.00 in. A_{nv} = 3⁄8 in.(9.00 in. – 3.00 in.) = 2.25 in.^{2} $\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ = 0.75(0.60)(58 ksi)(2.25 in.^{2}) = 58.7 kips > 42.0 kips **o.k.**	3 bolts(15⁄16 in. + 1⁄16 in.) = 3.00 in. A_{nv} = 3⁄8 in.(9.00 in. – 3.00 in.) = 2.25 in.^{2} $\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58\text{ ksi})(2.25\text{ in.}^2)}{2.00}$ = 39.2 kips > 28.0 kips **o.k.**

Block Shear Rupture Strength of the Web Plate (using AISC Specification Equation J4-5)

L_{eh} = 2 in.; L_{ev} = 1½ in.; U_{bs} = 1.0; n = 3

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv})$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt}$ = 1.0(65.3 kips/in.)(3⁄8 in.) Shear yielding component from AISC *Manual* Table 9-3b: $\phi 0.60 F_y A_{gv}$ = 121 kips/in.(3⁄8 in.) Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv}$ = 131 kips/in.(3⁄8 in.) ϕR_n = (65.3 kips/in. + 121 kips/in.)(3⁄8 in.) = 69.9 kips > 42.0 kips **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $U_{bs} F_u A_{nt}/\Omega$ = 1.0(43.5 kips/in.)(3⁄8 in.) Shear yielding component from AISC *Manual* Table 9-3b: $0.60 F_y A_{gv}/\Omega$ = 81.0 kips/in.(3⁄8 in.) Shear rupture component from AISC *Manual* Table 9-3c: $0.60 F_u A_{nv}/\Omega$ = 87.0 kips/in.(3⁄8 in.) $\dfrac{R_n}{\Omega}$ = (43.5 kips/in . + 81.0 kips/in.)(3⁄8 in.) = 46.7 kips > 28.0 kips **o.k.**

Web Plate to Column Flange Weld Shear Strength (using AISC Manual Part 8)

From AISC *Manual* Equation 8-2:

LRFD	ASD
$\phi R_n = 1.392 Dl(2)$ = 1.392(4 sixteenths)(9.00 in.)(2) = 100 kips > 42.0 kips **o.k.**	$\dfrac{R_n}{\Omega} = 0.928 Dl(2)$ = 0.928(4 sixteenths)(9.00 in.)(2) = 66.8 kips > 28.0 kips **o.k.**

Note: By inspection, the available shear yielding, shear rupture and block shear rupture strengths of the beam web are o.k.

Web Plate Rupture Strength at Welds (using AISC Manual Part 9)

$$t_{min} = \frac{0.6F_{EXX}\left(\frac{\sqrt{2}}{2}\right)\left(\frac{D}{16}\right)}{0.6F_u}$$

$$= \frac{3.09D}{F_u} \text{ for } F_{exx} = 70.0 \text{ ksi} \qquad (\textit{Manual} \text{ Eq. 9-2})$$

$$= \frac{(3.09)(4 \text{ sixteenths})}{65 \text{ ksi}} = 0.190 \text{ in.} < 0.780 \text{ in. column flange} \qquad \textbf{o.k.}$$

Tension Flange Plate and Connection

LRFD	ASD
$P_{uf} = \frac{M_u}{d} = \frac{252 \text{ kip-ft}(12 \text{ in./ft})}{18.0 \text{ in.}}$ $= 168$ kips Try a PL ¾×7. Determine critical bolt strength. Bolt shear using AISC *Manual* Table 7-1, $\phi r_n = 24.3$ kips/bolt Bearing on flange using AISC *Manual* Table 7-5, Edge distance = 1½ in. (Use 1¼ in. to account for possible underrun in beam length.) $\phi r_n = 45.7 \text{ kips/bolt/in.}(t_f)$ $= 45.7 \text{ kips/bolt/in.}(0.570 \text{ in.})$ $= 26.0$ kips/bolt Bearing on plate using AISC *Manual* Table 7-5, Edge distance = 1½ in. (Conservatively, use 1¼ in. value from table.) $\phi r_n = 40.8 \text{ kips/bolt/in.}(t_p)$ $= 40.8 \text{ kips/bolt/in.}(3/4 \text{ in.})$ $= 30.6$ kips/bolt Bolt shear controls, therefore the number of bolts required is as follows:	$P_{af} = \frac{M_a}{d} = \frac{168 \text{ kip-ft}(12 \text{ in./ft})}{18.0 \text{ in.}}$ $= 112$ kips Try a PL ¾×7. Determine critical bolt strength. Bolt shear using AISC *Manual* Table 7-1, $r_n / \Omega = 16.2$ kips/bolt Bearing on flange using AISC *Manual* Table 7-5, Edge distance = 1½ in. (Use 1¼ in. to account for possible underrun in beam length.) $r_n / \Omega = 30.5 \text{ kips/bolt/in.}(t_f)$ $= 30.5 \text{ kips/bolt/in.}(0.570 \text{ in.})$ $= 17.4$ kips/bolt Bearing on plate using AISC *Manual* Table 7-5, Edge distance = 1½ in. (Conservatively, use 1¼ in. value from table.) $r_n / \Omega = 27.2 \text{ kips/bolt/in.}(t_p)$ $= 27.2 \text{ kips/bolt/in.}(3/4 \text{ in.})$ $= 20.4$ kips/bolt Bolt shear controls, therefore the number of bolts required is as follows:

$n_{min} = \dfrac{P_{uf}}{\phi r_n} = \dfrac{168 \text{ kips}}{24.3 \text{ kips/bolt}}$ $= 6.91$ bolts **Use 8 bolts.**	$n_{min} = \dfrac{P_{af}}{r_n / \Omega} = \dfrac{112 \text{ kips}}{16.2 \text{ kips/bolt}}$ $= 6.91$ bolts **Use 8 bolts.**

Flange Plate Tensile Yielding

$$R_n = F_y A_g \qquad \text{(\textit{Spec}. Eq. J4-1)}$$
$$= 36 \text{ ksi}(7.00 \text{ in.})(\tfrac{3}{4} \text{ in.})$$
$$= 189 \text{ kips}$$

LRFD	ASD
$P_{uf} = \dfrac{M_u}{d + t_p} = \dfrac{252 \text{ kip-ft}(12 \text{ in./ft})}{(18.0 \text{ in.} + \tfrac{3}{4} \text{ in.})}$ $= 161$ kips $\phi = 0.90$ $\phi R_n = 0.90(189 \text{ kips})$ $= 170 \text{ kips} > 161 \text{ kips}$ **o.k.**	$P_{af} = \dfrac{M_a}{d + t_p} = \dfrac{168 \text{ kip-ft}(12 \text{ in./ft})}{(18.0 \text{ in.} + \tfrac{3}{4} \text{ in.})}$ $= 108$ kips $\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{189 \text{ kips}}{1.67}$ $= 113 \text{ kips} > 108 \text{ kips}$ **o.k.**

Flange Plate Tensile Rupture

$A_e = A_n$ from AISC *Specification* Section J4.1(b)

$$A_n = \left[B - 2(d_h + \tfrac{1}{16} \text{ in.})\right] t_p$$
$$= \left[(7.00 \text{ in.}) - 2(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{3}{4} \text{ in.})$$
$$= 3.75 \text{ in.}^2$$

$$A_e = 3.75 \text{ in.}^2$$

$$R_n = F_u A_e \qquad \text{(\textit{Spec}. Eq. J4-2)}$$
$$= 58 \text{ ksi}(3.75 \text{ in.}^2)$$
$$= 218 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(218 \text{ kips})$ $= 164 \text{ kips} > 161 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{218 \text{ kips}}{2.00}$ $= 109 \text{ kips} > 108 \text{ kips}$ **o.k.**

Flange Plate Block Shear Rupture

There are three cases for which block shear rupture must be checked (see Figure IIB-1). The first case involves the tearout of the two blocks outside the two rows of bolt holes in the flange plate; for this case L_{eh} = 1½ in. and L_{ev} = 1½ in. The second case involves the tearout of the block between the two rows of the holes in the flange plate. AISC *Manual* Tables 9-3a, 9-3b, and 9-3c may be adapted for this calculation by considering the 4 in. width to be comprised of two, 2 in. wide blocks where L_{eh} = 2 in. and L_{ev} = 1½ in. The first case is more critical than the second case because L_{eh} is smaller. The third case involves a shear failure through one row of bolts and a tensile failure through the two bolts closest to the column.

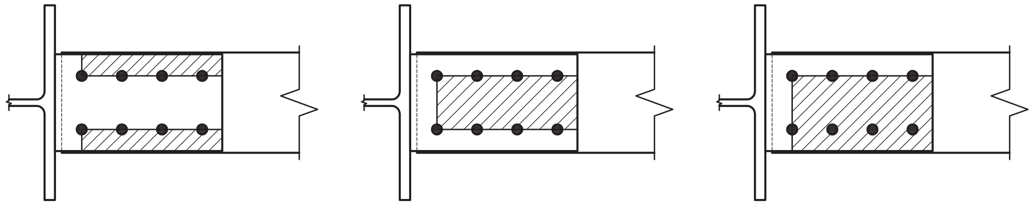


Fig. IIB-1. Three cases for block shear rupture.

LRFD	ASD
Case 1: From AISC *Specification* Equation J4-5: $\phi R_n = \phi U_{bs} F_u A_{nt} + \min\left(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv}\right)$ $U_{bs} = 1.0$, $L_{ev} = 1\frac{1}{2}$ in. Tension component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt} = 1.0(43.5 \text{ kips/in.})(\frac{3}{4} \text{ in.})(2)$ Shear yielding component from AISC *Manual* Table 9-3b: $\phi 0.60 F_y A_{gv} = 170 \text{ kips/in.}(\frac{3}{4} \text{ in.})(2)$ Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv} = 183 \text{ kips/in.}(\frac{3}{4} \text{ in.})(2)$ Shear yielding controls, thus, $\phi R_n = \left(\dfrac{43.5 \text{ kips}}{\text{in.}} + \dfrac{170 \text{ kips}}{\text{in.}}\right)\left(\frac{3}{4} \text{ in.}\right)(2)$ $= 320 \text{ kips} > 161 \text{ kips}$ **o.k.**	Case 1: From AISC *Specification* Equation J4-5: $\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $U_{bs} = 1.0$, $L_{ev} = 1\frac{1}{2}$ in. Tension component from AISC *Manual* Table 9-3a: $U_{bs} F_u A_{nt} / \Omega = 1.0(29.0 \text{ kips/in.})(\frac{3}{4} \text{ in.})(2)$ Shear yielding component from AISC *Manual* Table 9-3b: $0.60 F_y A_{gv} / \Omega = 113 \text{ kips/in.}(\frac{3}{4} \text{ in.})(2)$ Shear rupture component from AISC *Manual* Table 9-3c: $0.60 F_u A_{nv} / \Omega = 122 \text{ kips/in.}(\frac{3}{4} \text{ in.})(2)$ Shear yielding controls, thus, $\dfrac{R_n}{\Omega} = \left(\dfrac{29.0 \text{ kips}}{\text{in.}} + \dfrac{113 \text{ kips}}{\text{in.}}\right)\left(\frac{3}{4} \text{ in.}\right)(2)$ $= 213 \text{ kips} > 108 \text{ kips}$ **o.k.**

LRFD	ASD
Case 3: From AISC *Specification* Equation J4-5: $\phi R_n = \phi U_{bs} F_u A_{nt} + \min\left(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv}\right)$ $U_{bs} = 1.0$ Tension component: $A_{nt} = [5.50 \text{ in.} - 1.5(^{15}/_{16} \text{ in.} + ^{1}/_{16} \text{ in.})](^{3}/_{4} \text{ in.})$ $= 3.00 \text{ in.}^2$ $\phi U_{bs} F_u A_{nt} = 0.75(1.0)(58 \text{ ksi})(3.00 \text{ in.}^2)$ $= 131 \text{ kips}$ Shear yielding component from AISC *Manual* Table 9-3b with L_{ev} = 1½ in.: $\phi 0.60 F_y A_{gv} = 170 \text{ kips/in.}(^{3}/_{4} \text{ in.})$ Shear rupture component from AISC *Manual* Table 9-3c with L_{ev} = 1½ in.: $\phi 0.60 F_u A_{nv} = 183 \text{ kips/in.}(^{3}/_{4} \text{ in.})$ Shear yielding controls, thus, $\phi R_n = 131 \text{ kips} + 170 \text{ kips/in.}(^{3}/_{4} \text{ in.})$ $= 259 \text{ kips} > 161 \text{ kips}$ **o.k.**	Case 3: From AISC *Specification* Equation J4-5: $\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $U_{bs} = 1.0$ Tension component: $A_{nt} = [5.50 \text{ in.} - 1.5(^{15}/_{16} \text{ in.} + ^{1}/_{16} \text{ in.})](^{3}/_{4} \text{ in.})$ $= 3.00 \text{ in.}^2$ $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = \dfrac{1.0(58 \text{ ksi})(3.00 \text{ in.}^2)}{2.00}$ $= 87.0 \text{ kips}$ Shear yielding component from AISC *Manual* Table 9-3b with L_{ev} = 1½ in.: $0.60 F_y A_{gv} / \Omega = 113 \text{ kips/in.}(^{3}/_{4} \text{ in.})$ Shear rupture component from AISC *Manual* Table 9-3c with L_{ev} = 1½ in.: $0.60 F_u A_{nv} / \Omega = 122 \text{ kips/in.}(^{3}/_{4} \text{ in.})$ Shear yielding controls, thus, $\dfrac{R_n}{\Omega} = 87.0 \text{ kips} + 113 \text{ kips/in.}(^{3}/_{4} \text{ in.})$ $= 172 \text{ kips} > 108 \text{ kips}$ **o.k.**

Block Shear Rupture of the Beam Flange

This case involves the tearout of the two blocks outside the two rows of bolt holes in the flanges; for this case L_{eh} = 1¾ in. and L_{ev} = 1½ in. (Use 1¼ in. to account for possible underrun in beam length.)

LRFD	ASD
From AISC *Specification* Equation J4-5: $\phi R_n = \phi U_{bs} F_u A_{nt} + \min\left(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv}\right)$ $U_{bs} = 1.0$ Tension component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt} = 1.0(60.9 \text{ kips/in.})(0.570 \text{ in.})(2)$	From AISC *Specification* Equation J4-5: $\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $U_{bs} = 1.0$ Tension component from AISC *Manual* Table 9-3a: $U_{bs} F_u A_{nt} / \Omega = 1.0(40.6 \text{ kips/in.})(0.570 \text{ in.})(2)$

LRFD	ASD
Shear yielding component from AISC *Manual* Table 9-3b:	Shear yielding component from AISC *Manual* Table 9-3b:
$\phi 0.6F_y A_{gv} = (231\text{ kips/in.})(0.570\text{ in.})(2)$	$0.6F_y A_{gv} / \Omega = (154\text{ kips/in.})(0.570\text{ in.})(2)$
Shear rupture component from AISC *Manual* Table 9-3c:	Shear rupture component from AISC *Manual* Table 9-3c:
$\phi 0.6F_u A_{nv} = (197\text{ kips/in.})(0.570\text{ in.})(2)$	$0.6F_u A_{nv} / \Omega = (132\text{ kips/in.})(0.570\text{ in.})(2)$
Shear rupture controls, thus,	Shear rupture controls, thus,
$\phi R_n = \left(\frac{60.9\text{ kips}}{\text{in.}} + \frac{197\text{ kips}}{\text{in.}}\right)(0.570\text{ in.})(2)$	$\frac{R_n}{\Omega} = \left(\frac{40.6\text{ kips}}{\text{in.}} + \frac{132\text{ kips}}{\text{in.}}\right)(0.570\text{ in.})(2)$
$= 294\text{ kips} > 168\text{ kips}$ **o.k.**	$= 197\text{ kips} > 112\text{ kips}$ **o.k.**

Fillet Weld to Supporting Column Flange

The applied load is perpendicular to the weld length; therefore $\theta = 90°$ and $1.0 + 0.50\sin^{1.5}\theta = 1.5$.

From AISC *Manual* Equation 8-2:

LRFD	ASD
$D_{min} = \frac{P_{uf}}{2(1.5)(1.392)l}$	$D_{min} = \frac{P_{af}}{2(1.5)(0.928)l}$
$= \frac{161\text{ kips}}{2(1.5)(1.392)(7.00\text{ in.})}$	$= \frac{108\text{ kips}}{2(1.5)(0.928)(7.00\text{ in.})}$
$= 5.51\text{ sixteenths}$	$= 5.54\text{ sixteenths}$
Use ⅜-in. fillet welds, 6 > 5.51 **o.k.**	Use ⅜-in. fillet welds, 6 > 5.54 **o.k.**

Connecting Elements Rupture Strength at Welds (using AISC Manual Part 9)

$$t_{min} = \frac{3.09D}{F_u} \text{ for } F_{EXX} = 70\text{ ksi} \qquad \text{(\textit{Manual} Eq. 9-2)}$$

$$= \frac{3.09(5.54\text{ sixteenths})}{65\text{ ksi}}$$

$$= 0.263\text{ in.} < 0.780\text{ in. column flange} \qquad \textbf{o.k.}$$

Compression Flange Plate and Connection

Try PL ¾×7.

K = 0.65 from AISC *Specification Commentary* Table C-A-7.1
L = 2.00 in. (1½ in. edge distance and ½ in. setback)

$$\frac{KL}{r} = \frac{0.65(2.00\text{ in.})}{\left(\dfrac{3/4\text{ in.}}{\sqrt{12}}\right)}$$
$$= 6.00 \le 25$$

Therefore, $F_{cr} = F_y$ from AISC *Specification* Section J4.4.

$$A_g = 7.00\text{ in.}(3/4\text{ in.})$$
$$= 5.25\text{ in.}^2$$

From AISC *Specification* Equation J4-6:

LRFD	ASD
$\phi = 0.90$ $\phi P_n = \phi F_y A_g = 0.90(36\text{ ksi})(5.25\text{ in.}^2)$ $= 0.90\ (36\text{ ksi})(5.25\text{ in.}^2)$ $= 170\text{ kips} > 161\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{P_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{36\text{ ksi}(5.25\text{ in.}^2)}{1.67}$ $= 113\text{ kips} > 108\text{ kips}$ **o.k.**

The compression flange plate will be identical to the tension flange plate; a ¾-in.×7-in. plate with eight bolts in two rows of four bolts on a 4 in. gage and ⅜-in. fillet welds to the supporting column flange.

Note: The bolt bearing and shear checks are the same as for the tension flange plate and are o.k. by inspection. Tension due to load reversal must also be considered in the design of the fillet weld to the supporting column flange.

Flange Local Bending of Column (AISC Specification Section J10.1)

Assume the concentrated force to be resisted is applied at a distance from the member end that is greater than $10t_f$.

$$10t_f = 10(0.780\text{ in.})$$
$$= 7.80\text{ in.}$$

LRFD	ASD
$R_n = 6.25 F_{yf} t_f^{\,2}$ (*Spec*. Eq. J10-1) $= 6.25(50\text{ ksi})(0.780\text{ in.})^2$ $= 190\text{ kips}$ $\phi = 0.90$ $\phi R_n = 0.90(190\text{ kips})$ $= 171\text{ kips} > 161\text{ kips}$ **o.k.**	$R_n = 6.25 F_{yf} t_f^{\,2}$ (*Spec*. Eq. J10-1) $= 6.25(50\text{ ksi})(0.780\text{ in.})^2$ $= 190\text{ kips}$ $\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{190\text{ kips}}{1.67}$ $= 114\text{ kips} > 108\text{ kips}$ **o.k.**

Web Local Yielding of Column (AISC Specification Section J10.2)

Assume the concentrated force to be resisted is applied at a distance from the member that is greater than the depth of the member, *d*.

From AISC *Manual* Table 9-4:

LRFD	ASD
$\phi R_n = 2(\phi R_1) + l_b(\phi R_2)$ (*Manual* Eq. 9-46a) $= 2(83.7\text{ kips}) + 0.750\text{ in.}(24.3\text{ kips/in.})$ $= 186\text{ kips} > 161\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 2\left(\dfrac{R_1}{\Omega}\right) + l_b\left(\dfrac{R_2}{\Omega}\right)$ (*Manual* Eq. 9-46b) $= 2(55.8\text{ kips}) + 0.750\text{ in.}(16.2\text{ kips/in.})$ $= 124\text{ kips} > 108\text{ kips}$ **o.k.**

Web Crippling (AISC Specification Section J10.3)

Assume the concentrated force to be resisted is applied at a distance from the member end that is greater than or equal to $d/2$.

From AISC *Manual* Table 9-4:

LRFD	ASD
$\phi R_n = 2\left[(\phi R_3) + l_b(\phi R_4)\right]$ (*Manual* Eq. 9-49a) $= 2\left[(108\text{ kips}) + 0.750\text{ in.}(11.2\text{ kips/in.})\right]$ $= 233\text{ kips} > 161\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 2\left[\left(\dfrac{R_3}{\Omega}\right) + l_b\left(\dfrac{R_4}{\Omega}\right)\right]$ (*Manual* Eq. 9-49b) $= 2\left[(71.8\text{ kips}) + 0.750\text{ in.}(7.44\text{ kips/in.})\right]$ $= 155\text{ kips} > 108\text{ kips}$ **o.k.**

Note: Web compression buckling (AISC *Specification* Section J10.5) must be checked if another beam is framed into the opposite side of the column at this location.

Web panel zone shear (AISC *Specification* Section J10.6) should also be checked for this column.

For further information, see AISC Design Guide 13 *Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications* (Carter, 1999).

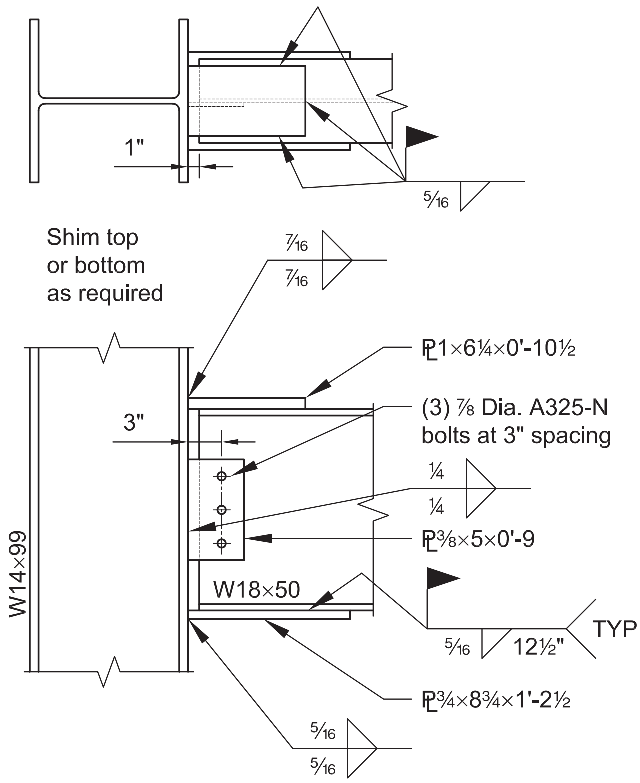
EXAMPLE II.B-2 WELDED FLANGE-PLATED FR MOMENT CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a welded flange-plated FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange to transfer the following vertical shear forces and strong-axis moments:

V_D = 7.0 kips M_D = 42 kip-ft
V_L = 21 kips M_L = 126 kip-ft

Use ⅞-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and 70-ksi electrodes. The flange plates are A36 material. Check the column for stiffening requirements.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×99
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
b_f = 7.50 in.
t_f = 0.570 in.
t_w = 0.355 in.
Z_x = 101 in.^{3}

Column
W14×99
d = 14.2 in.
b_f = 14.6 in.
t_f = 0.780 in.
t_w = 0.485 in.
k_{des} = 1.38 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(7.0 kips) + 1.6(21 kips) = 42.0 kips M_u = 1.2(42 kip-ft) + 1.6(126 kip-ft) = 252 kip-ft	R_a = 7.0 kips + 21 kips = 28.0 kips M_a = 42 kip-ft + 126 kip-ft = 168 kip-ft

The single-plate web connection is verified in Example II.B-1.

Note: By inspection, the available shear yielding, shear rupture and block shear rupture strengths of the beam web are o.k.

Tension Flange Plate and Connection

Determine the flange force using AISC *Manual* Part 12.

The top flange width, b_f = 7.50 in. Assume a shelf dimension of ⅝ in. on both sides of the plate. The plate width, then, is 7.50 in. – 2(⅝ in.) = 6.25 in. Try a 1 in. × 6 ¼ in. flange plate. Assume a ¾-in. bottom flange plate.

From AISC *Manual* Equation 12-1:

LRFD	ASD
$P_{uf} = \dfrac{M_u}{d + t_p}$ $= \dfrac{252 \text{ kip-ft}(12 \text{ in./ft})}{18.0 \text{ in.} + 0.875 \text{ in.}}$ $= 160 \text{ kips}$	$P_{af} = \dfrac{M_a}{d + t_p}$ $= \dfrac{168 \text{ kip-ft}(12 \text{ in./ft})}{18.0 \text{ in.} + 0.875 \text{ in.}}$ $= 107 \text{ kips}$

Flange Plate Tensile Yielding

$$R_n = F_y A_g \qquad \text{(Spec. Eq. J4-1)}$$
$$= 36 \text{ ksi}(6.25 \text{ in.})(1.00 \text{ in.})$$
$$= 225 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(225 \text{ kips})$ $= 203 \text{ kips}$ 203 kips > 160 kips **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{225 \text{ kips}}{1.67}$ $= 135 \text{ kips}$ 135 kips > 107 kips **o.k.**

Determine the force in the welds.

LRFD	ASD
$P_{uf} = \dfrac{M_u}{d}$ $= \dfrac{252 \text{ kip-ft}(12 \text{ in./ft})}{18.0 \text{ in.}}$ $= 168 \text{ kips}$	$P_{af} = \dfrac{M_a}{d}$ $= \dfrac{168 \text{ kip-ft}(12 \text{ in./ft})}{18.0 \text{ in.}}$ $= 112 \text{ kips}$

Required Weld Size and Length for Fillet Welds to Beam Flange (using AISC Manual Part 8)

Try a 5⁄16-in. fillet weld. The minimum length of weld, l_{min}, is determined as follows.

For weld compatibility, disregard the increased capacity due to perpendicular loading of the end weld (see Part 8 discussion under "Effect of Load Angle").

From AISC *Manual* Equation 8-2:

LRFD	ASD
$l_{min} = \dfrac{P_{uf}}{1.392D}$ $= \dfrac{168 \text{ kips}}{1.392(5)}$ $= 24.1 \text{ in.}$ Use 9 in. of weld along each side and 6 ¼ in. of weld along the end of the flange plate.	$l_{min} = \dfrac{P_{af}}{0.928D}$ $= \dfrac{112 \text{ kips}}{0.928(5)}$ $= 24.1 \text{ in.}$ Use 9 in. of weld along each side and 6¼ in. of weld along the end of the flange plate.

$l = 2(9.00 \text{ in.}) + 6.25 \text{ in.}$ $= 24.3 \text{ in.} > 24.1 \text{ in.}$ **o.k.**	$l = 2(9.00 \text{ in.}) + 6.25 \text{ in.}$ $= 24.3 \text{ in.} > 24.1 \text{ in.}$ **o.k.**

Connecting Elements Rupture Strength at Welds (Top Flange)

$$t_{min} = \frac{3.09D}{F_u} \quad \text{for } F_{EXX} = 70 \text{ ksi} \qquad (\textit{Manual} \text{ Eq. 9-2})$$
$$= \frac{3.09(5 \text{ sixteenths})}{65 \text{ ksi}}$$
$$= 0.238 \text{ in.} < 0.570 \text{ in. beam flange} \quad \textbf{o.k.}$$

$$t_{min} = \frac{3.09D}{F_u} \qquad (\textit{Manual} \text{ Eq. 9-2})$$
$$= \frac{3.09(5 \text{ sixteenths})}{58 \text{ ksi}}$$
$$= 0.266 \text{ in.} < 1.00 \text{ in. top flange plate} \quad \textbf{o.k.}$$

Required Fillet Weld Size at Top Flange Plate to Column Flange (using AISC Manual Part 8)

The applied tensile load is perpendicular to the weld, therefore,

$\theta = 90°$ and $1.0 + 0.50\sin^{1.5}\theta = 1.5$.

From AISC *Manual* Equation 8-2:

LRFD	ASD
$D_{min} = \frac{P_{uf}}{2(1.5)(1.392)l}$ $= \frac{160 \text{ kips}}{2(1.5)(1.392)(6.25 \text{ in.})}$ $= 6.13 \text{ sixteenths}$ Use 7⁄16-in. fillet welds, 7 > 6.13 **o.k.**	$D_{min} = \frac{P_{af}}{2(1.5)(0.928)l}$ $= \frac{107 \text{ kips}}{2(1.5)(0.928)(6.25 \text{ in.})}$ $= 6.15 \text{ sixteenths}$ Use 7⁄16-in. fillet welds, 7 > 6.15 **o.k.**

Connecting Elements Rupture Strength at Welds

$$t_{min} = \frac{3.09D}{F_u} \quad \text{for } F_{EXX} = 70 \text{ ksi} \qquad (\textit{Manual} \text{ Eq. 9-2})$$
$$= \frac{3.09(6.15 \text{ sixteenths})}{65 \text{ ksi}}$$
$$= 0.292 \text{ in.} < 0.780 \text{ in. column flange} \quad \textbf{o.k.}$$

Compression Flange Plate and Connection

Assume a shelf dimension of ⅝ in. The plate width, then, is 7.50 in. + 2(⅝ in.) = 8.75 in.

Try a ¾ in. × 8¾ in. compression flange plate.

Assume K = 0.65 from AISC *Specification* Commentary Table C-A-7.1, and L = 1.00 in (1-in. setback).

$$\frac{KL}{r} = \frac{0.65(1.00\text{ in.})}{\dfrac{3/4\text{ in.}}{\sqrt{12}}}$$
$$= 3.00 < 25$$

Therefore, $F_{cr} = F_y$ from AISC *Specification* Section J4.4.

$$A_g = 8\tfrac{3}{4}\text{ in.}(\tfrac{3}{4}\text{ in.})$$
$$= 6.56\text{ in.}^2$$

From AISC *Specification* Equation J4-6:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = \phi F_y A_g$ $= 0.90(36\text{ ksi})(6.56\text{ in.}^2)$ $= 213\text{ kips} > 160\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{36\text{ ksi}(6.56\text{ in.}^2)}{1.67}$ $= 141\text{ kips} > 107\text{ kips}$ **o.k.**

Required Weld Size and Length for Fillet Welds to Beam Flange (using AISC Manual Part 8)

Based upon the weld length required for the tension flange plate, use 5⁄16 in. fillet weld and 12½ in. of weld along each side of the beam flange.

Connecting Elements Rupture Strength at Welds (Bottom Flange)

$$t_{min} = \frac{3.09D}{F_u}\text{ for }F_{EXX} = 70\text{ ksi} \qquad (\textit{Manual}\text{ Eq. 9-2})$$
$$= \frac{3.09(5\text{ sixteenths})}{65\text{ ksi}}$$
$$= 0.238\text{ in.} < 0.570\text{ in. beam flange} \quad \textbf{o.k.}$$

$$t_{min} = \frac{3.09D}{F_u} \qquad (\textit{Manual}\text{ Eq. 9-2})$$
$$= \frac{3.09(5\text{ sixteenths})}{58\text{ ksi}}$$
$$= 0.266\text{ in.} < \tfrac{3}{4}\text{ in. bottom flange plate} \quad \textbf{o.k.}$$

Note: Tension due to load reversal must also be considered in the design of the fillet weld to the supporting column flange.

Required Fillet Weld Size at Bottom Flange Plate to Column Flange (using AISC Manual Part 8)

The applied tensile load is perpendicular to the weld; therefore

$\theta = 90°$ and $1.0 + 0.50\sin^{1.5}\theta = 1.5$.

From AISC *Manual* Equation 8-2:

LRFD	ASD
$D_{min} = \dfrac{P_{uf}}{2(1.5)(1.392)l}$ $= \dfrac{160 \text{ kips}}{2(1.5)(1.392)(8\frac{3}{4} \text{ in.})}$ $= 4.38$ sixteenths Use $\frac{5}{16}$-in. fillet welds, 5 > 4.38 **o.k.**	$D_{min} = \dfrac{P_{af}}{2(1.5)(0.928)l}$ $= \dfrac{107 \text{ kips}}{2(1.5)(0.928)(8\frac{3}{4} \text{ in.})}$ $= 4.39$ sixteenths Use $\frac{5}{16}$-in. fillet welds, 5 > 4.39 **o.k.**

Connecting Elements Rupture Strength at Welds

$$t_{max} = \frac{3.09D}{F_u} \text{ for } F_{EXX} = 70 \text{ ksi} \qquad (\textit{Manual} \text{ Eq. 9-2})$$

$$= \frac{3.09(4.39 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.209 \text{ in.} < 0.780 \text{ in. column flange} \quad \textbf{o.k.}$$

See Example II.B-1 for checks of the column under concentrated forces. For further information, see AISC Design Guide 13 *Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications*. (Carter, 1999).

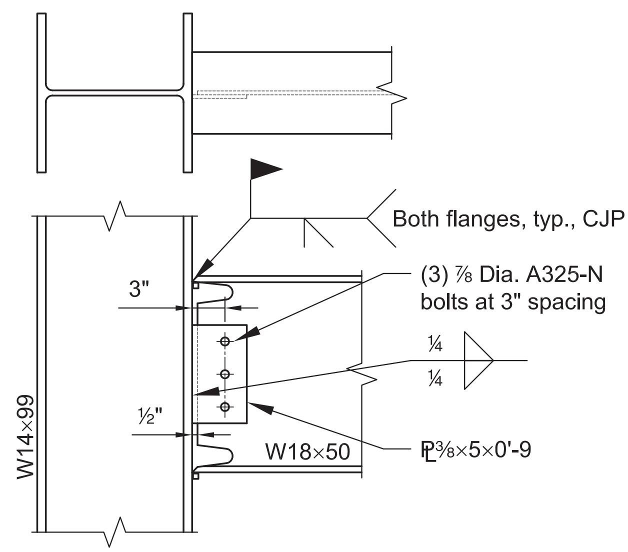
EXAMPLE II.B-3 DIRECTLY WELDED FLANGE FR MOMENT CONNECTION (BEAM-TO-COLUMN FLANGE)

Given:

Design a directly welded flange FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange to transfer the following vertical shear forces and strong-axis moments:

$V_D = 7.0$ kips $\quad M_D = 42$ kip-ft
$V_L = 21$ kips $\quad M_L = 126$ kip-ft

Use ⅞-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and 70-ksi electrodes. Check the column for stiffening requirements.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×50
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Column
W14×99
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Plate
ASTM A36
$F_y = 36$ ksi
$F_u = 58$ ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
b_f = 7.50 in.
t_f = 0.570 in.
t_w = 0.355 in.
Z_x = 101 in.^{3}

Column
W14×99
d = 14.2 in.
b_f = 14.6 in.
t_f = 0.780 in.
t_w = 0.485 in.
k_{des} = 1.38 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
R_u = 1.2(7.0 kips) + 1.6(21 kips) = 42.0 kips M_u = 1.2(42 kip-ft) + 1.6(126 kip-ft) = 252 kip-ft	R_a = 7.0 kips + 21 kips = 28.0 kips M_a = 42 kip-ft + 126 kip-ft = 168 kip-ft

The single-plate web connection is verified in Example II.B-1.

Note: By inspection, the available shear yielding, shear rupture, and block shear rupture strengths of the beam web are o.k.

Weld of Beam Flange to Column

A complete-joint-penetration groove weld will transfer the entire flange force in tension and compression. It is assumed that the beam is adequate for the applied moment and will carry the tension and compression forces through the flanges.

See Example II.B-1 for checks of the column under concentrated forces. For further information, see AISC Design Guide 13 *Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications*. (Carter, 1999).

Return to Table of Contents

EXAMPLE II.B-4 FOUR-BOLT UNSTIFFENED EXTENDED END-PLATE FR MOMENT CONNECTION (BEAM-TO-COLUMN FLANGE)

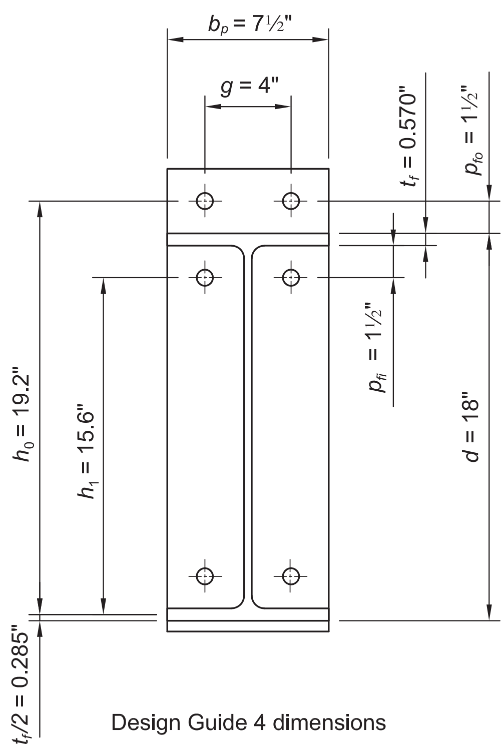
Given:

Design a four-bolt unstiffened extended end-plate FR moment connection between an ASTM A992 W18×50 beam and an ASTM A992 W14×99 column flange to transfer the following vertical shear forces and strong-axis moments:

$V_D = 7$ kips $M_D = 42$ kip-ft
$V_L = 21$ kips $M_L = 126$ kip-ft

Use ASTM A325-N snug-tight bolts in standard holes and 70-ksi electrodes. The plate is ASTM A36 material.

a. Use the design procedure from AISC Steel Design Guide 4 *Extended End-Plate Moment Connections Seismic and Wind Applications* (Murray and Sumner, 2003).
b. Use design procedure 2 (thin end-plate and larger diameter bolts) from AISC Design Guide 16, *Flush and Extended Multiple-Row Moment End-Plate Connections* (Murray and Shoemaker, 2002).



Design Guide 4 dimensions

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×50
ASTM A992
$F_y = 50$ ksi
$F_u = 65$ ksi

Column
W14×99
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
b_f = 7.50 in.
t_f = 0.570 in.
t_w = 0.355 in.
S_x = 88.9 in.^{3}

Column
W14×99
d = 14.2 in.
b_f = 14.6 in.
t_f = 0.780 in.
t_w = 0.485 in.
k_{des} = 1.38 in.

Solution a:

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$R_u = 1.2(7.0\text{ kips}) + 1.6(21\text{ kips})$ $= 42.0\text{ kips}$ $M_u = 1.2(42\text{ kip-ft}) + 1.6(126\text{ kip-ft})$ $= 252\text{ kip-ft}$	$R_a = 7.0\text{ kips} + 21\text{ kips}$ $= 28.0\text{ kips}$ $M_a = 42\text{ kip-ft} + 126\text{ kip-ft}$ $= 168\text{ kip-ft}$

Extended end-plate geometric properties are as follows:

b_p = 7½ in.
g = 4 in.
p_{fi} = 1½ in.
p_{fo} = 1½ in.

Additional dimensions are as follows:

$$h_0 = d + p_{fo} - \frac{t_f}{2}$$

$$= 18.0\text{ in.} + 1\tfrac{1}{2}\text{ in.} - \frac{0.570\text{ in.}}{2}$$

$$= 19.2\text{ in.}$$

$$h_1 = d - p_{fi} - t_f - \frac{t_f}{2}$$
$$= 18.0 \text{ in.} - 1\tfrac{1}{2} \text{ in.} - 0.570 \text{ in.} - \frac{0.570 \text{ in.}}{2}$$
$$= 15.6 \text{ in.}$$

Required Bolt Diameter Assuming No Prying Action

From AISC *Specification* Table J3.2, F_{nt} = 90 ksi, for ASTM A325-N bolts.

From Design Guide 4 Equation 3.5, determine the required bolt diameter:

LRFD	ASD
$\phi = 0.75$ $d_{b\ \text{Req'd}} = \sqrt{\dfrac{2M_u}{\pi\phi F_{nt}\left(h_0 + h_1\right)}}$ $= \sqrt{\dfrac{2\left(252 \text{ kip-ft}\right)\left(12 \text{ in./ft}\right)}{\pi\left(0.75\right)\left(90 \text{ ksi}\right)\left(19.2 \text{ in.} + 15.6 \text{ in.}\right)}}$ $= 0.905 \text{ in.}$ Use 1-in.-diameter ASTM A325-N snug-tightened bolts.	$\Omega = 2.00$ $d_{b\ \text{Req'd}} = \sqrt{\dfrac{2M_a\Omega}{\pi F_{nt}\left(h_0 + h_1\right)}}$ $= \sqrt{\dfrac{2\left(168 \text{ kip-ft}\right)\left(12 \text{ in./ft}\right)\left(2.00\right)}{\pi\left(90 \text{ ksi}\right)\left(19.2 \text{ in.} + 15.6 \text{ in.}\right)}}$ $= 0.905 \text{ in.}$ Use 1-in.-diameter ASTM A325-N snug-tightened bolts.

Required End-Plate Thickness

The end-plate yield line mechanism parameter is:

$$s = \frac{\sqrt{b_p g}}{2}$$
$$= \frac{\sqrt{7\tfrac{1}{2} \text{ in.}\left(4.00 \text{ in.}\right)}}{2}$$
$$= 2.74 \text{ in.}$$

$p_{fi} = 1.50 \text{ in.} \le s = 2.74 \text{ in.}$, therefore, use $p_{fi} = 1.50 \text{ in.}$

From Design Guide 4 Table 3.1:

$$Y_p = \frac{b_p}{2}\left[h_1\left(\frac{1}{p_{fi}} + \frac{1}{s}\right) + h_0\left(\frac{1}{p_{fo}}\right) - \tfrac{1}{2}\right] + \frac{2}{g}\left[h_1\left(p_{fi} + s\right)\right]$$
$$= \frac{7\tfrac{1}{2} \text{ in.}}{2}\left[15.6 \text{ in.}\left(\frac{1}{1\tfrac{1}{2} \text{ in.}} + \frac{1}{2.74 \text{ in.}}\right) + 19.2 \text{ in.}\left(\frac{1}{1\tfrac{1}{2} \text{ in.}}\right) - \tfrac{1}{2}\right] + \frac{2}{4.00 \text{ in.}}\left[15.6 \text{ in.}\left(1\tfrac{1}{2} \text{ in.} + 2.74 \text{ in.}\right)\right]$$
$$= 140 \text{ in.}$$

$$P_t = F_{nt}\left(\frac{\pi d_b^2}{4}\right) \qquad \text{(Design Guide 4 Eq. 3.9)}$$

$$= 90\text{ ksi}\left(\frac{\pi(1.00\text{ in.})^2}{4}\right)$$
$$= 70.7\text{ kips}$$

$$M_{np} = 2P_t(h_0 + h_1) \qquad \text{(Design Guide 4 Eq. 3.7)}$$
$$= 2(70.7\text{ kips})(19.2\text{ in.} + 15.6\text{ in.})$$
$$= 4{,}920\text{ kip-in.}$$

The no prying bolt available flexural strength is:

LRFD	ASD
$\phi = 0.75$ $\phi M_{np} = 0.75(4{,}920\text{ kip-in.})$ $= 3{,}690\text{ kip-in.}$	$\Omega = 2.00$ $\dfrac{M_{np}}{\Omega} = \dfrac{4{,}920\text{ kip-in.}}{2.00}$ $= 2{,}460\text{ kip-in.}$
$\phi_b = 0.90$ $t_{p\text{ Req'd}} = \sqrt{\dfrac{1.11\phi M_{np}}{\phi_b F_{yp} Y_p}}$ (Design Guide 4 Eq. 3.10) $= \sqrt{\dfrac{1.11(3{,}690\text{ kip-in.})}{0.90(36\text{ ksi})(140\text{ in.})}}$ $= 0.950\text{ in.}$ Use a 1-in.-thick end-plate.	$\Omega_b = 1.67$ $t_{p\text{ Req'd}} = \sqrt{\dfrac{1.11}{\left(\dfrac{F_{yp}}{\Omega_b}\right)Y_p}\left(\dfrac{M_{np}}{\Omega}\right)}$ (from Design Guide 4 Eq. 3.10) $= \sqrt{\dfrac{1.11}{\left(\dfrac{36\text{ ksi}}{1.67}\right)(140\text{ in.})}(2{,}460\text{ kip-in.})}$ $= 0.951\text{ in.}$ Use a 1-in.-thick end-plate.
With a 1-in.-thick end-plate, the design strength is: $\phi_b = 0.90$ $\phi_b M_{pl} = \dfrac{\phi_b F_{yp} t_p^2 Y_p}{1.11}$ $= \dfrac{0.90(36\text{ ksi})(1.00\text{ in.})^2(140\text{ in.})}{1.11}$ $= 4{,}090\text{ kip-in.}$	With a 1-in.-thick end-plate, the allowable strength is: $\Omega_b = 1.67$ $\dfrac{M_{pl}}{\Omega_b} = \dfrac{F_{yp} t_p^2 Y_p}{1.11\Omega_b}$ $= \dfrac{36\text{ ksi}(1.00\text{ in.})^2(140\text{ in.})}{1.11(1.67)}$ $= 2{,}720\text{ kip-in.}$

Beam Flange Force

The required force applied to the end plate through the beam flange is:

LRFD	ASD
$F_{fu} = \dfrac{M_u}{d - t_f}$ $= \dfrac{252 \text{ kip-ft}(12 \text{ in./ft})}{18.0 \text{ in.} - 0.570 \text{ in.}}$ $= 173 \text{ kips}$	$F_{fa} = \dfrac{M_a}{d - t_f}$ $= \dfrac{168 \text{ kip-ft}(12 \text{ in./ft})}{18.0 \text{ in.} - 0.570 \text{ in.}}$ $= 116 \text{ kips}$

Shear Yielding of the Extended End-Plate

The available strength due to shear yielding on the extended portion of the end-plate is determined as follows.

LRFD	ASD
$\phi = 0.90$ $\phi R_n = \phi 0.6 F_{yp} b_p t_p > \dfrac{F_{fu}}{2}$ (Design Guide 4 Eq. 3.12) $= 0.90(0.6)(36 \text{ ksi})(7\tfrac{1}{2} \text{ in.})(1.00 \text{ in.}) > \dfrac{173 \text{ kips}}{2}$ $= 146 \text{ kips} > 86.5 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{0.6 F_{yp} b_p t_p}{\Omega} > \dfrac{F_{fa}}{2}$ (from Design Guide 4 Eq. 3.12) $= \dfrac{0.6(36 \text{ ksi})(7\tfrac{1}{2} \text{ in.})(1.00 \text{ in.})}{1.67} > \dfrac{116 \text{ kips}}{2}$ $= 97.0 \text{ kips} > 58.0 \text{ kips}$ **o.k.**

Shear Rupture of the Extended End-Plate

The available strength due to shear rupture on the extended portion of the end-plate is determined as follows.

$$A_n = [7\tfrac{1}{2} \text{ in.} - 2(1\tfrac{1}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})](1.00 \text{ in.})$$
$$= 5.25 \text{ in.}^2$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.6 F_{up} A_n > \dfrac{F_{fu}}{2}$ (from Design Guide 4 Eq. 3.13) $= 0.75(0.6)(58 \text{ ksi})(5.25 \text{ in.}^2) > \dfrac{173 \text{ kips}}{2}$ $= 137 \text{ kips} > 86.5 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.6 F_{up} A_n}{\Omega} > \dfrac{F_{fa}}{2}$ (from Design Guide 4 Eq. 3.13) $= \dfrac{0.6(58 \text{ ksi})(5.25 \text{ in.}^2)}{2.00} > \dfrac{116 \text{ kips}}{2}$ $= 91.4 \text{ kips} > 58.0 \text{ kips}$ **o.k.**

Note: For the vertical shear forces, the shear yielding strength, shear rupture strength, and flexural yielding strength of the end-plate are all adequate by inspection.

Bolt Shear and Bearing

Try the minimum of four bolts at the tension flange and two bolts at the compression flange.

Note: Based on common practice, the compression bolts are assumed to resist all of the shear force.

LRFD	ASD
Bolt shear strength using AISC *Manual* Table 7-1: $\phi R_n = n\phi r_n = 2\text{ bolts}(31.8\text{ kips/bolt})$ $= 63.6\text{ kips} > 42.0\text{ kips}$ **o.k.** Bolt bearing on the end-plate ($L_e \geq L_{e\,full}$) using AISC Manual Table 7-5: $\phi R_n = n\phi r_n = (104\text{ kip/in./bolt})(1.00\text{ in.})$ $= 104\text{ kips/bolt} > 31.8\text{ kips/bolt}$ Bolt bearing on column flange ($L_e \geq L_{e\,full}$) using AISC *Manual* Table 7-5: $\phi R_n = n\phi r_n = (117\text{ kip/in./bolt})(0.780\text{ in.})$ $= 91.3\text{ kips/bolt} > 31.8\text{ kips/bolt}$ Bolt shear governs.	Bolt shear strength using AISC *Manual* Table 7-1: $R_n / \Omega = nr_n / \Omega = 2\text{ bolts}(21.2\text{ kips/bolt})$ $= 42.4\text{ kips} > 28.0\text{ kips}$ **o.k.** Bolt bearing on the end-plate ($L_e \geq L_{e\,full}$) using AISC Manual Table 7-5: $R_n / \Omega = nr_n / \Omega = (69.6\text{ kips/in./bolt})(1.00\text{ in.})$ $= 69.6\text{ kips/bolt} > 21.2\text{ kips/bolt}$ Bolt bearing on column flange ($L_e \geq L_{e\,full}$) using AISC *Manual* Table 7-5: $R_n / \Omega = nr_n / \Omega = (78.0\text{ kips/in./bolt})(0.780\text{ in.})$ $= 60.8\text{ kips/bolt} > 21.2\text{ kips/bolt}$ Bolt shear governs.

Determine the required size of the beam web to end-plate fillet weld in the tension-bolt region to develop the yield strength of the beam web. The minimum weld size required to match the shear rupture strength of the weld to the tension yield strength of the beam web, per unit length, is:

LRFD	ASD
$D_{min} = \dfrac{\phi F_y t_w (1\text{ in.})}{2(1.5)(1.392)(1\text{ in.})}$ $= \dfrac{0.90(50\text{ ksi})(0.355\text{ in.})(1\text{ in.})}{2(1.5)(1.392)(1\text{ in.})}$ $= 3.83$ Use ¼-in. fillet welds on both sides	$D_{min} = \dfrac{F_y t_w (1.0\text{ in.})}{\Omega(2)(1.5)(0.928)(1.0\text{ in.})}$ $= \dfrac{(50\text{ ksi})(0.355\text{ in.})(1.0\text{ in.})}{1.67(2)(1.5)(0.928)(1.0\text{ in.})}$ $= 3.82$ Use ¼-in. fillet welds on both sides

Use ¼-in. fillet welds on both sides of the beam web from the inside face of the beam tension flange to the centerline of the inside bolt holes plus two bolt diameters. Note that the 1.5 factor is from AISC *Specification* Section J2.4 and accounts for the increased strength of a transversely loaded fillet weld.

Weld Size Required for the End Reaction

The end reaction, R_u or R_a, is resisted by the lesser of the beam web-to-end-plate weld 1) between the mid-depth of the beam and the inside face of the compression flange, or 2) between the inner row of tension bolts plus two bolt diameters and the inside face of the beam compression flange. By inspection, the former governs for this example.

$$l = \frac{d}{2} - t_f$$
$$= \frac{18.0\text{ in.}}{2} - 0.570\text{ in.}$$
$$= 8.43\text{ in.}$$

From AISC *Manual* Equations 8-2:

LRFD	ASD
$D_{min} = \dfrac{R_u}{2(1.392)l}$ $= \dfrac{42.0\text{ kips}}{2(1.392)(8.43\text{ in.})}$ $= 1.79$ The minimum fillet weld size from AISC *Specification* Table J2.4 is 3⁄16 in. Use a 3⁄16-in. fillet weld on both sides of the beam web below the tension-bolt region.	$D_{min} = \dfrac{R_a}{2(0.928)l}$ $= \dfrac{28.0\text{ kips}}{2(0.928)(8.43\text{ in.})}$ $= 1.79$ The minimum fillet weld size from AISC *Specification* Table J2.4 is 3⁄16 in. Use a 3⁄16-in. fillet weld on both sides of the beam web below the tension-bolt region.

Connecting Elements Rupture Strength at Welds

$$t_{min} = \frac{6.19D}{F_u} \qquad \text{(*Manual* Eq. 9-3)}$$

$$= \frac{6.19(1.79\text{ sixteenths})}{65\text{ ksi}}$$

$$= 0.170\text{ in.} < 0.355\text{ in. beam web} \qquad \textbf{o.k.}$$

$$t_{min} = \frac{3.09D}{F_u} \qquad \text{(*Manual* Eq. 9-2)}$$

$$= \frac{3.09(1.79\text{ sixteenths})}{58\text{ ksi}}$$

$$= 0.0954\text{ in.} < 1.00\text{ in. end-plate} \qquad \textbf{o.k.}$$

Required Fillet Weld Size for the Beam Flange to End-Plate Connection

$$l = 2(b_f) - t_w$$
$$= 2(7.50\text{ in.}) - 0.355\text{ in.}$$
$$= 14.6\text{ in.}$$

From AISC *Manual* Part 8:

LRFD	ASD
$F_{fu} = 173$ kips $D_{min} = \dfrac{F_{fu}}{1.5(1.392)l}$ $= \dfrac{173\text{ kips}}{1.5(1.392)(14.6\text{ in.})}$ $= 5.67 \rightarrow 6$ sixteenths	$F_{fa} = 116$ kips $D_{min} = \dfrac{F_{fa}}{1.5(0.928)l}$ $= \dfrac{116\text{ kips}}{1.5(0.928)(14.6\text{ in.})}$ $= 5.71 \rightarrow 6$ sixteenths

Note that the 1.5 factor is from AISC *Specification* J2.4 and accounts for the increased strength of a transversely loaded fillet weld.

Use 3⁄8-in. fillet welds at the beam tension flange. Welds at the compression flange may be 1⁄4-in. fillet welds (minimum size per AISC *Specification* Table J2.4).

Connecting Elements Rupture Strength at Welds

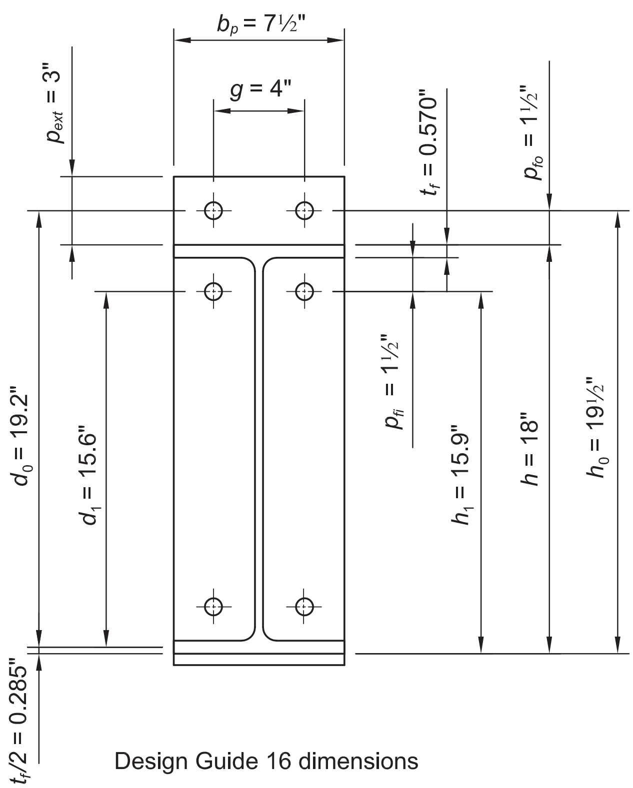
Shear rupture strength of base metal

$$t_{min} = \frac{3.09D}{F_u} \qquad (\textit{Manual}\text{ Eq. 9-2})$$

$$= \frac{3.09(5.71\text{ sixteenths})}{58\text{ ksi}}$$

$$= 0.304\text{ in.} < 1.00\text{ in. end-plate} \qquad \textbf{o.k.}$$

Solution b:



Design Guide 16 dimensions

Only those portions of the design that vary from the Solution "a" calculations are presented here.

Required End-Plate Thickness

LRFD	ASD
$\phi_b = 0.90$	$\Omega_b = 1.67$
$t_{p,req} = \sqrt{\dfrac{\gamma_r M_u}{\phi_b F_{py} Y}}$ (Design Guide 16 Eq. 2-9)	$t_{p,req} = \sqrt{\dfrac{\gamma_r M_a \Omega_b}{F_{py} Y}}$ (from Design Guide 16 Eq. 2-9)
$= \sqrt{\dfrac{1.0(252\text{ kip-ft})(12\text{ in./ft})}{0.90(36\text{ ksi})(140\text{ in.})}}$	$= \sqrt{\dfrac{1.0(168\text{ kip-ft})(12\text{ in./ft})(1.67)}{(36\text{ ksi})(140\text{ in.})}}$
$= 0.816$ in. Use t_p = ⅞ in.	$= 0.817$ in. Use t_p = ⅞ in.

Trial Bolt Diameter and Maximum Prying Forces

Try 1-in.-diameter bolts.

$$w' = \frac{b_p}{2} - (d_b + \tfrac{1}{16}\text{ in.}) \qquad \text{(Design Guide 16 Eq. 2-12)}$$

$$= \frac{7.50\text{ in.}}{2} - (1.00\text{ in.} + \tfrac{1}{16}\text{ in.})$$

$$= 2.69\text{ in.}$$

$$a_i = 3.62\left(\frac{t_p}{d_b}\right)^3 - 0.085 \qquad \text{(Design Guide 16 Eq. 2-13)}$$

$$= 3.62\left(\frac{\tfrac{7}{8}\text{ in.}}{1.00\text{ in.}}\right)^3 - 0.085$$

$$= 2.34\text{ in.}$$

$$F_i' = \frac{t_p^2 F_{py}\left[0.85\left(\dfrac{b_p}{2}\right) + 0.80w'\right] + \dfrac{\pi d_b^3 F_{nt}}{8}}{4p_{f,i}} \qquad \text{(Design Guide 16 Eq. 2-14)}$$

$$= \frac{(\tfrac{7}{8}\text{ in.})^2(36\text{ ksi})\left[0.85\left(\dfrac{7.50\text{ in.}}{2}\right) + 0.80(2.69\text{ in.})\right] + \dfrac{\pi(1.00\text{ in.})^3(90\text{ ksi})}{8}}{4(1\tfrac{1}{2}\text{ in.})}$$

$$= 30.4\text{ kips}$$

$$Q_{max,i} = \frac{w' t_p^2}{4a_i}\sqrt{F_{py}^2 - 3\left(\frac{F_i'}{w' t_p}\right)^2} \qquad \text{(Design Guide 16 Eq. 2-11)}$$

$$= \frac{2.69\text{ in.}(\tfrac{7}{8}\text{ in.})^2}{4(2.38\text{ in.})}\sqrt{(36\text{ ksi})^2 - 3\left(\frac{30.4\text{ kips}}{2.69\text{ in.}(\tfrac{7}{8}\text{ in.})}\right)^2}$$

$$= 6.10\text{ kips}$$

$$a_o = \min\left[a_i,\ p_{ext} - p_{f,o}\right] \qquad \text{(Design Guide 16 Eq. 2-16)}$$

$$= \min\left[2.34\text{ in., }(3.00\text{ in.} - 1\tfrac{1}{2}\text{ in.})\right]$$

$$= 1.50\text{ in.}$$

From AISC Design Guide 16 Equation 2-17:

$$F_o' = F_i'\left(\frac{p_{f,i}}{p_{f,o}}\right)$$

$$= 30.4\text{ kips}\left(\frac{1\tfrac{1}{2}\text{ in.}}{1\tfrac{1}{2}\text{ in.}}\right)$$

$$= 30.4\text{ kips}$$

$$Q_{max,o} = \frac{w' t_p^2}{4a_o}\sqrt{F_{py}^2 - 3\left(\frac{F_o'}{w' t_p}\right)^2} \qquad \text{(Design Guide 16 Eq. 2-15)}$$

$$= \frac{2.69\text{ in.}(7/8\text{ in.})^2}{4(1.50\text{ in.})}\sqrt{(36\text{ ksi})^2 - 3\left(\frac{30.4\text{ kips}}{2.69\text{ in.}(7/8\text{ in.})}\right)^2}$$

$$= 9.68\text{ kips}$$

Bolt Rupture with Prying Action

$$P_t = \frac{\pi d_b^2 F_{nt}}{4}$$

$$= \frac{\pi(1.00\text{ in.})^2(90\text{ ksi})}{4}$$

$$= 70.7\text{ kips}$$

From AISC *Specification* Table J3.1, the unmodified bolt pretension, T_{b0}, = 51 kips.

Modify bolt pretension for the snug-tight condition.

$T_b = 0.25(T_{b0})$ from AISC Design Guide 16 Table 4-1.

$$= 0.25(51\text{ kips})$$

$$= 12.8\text{ kips}$$

From AISC Design Guide Equation 2-19:

LRFD	ASD
$\phi = 0.75$ $\phi M_q = \max\begin{Bmatrix}\phi\left[2(P_t - Q_{max,o})d_0 + 2(P_t - Q_{max,i})d_1\right] \\ \phi\left[2(P_t - Q_{max,o})d_0 + 2(T_b)d_1\right] \\ \phi\left[2(P_t - Q_{max,i})d_1 + 2(T_b)d_0\right] \\ \phi\left[2(T_b)(d_0 + d_1)\right]\end{Bmatrix}$ $= \max\begin{Bmatrix}0.75\begin{bmatrix}2(70.7\text{ kips} - 9.68\text{ kips})(19.2\text{ in.}) \\ +2(70.7\text{ kips} - 6.10\text{ kips})(15.6\text{ in.})\end{bmatrix} \\ 0.75\begin{bmatrix}2(70.7\text{ kips} - 9.68\text{ kips})(19.2\text{ in.}) \\ +2(12.8\text{ kips})(15.6\text{ in.})\end{bmatrix} \\ 0.75\begin{bmatrix}2(70.7\text{ kips} - 6.10\text{ kips})(15.6\text{ in.}) \\ +2(12.8\text{ kips})(19.2\text{ in.})\end{bmatrix} \\ 0.75\left[2(12.8\text{ kips})(19.2\text{ in.} + 15.6\text{ in.})\right]\end{Bmatrix}$	$\Omega = 2.00$ $\frac{M_q}{\Omega} = \max\begin{Bmatrix}\frac{1}{\Omega}\left[2(P_t - Q_{max,o})d_0 + 2(P_t - Q_{max,i})d_1\right] \\ \frac{1}{\Omega}\left[2(P_t - Q_{max,o})d_0 + 2(T_b)d_1\right] \\ \frac{1}{\Omega}\left[2(P_t - Q_{max,i})d_1 + 2(T_b)d_0\right] \\ \frac{1}{\Omega}\left[2(T_b)(d_0 + d_1)\right]\end{Bmatrix}$ $= \max\begin{Bmatrix}\frac{1}{2.00}\begin{bmatrix}2(70.7\text{ kips} - 9.68\text{ kips})(19.2\text{ in.}) \\ +2(70.7\text{ kips} - 6.10\text{ kips})(15.6\text{ in.})\end{bmatrix} \\ \frac{1}{2.00}\begin{bmatrix}2(70.7\text{ kips} - 9.68\text{ kips})(19.2\text{ in.}) \\ +2(12.8\text{ kips})(15.6\text{ in.})\end{bmatrix} \\ \frac{1}{2.00}\begin{bmatrix}2(70.7\text{ kips} - 6.10\text{ kips})(15.6\text{ in.}) \\ +2(12.8\text{ kips})(19.2\text{ in.})\end{bmatrix} \\ \frac{1}{2.00}\left[2(12.8\text{ kips})(19.2\text{ in.} + 15.6\text{ in.})\right]\end{Bmatrix}$

LRFD	ASD
$= \max \begin{Bmatrix} 3{,}270 \text{ kip-in.} \\ 2{,}060 \text{ kip-in.} \\ 1{,}880 \text{ kip-in.} \\ 668 \text{ kip-in.} \end{Bmatrix} = 3{,}270 \text{ kip-in.}$	$= \max \begin{Bmatrix} 2{,}180 \text{ kip-in.} \\ 1{,}370 \text{ kip-in.} \\ 1{,}250 \text{ kip-in.} \\ 445 \text{ kip-in.} \end{Bmatrix} = 2{,}180 \text{ kip-in.}$
$\phi M_q = 3{,}270 \text{ kip-in.}$ $= 273 \text{ kip-ft} > 252 \text{ kip-ft}$ **o.k.**	$\dfrac{M_q}{\Omega} = 2{,}180 \text{ kip-in.}$ $= 182 \text{ kip-ft} > 168 \text{ kip-ft}$ **o.k.**

For Example II.B-4, the design procedure from Design Guide 4 produced a design with a 1-in.-thick end-plate and 1-in. diameter bolts. Design procedure 2 from Design Guide 16 produced a design with a ⅞-in.-thick end-plate and 1-in.-diameter bolts. Either design is acceptable. The first design procedure did not produce a smaller bolt diameter for this example, although in general it should result in a thicker plate and smaller diameter bolt than the second design procedure. Note that the bolt stress is lower in the first design procedure than in the second design procedure.

CHAPTER IIB DESIGN EXAMPLE REFERENCES

Carter, C.J. (1999), *Stiffening of Wide-Flange Columns at Moment Connections: Wind and Seismic Applications*, Design Guide 13, AISC, Chicago, IL.

Murray, T.M. and Sumner, E.A. (2003), *Extended End-Plate Moment Connections—Seismic and Wind Applications,* Design Guide 4, 2nd Ed., AISC, Chicago, IL.

Murray, T.M. and Shoemaker, W.L. (2002), *Flush and Extended Multiple-Row Moment End-Plate Connections,* Design Guide 16, AISC, Chicago, IL.

Chapter IIC
Bracing and Truss Connections

The design of bracing and truss connections is covered in Part 13 of the AISC *Steel Construction Manual*.

EXAMPLE II.C-1 TRUSS SUPPORT CONNECTION

Given:

Based on the configuration shown in Figure II.C-1-1, determine:

a. The connection requirements between the gusset and the column
b. The required gusset size and the weld requirements connecting the diagonal to the gusset

The reactions on the truss end connection are:

R_D = 16.6 kips
R_L = 53.8 kips

Use ⅞-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and 70-ksi electrodes. The top chord and column are ASTM A992 material. The diagonal member, gusset plate and clip angles are ASTM A36 material.

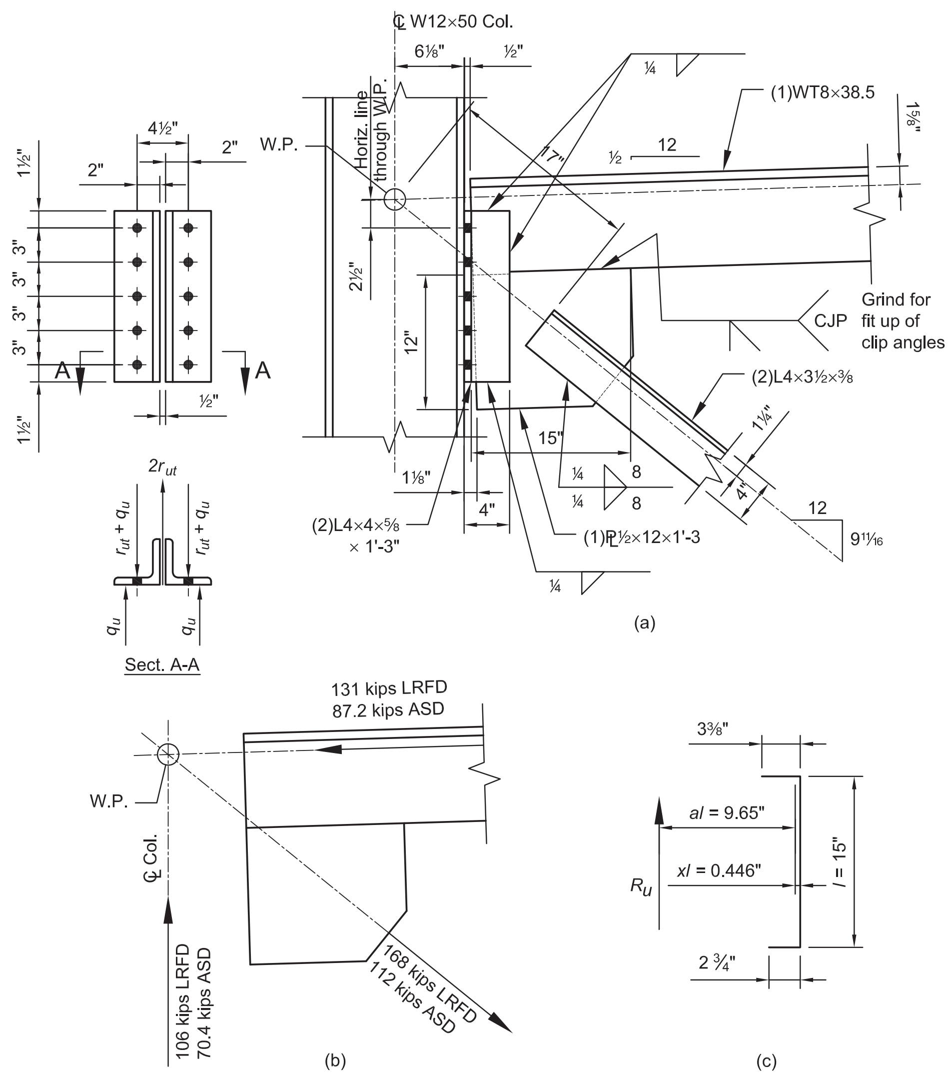


Fig. II.C-1-1. Truss support connection.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Top Chord
WT8×38.5
ASTM A992
F_y = 50 ksi
F_u = 65 ksi
Column
W12×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Diagonal
2L4×3½×⅜
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Gusset Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Clip Angles
2L4×4×⅝
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1, 1-8 and 1-15, the geometric properties are as follows:

Top Chord
WT8×38.5
d = 8.26 in.
t_w = 0.455 in.

Column
W12×50
d = 12.2 in.
t_f = 0.640 in.
b_f = 8.08 in.
t_w = 0.370 in.

Diagonal
2L4×3½×⅜
t = 0.375 in.
A = 5.36 in.^{2}
$\bar{x}$ = 0.947 in. for single angle

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
Brace axial load:	Brace axial load:
$R_u = 168\text{ kips}$	$R_a = 112\text{ kips}$
Truss end reaction:	Truss end reaction:
$R_u = 1.2(16.6\text{ kips}) + 1.6(53.8\text{ kips})$ $= 106\text{ kips}$	$R_a = 16.6\text{ kips} + 53.8\text{ kips}$ $= 70.4\text{ kips}$
Top chord axial load:	Top chord axial load:
$R_u = 131\text{ kips}$	$R_a = 87.2\text{ kips}$

Weld Connecting the Diagonal to the Gusset Plate

Note: AISC *Specification* Section J1.7 requiring that the center of gravity of the weld group coincide with the center of gravity of the member does not apply to end connections of statically loaded single angle, double angle and similar members.

For ⅜-in. angles, $D_{min} = 3$ from AISC *Specification* Table J2.4.

Try ¼-in. fillet welds, $D = 4$. From AISC *Manual* Equation 8-2, the required length is:

LRFD	ASD
$L_{req} = \dfrac{R_u}{4D(1.392)}$ $= \dfrac{168\text{ kips}}{4(4)(1.392)}$ $= 7.54\text{ in.}$	$L_{req} = \dfrac{R_a}{4D(0.928)}$ $= \dfrac{112\text{ kips}}{4(4)(0.928)}$ $= 7.54\text{ in.}$

Use 8 in. at the heel and 8 in. at the toe of each angle.

Tensile Yielding of Diagonal

$$R_n = F_y A_g$$
$$= 36\text{ ksi}(5.36\text{ in.}^2)$$
$$= 193\text{ kips}$$
(*Spec*. Eq. J4-1)

LRFD	ASD
$\phi = 0.90$	$\Omega = 1.67$
$\phi R_n = 0.90(193\text{ kips})$ $= 174\text{ kips} > 168\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{193\text{ kips}}{1.67}$ $= 116\text{ kips} > 112\text{ kips}$ **o.k.**

Tensile Rupture of Diagonal

$A_n = A_g = 5.36\text{ in.}^2$

$U = 1 - \dfrac{\bar{x}}{l}$ from AISC *Specification* Table D3.1 Case 2

$$= 1 - \frac{0.947\text{ in.}}{8.00\text{ in.}}$$
$$= 0.882$$

$A_e = A_n U$ (*Spec*. Eq. D3-1)
$= 5.36\text{ in.}^2(0.882)$
$= 4.73\text{ in.}^2$

$R_n = F_u A_e$ (*Spec*. Eq. J4-2)
$= 58\text{ ksi}(4.73\text{ in.}^2)$
$= 274\text{ kips}$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(274\text{ kips})$ $= 206\text{ kips} > 168\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{274\text{ kips}}{2.00}$ $= 137\text{ kips} > 112\text{ kips}$ **o.k.**

Use a ½-in. gusset plate. With the diagonal to gusset welds determined, a gusset plate layout as shown in Figure II.C-1-1(a) can be made.

Bolts Connecting Clip Angles to Column (Shear and Tension)

From AISC *Manual* Table 7-1, the number of ⅞-in.-diameter ASTM A325-N bolts required for shear only is:

LRFD	ASD
$n_{min} = \dfrac{R_u}{\phi r_n}$ $= \dfrac{106\text{ kips}}{24.3\text{ kips/bolt}}$ $= 4.36\text{ bolts}$	$n_{min} = \dfrac{R_u}{r_n / \Omega}$ $= \dfrac{70.4\text{ kips}}{16.2\text{ kips/bolt}}$ $= 4.35\text{ bolts}$

Try a clip angle thickness of ⅝ in. For a trial calculation, the number of bolts was increased to 10 in pairs at 3-in. spacing. This is done to "square off" the gusset plate.

With 10 bolts:

LRFD	ASD
$f_{rv} = \dfrac{R_u}{nA_b}$ $= \dfrac{106\text{ kips}}{10\text{ bolts}\left(0.601\text{ in.}^2\right)}$ $= 17.6\text{ ksi}$	$f_{rv} = \dfrac{R_a}{nA_b}$ $= \dfrac{70.4\text{ kips}}{10\text{ bolts}\left(0.601\text{ in.}^2\right)}$ $= 11.7\text{ ksi}$

The eccentric moment about the workpoint (WP) in Figure II.C-1-1 at the faying surface (face of column flange) is determined as follows. The eccentricity, e, is half of the column depth, d, equal to 12.1 in.

LRFD	ASD
$M_u = R_u e$ $= 106 \text{ kips}(6.10 \text{ in.})$ $= 647 \text{ kip-in.}$	$M_a = R_a e$ $= 70.4 \text{ kips}(6.10 \text{ in.})$ $= 429 \text{ kip-in.}$

For the bolt group, the Case II approach in AISC *Manual* Part 7 can be used. Thus, the maximum tensile force per bolt, T, is given by the following:

n' = number of bolts on tension side of the neutral axis (the bottom in this case) = 4 bolts
d_m = moment arm between resultant tensile force and resultant compressive force = 9.00 in.

From AISC *Manual* Equations 7-14a and 7-14b:

LRFD	ASD
$T_u = \dfrac{M_u}{n'd_m}$ $= \dfrac{647 \text{ kip-in.}}{4 \text{ bolts}(9.00 \text{ in.})}$ $= 18.0 \text{ kips/bolt}$	$T_a = \dfrac{M_a}{n'd_m}$ $= \dfrac{429 \text{ kip-in.}}{4 \text{ bolts}(9.00 \text{ in.})}$ $= 11.9 \text{ kips/bolt}$

Tensile strength of bolts:

From AISC *Specification* Table J3.2:

$F_{nt} = 90$ ksi
$F_{nv} = 54$ ksi

LRFD	ASD
$\phi = 0.75$ $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3a) $= 1.3(90 \text{ ksi}) - \dfrac{90 \text{ ksi}}{0.75(54 \text{ ksi})}(17.6 \text{ ksi})$ $= 77.9 \text{ ksi} < 90 \text{ ksi}$ **o.k.** $B = \phi F'_{nt} A_b$ $= 0.75(77.9 \text{ ksi})(0.601 \text{ in.}^2)$ $= 35.1 \text{ kips} > 18.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $F'_{nt} = 1.3F_{nt} - \dfrac{\Omega F_{nt}}{F_{nv}} f_{rv} \le F_{nt}$ (*Spec*. Eq. J3-3b) $= 1.3(90 \text{ ksi}) - \dfrac{2.00(90 \text{ ksi})}{54 \text{ ksi}}(11.7 \text{ ksi})$ $= 78.0 \text{ ksi} < 90 \text{ ksi}$ **o.k.** $B = \dfrac{F'_{nt}}{\Omega} A_b$ $= \dfrac{78.0 \text{ ksi}}{2.00}(0.601 \text{ in.}^2)$ $= 23.4 \text{ kips} > 11.9 \text{ kips}$ **o.k.**

Prying Action on Clip Angles (AISC Manual Part 9)

$$p = 3.00 \text{ in.}$$

$$b = 2.00 \text{ in.} - \frac{5/8 \text{ in.}}{2}$$
$$= 1.69 \text{ in.}$$

Note: 1⅜ in. entering and tightening clearance from AISC *Manual* Table 7-15 is accommodated and the column fillet toe is cleared.

$$a = \frac{8.08 \text{ in.} - 4\tfrac{1}{2} \text{ in.}}{2}$$
$$= 1.79 \text{ in.}$$

Note: a was calculated based on the column flange width in this case because it is less than the double angle width.

$$b' = b - \frac{d_b}{2} \qquad \text{(\textit{Manual} Eq. 9-21)}$$
$$= 1.69 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 1.25 \text{ in.}$$

$$a' = a + \frac{d_b}{2} \le 1.25b + \frac{d_b}{2} \qquad \text{(\textit{Manual} Eq. 9-27)}$$
$$= 1.79 \text{ in.} + \frac{\tfrac{7}{8} \text{ in.}}{2} \le 1.25(1.69 \text{ in.}) + \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 2.23 \text{ in.} \le 2.55 \text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad \text{(\textit{Manual} Eq. 9-26)}$$
$$= \frac{1.25 \text{ in.}}{2.23 \text{ in.}}$$
$$= 0.561$$

$$\delta = 1 - \frac{d'}{p} \qquad \text{(\textit{Manual} Eq. 9-24)}$$
$$= 1 - \frac{\tfrac{15}{16} \text{ in.}}{3.00 \text{ in.}}$$
$$= 0.688$$

LRFD	ASD
$\phi = 0.90$ $t_c = \sqrt{\dfrac{4Bb'}{\phi p F_u}}$ (*Manual* Eq. 9-30a) $= \sqrt{\dfrac{4(35.1 \text{ kips})(1.25 \text{ in.})}{0.90(3.00 \text{ in.})(58 \text{ ksi})}}$ $= 1.06$ in.	$\Omega = 1.67$ $t_c = \sqrt{\dfrac{\Omega 4Bb'}{p F_u}}$ (*Manual* Eq. 9-30b) $= \sqrt{\dfrac{1.67(4)(23.4 \text{ kips})(1.25 \text{ in.})}{3.00 \text{ in.}(58 \text{ ksi})}}$ $= 1.06$ in.

$$\alpha' = \frac{1}{\delta(1+\rho)}\left[\left(\frac{t_c}{t}\right)^2 - 1\right] \qquad \text{(\textit{Manual} Eq. 9-35)}$$
$$= \frac{1}{0.688(1+0.561)}\left[\left(\frac{1.06 \text{ in.}}{\tfrac{5}{8} \text{ in.}}\right)^2 - 1\right]$$

$= 1.75$

Because $\alpha' > 1$,

$$Q = \left(\frac{t}{t_c}\right)^2 (1+\delta) \qquad \text{(}\textit{Manual}\text{ Eq. 9-34)}$$

$$= \left(\frac{\text{5/8 in.}}{\text{1.06 in.}}\right)^2 (1+0.688)$$

$$= 0.587$$

LRFD	ASD
$T_{avail} = BQ$ (*Manual* Eq. 9-31) $= 35.1 \text{ kips}(0.587)$ $= 20.6 \text{ kips} > 18.0 \text{ kips}$ **o.k.**	$T_{avail} = BQ$ (*Manual* Eq. 9-31) $= 23.4 \text{ kips}(0.587)$ $= 13.7 \text{ kips} > 11.9 \text{ kips}$ **o.k.**

Shear Yielding of Clip Angles

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})[2(15.0 \text{ in.})(\text{5/8 in.})]$ $= 405 \text{ kips} > 106 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})[2(15.0 \text{ in.})(\text{5/8 in.})]}{1.50}$ $= 270 \text{ kips} > 70.4 \text{ kips}$ **o.k.**

Shear Rupture of Clip Angles

$$A_{nv} = 2[15.0 \text{ in.} - 5(\text{15/16 in.} + \text{1/16 in.})](\text{5/8 in.})$$
$$= 12.5 \text{ in.}^2$$

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58 \text{ ksi})(12.5 \text{ in.}^2)$ $= 326 \text{ kips} > 106 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58 \text{ ksi})(12.5 \text{ in.}^2)}{2.00}$ $= 218 \text{ kips} > 70.4 \text{ kips}$ **o.k.**

Block Shear Rupture of Clip Angles

Assume uniform tension stress, so use $U_{bs} = 1.0$.

$$A_{gv} = 2(15.0\text{ in.} - 1\tfrac{1}{2}\text{ in.})(\tfrac{5}{8}\text{ in.}) = 16.9\text{ in.}^2$$

$$A_{nv} = 16.9\text{ in.}^2 - 2\left[4.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{5}{8}\text{ in.})\right] = 11.3\text{ in.}^2$$

$$A_{nt} = 2\left[(2.00\text{ in.})(\tfrac{5}{8}\text{ in.}) - 0.5(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})(\tfrac{5}{8}\text{ in.})\right] = 1.88\text{ in.}^2$$

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi\left[U_{bs}F_uA_{nt} + \min\left\{0.60F_yA_{gv}, 0.60F_uA_{nv}\right\}\right]$ $= 0.75\left[1.0(58\text{ ksi})(1.88\text{ in.}^2) + \min\left\{\begin{matrix}0.60(36\text{ ksi})(16.9\text{ in.}^2)\\0.60(58\text{ ksi})(11.3\text{ in.}^2)\end{matrix}\right\}\right]$ $= 356\text{ kips} > 106\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{\left[U_{bs}F_uA_{nt} + \min\left\{0.60F_yA_{gv}, 0.60F_uA_{nv}\right\}\right]}{\Omega}$ $= \dfrac{\left[1.0(58\text{ ksi})(1.88\text{ in.}^2) + \min\left\{\begin{matrix}0.60(36\text{ ksi})(16.9\text{ in.}^2)\\0.60(58\text{ ksi})(11.3\text{ in.}^2)\end{matrix}\right\}\right]}{2.00}$ $= 237\text{ kips} > 70.4\text{ kips}$ **o.k.**

Bearing and Tearout on Clip Angles

The clear edge distance, l_c, for the top bolts is $l_c = L_e - d'/2$, where L_e is the distance to the center of the hole.

$$l_c = 1\tfrac{1}{2}\text{ in.} - \frac{\tfrac{15}{16}\text{ in.}}{2} = 1.03\text{ in.}$$

The available strength due to bearing/tearout of the top bolt is as follows:

From AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = \phi 1.2 l_c t F_u \leq \phi 2.4 d t F_u$ $= 0.75(1.2)(1.03\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$ $\leq 0.75(2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$ $= 33.6\text{ kips} \leq 57.1\text{ kips}$ 33.6 kips/bolt > 24.3 kips/bolt	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F}{\Omega} \leq \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{1.2(1.03\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $\leq \dfrac{2.4(\tfrac{7}{8}\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $= 22.4\text{ kips} \leq 38.1\text{ kips}$ 22.4 kips/bolt >16.2 kips/bolt

Bolt shear controls.

By inspection, the bearing capacity of the remaining bolts does not control.

Use 2L4x4x⅝ clip angles.

Prying Action on Column Flange (AISC Manual Part 9)

Using the same procedure as that for the clip angles, the tensile strength is:

LRFD	ASD
$T_{avail} = 18.7 \text{ kips} > 18.0 \text{ kips}$ **o.k.**	$T_{avail} = 12.4 \text{ kips} > 11.9 \text{ kips}$ **o.k.**

Bearing and Tearout on Column Flange

By inspection, these limit states will not control.

Clip Angle-to-Gusset Plate Connection

From AISC *Specification* Table J2.4, the minimum weld size is 3⁄16 in. with the top chord slope being ½ on 12, the horizontal welds are as shown in Figure II.C-1-1(c) due to the square cut end. Use the average length.

$$l = 15.0 \text{ in.}$$

$$\begin{aligned} kl &= \frac{3\tfrac{3}{8}\text{ in.} + 2\tfrac{3}{4}\text{ in.}}{2} \\ &= 3.06 \text{ in.} \end{aligned}$$

$$\begin{aligned} k &= \frac{kl}{l} \\ &= \frac{3.06 \text{ in.}}{15.0 \text{ in.}} \\ &= 0.204 \end{aligned}$$

$$\begin{aligned} xl &= \frac{(kl)^2}{(l + 2kl)} \\ &= \frac{(3.06 \text{ in.})^2}{15.0 \text{ in.} + 2(3.06 \text{ in.})} \\ &= 0.443 \text{ in.} \end{aligned}$$

$$\begin{aligned} al + xl &= 6.10 \text{ in.} + 4.00 \text{ in.} \\ &= 10.1 \text{ in.} \end{aligned}$$

$$\begin{aligned} al &= 10.1 \text{ in.} - xl \\ &= 10.1 \text{ in.} - 0.443 \text{ in.} \\ &= 9.66 \text{ in.} \end{aligned}$$

$$\begin{aligned} a &= \frac{al}{l} \\ &= \frac{9.66 \text{ in.}}{15.0 \text{ in.}} \\ &= 0.644 \end{aligned}$$

By interpolating AISC *Manual* Table 8-8 with $\theta = 0°$:

$C = 1.50$

From AISC *Manual* Table 8-8:

LRFD	ASD
$\phi = 0.75$ $D_{req} = \dfrac{R_u}{2(\phi C C_1 l)}$ $= \dfrac{106 \text{ kips}}{2(0.75)(1.50)(1.0)(15.0 \text{ in.})}$ $= 3.14 \rightarrow 4$ sixteenths	$\Omega = 2.00$ $D_{req} = \dfrac{\Omega R_a}{2(C C_1 l)}$ $= \dfrac{2.00(70.4 \text{ kips})}{2(1.50)(1.0)(15.0 \text{ in.})}$ $= 3.13 \rightarrow 4$ sixteenths

Use ¼-in. fillet welds.

Note: Using the average of the horizontal weld lengths provides a reasonable solution when the horizontal welds are close in length. A conservative solution can be determined by using the smaller of the horizontal weld lengths as effective for both horizontal welds. For this example, using $kl = 2.75$ in., $C = 1.43$ and $D_{req} = 3.29$ sixteenths.

Tensile Yielding of Gusset Plate on the Whitmore Section (AISC Manual Part 9)

The gusset plate thickness should match or slightly exceed that of the tee stem. This requirement is satisfied by the ½-in. plate previously selected.

The width of the Whitmore section is:

$$l_w = 4.00 \text{ in.} + 2(8.00 \text{ in.})\tan 30°$$
$$= 13.2 \text{ in.}$$

From AISC *Specification* Equation J4-1, the available strength due to tensile yielding is:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = \phi F_y A_g$ $= 0.90(36 \text{ ksi})(13.2 \text{ in.})(½ \text{ in.})$ $= 214 \text{ kips} > 168 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{36 \text{ ksi}(13.2 \text{ in.})(½ \text{ in.})}{1.67}$ $= 142 \text{ kips} > 112 \text{ kips}$ **o.k.**

Gusset Plate-to-Tee Stem Weld

The interface forces are:

LRFD	ASD
Horizontal shear between gusset and WT: H_{ub} = 131 kips – 4 bolts(18.0 kips/bolt) = 59.0 kips *Vertical tension between gusset and WT:* V_{ub} = 106 kips(4 bolts/10 bolts) = 42.4 kips *Compression between WT and column:* C_{ub} = 4 bolts(18.0 kips/bolt) =72.0 kips Summing moments about the face of the column at the workline of the top chord: $M_{ub} = C_{ub}(2\frac{1}{2}\text{ in.} + 1.50\text{ in.}) + H_{ub}(d - \bar{y})$ $- V_{ub}$ (gusset plate width/2 + setback) = 72.0 kips(4.00 in.) + 59.0 kips(8.26 in. – 1⅝ in.) – 42.4 kips(15.0 in./2 + ½ in.) = 340 kip-in.	*Horizontal shear between gusset and WT:* H_{ab} = 87.2 kips – 4 bolts(11.9 kips/bolt) = 39.6 kips *Vertical tension between gusset and WT:* V_{ab} = 70.4 kips(4 bolts/10 bolts) = 28.2 kips *Compression between WT and column:* C_{ab} = 4 bolts(11.9 kips/bolt) =47.6 kips Summing moments about the face of the column at the workline of the top chord: $M_{ub} = C_{ab}(2\frac{1}{2}\text{ in.} + 1.50\text{ in.}) + H_{ab}(d - \bar{y})$ $- V_{ab}$ (gusset plate width/2 + setback) = 47.6 kips(4.00 in.) + 39.6 kips(8.26 in. – 1⅝ in.) – 28.2 kips(15.0 in./2 + ½ in.) = 228 kip-in.

A CJP weld should be used along the interface between the gusset plate and the tee stem. The weld should be ground smooth under the clip angles.

The gusset plate width depends upon the diagonal connection. From a scaled layout, the gusset plate must be 1 ft 3 in. wide.

The gusset plate depth depends upon the connection angles. From a scaled layout, the gusset plate must extend 12 in. below the tee stem.

Use a PL½×12 in.×1 ft 3 in.

Return to Table of Contents

EXAMPLE II.C-2 BRACING CONNECTION

Given:

Design the diagonal bracing connection between the ASTM A992 W12×87 brace and the ASTM A992 W18×106 beam and the ASTM A992 W14×605 column.

Brace Axial Load	T_u = 675 kips	T_a = 450 kips
Beam End Reaction	R_u = 15 kips	R_a = 10 kips
Column Axial Load	P_u = 421 kips	P_a = 280 kips
Beam Axial Load	P_u = 528 kips	P_a = 352 kips

Use ⅞-in.-diameter ASTM A325-N or F1852-N bolts in standard holes and 70-ksi electrodes. The gusset plate and angles are ASTM A36 material.

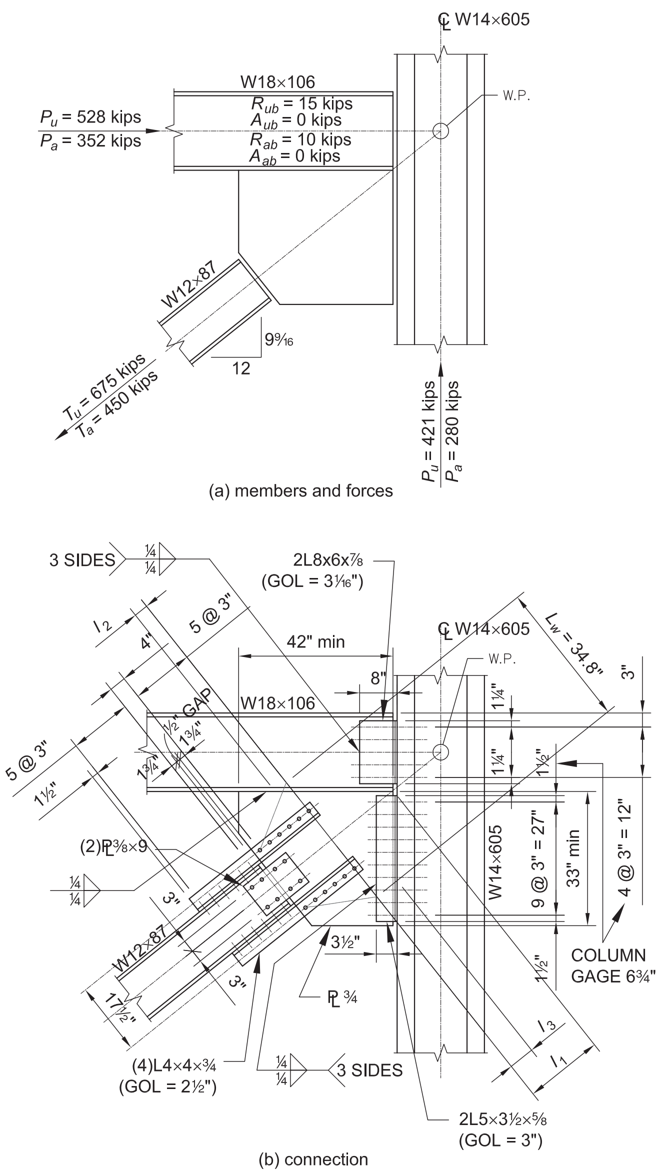


Fig. II.C-2-1. Diagonal bracing connection.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Brace
W12×87
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Beam
W18×106
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×605
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Gusset Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Angles
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Brace
W12×87
A = 25.6 in.^{2}
d = 12.5 in.
t_w = 0.515 in.
b_f = 12.1 in.
t_f = 0.810 in.

Beam
W18×106
d = 18.7 in.
t_w = 0.590 in.
b_f = 11.2 in.
t_f = 0.940 in.
k_{des} = 1.34

Column
W14×605
d = 20.9 in.
t_w = 2.60 in.
b_f = 17.4 in.

t_f =4.16 in.

Brace-to-Gusset Connection

Distribute brace force in proportion to web and flange areas.

LRFD	ASD
Force in one flange: $P_{uf} = \dfrac{P_u b_f t_f}{A}$ $= \dfrac{675 \text{ kips}(12.1 \text{ in.})(0.810 \text{ in.})}{25.6 \text{ in.}^2}$ $= 258 \text{ kips}$ Force in web: $P_{uw} = P_u - 2P_{uf}$ $= 675 \text{ kips} - 2(258 \text{ kips})$ $= 159 \text{ kips}$	Force in one flange: $P_{af} = \dfrac{P_a b_f t_f}{A}$ $= \dfrac{450 \text{ kips}(12.1 \text{ in.})(0.810 \text{ in.})}{25.6 \text{ in.}^2}$ $= 172 \text{ kips}$ Force in web: $P_{aw} = P_a - 2P_{af}$ $= 450 \text{ kips} - 2(172 \text{ kips})$ $= 106 \text{ kips}$

Brace-Flange-to-Gusset Connection

For short claw angle connections, eccentricity may be an issue. See AISC *Engineering Journal*, Vol. 33, No. 4, pp. 123-128, 1996.

Determine number of ⅞-in.-diameter ASTM A325-N bolts required on the brace side of the brace-flange-to-gusset connection for single shear.

From AISC *Manual* Table 7-1:

LRFD	ASD
$n_{min} = \dfrac{P_{uf}}{\phi r_n}$ $= \dfrac{258 \text{ kips}}{24.3 \text{ kips/bolt}}$ $= 10.6 \text{ bolts} \rightarrow$ use 12 bolts	$n_{min} = \dfrac{P_{af}}{r_n / \Omega}$ $= \dfrac{172 \text{ kips}}{16.2 \text{ kips/bolt}}$ $= 10.6 \text{ bolts} \rightarrow$ use 12 bolts

On the gusset side, since the bolts are in double shear, half as many bolts will be required. Try six rows of two bolts through each flange, six bolts per flange through the gusset, and 2L4x4x¾ angles per flange.

From AISC *Manual* Tables 1-7 and 1-15, the geometric properties are as follows:

$A = 10.9 \text{ in.}^2$
$\bar{x} = 1.27 \text{ in.}$

Tensile Yielding of Angles

From AISC *Specification* Equation J4-1:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = \phi F_y A_g$ $= 0.90(36\text{ ksi})(10.9\text{ in.}^2)$ $= 353\text{ kips} > 258\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{1.67}$ $= \dfrac{36\text{ ksi}(10.9\text{ in.}^2)}{1.67}$ $= 235\text{ kips} > 172\text{ kips}$ **o.k.**

Tensile Rupture of Angles

$$U = 1 - \frac{\bar{x}}{l} \text{ from AISC } \textit{Specification} \text{ Table D3.1 Case 2}$$

$$= 1 - \frac{1.27\text{ in.}}{15.0\text{ in.}}$$

$$= 0.915$$

$$A_e = A_n U \qquad \text{(}\textit{Spec}\text{. Eq. D3-1)}$$

$$= \left[10.9\text{ in.}^2 - 2(\tfrac{3}{4}\text{ in.})(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](0.915)$$

$$= 8.60\text{ in.}^2$$

From AISC *Specification* Equation J4-2:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi F_u A_e$ $= 0.75(58\text{ ksi})(8.60\text{ in.}^2)$ $= 374\text{ kips} > 258\text{ kips}$ **o.k.**	$\Omega = 2.00$ $R_n / \Omega = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{(58\text{ ksi})(8.60\text{ in.}^2)}{2.00}$ $= 249\text{ kips} > 172\text{ kips}$ **o.k.**

Block Shear Rupture of Angles

Use $n = 6$, $L_{ev} = 1\tfrac{1}{2}$ in., $L_{eh} = 1\tfrac{1}{2}$ in. and $U_{bs} = 1.0$.

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min\left(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv}\right)$	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$
Tension rupture component from AISC *Manual* Table 9-3a:	Tension rupture component from AISC *Manual* Table 9-3a:
$\phi U_{bs} F_u A_{nt} = 1.0(43.5 \text{ kips/in.})(\tfrac{3}{4} \text{ in.})(2)$	$\dfrac{U_{bs} F_u A_{nt}}{\Omega} = 1.0(29.0 \text{ kips/in.})(\tfrac{3}{4} \text{ in.})(2)$
Shear yielding component from AISC *Manual* Table 9-3b:	Shear yielding component from AISC *Manual* Table 9-3b:
$\phi 0.60 F_y A_{gv} = 267 \text{ kips/in.}(\tfrac{3}{4} \text{ in.})(2)$	$\dfrac{0.60 F_y A_{gv}}{\Omega} = 178 \text{ kips/in.}(\tfrac{3}{4} \text{ in.})(2)$
Shear rupture component from AISC *Manual* Table 9-3c:	Shear rupture component from AISC *Manual* Table 9-3c:
$\phi 0.60 F_u A_{nv} = 287 \text{ kips/in.}(\tfrac{3}{4} \text{ in.})(2)$	$\dfrac{0.60 F_u A_{nv}}{\Omega} = 191 \text{ kips/in.}(\tfrac{3}{4} \text{ in.})(2)$
$\phi R_n = (43.5 \text{ kips} + 267 \text{ kips})(\tfrac{3}{4} \text{ in.})(2)$ $= 466 \text{ kips} > 258 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = (29.0 \text{ kips} + 178 \text{ kips})(\tfrac{3}{4} \text{ in.})(2)$ $= 311 \text{ kips} > 172 \text{ kips}$ **o.k.**

The flange thickness is greater than the angle thickness, the yield and tensile strengths of the flange are greater than that of the angles, L_{ev} = 1¾ in. for the brace is greater than 1½ in. for the angles and L_{eh} = 3³⁄₁₆ in. for the brace is greater than 1½ in. for the angles.

Therefore, by inspection, the block shear rupture strength of the brace flange is o.k.

Brace-Web-to-Gusset Connection

Determine number of ⅞-in.-diameter ASTM A325-N bolts required on the brace side (double shear) for shear.

From AISC *Manual* Table 7-1:

LRFD	ASD
$n_{min} = \dfrac{P_{uw}}{\phi r_n}$ $= \dfrac{159 \text{ kips}}{48.7 \text{ kips/bolt}}$ $= 3.26 \text{ bolts} \rightarrow 4 \text{ bolts}$	$n_{min} = \dfrac{P_{aw}}{r_n/\Omega}$ $= \dfrac{106 \text{ kips}}{32.5 \text{ kips/bolt}}$ $= 3.26 \text{ bolts} \rightarrow 4 \text{ bolts}$

On the gusset side, the same number of bolts are required. Try two rows of two bolts and two PL⅜ × 9.

Tensile Yielding of Plates

From AISC *Specification* Equation J4-1:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = \phi F_y A_g$ $= 0.90(36 \text{ ksi})(2)(3/8 \text{ in.})(9.00 \text{ in.})$ $= 219 \text{ kips} > 159 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{36 \text{ ksi}(2)(3/8 \text{ in.})(9.00 \text{ in.})}{1.67}$ $= 146 \text{ kips} > 106 \text{ kips}$ **o.k.**

Tensile Rupture of Plates

From AISC *Specification* Section J4.1, take A_e as the lesser of A_n and $0.85A_g$.

$$\begin{aligned}A_e &= \min\left(A_n, 0.85A_g\right)\\ &= \min\left\{(3/8 \text{ in.})\left[2(9.00 \text{ in.}) - 4(1.00 \text{ in.})\right],\ 0.85(2)(3/8 \text{ in.})(9.00 \text{ in.})\right\}\\ &= 5.25 \text{ in.}^2\end{aligned}$$

From AISC *Specification* Equation J4-2:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi F_u A_e$ $= 0.75(58 \text{ ksi})(5.25 \text{ in.}^2)$ $= 228 \text{ kips} > 159 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{58 \text{ ksi}(5.25 \text{ in.}^2)}{2.00}$ $= 152 \text{ kips} > 106 \text{ kips}$ **o.k.**

Block Shear Rupture of Plates (Outer Blocks)

Use $n = 2$, $L_{ev} = 1\frac{1}{2}$ in., $L_{eh} = 1\frac{1}{2}$ in. and $U_{bs} = 1.0$.

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min\left(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv}\right)$ Tension rupture component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt} = 1.0\left(43.5 \text{ kips/in.}\right)\left(\tfrac{3}{8} \text{ in.}\right)\left(4\right)$ Shear yielding component from AISC *Manual* Table 9-3b: $\phi 0.60 F_y A_{gv} = 72.9 \text{ kips/in.}\left(\tfrac{3}{8} \text{ in.}\right)\left(4\right)$ Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv} = 78.3 \text{ kips/in.}\left(\tfrac{3}{8} \text{ in.}\right)\left(4\right)$ $\phi R_n = \left(43.5 \text{ kips} + 72.9 \text{ kips}\right)\left(\tfrac{3}{8} \text{ in.}\right)\left(4\right)$ $= 175 \text{ kips} > 159 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = 1.0\left(29.0 \text{ kips/in.}\right)\left(\tfrac{3}{8} \text{ in.}\right)\left(4\right)$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60 F_y A_{gv}}{\Omega} = 48.6 \text{ kips/in.}\left(\tfrac{3}{8} \text{ in.}\right)\left(4\right)$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60 F_u A_{nv}}{\Omega} = 52.2 \text{ kips/in.}\left(\tfrac{3}{8} \text{ in.}\right)\left(4\right)$ $\dfrac{R_n}{\Omega} = \left(29.0 \text{ kips} + 48.6 \text{ kips}\right)\left(\tfrac{3}{8} \text{ in.}\right)\left(4\right)$ $= 116 \text{ kips} > 106 \text{ kips}$ **o.k.**

Similarly, by inspection, because the tension area is larger for the interior blocks, the block shear rupture strength of the interior blocks of the brace-web plates is o.k.

Block Shear Rupture of Brace Web

Use n = 2, L_{ev} = 1¾ in., but use 1½ in. for calculations to account for possible underrun in brace length, and L_{eh} = 3 in.

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min\left(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv}\right)$ Tension rupture component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt} = 1.0\left(122 \text{ kips/in.}\right)\left(0.515 \text{ in.}\right)\left(2\right)$ Shear yielding component from AISC *Manual* Table 9-3b:	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = 1.0\left(81.3 \text{ kips/in.}\right)\left(0.515 \text{ in.}\right)\left(2\right)$ Shear yielding component from AISC *Manual* Table 9-3b:

$\phi 0.60 F_y A_{gv} = 101 \text{ kips/in.}(0.515 \text{ in.})(2)$ Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv} = 87.8 \text{ kips/in.}(0.515 \text{ in.})(2)$ $\phi R_n = (122 \text{ kips} + 87.8 \text{ kips})(0.515 \text{ in.})(2)$ $= 216 \text{ kips} > 159 \text{ kips}$ **o.k.**	$\dfrac{0.6 F_y A_{gv}}{\Omega} = 67.5 \text{ kips/in.}(0.515 \text{ in.})(2)$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.6 F_u A_{nv}}{\Omega} = 58.5 \text{ kips/in.}(0.515 \text{ in.})(2)$ $\dfrac{R_n}{\Omega} = (81.3 \text{ kips} + 58.5 \text{ kips})(0.515 \text{ in.})(2)$ $= 144 \text{ kips} > 106 \text{ kips}$ **o.k.**

Tensile Yielding of Brace

From AISC *Specification* Equation J4-1:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = \phi F_y A_g$ $= 0.90(50 \text{ ksi})(25.6 \text{ in.}^2)$ $= 1150 \text{ kips} > 675 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{50 \text{ ksi}(25.6 \text{ in.}^2)}{1.67}$ $= 766 \text{ kips} > 450 \text{ kips}$ **o.k.**

Tensile Rupture of Brace

Because the load is transmitted to all of the cross-sectional elements, $U = 1.0$ and $A_e = A_n$.

$$A_e = A_n = 25.6 \text{ in.}^2 - \left[4(0.810 \text{ in.}) + 2(0.515 \text{ in.})\right]\left(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.}\right)$$
$$= 21.3 \text{ in.}^2$$

From AISC *Specification* Equation J4-2:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi F_u A_e$ $= 0.75(65 \text{ ksi})(21.3 \text{ in.}^2)$ $= 1040 \text{ kips} > 675 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{F_u A_e}{\Omega}$ $= \dfrac{65 \text{ ksi}(21.3 \text{ in.}^2)}{2.00}$ $= 692 \text{ kips} > 450 \text{ kips}$ **o.k.**

Gusset Plate

From edge distance, spacing and thickness requirements of the angles and web plates, try PL ¾.

For the bolt layout in this example, because the gusset plate thickness is equal to the sum of the web plate thicknesses, block shear rupture of the gusset plate for the web force is o.k., by inspection.

Block Shear Rupture of Gusset Plate for Total Brace Force

U_{bs} = 1.0

From gusset plate geometry:

A_{gv} = 25.1 in.^{2}
A_{nv} = 16.9 in.^{2}
A_{nt} = 12.4 in.^{2}

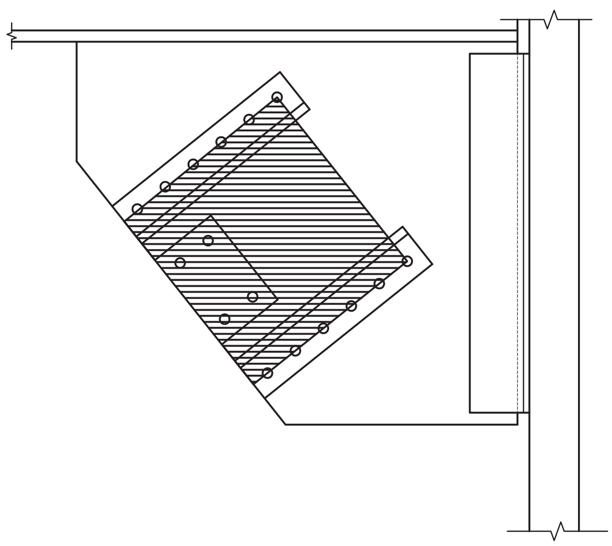


Fig. II.C-2-2. Block shear rupture area for gusset.

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi U_{bs} F_u A_{nt} + \min\left(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv}\right)$ Tension rupture component: $\phi U_{bs} F_u A_{nt} = 0.75(1.0)(58\text{ ksi})(12.4\text{ in.}^2)$ $= 539\text{ kips}$ Shear yielding component: $\phi 0.60 F_y A_{gv} = 0.75(0.60)(36\text{ ksi})(25.1\text{ in.}^2)$ $= 407\text{ kips}$ Shear rupture component: $\phi 0.60 F_u A_{nv} = 0.75(0.60)(58\text{ ksi})(16.9\text{ in.}^2)$ $= 441\text{ kips}$ $\phi R_n = 539\text{ kips} + \min(407\text{ kips},\ 441\text{ kips})$ $= 946\text{ kips} > 675\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ Tension rupture component: $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = \dfrac{1.0(58\text{ ksi})(12.4\text{ in.}^2)}{2.00}$ $= 360\text{ kips}$ Shear yielding component: $\dfrac{0.60 F_y A_{gv}}{\Omega} = \dfrac{0.60(36\text{ ksi})(25.1\text{ in.}^2)}{2.00}$ $= 271\text{ kips}$ Shear rupture component: $\dfrac{0.60 F_u A_{nv}}{\Omega} = \dfrac{0.60(58\text{ ksi})(16.9\text{ in.}^2)}{2.00}$ $= 294\text{ kips}$ $\dfrac{R_n}{\Omega} = 360\text{ kips} + \min(271\text{ kips},\ 294\text{ kips})$ $= 631\text{ kips} > 450\text{ kips}$ **o.k.**

Tensile Yielding on Whitmore Section of Gusset Plate

The Whitmore section, as illustrated with dashed lines in Figure II.C-2-1(b), is 34.8 in. long; 30.9 in. occurs in the gusset and 3.90 in. occurs in the beam web.

From AISC *Specification* Equation J4-1:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = \phi F_y A_g$ $= 0.90\begin{bmatrix}(36\text{ ksi})(30.9\text{ in.})(0.750\text{ in.}) + \\ (50\text{ ksi})(3.90\text{ in.})(0.590\text{ in.})\end{bmatrix}$ $= 854\text{ kips} > 675\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{\begin{bmatrix}(36\text{ ksi})(30.9\text{ in.})(0.750\text{ in.}) + \\ (50\text{ ksi})(3.90\text{ in.})(0.590\text{ in.})\end{bmatrix}}{1.67}$ $= 568\text{ kips} > 450\text{ kips}$ **o.k.**

Note: The beam web thickness is used, conservatively ignoring the larger thickness in the beam flange and the flange-to-web fillet area.

Bolt Bearing Strength of Angles, Brace Flange and Gusset Plate

By inspection, bolt bearing on the gusset plate controls.

For an edge bolt,

$$\begin{aligned} l_c &= 1\tfrac{3}{4}\text{ in.} - (0.5)\left(\tfrac{15}{16}\text{ in.}\right) \\ &= 1.28\text{ in.} \end{aligned}$$

From AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = \phi 1.2 l_c t F_u \le \phi 2.4 d t F_u$ $= 0.75(1.2)(1.28\text{ in.})(\tfrac{3}{4}\text{ in.})(58\text{ ksi})$ $\le (0.75)2.4(\tfrac{7}{8}\text{ in.})(\tfrac{3}{4}\text{ in.})(58\text{ ksi})$ $= 50.1\text{ kips} \le 68.5\text{ kips}$ $= 50.1\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega} \le \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{1.2(1.28\text{ in.})(\tfrac{3}{4}\text{ in.})(58\text{ ksi})}{2.00}$ $\le \dfrac{2.4(\tfrac{7}{8}\text{ in.})(\tfrac{3}{4}\text{ in.})(58\text{ ksi})}{2.00}$ $= 33.4\text{ kips} \le 45.7\text{ kips}$ $= 33.4\text{ kips/bolt}$

For an interior bolt,

$$\begin{aligned} l_c &= 3.00\text{ in.} - (1)\left(\tfrac{15}{16}\text{ in.}\right) \\ &= 2.06\text{ in.} \end{aligned}$$

From AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = \phi 1.2 l_c t F_u \le \phi 2.4 d t F_u$ $= 0.75(1.2)(2.06\text{ in.})(\tfrac{3}{4}\text{ in.})(58\text{ ksi})$ $\le 0.75(2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{3}{4}\text{ in.})(58\text{ ksi})$ $= 80.6\text{ kips} \le 68.5\text{ kips}$ $= 68.5\text{ kips/bolt}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega} \le \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{1.2(2.06\text{ in.})(\tfrac{3}{4}\text{ in.})(58\text{ ksi})}{2.00}$ $\le \dfrac{2.4(\tfrac{7}{8}\text{ in.})(\tfrac{3}{4}\text{ in.})(58\text{ ksi})}{2.00}$ $= 53.8\text{ kips} \le 45.7\text{ kips}$ $= 45.7\text{ kips/bolt}$

The total bolt bearing strength is:

LRFD	ASD
$\phi R_n = 1\text{ bolt}(50.1\text{ kips/bolt}) + 5\text{ bolts}(68.5\text{ kips/bolt})$ $= 393\text{ kips} > 258\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 1\text{ bolt}(33.4\text{ kips/bolt}) + 5\text{ bolts}(45.7\text{ kips/bolt})$ $= 262\text{ kips} > 172\text{ kips}$ **o.k.**

Note: If any of these bearing strengths were less than the bolt double shear strength; the bolt shear strength would need to be rechecked.

Bolt Bearing Strength of Brace Web, Web Plates and Gusset Plate

The total web plate thickness is the same as the gusset plate thickness, but since the web plates and brace web have a smaller edge distance due to possible underrun in brace length they control over the gusset plate for bolt bearing strength.

Accounting for a possible ¼ in. underrun in brace length, the brace web and web plates have the same edge distance. Therefore, the controlling element can be determined by finding the minimum tF_u. For the brace web, 0.515 in.(65 ksi) = 33.5 kip/in. For the web plates, ⅜ in.(2)(58 ksi) = 43.5 kip/in. The brace web controls for bolt bearing strength.

For an edge bolt,

$$\begin{aligned} l_c &= 1\tfrac{3}{4}\text{ in.} - \tfrac{1}{4}\text{ in.} - 0.5(\tfrac{15}{16}\text{ in.}) \\ &= 1.03\text{ in.} \end{aligned}$$

From AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = \phi 1.2 l_c t F_u \leq \phi 2.4 d t F_u$ $= 0.75(1.2)(1.03\text{ in.})(0.515\text{ in.})(65\text{ ksi})$ $\leq 0.75(2.4)(\tfrac{7}{8}\text{ in.})(0.515\text{ in.})(65\text{ ksi})$ $= 31.0\text{ kips} \leq 52.7\text{ kips}$ $= 31.0\text{ kips}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega} \leq \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{1.2(1.03\text{ in.})(0.515\text{ in.})(65\text{ ksi})}{2.00}$ $\leq \dfrac{2.4(\tfrac{7}{8}\text{ in.})(0.515\text{ in.})(65\text{ ksi})}{2.00}$ $= 20.7\text{ kips} \leq 35.1\text{ kips}$ $= 20.7\text{ kips}$

For an interior bolt,

$$\begin{aligned} l_c &= 3.00\text{ in.} - 1.0(\tfrac{15}{16}\text{ in.}) \\ &= 2.06\text{ in.} \end{aligned}$$

From AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = \phi 1.2 l_c t F_u \leq \phi 2.4 d t F_u$ $= 0.75(1.2)(2.06\text{ in.})(0.515\text{ in.})(65\text{ ksi})$ $\leq 0.75(2.4)(\tfrac{7}{8}\text{ in.})(0.515\text{ in.})(65\text{ ksi})$ $= 62.1\text{ kips} \leq 52.7\text{ kips}$ $= 52.7\text{ kips}$	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega} \leq \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{1.2(2.06\text{ in.})(0.515\text{ in.})(65\text{ ksi})}{2.00}$ $\leq \dfrac{2.4(\tfrac{7}{8}\text{ in.})(0.515\text{ in.})(65\text{ ksi})}{2.00}$ $= 41.4\text{ kips} \leq 35.1\text{ kips}$ $= 35.1\text{ kips}$

The total bolt bearing strength is:

LRFD	ASD
$\phi R_n = 2\text{ bolts}(31.0\text{ kips/bolt}) + 2\text{ bolts}(52.7\text{ kips/bolt})$ $= 167\text{ kips} > 159\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 2\text{ bolts}(20.7\text{ kips/bolt}) + 2\text{ bolts}(35.1\text{ kips/bolt})$ $= 112\text{ kips} > 106\text{ kips}$ **o.k.**

Note: The bearing strength for the edge bolts is less than the double shear strength of the bolts; therefore, the bolt group strength must be rechecked. Using the minimum of the bearing strength and the bolt shear strength for each bolt the revised bolt group strength is:

LRFD	ASD
$\phi R_n = 2\text{ bolts}(31.0\text{ kips/bolt}) + 2\text{ bolts}(48.7\text{ kips/bolt})$ $= 159\text{ kips} \geq 159\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = 2\text{ bolts}(20.7\text{ kips/bolt}) + 2\text{ bolts}(32.5\text{ kips/bolt})$ $= 106\text{ kips} \geq 106\text{ kips}$ **o.k.**

Note: When a brace force is compressive, gusset plate buckling would have to be checked. Refer to the comments at the end of this example.

Distribution of Brace Force to Beam and Column (AISC Manual Part 13)

From the member geometry:

$$e_b = \frac{d_{beam}}{2} = \frac{18.7\text{ in.}}{2} = 9.35\text{ in.}$$

$$e_c = \frac{d_{column}}{2} = \frac{20.9\text{ in.}}{2} = 10.5\text{ in.}$$

$$\tan\theta = \frac{12}{9\tfrac{9}{16}} = 1.25$$

$$e_b\tan\theta - e_c = 9.35\text{ in.}(1.25) - 10.5\text{ in.} = 1.19\text{ in.}$$

Try gusset PL¾ × 42 in. horizontally × 33 in. vertically (several intermediate gusset dimensions were inadequate). Place connection centroids at the midpoint of the gusset plate edges.

$$\bar{\alpha} = \frac{42.0\text{ in.}}{2} + 0.500\text{ in.} = 21.5\text{ in.}$$

½ in. is allowed for the setback between the gusset plate and the column.

$$\bar{\beta} = \frac{33.0\text{ in.}}{2}$$
$$= 16.5\text{ in.}$$

Choosing $\beta = \bar{\beta}$, the α required for the uniform forces from AISC *Manual* Equation 13-1 is:

$$\alpha = e_b \tan\theta - e_c + \beta\tan\theta$$
$$= 1.19\text{ in.} + 16.5\text{ in.}(1.25)$$
$$= 21.8\text{ in.}$$

The resulting eccentricity is $\alpha - \bar{\alpha}$.

$$\alpha - \bar{\alpha} = 21.8\text{ in.} - 21.5\text{ in.}$$
$$= 0.300\text{ in.}$$

Since this slight eccentricity is negligible, use α = 21.8 in. and β = 16.5 in.

Gusset Plate Interface Forces

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \qquad (\textit{Manual}\text{ Eq. 13-6})$$
$$= \sqrt{(21.8\text{ in.} + 10.5\text{ in.})^2 + (16.5\text{ in.} + 9.35\text{ in.})^2}$$
$$= 41.4\text{ in.}$$

On the gusset-to-column connection,

LRFD	ASD
$H_{uc} = \frac{e_c P_u}{r}$ (from *Manual* Eq. 13-3) $= \frac{10.5\text{ in.}(675\text{ kips})}{41.4\text{ in.}}$ $= 171\text{ kips}$ $V_{uc} = \frac{\beta P_u}{r}$ (from *Manual* Eq. 13-2) $= \frac{16.5\text{ in.}(675\text{ kips})}{41.4\text{ in.}}$ $= 269\text{ kips}$	$H_{ac} = \frac{e_c P_a}{r}$ (from *Manual* Eq. 13-3) $= \frac{10.5\text{ in.}(450\text{ kips})}{41.4\text{ in.}}$ $= 114\text{ kips}$ $V_{ac} = \frac{\beta P_a}{r}$ (from *Manual* Eq. 13-2) $= \frac{16.5\text{ in.}(450\text{ kips})}{41.4\text{ in.}}$ $= 179\text{ kips}$

On the gusset-to-beam connection,

LRFD	ASD
$H_{ub} = \frac{\alpha P_u}{r}$ (from *Manual* Eq. 13-5) $= \frac{21.8\text{ in.}(675\text{ kips})}{41.4\text{ in.}}$ $= 355\text{ kips}$	$H_{ab} = \frac{\alpha P_a}{r}$ (from *Manual* Eq. 13-5) $= \frac{21.8\text{ in.}(450\text{ kips})}{41.4\text{ in.}}$ $= 237\text{ kips}$

Return to Table of Contents

LRFD	ASD
$V_{ub} = \dfrac{e_b P_u}{r}$ (from *Manual* Eq. 13-4) $= \dfrac{9.35 \text{ in.}(675 \text{ kips})}{41.4 \text{ in.}}$ $= 152 \text{ kips}$	$V_{ab} = \dfrac{e_b P_a}{r}$ (from *Manual* Eq. 13-4) $= \dfrac{9.35 \text{ in.}(450 \text{ kips})}{41.4 \text{ in.}}$ $= 102 \text{ kips}$

Gusset Plate-to-Column Connection

The forces involved are:

V_{uc} = 269 kips and V_{ac} = 179 kips shear
H_{uc} = 171 kips and H_{ac} = 114 kips tension

Try 2L5×3½×⅝×2 ft 6 in. welded to the gusset plate and bolted with 10 rows of ⅞-in.-diameter A325-N bolts in standard holes to the column flange.

The required tensile strength per bolt is:

LRFD	ASD
$T_u = \dfrac{H_{uc}}{n}$ $= \dfrac{171 \text{ kips}}{20 \text{ bolts}}$ $= 8.55 \text{ kips/bolt}$	$T_a = \dfrac{H_{ac}}{n}$ $= \dfrac{114 \text{ kips}}{20 \text{ bolts}}$ $= 5.70 \text{ kips/bolt}$
Design strength of bolts for tension-shear interaction is determined from AISC *Specification* Section J3.7 as follows:	Allowable strength of bolts for tension-shear interaction is determined from AISC *Specification* Section J3.7 as follows:
$r_{uv} = \dfrac{V_{uc}}{n}$ $= \dfrac{269 \text{ kips}}{20 \text{ bolts}}$ $= 13.5 \text{ kips/bolt}$	$r_{av} = \dfrac{V_{ac}}{n}$ $= \dfrac{179 \text{ kips}}{20 \text{ bolts}}$ $= 8.95 \text{ kips/bolt}$
From AISC *Manual* Table 7-1, the bolt available shear strength is:	From AISC *Manual* Table 7-1, the bolt available shear strength is:
24.3 kips/bolt > 13.5 kips/bolt **o.k.**	16.2 kips/bolt > 8.95 kips/bolt **o.k.**
$f_{uv} = \dfrac{r_{uv}}{A_b}$ $= \dfrac{13.5 \text{ kips}}{0.601 \text{ in.}^2}$ $= 22.5 \text{ ksi}$	$f_{av} = \dfrac{r_{av}}{A_b}$ $= \dfrac{8.95 \text{ kips}}{0.601 \text{ in.}^2}$ $= 14.9 \text{ ksi}$
$\phi = 0.75$	$\Omega = 2.00$
From AISC *Specification* Table J3.2:	From AISC *Specification* Table J3.2:

LRFD	ASD
F_{nv} = 54 ksi and F_{nt} = 90 ksi $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}}{\phi F_{nv}} f_{uv} \le F_{nt}$ (*Spec*. Eq. J3-3a) $= 1.3(90\text{ ksi}) - \dfrac{90\text{ ksi}}{0.75(54\text{ ksi})}(22.5\text{ ksi})$ $= 67.0\text{ ksi} \le 90\text{ ksi}$ $B_u = \phi F'_{nt} A_b$ (*Spec*. Eq. J3-2) $= 0.75(67.0\text{ ksi})(0.601\text{ in.}^2)$ $= 30.2\text{ kips/bolt} > 8.55\text{ kips/bolt}$ **o.k.**	F_{nv} = 54 ksi and F_{nt} = 90 ksi $F'_{nt} = 1.3F_{nt} - \dfrac{F_{nt}\Omega}{F_{nv}} f_{av} \le F_{nt}$ (*Spec*. Eq. J3-3b) $= 1.3(90\text{ ksi}) - \dfrac{90\text{ ksi}(2.00)}{54\text{ ksi}}(14.9\text{ ksi})$ $= 67.3\text{ ksi} \le 90\text{ ksi}$ $B_a = \dfrac{F'_{nt} A_b}{\Omega}$ (*Spec*. Eq. J3-2) $= \dfrac{67.3\text{ ksi}(0.601\text{ in.}^2)}{2.00}$ $= 20.2\text{ kips/bolt} > 5.70\text{ kips/bolt}$ **o.k.**

Bolt Bearing Strength on Double Angles at Column Flange

From AISC *Specification* Equation J3-6a using L_c = 1½ in. – 0.5(15⁄16 in.) = 1.03 in. for an edge bolt.

LRFD	ASD
$\phi = 0.75$ $\phi r_n = \phi 1.2 l_c t F_u \le \phi 2.4 d t F_u$ $= 0.75(1.2)(1.03\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$ $\le 0.75(2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})$ $= 33.6\text{ kips} \le 57.1\text{ kips}$ $= 33.6\text{ kips/bolt}$ Since this edge bolt value exceeds the single bolt shear strength of 24.3 kips, and the actual shear per bolt of 13.5 kips, bolt shear and bolt bearing strengths are o.k.	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega} \le \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{1.2(1.03\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $\le \dfrac{2.4(\tfrac{7}{8}\text{ in.})(\tfrac{5}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $= 22.4\text{ kips} \le 38.1\text{ kips}$ $= 22.4\text{ kips/bolt}$ Since this edge bolt value exceeds the single bolt shear strength of 16.2 kips, and the actual shear per bolt of 8.95 kips, bolt shear and bolt bearing strengths are o.k.

The bearing strength of the interior bolts on the double angle will not control.

Prying Action on Double Angles (AISC Manual Part 9)

$$b = g - \frac{t}{2}$$
$$= 3.00\text{ in.} - \frac{\tfrac{5}{8}\text{ in.}}{2}$$
$$= 2.69\text{ in.}$$

$$a = 5.00\text{ in.} - g$$
$$= 5.00\text{ in.} - 3.00\text{ in.}$$
$$= 2.00\text{ in.}$$

$$b' = b - \frac{d_b}{2} \qquad \text{(\textit{Manual} Eq. 9-21)}$$
$$= 2.69 \text{ in.} - \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 2.25 \text{ in.}$$

$$a' = a + \frac{d_b}{2} \le \left(1.25b \text{ in.} + \frac{d_b}{2}\right) \qquad \text{(\textit{Manual} Eq. 9-27)}$$
$$= 2.00 \text{ in.} + \frac{\tfrac{7}{8} \text{ in.}}{2} \le 1.25(2.69 \text{ in.}) + \frac{\tfrac{7}{8} \text{ in.}}{2}$$
$$= 2.44 \text{ in.} \le 3.80 \text{ in.}$$

$$\rho = \frac{b'}{a'} \qquad \text{(\textit{Manual} Eq. 9-26)}$$
$$= \frac{2.25 \text{ in.}}{2.44 \text{ in.}}$$
$$= 0.922$$

LRFD	ASD
$\beta = \frac{1}{\rho}\left(\frac{B_u}{T_u} - 1\right)$ (*Manual* Eq. 9-25) $= \frac{1}{0.922}\left(\frac{30.2 \text{ kips/bolt}}{8.55 \text{ kips/bolt}} - 1\right)$ $= 2.75$	$\beta = \frac{1}{\rho}\left(\frac{B_a}{T_a} - 1\right)$ (*Manual* Eq. 9-25) $= \frac{1}{0.922}\left(\frac{20.2 \text{ kips/bolt}}{5.70 \text{ kips/bolt}} - 1\right)$ $= 2.76$

Because $\beta > 1$, set $\alpha' = 1.0$.

$$\delta = 1 - \frac{d'}{p} \qquad \text{(\textit{Manual} Eq. 9-24)}$$
$$= 1 - \frac{\tfrac{15}{16} \text{ in.}}{3.00 \text{ in.}}$$
$$= 0.688$$

LRFD	ASD
$\phi = 0.90$ $t_{req} = \sqrt{\frac{4T_u b'}{\phi p F_u (1 + \delta\alpha')}}$ (*Manual* Eq. 9-23a) $= \sqrt{\frac{4(8.55 \text{ kips/bolt})(2.25 \text{ in.})}{0.90(3.00 \text{ in.})(58 \text{ ksi})\left[1 + 0.688(1.0)\right]}}$ $= 0.540 \text{ in.} < 0.625 \text{ in.}$ **o.k.**	$\Omega = 1.67$ $t_{req} = \sqrt{\frac{\Omega 4T_a b'}{p F_u (1 + \delta\alpha')}}$ (*Manual* Eq. 9-23b) $= \sqrt{\frac{1.67(4)(5.70 \text{ kips/bolt})(2.25 \text{ in.})}{3.00 \text{ in.}(58 \text{ ksi})\left[1 + 0.688(1.0)\right]}}$ $= 0.540 \text{ in.} < 0.625 \text{ in.}$ **o.k.**

Use the 2L5x3½x⅝ for the gusset plate-to-column connection.

Weld Design

Try fillet welds around the perimeter (three sides) of both angles.

The resultant required force on the welds is:

LRFD	ASD
$P_{uc} = \sqrt{H_{uc}^2 + V_{uc}^2}$ $= \sqrt{(171\text{ kips})^2 + (269\text{ kips})^2}$ $= 319\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{H_{uc}}{V_{uc}}\right)$ $= \tan^{-1}\left(\dfrac{171\text{ kips}}{269\text{ kips}}\right)$ $= 32.4°$	$P_{ac} = \sqrt{H_{ac}^2 + V_{ac}^2}$ $= \sqrt{(114\text{ kips})^2 + (179\text{ kips})^2}$ $= 212\text{ kips}$ $\theta = \tan^{-1}\left(\dfrac{H_{ac}}{V_{ac}}\right)$ $= \tan^{-1}\left(\dfrac{114\text{ kips}}{179\text{ kips}}\right)$ $= 32.5°$

$l = 30.0\text{ in.}$

$kl = 3.00$ in., therefore, $k = 0.100$

$$xl = \frac{(kl)^2}{(l + 2kl)}$$
$$= \frac{(3.00\text{ in.})^2}{30.0\text{ in.} + 2(3.00\text{ in.})}$$
$$= 0.250\text{ in.}$$

$$al = 3.50\text{ in.} - xl$$
$$= 3.50\text{ in.} - 0.250\text{ in.}$$
$$= 3.25\text{ in.}$$

$a = 0.108$

By interpolating AISC *Manual* Table 8-8 with θ = 30°,

$C = 2.55$

LRFD	ASD
$\phi = 0.75$ $D_{req} = \dfrac{P_{uc}}{\phi C C_1 l}$ $= \dfrac{319\text{ kips}}{0.75(2.55)(1.0)(2\text{ welds})(30.0\text{ in.})}$ $= 2.78 \rightarrow 3\text{ sixteenths}$	$\Omega = 2.00$ $D_{req} = \dfrac{\Omega P_{ac}}{C C_1 l}$ $= \dfrac{(2.00)212\text{ kips}}{2.55(1.0)(2\text{ welds})(30.0\text{ in.})}$ $= 2.77 \rightarrow 3\text{ sixteenths}$

From AISC *Specification* Table J2.4, minimum fillet weld size is ¼ in. Use ¼-in. fillet welds.

Gusset Plate Thickness

$$t_{min} = \frac{6.19D}{F_u} \quad \text{(\textit{Manual} Eq. 9-3)}$$

$$= \frac{6.19(2.78 \text{ sixteenths})}{58 \text{ ksi}}$$

$$= 0.297 \text{ in.} < 3/4 \text{ in.} \quad \textbf{o.k.}$$

Shear Yielding of Angles (due to V_{uc} or V_{ac})

$$A_{gv} = 2(30.0 \text{ in.})(5/8 \text{ in.})$$

$$= 37.5 \text{ in.}^2$$

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(37.5 \text{ in.}^2)$ $= 810 \text{ kips} > 269 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{0.60 F_y A_{gv}}{\Omega}$ $= \frac{0.60(36 \text{ ksi})(37.5 \text{ in.}^2)}{1.50}$ $= 540 \text{ kips} > 179 \text{ kips}$ **o.k.**

Similarly, shear yielding of the angles due to H_{uc} and H_{ac} is not critical.

Shear Rupture of Angles

$$A_{nv} = 5/8 \text{ in.}\left[2(30.0 \text{ in.}) - 20(15/16 \text{ in.} + 1/16 \text{ in.})\right]$$

$$= 25.0 \text{ in.}^2$$

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58 \text{ ksi})(25.0 \text{ in.}^2)$ $= 653 \text{ kips} > 269 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\frac{R_n}{\Omega} = \frac{0.60 F_u A_{nv}}{\Omega}$ $= \frac{0.60(58 \text{ ksi})(25.0 \text{ in.}^2)}{2.00}$ $= 435 \text{ kips} > 179 \text{ kips}$ **o.k.**

Block Shear Rupture of Angles

Use $n = 10$, $L_{ev} = 1\frac{1}{2}$ in. and $L_{eh} = 2$ in.

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min\left(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv}\right)$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\phi U_{bs} F_u A_{nt} = 1.0\left(65.3\text{ kips/in.}\right)\left(\text{⅝ in.}\right)\left(2\right)$ Shear yielding component from AISC *Manual* Table 9-3b: $\phi 0.60 F_y A_{gv} = 462\text{ kips/in.}\left(\text{⅝ in.}\right)\left(2\right)$ Shear rupture component from AISC *Manual* Table 9-3c: $\phi 0.60 F_u A_{nv} = 496\text{ kips/in.}\left(\text{⅝ in.}\right)\left(2\right)$ $\phi R_n = \left(65.3\text{ kips} + 462\text{ kips}\right)\left(\text{⅝ in.}\right)\left(2\right)$ $= 659\text{ kips} > 269\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega},\ \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $U_{bs} = 1.0$ Tension rupture component from AISC *Manual* Table 9-3a: $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = 1.0\left(43.5\text{ kips/in.}\right)\left(\text{⅝ in.}\right)\left(2\right)$ Shear yielding component from AISC *Manual* Table 9-3b: $\dfrac{0.60 F_y A_{gv}}{\Omega} = 308\text{ kips/in.}\left(\text{⅝ in.}\right)\left(2\right)$ Shear rupture component from AISC *Manual* Table 9-3c: $\dfrac{0.60 F_u A_{nv}}{\Omega} = 331\text{ kips/in.}\left(\text{⅝ in.}\right)\left(2\right)$ $\dfrac{R_n}{\Omega} = \left(43.5\text{ kips} + 308\text{ kips}\right)\left(\text{⅝ in.}\right)\left(2\right)$ $= 439\text{ kips} > 179\text{ kips}$ **o.k.**

Column Flange

By inspection, the 4.16-in.-thick column flange has adequate flexural strength, stiffness and bearing strength.

Gusset Plate-to-Beam Connection

The forces involved are:

$V_{ub} = 152$ kips and $V_{ab} = 102$ kips
$H_{ub} = 355$ kips and $H_{ab} = 237$ kips

This edge of the gusset plate is welded to the beam. The distribution of force on the welded edge is known to be nonuniform. The Uniform Force Method, as shown in AISC *Manual* Part 13, used here assumes a uniform distribution of force on this edge. Fillet welds are known to have limited ductility, especially if transversely loaded. To account for this, the required strength of the gusset edge weld is amplified by a factor of 1.25 to allow for the redistribution of forces on the weld as discussed in Part 13 of the AISC *Manual*.

The stresses on the gusset plate at the welded edge are as follows:

From AISC *Specification* Sections J4.1(a) and J4.2:

LRFD	ASD
$f_{ua} = \dfrac{V_{ub}}{tl} \le \phi F_y$ $= \dfrac{152 \text{ kips}}{\frac{3}{4}\text{ in.}(42.0\text{ in.})} \le 0.90(36\text{ ksi})$ $= 4.83\text{ ksi} < 32.4\text{ ksi}$ **o.k.** $f_{uv} = \dfrac{H_{ub}}{tl} \le \phi 0.60 F_y$ $= \dfrac{355 \text{ kips}}{\frac{3}{4}\text{ in.}(42.0\text{ in.})} \le 1.00(0.60)(36\text{ ksi})$ $= 11.3\text{ ksi} < 21.6\text{ ksi}$ **o.k.**	$f_{aa} = \dfrac{V_{ab}}{tl} \le \dfrac{F_y}{\Omega}$ $= \dfrac{102 \text{ kips}}{\frac{3}{4}\text{ in.}(42.0\text{ in.})} \le \dfrac{36\text{ ksi}}{1.67}$ $= 3.24\text{ ksi} < 21.6\text{ ksi}$ **o.k.** $f_{av} = \dfrac{H_{ab}}{tl} \le \dfrac{0.60 F_y}{\Omega}$ $= \dfrac{237 \text{ kips}}{\frac{3}{4}\text{ in.}(42.0\text{ in.})} \le \dfrac{0.60(36\text{ ksi})}{1.50}$ $= 7.52\text{ ksi} < 14.4\text{ ksi}$ **o.k.**

LRFD	ASD
$\theta = \tan^{-1}\left(\dfrac{V_{ub}}{H_{ub}}\right)$ $= \tan^{-1}\left(\dfrac{152\text{ kips}}{355\text{ kips}}\right)$ $= 23.2°$	$\theta = \tan^{-1}\left(\dfrac{V_{ab}}{H_{ab}}\right)$ $= \tan^{-1}\left(\dfrac{102\text{ kips}}{237\text{ kips}}\right)$ $= 23.3°$

From AISC *Specification* Equation J2-5 and AISC *Manual* Part 8:

$$\mu = 1.0 + 0.50\sin^{1.5}\theta$$
$$= 1.0 + 0.50\sin^{1.5}(23.2°)$$
$$= 1.12$$

LRFD	ASD
The weld strength per 1⁄16 in. is as follows from AISC *Manual* Equation 8-2a: $\phi r_w = 1.392\text{ kips/in.}(1.12)$ $= 1.56\text{ kips/in.}$ The peak weld stress is: $f_{u\,peak} = \left(\dfrac{t}{2}\right)\sqrt{(f_{ua}+f_{ub})^2 + f_{uv}^{\,2}}$ $= \left(\dfrac{\frac{3}{4}\text{ in.}}{2}\right)\sqrt{(4.83\text{ ksi}+0\text{ ksi})^2 + (11.3\text{ ksi})^2}$ $= 4.61\text{ kips/in.}$	The weld strength per 1⁄16 in. is as follows from AISC *Manual* Equation 8-2b: $\dfrac{r_w}{\Omega} = 0.928\text{ kips/in.}(1.12)$ $= 1.04\text{ kips/in.}$ The peak weld stress is: $f_{a\,peak} = \left(\dfrac{t}{2}\right)\sqrt{(f_{aa}+f_{ab})^2 + f_{av}^{\,2}}$ $= \left(\dfrac{\frac{3}{4}\text{ in.}}{2}\right)\sqrt{(3.24\text{ ksi}+0\text{ ksi})^2 + (7.52\text{ ksi})^2}$ $= 3.07\text{ kips/in.}$

LRFD	ASD
The average stress is: $$f_{u\ ave}=\frac{\frac{t}{2}\left[\sqrt{\left(f_{ua}-f_{ub}\right)^2+f_{uv}{}^2}+\sqrt{\left(f_{ua}+f_{ub}\right)^2+f_{uv}{}^2}\right]}{2}$$ Because f_{ub} = 0 ksi (there is no moment on the edge), $f_{u\ ave}=f_{u\ peak}$ = 4.61 kips/in. The design weld stress is: $$\begin{aligned}f_{u\ weld}&=\max\left\{f_{u\ peak},\ 1.25f_{u\ ave}\right\}\\&=\max\left\{\begin{matrix}4.61\text{ kips/in.},\\1.25\left(4.61\text{ kips/in.}\right)\end{matrix}\right\}\\&=5.76\text{ kips/in.}\end{aligned}$$ The required weld size is: $$\begin{aligned}D_{req}&=\frac{f_{u\ weld}}{\phi r_w}\\&=\frac{5.76\text{ kips/in.}}{1.56\text{ kips/in.}}\\&=3.69\rightarrow 4\text{ sixteenths}\end{aligned}$$	The average stress is: $$f_{a\ ave}=\frac{\frac{t}{2}\left[\sqrt{\left(f_{aa}-f_{ab}\right)^2+f_{av}{}^2}+\sqrt{\left(f_{aa}+f_{ab}\right)^2+f_{av}{}^2}\right]}{2}$$ Because f_{ab} = 0 ksi (there is no moment on the edge), $f_{a\ ave}=f_{a\ peak}$ = 3.07 kips/in. The design weld stress is: $$\begin{aligned}f_{a\ weld}&=\max\left\{f_{a\ peak},\ 1.25f_{a\ ave}\right\}\\&=\max\left\{\begin{matrix}3.07\text{ kips/in.},\\1.25\left(3.07\text{ kips/in.}\right)\end{matrix}\right\}\\&=3.84\text{ kips/in.}\end{aligned}$$ The required weld size is: $$\begin{aligned}D_{req}&=\frac{f_{a\ weld}}{r_w/\Omega}\\&=\frac{3.84\text{ kips/in.}}{1.04\text{ kips/in.}}\\&=3.69\rightarrow 4\text{ sixteenths}\end{aligned}$$

From AISC *Specification* Table J2.4, the minimum fillet weld size is ¼ in. Use a ¼-in. fillet weld.

Web Local Yielding of Beam

From AISC *Manual* Table 9-4:

LRFD	ASD
$$\begin{aligned}\phi R_n&=\phi R_1+l_b(\phi R_2)\\&=98.8\text{ kips}+42.0\text{ in.}(29.5\text{ kips/in.})\\&=1{,}340\text{ kips}>152\text{ kips}\end{aligned}$$ **o.k.**	$$\begin{aligned}\frac{R_n}{\Omega}&=\frac{R_1}{\Omega}+l_b\left(\frac{R_2}{\Omega}\right)\\&=65.9\text{ kips}+42.0\text{ in.}(19.7\text{ kips/in.})\\&=893\text{ kips}>102\text{ kips}\end{aligned}$$ **o.k.**

Web Crippling of Beam

$$\begin{aligned}\frac{l_b}{d}&=\frac{42.0\text{ in.}}{18.7\text{ in.}}\\&=2.25>0.2\end{aligned}$$

From AISC *Manual* Table 9-4:

LRFD	ASD
$\phi R_n = \phi R_5 + l_b(\phi R_6)$ $= 143 \text{ kips} + 42.0 \text{ in.}(16.9 \text{ kips/in.})$ $= 853 \text{ kips} > 152 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{R_5}{\Omega} + l_b\left(\dfrac{R_6}{\Omega}\right)$ $= 95.3 \text{ kips} + 42.0 \text{ in.}(11.3 \text{ kips/in.})$ $= 570 \text{ kips} > 102 \text{ kips}$ **o.k.**

Beam-to-Column Connection

Since the brace is in tension, the required strength of the beam-to-column connection is as follows.

LRFD	ASD
The required shear strength is: $R_{ub} + V_{ub} = 15 \text{ kips} + 152 \text{ kips}$ $= 167 \text{ kips}$ $H_u - H_{ub} = H_{uc}$ $= 171 \text{ kips}$ The required axial strength is determined as follows: $A_{ub} + (H_u - H_{ub}) = 0 \text{ kips} + 171 \text{ kips}$ $= 171 \text{ kips compression}$	The required shear strength is: $R_{ab} + V_{ab} = 10 \text{ kips} + 102 \text{ kips}$ $= 112 \text{ kips}$ $H_a - H_{ab} = H_{ac}$ $= 114 \text{ kips}$ The required axial strength is determined as follows: $A_{ab} + (H_a - H_{ab}) = 0 \text{ kips} + 114 \text{ kips}$ $= 114 \text{ kips compression}$

Try 2L8×6×⅞×1 ft 2½ in. LLBB (leg gage = 3 1⁄16 in.) welded to the beam web, bolted with five rows of ⅞-in.-diameter A325-N bolts in standard holes to the column flange.

Since the connection is in compression in this example, the bolts resist shear only, no tension. If the bolts were in tension, the angles would also have to be checked for prying action.

Bolt Shear

LRFD	ASD
$r_u = \dfrac{167 \text{ kips}}{10 \text{ bolts}}$ $= 16.7 \text{ kips/bolt}$ From AISC *Manual* Table 7-1: $\phi r_n = 24.3 \text{ kips/bolt} > 16.7 \text{ kips/bolt}$ **o.k.**	$r_a = \dfrac{112 \text{ kips}}{10 \text{ bolts}}$ $= 11.2 \text{ kips/bolt}$ From AISC *Manual* Table 7-1: $\dfrac{r_n}{\Omega} = 16.2 \text{ kips/bolt} > 11.2 \text{ kips/bolt}$ **o.k.**

Bolt Bearing

Bearing on the angles controls over bearing on the column flange.

$$l_c = 1\tfrac{1}{4} \text{ in.} - \frac{\tfrac{15}{16} \text{ in.}}{2}$$

$= 0.781$ in.

From AISC *Specification* Equation J3-6a:

LRFD	ASD
$\phi = 0.75$ $\phi r_n = \phi 1.2 l_c t F_u < \phi 2.4 d t F_u$ $= 0.75(1.2)(0.781\text{ in.})(\tfrac{7}{8}\text{ in.})(58\text{ ksi})$ $\leq 0.75(2.4)(\tfrac{7}{8}\text{ in.})(\tfrac{7}{8}\text{ in.})(58\text{ ksi})$ $= 35.7\text{ kips} < 79.9\text{ kips}$ $= 35.7$ kips/bolt Since this edge bolt value exceeds the single bolt shear strength of 24.3 kips, bearing does not control.	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{1.2 l_c t F_u}{\Omega} \leq \dfrac{2.4 d t F_u}{\Omega}$ $= \dfrac{1.2(0.781\text{ in.})(\tfrac{7}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $\leq \dfrac{2.4(\tfrac{7}{8}\text{ in.})(\tfrac{7}{8}\text{ in.})(58\text{ ksi})}{2.00}$ $= 23.8\text{ kips} < 53.3\text{ kips}$ $= 23.8$ kips/bolt Since this edge bolt value exceeds the single bolt shear strength of 16.2 kips, bearing does not control.

Weld Design

Try fillet welds around perimeter (three sides) of both angles.

LRFD	ASD
$P_{uc} = \sqrt{(171\text{ kips})^2 + (167\text{ kips})^2}$ $= 239$ kips $\theta = \tan^{-1}\left(\dfrac{171\text{ kips}}{167\text{ kips}}\right)$ $= 45.7°$	$P_{ac} = \sqrt{(114\text{ kips})^2 + (112\text{ kips})^2}$ $= 160$ kips $\theta = \tan^{-1}\left(\dfrac{114\text{ kips}}{112\text{ kips}}\right)$ $= 45.5°$

$l = 14.5$ in., $kl = 7.50$ in. and $k = 0.517$

$$xl = \frac{(kl)^2}{l + 2kl}$$
$$= \frac{(7.50\text{ in.})^2}{14.5\text{ in.} + 2(7.50\text{ in.})}$$
$$= 1.91\text{ in.}$$

$$al = 8.00\text{ in.} - xl$$
$$= 8.00\text{ in.} - 1.91\text{ in.}$$
$$= 6.09\text{ in.}$$

$a = 0.420$

By interpolating AISC *Manual* Table 8-8 for $\theta = 45°$:

$C = 3.55$

LRFD	ASD
$\phi = 0.75$ $D_{req} = \dfrac{P_{uc}}{\phi C C_1 l}$ $= \dfrac{239 \text{ kips}}{0.75(3.55)(1.0)(2 \text{ welds})(14.5 \text{ in.})}$ $= 3.10 \rightarrow 4 \text{ sixteenths}$	$\Omega = 2.00$ $D_{req} = \dfrac{\Omega P_{ac}}{C C_1 l}$ $= \dfrac{2.00(160 \text{ kips})}{3.55(1.0)(2 \text{ welds})(14.5 \text{ in.})}$ $= 3.11 \rightarrow 4 \text{ sixteenths}$

From AISC *Specification* Table J2.4, the minimum fillet weld size is ¼ in. Use ¼-in. fillet welds.

Beam Web Thickness

$$t_{min} = \frac{6.19D}{F_u} \qquad (\textit{Manual} \text{ Eq. 9-3})$$

$$= \frac{6.19(3.11 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.296 \text{ in.} < 0.590 \text{ in.} \qquad \textbf{o.k.}$$

Shear Yielding of Angles

$$A_{gv} = 2(14.5 \text{ in.})(\tfrac{7}{8} \text{ in.})$$

$$= 25.4 \text{ in.}^2$$

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36 \text{ ksi})(25.4 \text{ in.}^2)$ $= 549 \text{ kips} > 167 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36 \text{ ksi})(25.4 \text{ in.}^2)}{1.50}$ $= 366 \text{ kips} > 112 \text{ kips}$ **o.k.**

Similarly, shear yielding of the angles due to H_{uc} and H_{ac} is not critical.

Shear Rupture of Angles

$$A_{nv} = \tfrac{7}{8} \text{ in.}\left[2(14.5 \text{ in.}) - 10(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right]$$

$$= 16.6 \text{ in.}^2$$

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58\text{ ksi})(16.6\text{ in.}^2)$ $= 433\text{ kips} > 167\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58.0\text{ ksi})(16.6\text{ in.}^2)}{2.00}$ $= 289\text{ kips} > 112\text{ kips}$ **o.k.**

Block Shear Rupture of Angles

Use $n = 5$, $L_{ev} = 1\frac{1}{4}$ in. and $L_{eh} = 2.94$ in.

$$\begin{aligned}A_{nt} &= \left[(2.94\text{ in.}) - (0.5)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{7}{8}\text{ in.})(2) \\ &= 4.27\text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_{gv} &= 13.3\text{ in.}(\tfrac{7}{8}\text{ in.})(2) \\ &= 23.3\text{ in.}^2\end{aligned}$$

$$\begin{aligned}A_{nv} &= \left[(13.3\text{ in.}) - (4.50)(\tfrac{15}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](\tfrac{7}{8}\text{ in.})(2) \\ &= 15.4\text{ in.}^2\end{aligned}$$

$$\begin{aligned}U_{bs} F_u A_{nt} &= 1.0(58\text{ ksi})(4.27\text{ in.}^2) \\ &= 248\text{ kips}\end{aligned}$$

$$\begin{aligned}0.60 F_y A_{gv} &= 0.60(36\text{ ksi})(23.3\text{ in.}^2) \\ &= 503\text{ kips} \qquad \textbf{controls}\end{aligned}$$

$$\begin{aligned}0.60 F_u A_{nv} &= 0.60(58\text{ ksi})(15.4\text{ in.}^2) \\ &= 536\text{ kips}\end{aligned}$$

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi = 0.75$	$\Omega = 2.00$
$\phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv},\ \phi 0.60 F_u A_{nv})$ $= 0.75(248\text{ kips} + 503\text{ kips})$ $= 563\text{ kips} > 167\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega}, \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ $= \dfrac{248\text{ kips} + 503\text{ kips}}{2.00}$ $= 376\text{ kips} > 112\text{ kips}$ **o.k.**

Column Flange

By inspection, the 4.16-in.-thick column flange has adequate flexural strength, stiffness and bearing strength.

Note: When the brace is in compression, the buckling strength of the gusset would have to be checked as follows:

LRFD	ASD
$\phi R_n = \phi_c F_{cr} A_w$	$\dfrac{R_n}{\Omega} = \dfrac{F_{cr} A_w}{\Omega_c}$

In the preceding equation, $\phi_c F_{cr}$ or F_{cr}/Ω_c may be determined with Kl_1/r from AISC *Specification* Section J4.4, where l_1 is the perpendicular distance from the Whitmore section to the interior edge of the gusset plate. Alternatively, the average value of $l = (l_1 + l_2 + l_3)/3$ may be substituted, where these quantities are illustrated in the figure. Note that for this example, l_2 is negative since part of the Whitmore section is in the beam web.

The effective length factor K has been established as 0.5 by full scale tests on bracing connections (Gross, 1990). It assumes that the gusset plate is supported on both edges. In cases where the gusset plate is supported on one edge only, such as illustrated in Example II.C-3, Figure (d), the brace can more readily move out-of-plane and a sidesway mode of buckling can occur in the gusset. For that case, K should be taken as 1.2.

Gusset Plate Buckling

The area of the Whitmore section is:

$$A_w = 30.9 \text{ in.}(\tfrac{3}{4} \text{ in.}) + 3.90 \text{ in.}(0.590 \text{ in.})\left(\frac{50 \text{ ksi}}{36 \text{ ksi}}\right)$$
$$= 26.4 \text{ in.}^2$$

In the preceding equation, the area in the beam web is multiplied by the ratio 50/36 to convert the area to an equivalent area of ASTM A36 plate. Assume l_1 = 17.0 in.

$$\frac{Kl_1}{r} = \frac{0.5(17.0 \text{ in.})\left(\sqrt{12}\right)}{\tfrac{3}{4} \text{ in.}}$$
$$= 39.3$$

Because $Kl_1/r > 25$, use AISC *Specification* Section E3:

$$F_e = \frac{\pi^2 E}{\left(\dfrac{Kl}{r}\right)^2} \qquad \text{(}\textit{Spec}\text{. Eq. E3-4)}$$
$$= \frac{\pi^2(29{,}000 \text{ ksi})}{39.3^2}$$
$$= 185 \text{ ksi}$$

$$4.71\sqrt{\frac{E}{F_y}} = 4.71\sqrt{\frac{29{,}000 \text{ ksi}}{36 \text{ ksi}}}$$
$$= 134$$

$$F_{cr} = \left[0.658^{\frac{F_y}{F_e}}\right]F_y$$ (*Spec*. Eq. E3-2)

$$= \left[0.658^{\frac{36\text{ ksi}}{185\text{ ksi}}}\right](36\text{ ksi})$$

$$= 33.2\text{ ksi}$$

$$R_n = F_{cr}A_g$$ (*Spec*. Eq. E3-1)

$$= 33.2\text{ ksi}\left(26.4\text{ in.}^2\right)$$

$$= 876\text{ kips}$$

From AISC *Specification* Section E1:

LRFD	ASD
$\phi_c = 0.90$ $\phi_c R_n = 0.90(876\text{ kips})$ $= 788\text{ kips} > 675\text{ kips}$ **o.k.**	$\Omega_c = 1.67$ $\dfrac{R_n}{\Omega_c} = \dfrac{876\text{ kips}}{1.67}$ $= 525\text{ kips} > 450\text{ kips}$ **o.k.**

Reference:

Gross, J.L. (1990), "Experimental Study of Gusseted Connections," *Engineering Journal*, AISC, Vol. 27, No. 3, 3rd quarter, pp.89-97.

EXAMPLE II.C-3 BRACING CONNECTION

Given:

Each of the four designs shown for the diagonal bracing connection between the W14×68 brace, W24×55 beam and W14×211 column web have been developed using the Uniform Force Method (the General Case and Special Cases 1, 2, and 3).

For the given values of α and β, determine the interface forces on the gusset-to-column and gusset-to-beam connections for the following:

a. General Case of Figure (a)
b. Special Case 1 of Figure (b)
c. Special Case 2 of Figure (c)
d. Special Case 3 of Figure (d)

Brace Axial Load	P_u = ±195 kips	P_a = ±130 kips
Beam End Reaction	R_u = 44 kips	R_a = 29 kips
Beam Axial Load	A_u = 26 kips	A_a = 17 kips

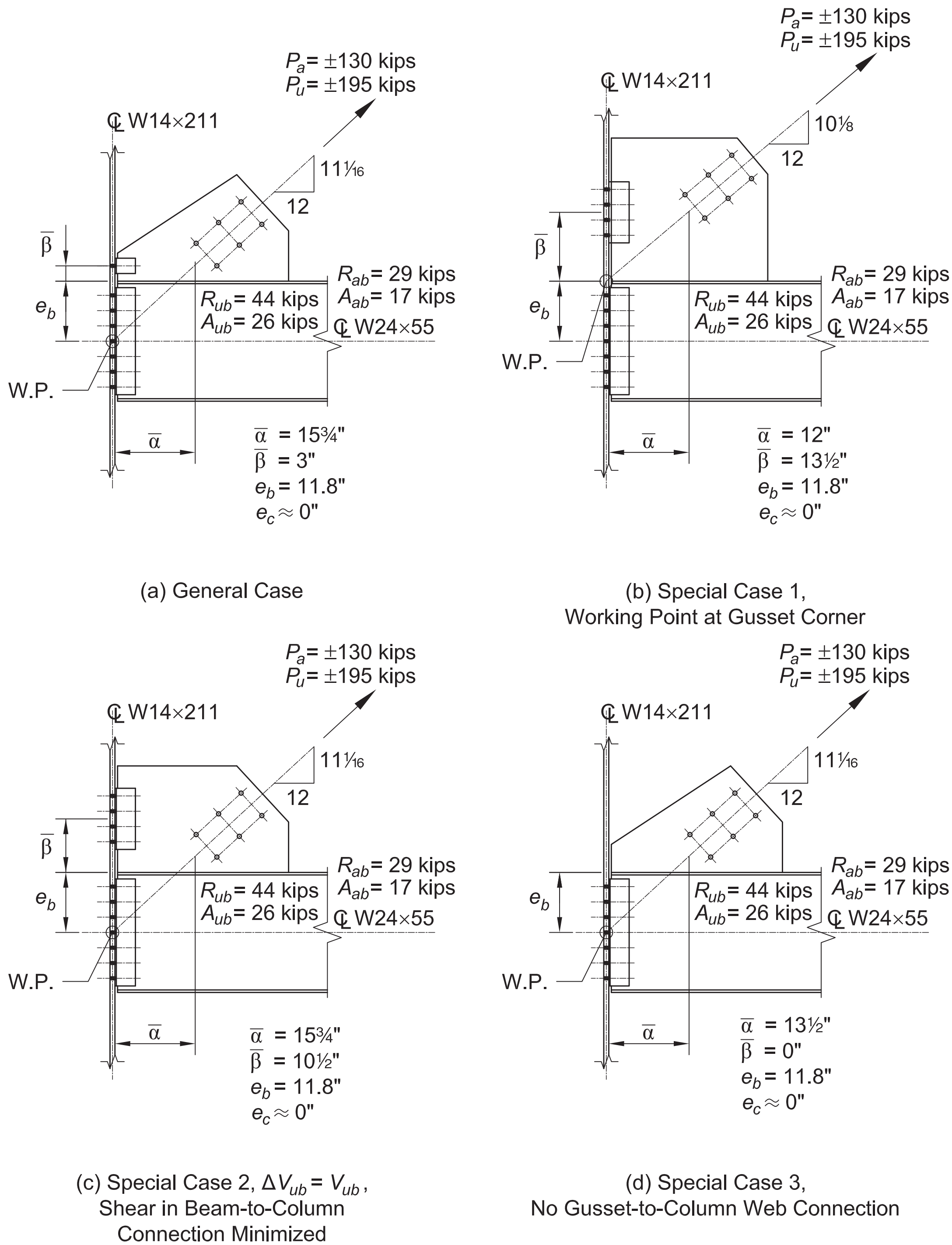


Fig. II.C-3-1. Bracing connection configurations for Example II.C-3.

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Brace
W14×68
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Beam
W24×55
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Column
W14×211
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Gusset Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Brace
W14×68
A = 20.0 in.^{2}
d = 14.0 in.
t_w = 0.415 in.
b_f = 10.0 in.
t_f = 0.720 in.

Beam
W24×55
d = 23.6 in.
t_w = 0.395 in.
b_f = 7.01 in.
t_f = 0.505 in.
k_{des} = 1.01 in.

Column
W14×211
d = 15.7 in.
t_w = 0.980 in.
b_f = 15.8 in.
t_f = 1.56 in.

Solution A (General Case):

Assume $\beta = \bar{\beta} = 3.00$ in.

From AISC *Manual* Equation 13-1:

$$\alpha = e_b \tan\theta - e_c + \beta \tan\theta$$
$$= 11.8 \text{ in.}\left(\frac{12}{11^1\!/\!_{16}}\right) - 0 + 3.00 \text{ in.}\left(\frac{12}{11^1\!/\!_{16}}\right)$$
$$= 16.1 \text{ in.}$$

Since $\alpha \neq \bar{\alpha}$, an eccentricity exists on the gusset-to-beam connection.

Interface Forces

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \qquad (\textit{Manual}\text{ Eq. 13-6})$$
$$= \sqrt{(16.1 \text{ in.} + 0 \text{ in.})^2 + (3.00 \text{ in.} + 11.8 \text{ in.})^2}$$
$$= 21.9 \text{ in.}$$

On the gusset-to-column connection:

LRFD	ASD
$V_{uc} = \dfrac{\beta}{r}P_u$ (*Manual* Eq. 13-2) $= \dfrac{3.00 \text{ in.}}{21.9 \text{ in.}}(195 \text{ kips})$ $= 26.7 \text{ kips}$ $H_{uc} = \dfrac{e_c}{r}P_u$ (*Manual* Eq. 13-3) $= 0 \text{ kips}$	$V_{ac} = \dfrac{\beta}{r}P_a$ (*Manual* Eq. 13-2) $= \dfrac{3.00 \text{ in.}}{21.9 \text{ in.}}(130 \text{ kips})$ $= 17.8 \text{ kips}$ $H_{ac} = \dfrac{e_c}{r}P_a$ (*Manual* Eq. 13-3) $= 0 \text{ kips}$

On the gusset-to-beam connection:

LRFD	ASD
$V_{ub} = \dfrac{e_b}{r}P_u$ (*Manual* Eq. 13-4) $= \dfrac{11.8 \text{ in.}}{21.9 \text{ in.}}(195 \text{ kips})$ $= 105 \text{ kips}$ $H_{ub} = \dfrac{\alpha}{r}P_u$ (*Manual* Eq. 13-5) $= \dfrac{16.1 \text{ in.}}{21.9 \text{ in.}}(195 \text{ kips})$ $= 143 \text{ kips}$ $M_{ub} = V_{ub}(\alpha - \bar{\alpha})$ $= \dfrac{105 \text{ kips}(16.1 \text{ in.} - 15\frac{3}{4} \text{ in.})}{12 \text{ in./ft}}$ $= 3.06 \text{ kip-ft}$	$V_{ab} = \dfrac{e_b}{r}P_a$ (*Manual* Eq. 13-4) $= \dfrac{11.8 \text{ in.}}{21.9 \text{ in.}}(130 \text{ kips})$ $= 70.0 \text{ kips}$ $H_{ab} = \dfrac{\alpha}{r}P_a$ (*Manual* Eq. 13-5) $= \dfrac{16.1 \text{ in.}}{21.9 \text{ in.}}(130 \text{ kips})$ $= 95.6 \text{ kips}$ $M_{ab} = V_{ab}(\alpha - \bar{\alpha})$ $= \dfrac{70.0 \text{ kips}(16.1 \text{ in.} - 15\frac{3}{4} \text{ in.})}{12 \text{ in./ft}}$ $= 2.04 \text{ kip-ft}$

In this case, this small moment is negligible.

On the beam-to-column connection, the required shear strength is:

LRFD	ASD
$R_{ub} + V_{ub} = 44.0 \text{ kips} + 105 \text{ kips}$ $= 149 \text{ kips}$	$R_{ab} + V_{ab} = 29.0 \text{ kips} + 70.0 \text{ kips}$ $= 99.0 \text{ kips}$

The required axial strength is

LRFD	ASD
$A_{ub} + H_{uc} = 26.0 \text{ kips} + 0 \text{ kips}$ $= 26.0 \text{ kips}$	$A_{ab} + H_{ac} = 17.0 \text{ kips} + 0 \text{ kips}$ $= 17.0 \text{ kips}$

For a discussion of the sign use between A_{ub} and H_{uc} (A_{ab} and H_{ac} for ASD), refer to Part 13 of the AISC *Manual*.

Solution B (Special Case 1):

In this case, the centroidal positions of the gusset edge connections are irrelevant; $\bar{\alpha}$ and $\bar{\beta}$ are given to define the geometry of the connection, but are not needed to determine the gusset edge forces.

The angle of the brace from the vertical is

$$\theta = \tan^{-1}\left(\frac{12}{10\frac{1}{8}}\right)$$
$$= 49.8°$$

The horizontal and vertical components of the brace force are:

LRFD	ASD
$H_u = P_u \sin\theta$ (*Manual* Eq. 13-9) $= (195 \text{ kips})\sin 49.8°$ $= 149 \text{ kips}$ $V_u = P_u \cos\theta$ (*Manual* Eq. 13-7) $= (195 \text{ kips})\cos 49.8°$ $= 126 \text{ kips}$	$H_a = P_a \sin\theta$ (*Manual* Eq. 13-9) $= (130 \text{ kips})\sin 49.8°$ $= 99.3 \text{ kips}$ $V_a = P_a \cos\theta$ (*Manual* Eq. 13-7) $= (130 \text{ kips})\cos 49.8°$ $= 83.9 \text{ kips}$

On the gusset-to-column connection:

LRFD	ASD
$V_{uc} = V_u = 126 \text{ kips}$ $H_{uc} = 0 \text{ kips}$ (*Manual* Eq. 13-10)	$V_{ac} = V_a = 83.9 \text{ kips}$ $H_{ac} = 0 \text{ kips}$ (*Manual* Eq. 13-10)

On the gusset-to-beam connection:

LRFD	ASD
$V_{ub} = 0 \text{ kips}$ (*Manual* Eq. 13-8) $H_{ub} = H_u = 149 \text{ kips}$	$V_{ab} = 0 \text{ kips}$ (*Manual* Eq. 13-8) $H_{ab} = H_a = 99.3 \text{ kips}$

On the beam-to-column connection:

LRFD	ASD
R_{ub} = 44.0 kips (shear) A_{ub} = 26.0 kips (axial transfer force)	R_{ab} = 29.0 kips (shear) A_{ab} = 17.0 kips (axial transfer force)

In addition to the forces on the connection interfaces, the beam is subjected to a moment M_{ub} or M_{ab}.

LRFD	ASD
$M_{ub} = H_{ub}e_b$ (*Manual* Eq. 13-11) $= \dfrac{149\text{ kips}(11.8\text{ in.})}{12\text{ in./ft}}$ $= 147$ kip-ft	$M_{ab} = H_{ab}e_b$ (*Manual* Eq. 13-11) $= \dfrac{99.3\text{ kips}(11.8\text{ in.})}{12\text{ in./ft}}$ $= 97.6$ kip-ft

This moment, as well as the beam axial load H_{ub} = 149 kips or H_{ab} = 99.3 kips and the moment and shear in the beam associated with the end reaction R_{ub} or R_{ab}, must be considered in the design of the beam.

Solution C (Special Case 2):

Assume $\beta = \bar{\beta} = 10\frac{1}{2}$ in.

From AISC *Manual* Equation 13-1:

$$\begin{aligned}\alpha &= e_b \tan\theta - e_c + \beta\tan\theta \\ &= 11.8\text{ in.}\left(\frac{12}{11\frac{1}{16}}\right) - 0 + 10\tfrac{1}{2}\text{ in.}\left(\frac{12}{11\frac{1}{16}}\right) \\ &= 24.2\text{ in.}\end{aligned}$$

Calculate the interface forces for the general case before applying Special Case 2.

$$r = \sqrt{(\alpha + e_c)^2 + (\beta + e_b)^2} \qquad (\textit{Manual}\text{ Eq. 13-6})$$
$$\begin{aligned} &= \sqrt{(24.2\text{ in.} + 0\text{ in.})^2 + (10\tfrac{1}{2}\text{ in.} + 11.8\text{ in.})^2} \\ &= 32.9\text{ in.}\end{aligned}$$

On the gusset-to-column connection:

LRFD	ASD
$V_{uc} = \dfrac{\beta}{r}P_u$ (*Manual* Eq. 13-2) $= \dfrac{10\frac{1}{2}\text{ in.}}{32.9\text{ in.}}(195\text{ kips})$ $= 62.2$ kips $H_{uc} = \dfrac{e_c}{r}P_u$ (*Manual* Eq. 13-3) $= 0$ kips	$V_{ac} = \dfrac{\beta}{r}P_a$ (*Manual* Eq. 13-2) $= \dfrac{10\frac{1}{2}\text{ in.}}{32.9\text{ in.}}(130\text{ kips})$ $= 41.5$ kips $H_{ac} = \dfrac{e_c}{r}P_a$ (*Manual* Eq. 13-3) $= 0$ kips

On the gusset-to-beam connection:

LRFD	ASD
$V_{ub} = \frac{e_b}{r} P_u$ (*Manual* Eq. 13-4) $= \frac{11.8\text{ in.}}{32.9\text{ in.}}(195\text{ kips})$ $= 69.9\text{ kips}$ $H_{ub} = \frac{\alpha}{r} P_u$ (*Manual* Eq. 13-5) $= \frac{24.2\text{ in.}}{32.9\text{ in.}}(195\text{ kips})$ $= 143\text{ kips}$	$V_{ab} = \frac{e_b}{r} P_a$ (*Manual* Eq. 13-4) $= \frac{11.8\text{ in.}}{32.9\text{ in.}}(130\text{ kips})$ $= 46.6\text{ kips}$ $H_{ab} = \frac{\alpha}{r} P_a$ (*Manual* Eq. 13-5) $= \frac{24.2\text{ in.}}{32.9\text{ in.}}(130\text{ kips})$ $= 95.6\text{ kips}$

On the beam-to-column connection, the shear is:

LRFD	ASD
$R_{ub} + V_{ub} = 44.0\text{ kips} + 69.9\text{ kips}$ $= 114\text{ kips}$	$R_{ab} + V_{ab} = 29.0\text{ kips} + 46.6\text{ kips}$ $= 75.6\text{ kips}$

The axial force is:

LRFD	ASD
$A_{ub} + H_{uc} = 26.0\text{ kips} + 0\text{ kips}$ $= 26.0\text{ kips}$	$A_{ab} + H_{ac} = 17.0\text{ kips} + 0\text{ kips}$ $= 17.0\text{ kips}$

Next, applying Special Case 2 with $\Delta V_{ub} = V_{ub} = 69.9$ kips ($\Delta V_{ab} = V_{ab} = 46.6$ kips for ASD), calculate the interface forces.

On the gusset-to-column connection (where V_{uc} is replaced by $V_{uc} + \Delta V_{ub}$) or (where V_{ac} is replaced by $V_{ac} + \Delta V_{ab}$ for ASD):

LRFD	ASD
$V_{uc} = 62.2\text{ kips} + 69.9\text{ kips}$ $= 132\text{ kips}$ $H_{uc} = 0\text{ kips}$ (unchanged)	$V_{ac} = 41.5\text{ kips} + 46.6\text{ kips}$ $= 88.1\text{ kips}$ $H_{ac} = 0\text{ kips}$ (unchanged)

On the gusset-to-beam connection (where V_{ub} is replaced by $V_{ub} - \Delta V_{ub}$) or (where V_{ab} is replaced by $V_{ab} - \Delta V_{ab}$):

LRFD	ASD
$H_{ub} = 143\text{ kips}$ (unchanged) $V_{ub} = 69.9\text{ kips} - 69.9\text{ kips}$ $= 0\text{ kips}$ $M_{ub} = (\Delta V_{ub})\alpha$ (*Manual* Eq. 13-13) $= \frac{69.9\text{ kips}(24.2\text{ in.})}{12\text{ in./ft}}$ $= 141\text{ kip-ft}$	$H_{ab} = 95.6\text{ kips}$ (unchanged) $V_{ab} = 46.6\text{ kips} - 46.6\text{ kips}$ $= 0\text{ kips}$ $M_{ab} = (\Delta V_{ab})\alpha$ (*Manual* Eq. 13-13) $= \frac{46.6\text{ kips}(24.2\text{ in.})}{12\text{ in./ft}}$ $= 94.0\text{ kip-ft}$

On the beam-to-column connection, the shear is:

LRFD	ASD
$R_{ub} + V_{ub} - \Delta V_{ub}$ $= 44.0 \text{ kips} + 69.9 \text{ kips} - 69.9 \text{ kips}$ $= 44.0 \text{ kips}$	$R_{ab} + V_{ab} - \Delta V_{ab}$ $= 29.0 \text{ kips} + 46.6 \text{ kips} - 46.6 \text{ kips}$ $= 29.0 \text{ kips}$

The axial force is:

LRFD	ASD
$A_{ub} + H_{uc} = 26.0 \text{ kips} \pm 0 \text{ kips}$ $= 26.0 \text{ kips}$	$A_{ab} + H_{ac} = 17.0 \text{ kips} \pm 0 \text{ kips}$ $= 17.0 \text{ kips}$

Solution D (Special Case 3):

Set $\beta = \bar{\beta} = 0$ in.

$$\alpha = e_b \tan\theta$$
$$= 11.8 \text{ in.}\left(\frac{12}{11\frac{1}{16}}\right)$$
$$= 12.8 \text{ in.}$$

Since, $\alpha \neq \bar{\alpha}$, an eccentricity exists on the gusset-to-beam connection.

Interface Forces

From AISC *Manual* Equation 13-6:

$$r = \sqrt{\alpha^2 + e_b{}^2}$$
$$= \sqrt{(12.8 \text{ in.})^2 + (11.8 \text{ in.})^2}$$
$$= 17.4 \text{ in.}$$

On the gusset-to-beam connection:

LRFD	ASD
$H_{ub} = \dfrac{\alpha}{r} P_u$ (*Manual* Eq. 13-5) $= \dfrac{12.8 \text{ in.}}{17.4 \text{ in.}}(195 \text{ kips})$ $= 143 \text{ kips}$	$H_{ab} = \dfrac{\alpha}{r} P_a$ (*Manual* Eq. 13-5) $= \dfrac{12.8 \text{ in.}}{17.4 \text{ in.}}(130 \text{ kips})$ $= 95.6 \text{ kips}$
$V_{ub} = \dfrac{e_b}{r} P_u$ (*Manual* Eq. 13-4) $= \dfrac{11.8 \text{ in.}}{17.4 \text{ in.}}(195 \text{ kips})$ $= 132 \text{ kips}$	$V_{ab} = \dfrac{e_b}{r} P_a$ (*Manual* Eq. 13-4) $= \dfrac{11.8 \text{ in.}}{17.4 \text{ in.}}(130 \text{ kips})$ $= 88.2 \text{ kips}$
$M_{ub} = V_{ub}(\alpha - \bar{\alpha})$ (*Manual* Eq. 13-14) $= \dfrac{132 \text{ kips}(12.8 \text{ in.} - 13\frac{1}{2} \text{ in.})}{12 \text{ in./ft}}$	$M_{ab} = V_{ab}(\alpha - \bar{\alpha})$ (*Manual* Eq. 13-14) $= \dfrac{88.2 \text{ kips}(12.8 \text{ in.} - 13\frac{1}{2} \text{ in.})}{12 \text{ in./ft}}$

LRFD	ASD
= -7.70 kip-ft	= -5.15 kip-ft

In this case, this small moment is negligible.

On the beam-to-column connection, the shear is:

LRFD	ASD
$R_{ub} + V_{ub}$ = 44.0 kips + 132 kips = 176 kips	$R_{ab} + V_{ab}$ = 29.0 kips + 88.2 kips = 117 kips

The axial force is:

LRFD	ASD
$A_{ub} + H_{uc}$ = 26 kips + 0 kips = 26.0 kips	$A_{ab} + H_{ac}$ = 17 kips + 0 kips = 17.0 kips

Note: Designs by Special Case 1 result in moments on the beam and/or column that must be considered.

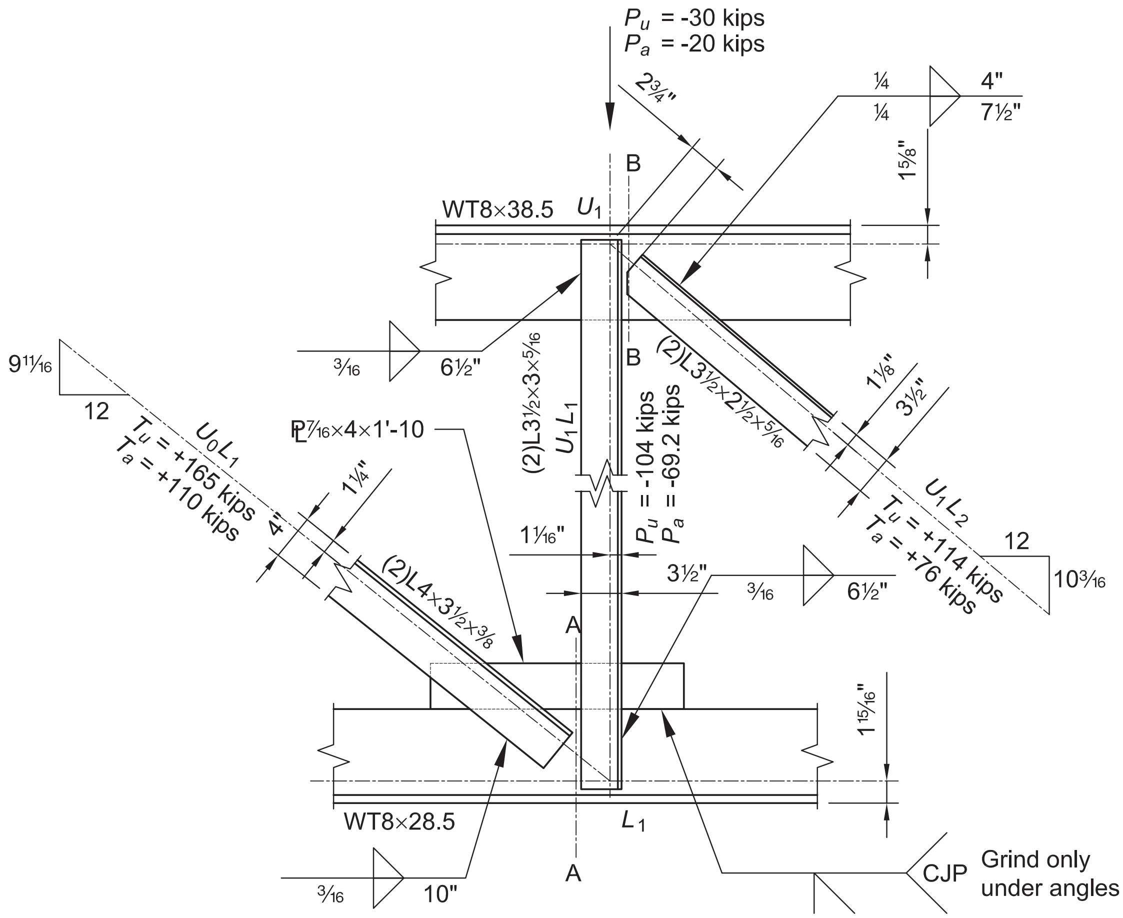
EXAMPLE II.C-4 TRUSS SUPPORT CONNECTION

Given:

Design the truss support connections at the following joints:

a. Joint L_1
b. Joint U_1

Use 70-ksi electrodes, ASTM A36 plate, ASTM A992 bottom and top chords, and ASTM A36 double angles.



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Top Chord
WT8×38.5
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Bottom Chord
WT8×28.5
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Diagonal U_0L_1
2L4×3½×⅜
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Web U_1L_1
2L3½×3×5⁄16
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Diagonal U_1L_2
2L3½×2½×5⁄16
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Plate
PL7⁄16×4×1'–10"
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-7, 1-8 and 1-15, the geometric properties are as follows:

Top Chord
WT8×38.5
t_w = 0.455 in.
d = 8.26 in.

Bottom Chord
WT8×28.5
t_w = 0.430 in.
d = 8.22 in.

Diagonal U_0L_1
2L4×3½×⅜
A = 5.36 in.^{2}
$\bar{x}$ = 0.947 in.

Web U_1L_1
2L3½×3×5⁄16
A = 3.90 in.^{2}

Diagonal U_1L_2
2L3½×2½×5⁄16
$A = 3.58$ in.^{2}
$\bar{x} = 0.632$ in.

LRFD	ASD
Web U_1L_1 load:	Web U_1L_1 load:
R_u = -104 kips	R_a = -69.2 kips
Diagonal U_0L_1 load:	Diagonal U_0L_1 load:
R_u = +165 kips	R_a = +110 kips
Diagonal U_1L_2 load:	Diagonal U_1L_2 load:
R_u = +114 kips	R_a = +76.0 kips

Solution a:

Shear Yielding of Bottom Chord Tee Stem (on Section A-A)

$$R_n = 0.60F_yA_{gv}$$ (*Spec*. Eq. J4-3)
$$= 0.60(50 \text{ ksi})(8.22 \text{ in.})(0.430 \text{ in.})$$
$$= 106 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$	$\Omega = 1.50$
$\phi R_n = 1.00(106 \text{ kips})$ $= 106 \text{ kips} > 104 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{106 \text{ kips}}{1.50}$ $= 70.7 \text{ kips} > 69.2 \text{ kips}$ **o.k.**

Welds for Member U_1L_1

Note: AISC *Specification* Section J1.7 requiring that the center of gravity of the weld group coincide with the center of gravity of the member does not apply to end connections of statically loaded single angle, double angle and similar members.

From AISC *Specification* Table J2.4, the minimum weld size is w_{min} = 3⁄16 in. The maximum weld size is w_{max} = thickness – 1⁄16 in. = ¼ in.

Using AISC *Manual* Part 8, Equations 8-2, the minimum length of 3⁄16-in. fillet weld is:

LRFD	ASD
$L_{min} = \dfrac{R_u}{1.392D}$ $= \dfrac{104 \text{ kips}}{1.392(3 \text{ sixteenths})}$ $= 24.9 \text{ in.}$	$L_{min} = \dfrac{R_a}{0.928D}$ $= \dfrac{69.2 \text{ kips}}{0.928(3 \text{ sixteenths})}$ $= 24.9 \text{ in.}$

Use 6½ in. of 3⁄16-in. weld at the heel and toe of both angles for a total of 26 in.

Minimum Angle Thickness to Match the Required Shear Rupture Strength of Welds

$$t_{min} = \frac{3.09D}{F_u}$$ (*Manual* Eq. 9-2)

$$= \frac{3.09(3 \text{ sixteenths})}{58 \text{ ksi}}$$

$$= 0.160 \text{ in.} < 5/16 \text{ in.} \quad \textbf{o.k.}$$

Minimum Stem Thickness to Match the Required Shear Rupture Strength of Welds

$$t_{min} = \frac{6.19D}{F_u}$$ (*Manual* Eq. 9-3)

$$= \frac{6.19(3 \text{ sixteenths})}{65 \text{ ksi}}$$

$$= 0.286 \text{ in.} < 0.430 \text{ in.} \quad \textbf{o.k.}$$

Top and bottom chords are o.k.

Welds for Member U_0L_1

From AISC *Specification* Table J2.4, the minimum weld size is $w_{min} = 3/16$ in. The maximum weld size is $w_{max} = \text{thickness} - 1/16 \text{ in.} = 5/16$ in.

Using AISC *Manual* Part 8, Equations 8-2, the minimum length of 3⁄16-in. fillet weld is:

LRFD	ASD
$L_{min} = \frac{R_u}{1.392D}$ $= \frac{165 \text{ kips}}{1.392(3 \text{ sixteenths})}$ $= 39.5$ in.	$L_{min} = \frac{R_a}{0.928D}$ $= \frac{110 \text{ kips}}{0.928(3 \text{ sixteenths})}$ $= 39.5$ in.

Use 10 in. of 3⁄16-in. weld at the heel and toe of both angles for a total of 40 in.

Note: A plate will be welded to the stem of the WT to provide room for the connection. Based on the preceding calculations for the minimum angle and stem thicknesses, by inspection the angles, stems, and stem plate extension have adequate strength.

Tensile Yielding of Diagonal U_0L_1

$$R_n = F_y A_g$$ (*Spec*. Eq. J4-1)

$$= 36 \text{ ksi}\left(5.36 \text{ in.}^2\right)$$

$$= 193 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(193\text{ kips})$ $= 174\text{ kips} > 165\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{193\text{ kips}}{1.67}$ $= 116\text{ kips} > 110\text{ kips}$ **o.k.**

Tensile Rupture of Diagonal U_0L_1

$U = 1 - \dfrac{\bar{x}}{l}$ from AISC *Specification* Table D3.1 Case 2

$$= 1 - \frac{0.947\text{ in.}}{10.0\text{ in.}}$$
$$= 0.905$$

$$R_n = F_u A_e \qquad (Spec.\ \text{Eq. J4-2})$$
$$= 58\text{ ksi}(0.905)(5.36\text{ in.}^2)$$
$$= 281\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(281\text{ kips})$ $= 211\text{ kips} > 165\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{281\text{ kips}}{2.00}$ $=141\text{ kips} > 110\text{ kips}$ **o.k.**

Block Shear Rupture of Bottom Chord

$$A_{nt} = 4.0\text{ in.}(0.430\text{ in.})$$
$$= 1.72\text{ in.}^2$$

$$A_{gv} = 10.0\text{ in.}(0.430\text{ in.})(2)$$
$$= 8.60\text{ in.}^2$$

From AISC *Specification* Equation J4-5:

$$R_n = U_{bs} F_u A_{nt} + 0.60 F_y A_{gv}$$
$$= 1.0(65\text{ ksi})(1.72\text{ in.}^2) + 0.60(36\text{ ksi})(8.60\text{ in.}^2)$$
$$= 298\text{ kips}$$

Because an ASTM A36 plate is used for the stem extension plate, use $F_y = 36$ ksi.

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(298\text{ kips})$ $= 224\text{ kips} > 165\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{298\text{ kips}}{2.00}$ $= 149\text{ kips} > 110\text{ kips}$ **o.k.**

Solution b:

Shear Yielding of Top Chord Tee Stem (on Section B-B)

$$R_n = 0.60F_y A_{gv} \qquad \text{(Spec. Eq. J4-3)}$$
$$= 0.60(50 \text{ ksi})(8.26 \text{ in.})(0.455 \text{ in.})$$
$$= 113 \text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(113 \text{ kips})$ $= 113 \text{ kips} > 74.0 \text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{113 \text{ kips}}{1.50}$ $= 75.3 \text{ kips} > 49.2 \text{ kips}$ **o.k.**

Welds for Member U_1L_1

As calculated previously in Solution a, use 6½ in. of 3⁄16-in. weld at the heel and toe of both angles for a total of 26 in.

Welds for Member U_1L_2

From AISC *Specification* Table J2.4, the minimum weld size is w_{min} = 3⁄16 in. The maximum weld size is w_{max} = ¼ in.

Using AISC *Manual* Part 8, Equations 8-2, the minimum length of ¼-in. fillet weld is:

LRFD	ASD
$L_{min} = \dfrac{R_u}{1.392D}$ $= \dfrac{114 \text{ kips}}{1.392(4 \text{ sixteenths})}$ $= 20.5 \text{ in.}$	$L_{min} = \dfrac{R_a}{0.928D}$ $= \dfrac{76.0 \text{ kips}}{0.928(4 \text{ sixteenths})}$ $= 20.5 \text{ in.}$

Use 7½ in. of ¼-in. fillet weld at the heel and 4 in. of ¼-in. fillet weld at the toe of each angle for a total of 23 in.

Minimum Angle Thickness to Match the Required Shear Rupture Strength of Welds

$$t_{min} = \frac{3.09D}{F_u} \qquad \text{(Manual Eq. 9-2)}$$
$$= \frac{3.09(4 \text{ sixteenths})}{58 \text{ ksi}}$$
$$= 0.213 \text{ in.} < 5/16 \text{ in.} \quad \textbf{o.k.}$$

Minimum Stem Thickness to Match the Required Shear Rupture Strength of Welds

$$t_{min} = \frac{6.19D}{F_u} \qquad \text{(Manual Eq. 9-3)}$$
$$= \frac{6.19(4 \text{ sixteenths})}{65 \text{ ksi}}$$

$= 0.381 \text{ in.} < 0.455 \text{ in.}$ **o.k.**

Tensile Yielding of Diagonal U_1L_2

$$
\begin{aligned}
R_n &= F_y A_g \\
&= 36 \text{ ksi}\left(3.58 \text{ in.}^2\right) \\
&= 129 \text{ kips}
\end{aligned}
$$
(*Spec*. Eq. J4-1)

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90\left(129 \text{ kips}\right)$ $= 116 \text{ kips} > 114 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{129 \text{ kips}}{1.67}$ $= 77.2 \text{ kips} > 76.0 \text{ kips}$ **o.k.**

Tensile Rupture of Diagonal U_1L_2

$U = 1 - \dfrac{\bar{x}}{l}$ from AISC *Specification* Table D3.1 Case 2

$$
\begin{aligned}
&= 1 - \frac{0.632 \text{ in.}}{\left(\dfrac{7.50 \text{ in.} + 4.00 \text{ in.}}{2}\right)} \\
&= 0.890
\end{aligned}
$$

$$
\begin{aligned}
R_n &= F_u A_e \\
&= 58 \text{ ksi}\left(0.890\right)\left(3.58 \text{ in.}^2\right) \\
&= 185 \text{ kips}
\end{aligned}
$$
(*Spec*. Eq. J4-2)

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75\left(185 \text{ kips}\right)$ $= 139 \text{ kips} > 114 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{185 \text{ kips}}{2.00}$ $= 92.5 \text{ kips} > 76.0 \text{ kips}$ **o.k.**

Example II.C-5 HSS Chevron Brace Connection

Given:

Verify that the chevron brace connection shown in Figure II.C-5-1 is adequate for the loading shown. The ASTM A36, ¾-in.-thick gusset plate is welded with 70-ksi electrode welds to an ASTM A992 W18×35 beam. The braces are ASTM A500 Grade B HSS6×6×½.

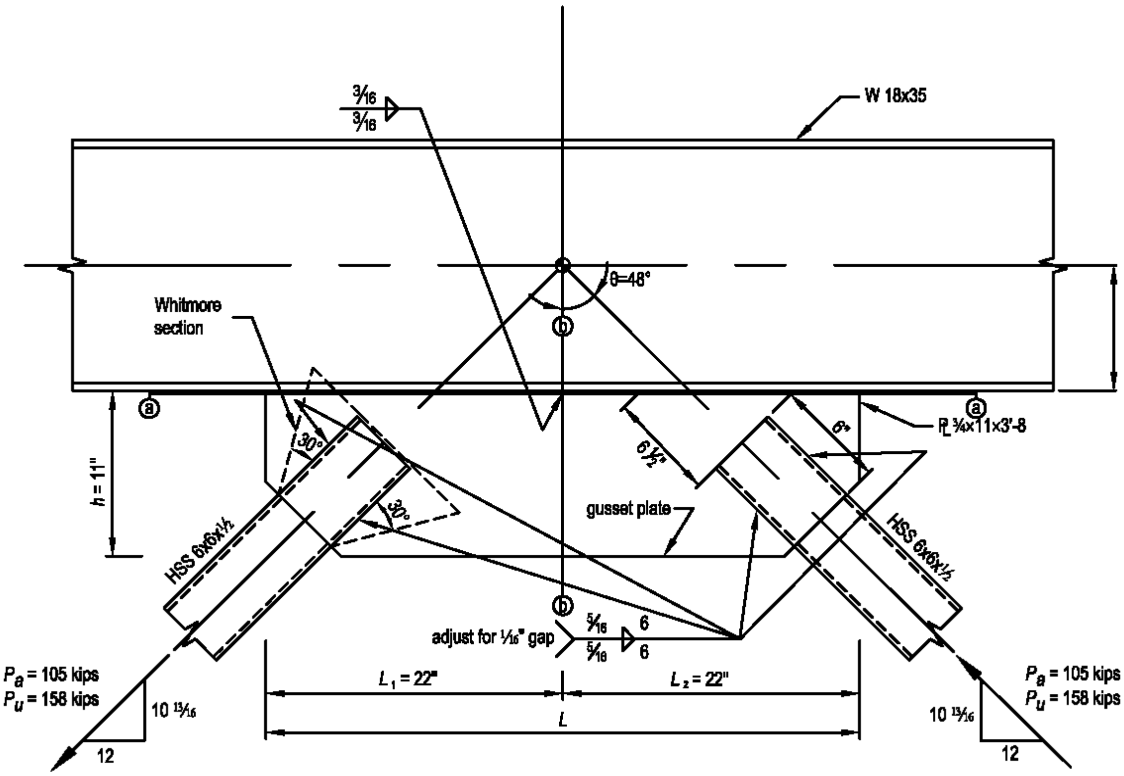


Fig. II.C-5-1. Layout for chevron brace connection.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×35
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Brace
HSS 6×6×½
ASTM A500 Grade B
F_y = 46 ksi
F_u = 58 ksi

Gusset Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

t_p = ¾ in.

From the AISC *Manual* Tables 1-1 and 1-12, the geometric properties are as follows:

Beam
W18×35
d = 17.7 in.
t_w = 0.300 in.
t_f = 0.425 in.
k_{des} = 0.827 in.
b_f = 6.00 in.

Brace
HSS 6×6×½
$H = B$ = 6.00 in.
A = 9.74 in.^{2}
t = 0.465 in.

Solution:

Calculate the interface forces (at the beam-gusset plate interface).

$\Delta = \frac{1}{2}(L_2 - L_1) = 0$ (Note: Δ is negative if $L_2 < L_1$; see Figure II.C-5-2.)

As shown in Figure II.C-5-1, the work point is at the concentric location at the beam gravity axis, e_b = 8.85 in. The brace bevels and loads are equal, thus the gusset will be symmetrical and Δ = 0.

Brace forces may both act in tension or compression, but the most common case is for one to be in tension and the other to be in compression, as shown for this example in Figure II.C-5-1.

From Figure II.C-5-1:

$$e_b = \frac{d}{2}$$
$$= \frac{17.7 \text{ in.}}{2}$$
$$= 8.85 \text{ in.}$$
$$\theta = \tan^{-1}\left(\frac{12}{10\,{}^{13}\!/_{16}}\right)$$
$$= 48.0^\circ$$
L = 44.0 in.
$L_1 = L_2$ = 22.0 in.
h = 11.0 in.

Determine the brace component forces and moments as indicated in the general case in Figure II.C-5-2.

LRFD	ASD
$P_{u1} = 158 \text{ kips}$ $H_{u1} = 158\sin 48^\circ$ $= 117 \text{ kips}$ $V_{u1} = 158\cos 48^\circ$ $= 106 \text{ kips}$ $P_{u2} = -158 \text{ kips}$ $H_{u2} = -117 \text{ kips}$ $V_{u2} = -106 \text{ kips}$ $M_{u1} = H_{u1}e_b + V_{u1}\Delta$ $= 117 \text{ kips}(8.85 \text{ in.}) + 106 \text{ kips}(0 \text{ in.})$ $= 1{,}040 \text{ kip-in.}$ $M_{u2} = H_{u2}e_b - V_{u2}\Delta$ $= -1{,}040 \text{ kip-in.}$ $M_{u1}' = \frac{1}{8}V_{u1}L - \frac{1}{4}H_{u1}h - \frac{1}{2}M_{u1}$ $= \frac{1}{8}(106 \text{ kips})(44.0 \text{ in.})$ $- \frac{1}{4}(117 \text{ kips})(11.0 \text{ in.})$ $- \frac{1}{2}(1{,}040 \text{ kip-in.})$ $= -259 \text{ kip-in.}$ $M_{u2}' = \frac{1}{8}V_{u2}L - \frac{1}{4}H_{u2}h - \frac{1}{2}M_{u2}$ $= \frac{1}{8}(-106 \text{ kips})(44.0 \text{ in.})$ $- \frac{1}{4}(-117 \text{ kips})(11.0 \text{ in.})$ $- \frac{1}{2}(-1{,}040 \text{ kip-in.})$ $= 259 \text{ kip-in.}$	$P_{a1} = 105 \text{ kips}$ $H_{a1} = 105\sin 48^\circ$ $= 78.0 \text{ kips}$ $V_{a1} = 105\cos 48^\circ$ $= 70.3 \text{ kips}$ $P_{a2} = -105 \text{ kips}$ $H_{a2} = -78.0 \text{ kips}$ $V_{a2} = -70.3 \text{ kips}$ $M_{a1} = H_{a1}e_b + V_{a1}\Delta$ $= 78.0 \text{ kips}(8.85 \text{ in.}) + 70.3 \text{ kips}(0 \text{ in.})$ $= 690 \text{ kip-in.}$ $M_{a2} = H_{a2}e_b - V_{a2}\Delta$ $= -690 \text{ kip-in.}$ $M_{a1}' = \frac{1}{8}V_{a1}L - \frac{1}{4}H_{a1}h - \frac{1}{2}M_{a1}$ $= \frac{1}{8}(70.3 \text{ kips})(44.0 \text{ in.})$ $- \frac{1}{4}(78.0 \text{ kips})(11.0 \text{ in.})$ $- \frac{1}{2}(690 \text{ kip-in.})$ $= -173 \text{ kip-in.}$ $M_{a2}' = \frac{1}{8}V_{a2}L - \frac{1}{4}H_{a2}h - \frac{1}{2}M_{a2}$ $= \frac{1}{8}(-70.3 \text{ kips})(44.0 \text{ in.})$ $- \frac{1}{4}(-78.0 \text{ kips})(11.0 \text{ in.})$ $- \frac{1}{2}(-690 \text{ kip-in.})$ $= 173 \text{ kip-in.}$

Note: The signs on the variables represent the directions of the forces shown on the Figures IIC-5-2,3 and 4. Forces are positive if in the directions shown in these figures, the forces are negative otherwise, and the signs must be used consistently in the formulas.

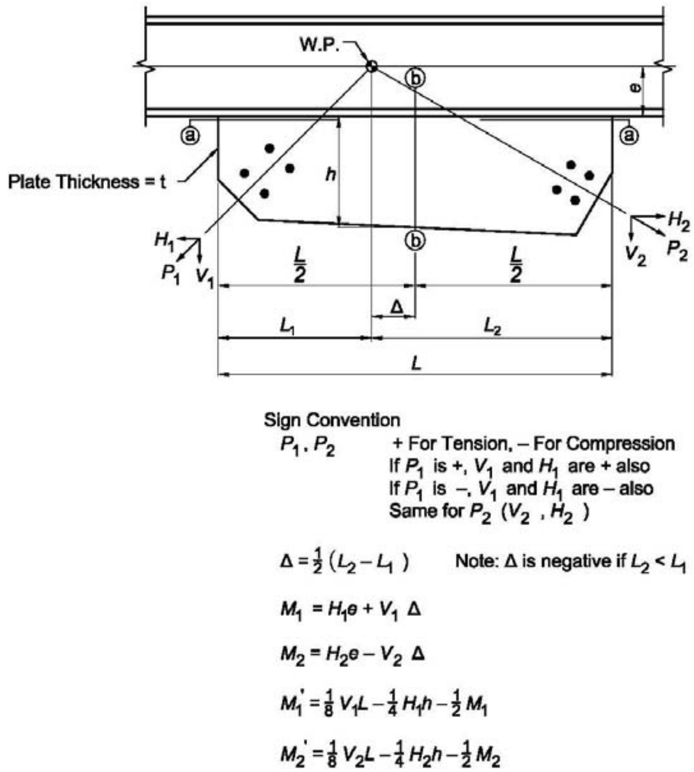


Fig. II.C-5-2. Chevron brace gusset forces—general case.
(Also see Figures II.C-5-3 and II.C-5-4 for forces and moments at Sections a-a and b-b.)

Forces for Section a-a (Gusset Edge Forces),

Determine the forces and moments at Section a-a in Figure II.C-5-1, as indicated in the general case in Figure II.C-5-3.

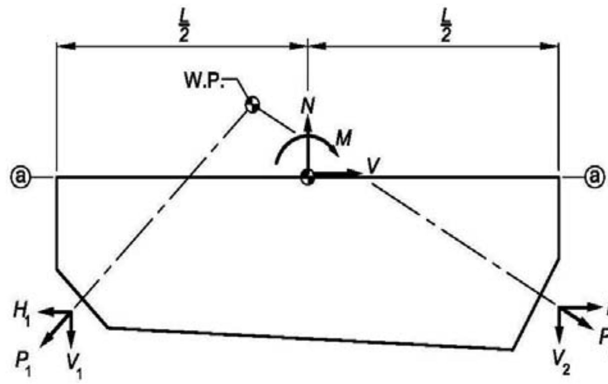
LRFD	ASD
Axial $N_u = V_{u1} + V_{u2}$ $= 106\text{ kips} + (-106\text{ kips})$ $= 0\text{ kips}$ Shear $V_u = H_{u1} - H_{u2}$ $= 117\text{ kips} - (-117\text{ kips})$ $= 234\text{ kips}$ Moment	Axial $N_a = V_{a1} + V_{a2}$ $= 70.3\text{ kips} + (-70.3\text{ kips})$ $= 0\text{ kips}$ Shear $V_a = H_{a1} - H_{a2}$ $= 78.0\text{ kips} - (-78.0\text{ kips})$ $= 156\text{ kips}$ Moment

$M_u = M_{u1} - M_{u2}$ $= 1{,}040 \text{ kip-in.} - (-1{,}040 \text{ kip-in.})$ $= 2{,}080 \text{ kip-in.}$	$M_a = M_{a1} - M_{a2}$ $= 690 \text{ kip-in.} - (-690 \text{ kip-in.})$ $= 1{,}380 \text{ kip-in.}$

Forces for Section b-b (Gusset Internal Forces)

Determine the forces and moments at Section b-b in Figure II.C-5-1, as indicated in the general case in Figure II.C-5-4.

LRFD	ASD
Axial $N'_u = \frac{1}{2}(H_{u1} + H_{u2})$ $= \frac{1}{2}[117 \text{ kips} + (-117 \text{ kips})]$ $= 0 \text{ kips}$ Shear $V'_u = \frac{1}{2}(V_{u1} - V_{u2}) - \frac{2M_u}{L}$ $= \frac{1}{2}[106 \text{ kips} - (-106 \text{ kips})]$ $- \frac{2(2{,}080 \text{ kip-in.})}{44.0 \text{ in.}}$ $= 11.5 \text{ kips}$ Moment $M'_u = M'_{u1} + M'_{u2}$ $= -259 \text{ kip-in.} + 259 \text{ kip-in.}$ $= 0 \text{ kip-in.}$	Axial $N'_a = \frac{1}{2}(H_{a1} + H_{a2})$ $= \frac{1}{2}[78.0 \text{ kips} + (-78.0 \text{ kips})]$ $= 0 \text{ kips}$ Shear $V'_a = \frac{1}{2}(V_{a1} - V_{a2}) - \frac{2M_a}{L}$ $= \frac{1}{2}[70.3 \text{ kips} - (-70.3 \text{ kips})]$ $- \frac{2(1{,}380 \text{ kip-in.})}{44.0 \text{ in.}}$ $= 7.57 \text{ kips}$ Moment $M'_a = M'_{a1} + M'_{a2}$ $= -173 \text{ kip-in.} + 173 \text{ kip-in.}$ $= 0 \text{ kip-in.}$



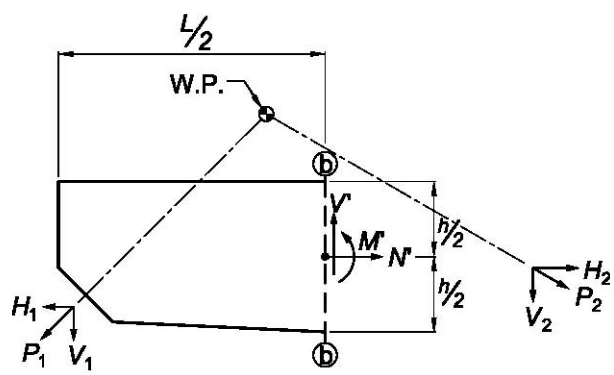
Forces on Section a-a

Axial: $N = V_1 + V_2$

Shear: $V = H_1 - H_2$

Moment: $M = M_1 - M_2$

Fig. II.C-5-3. Forces on Section a-a (positive directions shown)—general case.



Forces on Section b-b:

Axial: $N' = \frac{1}{2}(H_1 + H_2)$

Shear: $V' = \frac{1}{2}(V_1 - V_2) - \frac{2}{L}(M)$

Moment: $M' = M_1' + M_2'$

Fig. II.C-5-4. Forces on Section b-b (positive directions shown)—general case.

Return to Table of Contents

Design Brace-to-Gusset Connection

This part of the connection should be designed first because it will give a minimum required size of the gusset plate.

Brace Gross Tension Yielding

From AISC *Specification* Equation J4-1, determine the available strength due to tensile yielding in the gross section as follows:

LRFD	ASD
$\phi R_n = \phi F_y A_g$ $= 0.90(46\text{ ksi})(9.74\text{ in.}^2)$ $= 403\text{ kips} > 158\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{(46\text{ ksi})(9.74\text{ in.}^2)}{1.67}$ $= 268\text{ kips} > 105\text{ kips}$ **o.k.**

Brace Shear Rupture

Because net tension rupture involves shear lag, first determine the weld length, l, required for shear rupture of the brace.

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi R_n = \phi 0.6 F_u A_{nv}$ 158 kips = 0.75(0.60) (58 ksi) (0.465 in.) (l)(4) Therefore, l = 3.25 in.	$\dfrac{R_n}{\Omega} = \dfrac{0.6 F_u A_{nv}}{\Omega}$ $105\text{ kips} = \dfrac{0.60(58\text{ ksi})(0.465\text{ in.})(l)(4)}{2.00}$ Therefore, $l = 3.24\text{ in.}$

Assume a 5⁄16-in. fillet weld. The weld length, l_w, required is determined from AISC *Manual* Equation 8-2:

LRFD	ASD
$l_w = \dfrac{\phi R_n}{1.392(D)(4)}$ $= \dfrac{158\text{ kips}}{1.392(5)(4)}$ $= 5.68\text{ in.}$	$l_w = \dfrac{R_n / \Omega}{0.928(D)(4)}$ $= \dfrac{105\text{ kips}}{0.928(5)(4)}$ $= 5.66\text{ in.}$

Use 6-in.-long 5⁄16-in. fillet welds.

Brace Tension Rupture (Assume ¾-in.-thick gusset plate)

Determine the shear lag factor, U, from AISC *Specification* Table D3.1, Case 6. For a single concentric gusset plate:

$$\bar{x} = \frac{B^2 + 2BH}{4(B+H)}$$
$$= \frac{(6.00 \text{ in.})^2 + 2(6.00 \text{ in.})(6.00 \text{ in.})}{4(6.00 \text{ in.} + 6.00 \text{ in.})}$$
$$= 2.25 \text{ in.}$$

$$U = 1 - \frac{\bar{x}}{l}$$
$$= 1 - \frac{2.25 \text{ in.}}{6.00 \text{ in.}}$$
$$= 0.625$$

$$A_n = A_g - 2td_{slot} \qquad d_{slot} = \text{slot width}$$
$$A_n = 9.74 \text{ in.}^2 - 2(0.465 \text{ in.})(\tfrac{3}{4} \text{ in.} + \tfrac{1}{8} \text{ in.})$$
$$= 8.93 \text{ in.}^2$$

$$A_e = A_n U$$
$$= (8.93 \text{ in.}^2)(0.625)$$
$$= 5.58 \text{ in.}^2$$

The nominal tensile rupture of the brace is:

$$R_n = F_u A_e \qquad (Spec.\text{ Eq. J4-2})$$
$$= 58 \text{ ksi}(5.58 \text{ in.}^2)$$
$$= 324 \text{ kips}$$

The available tensile rupture strength of the brace is:

LRFD	ASD
$\phi R_n = 0.75(324 \text{ kips})$ $= 243 \text{ kips} > 158 \text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = \frac{324 \text{ kips}}{2.00}$ $= 162 \text{ kips} > 105 \text{ kips}$ **o.k.**

Check Block Shear on Gusset

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec.\text{ Eq. J4-5})$$

$$U_{bs} = 1.0$$

$$A_{gv} = A_{nv}$$
$$= 2t_p l_w$$
$$= 2(\tfrac{3}{4} \text{ in.})(6.00 \text{ in.})$$
$$= 9.00 \text{ in.}^2$$

$$A_{gt} = A_{nt}$$
$$= t_p B$$
$$= \tfrac{3}{4} \text{ in.}(6.00 \text{ in.})$$
$$= 4.50 \text{ in.}^2$$

$$F_u A_{nt} = 58 \text{ ksi}\left(4.50 \text{ in.}^2\right)$$
$$= 261 \text{ kips}$$
$$F_y A_{gv} = 36 \text{ ksi}\left(9.00 \text{ in.}^2\right)$$
$$= 324 \text{ kips}$$
$$F_u A_{nv} = 58 \text{ ksi}\left(9.00 \text{ in.}^2\right)$$
$$= 522 \text{ kips}$$

Shear Yielding:

$$R_n = 0.60 F_y A_{gv}$$
$$= 0.60\left(324 \text{ kips}\right)$$
$$= 194 \text{ kips} \quad \textbf{governs}$$

Shear Rupture:

$$R_n = 0.60 F_u A_{nv}$$
$$= 0.60\left(522 \text{ kips}\right)$$
$$= 313 \text{ kips}$$

The available block shear rupture strength is:

LRFD	ASD
$\phi R_n = 0.75\left(0.60 F_y A_{gv} + U_{bs} F_u A_{nt}\right)$ $= 0.75\left[194 \text{ kips} + 1.0\left(261 \text{ kips}\right)\right]$ $= 341 \text{ kips} > 158 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{1}{2.00}\left(0.60 F_y A_{gv} + U_{bs} F_u A_{nt}\right)$ $= \dfrac{1}{2.00}\left[194 \text{ kips} + 1.0\left(261 \text{ kips}\right)\right]$ $= 228 \text{ kips} > 105 \text{ kips}$ **o.k.**

Whitmore Tension Yield and Compression Buckling of Gusset Plate (AISC Manual Part 9)

Determine whether AISC *Specification* Equation J4-6 is applicable ($KL/r \le 25$).

$$r = \frac{t_p}{\sqrt{12}}$$
$$= \frac{\text{¾ in.}}{\sqrt{12}}$$
$$= 0.217 \text{ in.}$$

Assume K = 0.65, from Dowswell (2012).

From geometry, the unbraced gusset plate length is L = 6.50 in.

$$\frac{KL}{r} = \frac{0.65\left(6.50 \text{ in.}\right)}{0.217 \text{ in.}}$$
$$= 19.5$$

Return to Table of Contents

Based on AISC *Specification* Section J4.4, Equation J4-6 is applicable because $KL/r \leq 25$.

Determine the length of the Whitmore Section

$$
\begin{aligned}
l_w &= B + 2(\text{connection length})\tan 30^\circ \\
&= 6.00\text{ in.} + 2(6.00\text{ in.})\tan 30^\circ \\
&= 12.9\text{ in.}
\end{aligned}
$$

$$
\begin{aligned}
A_w &= l_w t_p \\
&= 12.9\text{ in.}(\text{w in.}) \\
&= 9.68\text{ in.}^2 \qquad \text{(Whitmore section is assumed to be entirely in gusset.)}
\end{aligned}
$$

$$
\begin{aligned}
P_n &= F_y A_w \qquad \text{(from } \textit{Spec.}\text{ Eq. J4-6)} \\
&= 36\text{ ksi}(9.68\text{ in.}^2) \\
&= 348\text{ kips}
\end{aligned}
$$

LRFD	ASD
$\phi P_n = 0.90(348\text{ kips})$ $= 313\text{ kips} > 158\text{ kips}$ **o.k.**	$\dfrac{P_n}{\Omega} = \dfrac{348\text{ kips}}{1.67}$ $= 208\text{ kips} > 105\text{ kips}$ **o.k.**

Gusset-to-Beam Connection

As determined previously, the forces and moment at Section a-a of Figure II.C-5-1:

LRFD		ASD	
Shear:	$V_u = 234\text{ kips}$	Shear:	$V_a = 156\text{ kips}$
Axial:	$N_u = 0\text{ kips}$	Axial:	$N_a = 0\text{ kips}$
Moment:	$M_u = 2{,}080\text{ kip-in.}$	Moment:	$M_a = 1{,}380\text{ kip-in.}$

The gusset stresses are:

LRFD	ASD
Shear: $f_v = \dfrac{V_u}{t_p L}$ $= \dfrac{234\text{ kips}}{\frac{3}{4}\text{ in.}(44.0\text{ in.})}$ $= 7.09\text{ ksi}$	Shear: $f_v = \dfrac{V_a}{t_p L}$ $= \dfrac{156\text{ kips}}{\frac{3}{4}\text{ in.}(44.0\text{ in.})}$ $= 4.73\text{ ksi}$

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi R_n = 1.00(0.60)(36\text{ ksi})$ $= 21.6\text{ ksi} > 7.09\text{ ksi}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60(36\text{ ksi})}{1.50}$ $= 14.4\text{ ksi} > 4.73\text{ ksi}$ **o.k.**

Axial: $$f_a = \frac{N_u}{t_p L} = \frac{0 \text{ kips}}{3/4 \text{ in.}(44.0 \text{ in.})} = 0 \text{ kips}$$ Moment: $$f_b = \frac{M_u}{Z_x} = \frac{M_u}{t_p L^2/4} = \frac{4(2{,}080 \text{ kip-in.})}{3/4 \text{ in.}(44.0 \text{ in.})^2} = 5.73 \text{ ksi}$$ Total Axial Stress: $$f_n = f_a + f_b = 0 \text{ ksi} + 5.73 \text{ ksi} = 5.73 \text{ ksi}$$	Axial: $$f_a = \frac{N_a}{t_p L} = \frac{0 \text{ kips}}{3/4 \text{ in.}(44.0 \text{ in.})} = 0 \text{ kips}$$ Moment: $$f_b = \frac{M_a}{Z_x} = \frac{M_a}{t_p L^2/4} = \frac{4(1{,}380 \text{ kip-in.})}{3/4 \text{ in.}(44.0 \text{ in.})^2} = 3.80 \text{ ksi}$$ Total Axial Stress: $$f_n = f_a + f_b = 0 \text{ ksi} + 3.80 \text{ ksi} = 3.80 \text{ ksi}$$

LRFD	ASD
Compare the total axial stress to the available stress from AISC *Specification* Section J4.1(a) for the limit state of tensile yielding. $5.73 \text{ ksi} \le 0.90\,(36 \text{ ksi}) = 32.4 \text{ ksi}$ **o.k.**	Compare the total axial stress to the available stress from AISC *Specification* Section J4.1(a) for the limit state of tensile yielding. $3.80 \text{ ksi} \le \frac{36 \text{ ksi}}{1.67} = 21.6 \text{ ksi}$ **o.k.**

Weld of Gusset to Beam

LRFD	ASD
$$R = \sqrt{V_u^2 + N_u^2} = \sqrt{(234 \text{ kips})^2 + (0 \text{ kips})^2} = 234 \text{ kips}$$	$$R = \sqrt{V_a^2 + N_a^2} = \sqrt{(156 \text{ kips})^2 + (0 \text{ kips})^2} = 156 \text{ kips}$$

The 234 kips (LRFD) and 156 kips (ASD) shear force is actually at the center of the beam; this is a function of the moment (2,080 kip-in. (LRFD) and 1,380 kips (ASD)). The effective eccentricity of V is:

LRFD	ASD
$e = \dfrac{M_u}{V_u} = \dfrac{2{,}080 \text{ kip-in.}}{234 \text{ kips}} = 8.89 \text{ in.}$	$e = \dfrac{M_a}{V_a} = \dfrac{1{,}380 \text{ kip-in.}}{156 \text{ kips}} = 8.85 \text{ in.}$

Note that e is e_b. The LRFD and ASD values for e differ due to rounding. Continue the problem with e = 8.89 in.

$$e = 8.89 \text{ in.}$$
$$\theta = 0°$$
$$k = 0$$
$$a = \frac{e}{L} = \frac{8.89 \text{ in.}}{44.0 \text{ in.}} = 0.202$$

From AISC *Manual* Table 8-4: $C = 3.50,\ C_1 = 1.00$

Applying a ductility factor of 1.25 as discussed in Part 13 of the AISC *Manual* to AISC *Manual* Equation 8-13, the weld required is determined as follows:

LRFD	ASD
$D_{req'd} = \dfrac{V_u(1.25)}{\phi C C_1 L} = \dfrac{234 \text{ kips}(1.25)}{0.75(3.50)(1.00)(44 \text{ in.})} = 2.53$	$D_{req'd} = \dfrac{\Omega V_a(1.25)}{C C_1 L} = \dfrac{2.00(156 \text{ kips})(1.25)}{3.50(1.00)(44 \text{ in.})} = 2.53$

A 3⁄16-in. fillet weld is required.

Verifying this with AISC *Specification* Table J2.4, the material thickness of the thinner part joined is t_f = 0.425 in. and the minimum size fillet weld is 3⁄16 in.

Therefore, use a 3⁄16-in. fillet weld.

Gusset Internal Stresses

The gusset internal forces and moment at Section b-b of Figure II.C-5-1, as determined previously, are:

LRFD		ASD	
Shear:	$V'_u = 11.5$ kips	Shear:	$V'_a = 7.57$ kips
Axial:	$N'_u = 0$ kips	Axial:	$N'_a = 0$ kips
Moment:	$M'_u = 0$ kip-in.	Moment:	$M'_a = 0$ kip-in.

Gusset Stresses

The limit of shear yielding of the gusset is checked using AISC *Specification* Equation J4-3 as follows:

LRFD	ASD
$f_v = \dfrac{V_u'}{t_p h}$ $= \dfrac{11.5\text{ kips}}{\text{w in.}(11.0\text{ in.})}$ $= 1.39\text{ ksi}$ $\le 1.00(0.60)(36\text{ ksi}) = 21.6\text{ ksi}$ **o.k.** $N_u' = M_u' = 0$ No check is required	$f_v = \dfrac{V_a'}{t_p h}$ $= \dfrac{7.57\text{ kips}}{\text{w in.}(11.0\text{ in.})}$ $= 0.918\text{ ksi}$ $\le \dfrac{0.60(36\text{ ksi})}{1.50} = 14.4\text{ ksi}$ **o.k.** $N_a' = M_a' = 0$ No check is required

If N' and M' are greater than zero, it is possible for a compressive stress to exist on the gusset free edge at Section b-b. In this case, the gusset should be checked for buckling under this stress. The procedure in AISC *Manual* Part 9 for buckling of a coped beam can be used. If gusset plate buckling controls, an edge stiffener could be added or a thicker plate used.

Check Web Local Yielding of Beam Under Normal Force

The limit state of web local yielding is checked using AISC *Specification* Equation J10-2, with $l_b = L = 44$ in. and $k = k_{des}$, as follows:

LRFD	ASD
$N_{max} = \lvert N_u \rvert + \left\lvert \dfrac{4M_u}{L} \right\rvert$ $= \lvert 0\text{ kips} \rvert + \left\lvert \dfrac{4(2{,}080\text{ kip-in.})}{44.0\text{ in.}} \right\rvert$ $= 189\text{ kips}$ $\phi R_n = \phi F_{yw} t_w (5k + l_b)$ $= 1.00(50\text{ ksi})(0.300\text{ in.})[5(0.827\text{ in.}) + 44.0\text{ in.}]$ $= 722\text{ kips} \ge 189\text{ kips}$ **o.k.**	$N_{max} = \lvert N_a \rvert + \left\lvert \dfrac{4M_a}{L} \right\rvert$ $= \lvert 0\text{ kips} \rvert + \left\lvert \dfrac{4(1{,}380\text{ kip-in.})}{44.0\text{ in.}} \right\rvert$ $= 125\text{ kips}$ $\dfrac{R_n}{\Omega} = \dfrac{F_{yw} t_w}{\Omega}(5k + l_b)$ $= \dfrac{(50\text{ ksi})(0.300\text{ in.})}{1.50}[5(0.827\text{ in.}) + 44.0\text{ in.}]$ $= 481\text{ kips} \ge 125\text{ kips}$ **o.k.**

Web Crippling Under Normal Load

From AISC *Specification* Equation J10-4:

LRFD	ASD
$\phi R_n = \phi 0.80 t_w^2 \left[1+3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_y t_f}{t_w}}$ $= 0.75(0.80)(0.300 \text{ in.})^2\left[1+3\left(\dfrac{44.0 \text{ in.}}{17.7 \text{ in.}}\right)\left(\dfrac{0.300 \text{ in.}}{0.425 \text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\dfrac{29{,}000(50 \text{ ksi})(0.425 \text{ in.})}{0.300 \text{ in.}}}$ $= 420 \text{ kips} \geq 189 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.80 t_w^2}{\Omega}\left[1+3\left(\dfrac{l_b}{d}\right)\left(\dfrac{t_w}{t_f}\right)^{1.5}\right]\sqrt{\dfrac{EF_y t_f}{t_w}}$ $= \dfrac{(0.80)(0.300 \text{ in.})^2}{2.00}\left[1+3\left(\dfrac{44.0 \text{ in.}}{17.7 \text{ in.}}\right)\left(\dfrac{0.300 \text{ in.}}{0.425 \text{ in.}}\right)^{1.5}\right]$ $\times\sqrt{\dfrac{29{,}000(50 \text{ ksi})(0.425 \text{ in.})}{0.300 \text{ in.}}}$ $= 280 \text{ kips} \geq 125 \text{ kips}$ **o.k.**

Check of Beam for Horizontal Shear

The length of beam web effective in carrying the shear on Section a-a can be determined based on the assumption that the critical section in the web is $L + 5k$ plus some web area length over which the force in the beam flange is transferred into the web area, that is determined as follows. If the flange area of the beam is $A_f = b_f t_f$, the nominal tensile yielding strength of the beam flange is $2F_yA_f$ from AISC *Specification* Equation J4-1, where the factor of 2 comes from the two effective flange areas, one on each side of the chevron gusset coinciding with the two braces. This force can be taken out of the flange and into the web shear area, where the length of the web, L_w, is determined by setting the tensile yielding strength of the flange equal to the shear yielding strength of the web from AISC *Specification* Equation J4-3 and solving for L_w, as follows:

LRFD	ASD
$2\phi_t F_y b_f t_f = \phi_v 0.60 F_y t_w L_w$	$\dfrac{2F_y b_f t_f}{\Omega_t} = \dfrac{0.60 F_y t_w L_w}{\Omega_v}$

and therefore:

LRFD	ASD
$L_w = \dfrac{2\phi_t b_f t_f}{\phi_v 0.60 t_w}$	$L_w = \dfrac{2\Omega_v b_f t_f}{\Omega_t 0.60 t_w}$

The total web length effective in transferring the shear into the beam is:

$$L_{eff} = L + 5k + L_w$$

LRFD	ASD
$L_{eff} = L + 5k + \dfrac{2\phi_t b_f t_f}{\phi_v 0.60 t_w}$ $= 44.0 \text{ in.} + 5(0.827 \text{ in.})$ $+\dfrac{2(0.90)(6.00 \text{ in})(0.425 \text{ in.})}{1.00(0.60)(0.300 \text{ in.})}$ $= 73.6 \text{ in.}$	$L_{eff} = L + 5k + \dfrac{2\Omega_v b_f t_f}{\Omega_t 0.60 t_w}$ $= 44.0 \text{ in.} + 5(0.827 \text{ in.})$ $+\dfrac{2(1.50)(6.00 \text{ in})(0.425 \text{ in.})}{1.67(0.60)(0.300 \text{ in.})}$ $= 73.6 \text{ in.}$

From AISC *Specification* Equation J4-3, the available web shear strength is:

LRFD	ASD

$\phi R_n = \phi 0.60 F_y t_w L_{eff}$ $= 1.00(0.60)(50\text{ ksi})(0.300\text{ in.})(73.6\text{ in.})$ $= 662\text{ kips} > V_u = 234\text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y t_w L_{eff}}{\Omega}$ $= \dfrac{(0.60)(50\text{ ksi})(0.300\text{ in.})(73.6\text{ in.})}{1.50}$ $= 442\text{ kips} > V_a = 156\text{ kips}$ **o.k.**

Transverse Section Web Yielding

The transverse shear induced in the beam at the centerline of the gusset (Section b-b) is calculated and compared to the available shear yielding limit state determined from AISC *Specification* Equation G2-1, with $C_v = 1.0$.

LRFD	ASD
$V_u = 106\text{ kips} - 11.5\text{ kips}$ $= 94.5\text{ kips}$ $\phi R_n = \phi_v 0.6 F_y d t_w C_v$ $= 1.00(0.6)(50\text{ ksi})(17.7\text{ in.})(0.300\text{ in.})(1.0)$ $= 159\text{ kips} \geq 94.5\text{ kips}$ **o.k.**	$V_a = 70.3\text{ kips} - 7.57\text{ kips}$ $= 62.7\text{ kips}$ $\dfrac{R_n}{\Omega_v} = \dfrac{0.60 F_y d t_w C_v}{\Omega_v}$ $= \dfrac{0.60(50\text{ ksi})(17.7\text{ in.})(0.300\text{ in.})(1.0)}{1.50}$ $= 106\text{ kips} \geq 62.7\text{ kips}$ **o.k.**

Reference

Dowswell, B. (2012), "Effective Length Factors for Gusset Plates in Chevron Braced Frames," *Engineering Journal*, AISC, Vol. 49, No. 3, 3rd Quarter, pp. 115–117.

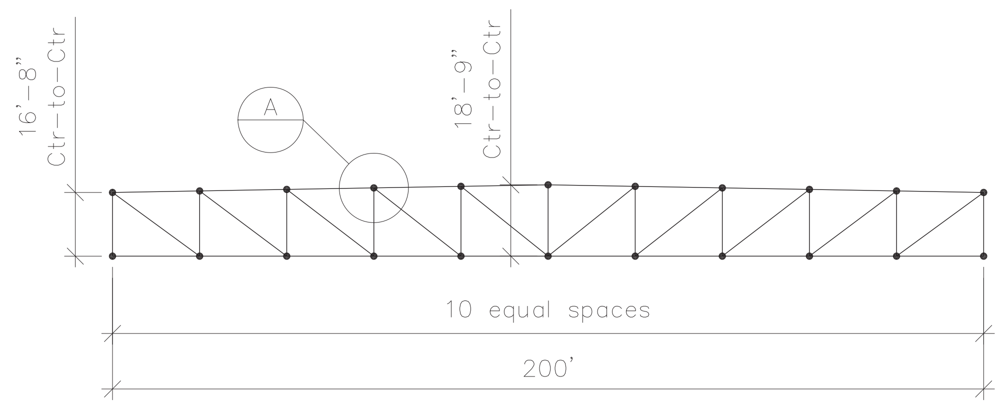
Return to Table of Contents

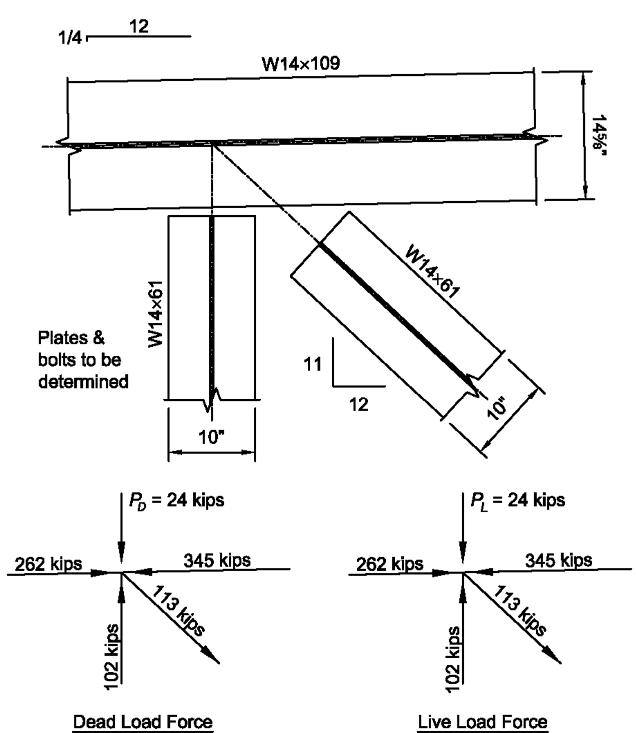
EXAMPLE II.C-6 HEAVY WIDE FLANGE COMPRESSION CONNECTION (FLANGES ON THE OUTSIDE)

Given:

This truss has been designed with nominal 14-in. ASTM A992 W-shapes, with the flanges to the outside of the truss. Beams framing into the top chord and lateral bracing are not shown but can be assumed to be adequate.

Based on multiple load cases, the critical dead and live load forces for this connection were determined. A typical top chord connection and the dead load and live load forces are shown as follows in detail A. Design this typical connection using 1-in.-diameter ASTM A325 slip-critical bolts in standard holes with a Class A faying surface and ASTM A36 gusset plates.





Detail A

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

W-shapes
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Gusset Plates
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W14×109
d = 14.3 in.
b_f = 14.6 in.
t_f = 0.860 in.

W14×61
d = 13.9 in.
b_f = 10.0 in.
t_f = 0.645 in.

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
Left top chord: P_u = 1.2(262 kips) + 1.6(262 kips) = 734 kips Right top chord: P_u = 1.2(345 kips) + 1.6(345 kips) = 966 kips Vertical Web: P_u = 1.2(102 kips) + 1.6(102 kips) = 286 kips Diagonal Web: T_u = 1.2(113 kips) + 1.6(113 kips) = 316 kips	Left top chord: P_a = 262 kips + 262 kips = 524 kips Right top chord: P_a = 345 kips + 345 kips = 690 kips Vertical Web: P_a = 102 kips + 102 kips = 204 kips Diagonal Web: T_a = 113 kips + 113 kips = 226 kips

Single Bolt Shear Strength (AISC Specification Section J3.8)

d = 1.00 in.
ASTM A325-SC bolts
Class A faying surface

μ = 0.30
d_h = 1$\frac{1}{16}$ in. (diameter of holes at gusset plates)
h_f = 1.0 (factor for fillers)
T_b = 51 kips from AISC *Specification* Table J3.1
D_u = 1.13

$R_n = \mu D_u h_f T_b n_s$ (*Spec*. Eq. J3-4)
= 0.30(1.13)(1.0)(51 kips)(1)
= 17.3 kips

For standard holes, determine the available slip resistance and available bolt shear rupture strength:

LRFD	ASD
ϕ = 1.00 ϕR_n = 1.00(17.3 kips) = 17.3 kips/bolt From AISC *Manual* Table 7-1, the shear strength of an ASTM A325-N bolt is: ϕr_n = 31..8 kips > 17.3 kips **o.k.**	Ω = 1.50 $\frac{R_n}{\Omega} = \frac{17.3 \text{ kips}}{1.50}$ = 11.5 kips/bolt From AISC *Manual* Table 7-1, the shear strength of an ASTM A325-N bolt is: $\frac{r_n}{\Omega}$ = 21.2 kips > 11.5 kips **o.k.**

Note: Standard holes are used in both plies for this example. Other hole sizes may be used and should be considered based on the preferences of the fabricator or erector on a case-by-case basis.

Diagonal Connection

LRFD	ASD
P_u = 316 kips 316 kips / 17.3 kips/bolt = 18.3 bolts 2 lines both sides = 18.3 bolts / 4 = 4.58 Therefore, use 5 rows at min. 3-in. spacing.	P_a = 226 kips 226 kips / 11.5 kips/bolt = 19.7 bolts 2 lines both sides = 19.7 bolts / 4 = 4.93 Therefore, use 5 rows at min. 3-in. spacing.

Whitmore Section in Gusset Plate (AISC Manual Part 9)

Whitmore section = gage of the bolts + tan 30°(length of the bolt group)(2)
= 5½ in. + tan 30°[(4 spaces)(3.00 in.)](2)
= 19.4 in.

Try ⅜-in.-thick plate

A_g = ⅜ in.(19.4 in.)
= 7.28 in.^{2}

Tensile Yielding of Gusset Plate

From AISC *Specification* Equation J4-1:

LRFD	ASD
$\phi = 0.90$ $\phi R_n = \phi F_y A_g$ $= 0.90(36\text{ ksi})(7.28\text{ in.}^2)(2)$ $= 472\text{ kips} > 316\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{F_y A_g}{\Omega}$ $= \dfrac{36\text{ ksi}\left(7.28\text{ in.}^2\right)(2)}{1.67}$ $= 314\text{ kips} > 226\text{ kips}$ **o.k.**

Block Shear Rupture of Gusset Plate

Tension stress is uniform, therefore, U_{bs} = 1.0. Assume a 2-in. edge distance on the diagonal gusset plate connection.

t_p = ⅜ in.
A_{gv} = ⅜ in.{2 lines[(4 spaces)(3 in.) + 2 in.]}
= 10.5 in.^{2}
A_{nv} = 10.5 in.^{2} – (⅜ in.)(2 lines)(4.5 bolts)(1¹⁄₁₆ in. + ¹⁄₁₆ in.)
= 6.70 in.^{2}
A_{nt} = ⅜ in.[5.50 in. – (1¹⁄₁₆ in. + ¹⁄₁₆ in)]
= 1.64 in.^{2}

From AISC *Specification* Equation J4-5:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi U_{bs} F_u A_{nt} + \min(\phi 0.60 F_y A_{gv}, \phi 0.6 F_u A_{nv})$ Tension rupture component: $\phi U_{bs} F_u A_{nt} = 0.75(1.0)(58\text{ ksi})(1.64\text{ in.}^2)$ $= 71.3\text{ kips}$ Shear yielding component: $\phi 0.60 F_y A_{gv} = 0.75(0.6)(36\text{ ksi})(10.5\text{ in.}^2)$ $= 170\text{ kips}$ Shear rupture component: $\phi 0.6 F_u A_{nv} = 0.75(0.6)(58\text{ ksi})(6.70\text{ in.}^2)$ $= 175\text{ kips}$	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{U_{bs} F_u A_{nt}}{\Omega} + \min\left(\dfrac{0.60 F_y A_{gv}}{\Omega}, \dfrac{0.60 F_u A_{nv}}{\Omega}\right)$ Tension rupture component: $\dfrac{U_{bs} F_u A_{nt}}{\Omega} = \dfrac{1.0(58\text{ ksi})\left(1.64\text{ in.}^2\right)}{2.00}$ $= 47.6\text{ kips}$ Shear yielding component: $\dfrac{0.6 F_y A_{gv}}{\Omega} = \dfrac{0.6(36\text{ ksi})\left(10.5\text{ in.}^2\right)}{2.00}$ $= 113\text{ kips}$ Shear rupture component: $\dfrac{0.6 F_u A_{nv}}{\Omega} = \dfrac{0.6(58\text{ ksi})\left(6.70\text{ in.}^2\right)}{2.00}$ $= 117\text{ kips}$

LRFD	ASD
$\phi R_n = 71.3 \text{ kips} + 170 \text{ kips/in.}$ $= 241 \text{ kips} > 316 \text{ kips}/2 = 158 \text{ kips}$ **o.k.**	$\frac{R_n}{\Omega} = 47.6 \text{ kips} + 113 \text{ kips}$ $= 161 \text{ kips} > 226 \text{ kips}/2 = 113 \text{ kips}$ **o.k.**

Block Shear Rupture of Diagonal Flange

By inspection, block shear rupture on the diagonal flange will not control.

Bolt Bearing on Gusset Plate

LRFD	ASD
From AISC *Manual* Table 7-4 with s = 3 in. and standard holes, $\phi r_n = 101 \text{ kips/in.}(\tfrac{3}{8} \text{ in.})$ $= 37.9 \text{ kips} > 17.3 \text{ kips}$ **o.k.** From AISC *Manual* Table 7-5 with L_e = 2 in. and standard holes, $\phi r_n = 76.7 \text{ kips/in.}(\tfrac{3}{8} \text{ in.})$ $= 28.8 \text{ kips} > 17.3 \text{ kips}$ **o.k.**	From AISC *Manual* Table 7-4 with s = 3 in. and standard holes, $\frac{r_n}{\Omega} = 67.4 \text{ kips/in.}\left(\tfrac{3}{8} \text{ in.}\right)$ $= 25.3 \text{ kips} > 11.5 \text{ kips}$ **o.k.** From AISC *Manual* Table 7-5 with L_e = 2 in. and standard holes, $\frac{r_n}{\Omega} = 51.1 \text{ kips/in.}\left(\tfrac{3}{8} \text{ in.}\right)$ $= 19.2 \text{ kips} > 11.5 \text{ kips}$ **o.k.**

Bolt Bearing on Diagonal Flange

LRFD	ASD
From AISC *Manual* Table 7-4 with s = 3 in. and standard holes, $\phi r_n = 113 \text{ kips/in.}(0.645 \text{ in.})$ $= 72.9 \text{ kips} > 17.3 \text{ kips}$ **o.k.** From AISC *Manual* Table 7-5 with L_e = 2 in. and standard holes, $\phi r_n = 85.9 \text{ kips/in.}(0.645 \text{ in.})$ $= 55.4 \text{ kips} > 17.3 \text{ kips}$ **o.k.**	From AISC *Manual* Table 7-4 with s = 3 in. and standard holes, $\frac{r_n}{\Omega} = 75.6 \text{ kips/in.}\left(0.645 \text{ in.}\right)$ $= 48.8 \text{ kips} > 11.5 \text{ kips}$ **o.k.** From AISC *Manual* Table 7-5 with L_e = 2 in. and standard holes, $\frac{r_n}{\Omega} = 57.3 \text{ kips/in.}\left(0.645 \text{ in.}\right)$ $= 37.0 \text{ kips} > 11.5 \text{ kips}$ **o.k.**

Horizontal Connection

LRFD	ASD
Required strength: $P_u = 966 \text{ kips} - 734 \text{ kips}$ $= 232 \text{ kips}$ As determined previously, the design bolt shear strength is 17.3 kips/bolt.	Required strength: $P_a = 690 \text{ kips} - 524 \text{ kips}$ $= 166 \text{ kips}$ As determined previously, the allowable bolt shear strength is 11.5 kips/bolt.

232 kips / 17.3 kips/bolt = 13.4 bolts 2 lines both sides = 13.4 bolts / 4 = 3.35 Use 4 rows on each side.	166 kips / 11.5 kips/bolt = 14.4 bolts 2 lines both sides = 14.4 bolts / 4 = 3.60 Use 4 rows on each side.

For members not subject to corrosion the maximum bolt spacing is calculated using AISC *Specification* Section J3.5(a):

$$\begin{aligned}\text{Maximum bolt spacing} &= 24(\tfrac{3}{8}\text{ in.})\\ &= 9.00\text{ in.}\end{aligned}$$

Due to the geometry of the gusset plate, the use of 4 rows of bolts in the horizontal connection will exceed the maximum bolt spacing; instead use 5 rows of bolts in two lines.

Shear Yielding of Plate

Try plate with, $t_p = \tfrac{3}{8}$ in.

$$\begin{aligned}A_{gv} &= \tfrac{3}{8}\text{ in.}(32\text{ in.})\\ &= 12.0\text{ in.}^2\end{aligned}$$

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(12.0\text{ in.}^2)$ $= 259\text{ kips} > 232\text{ kips}/2 = 116\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(12.0\text{ in.}^2)}{1.50}$ $= 173\text{ kips} > 166\text{ kips}/2 = 83.0\text{ kips}$ **o.k.**

Shear Rupture of Plate

$$\begin{aligned}A_{nv} &= 12.0\text{ in.}^2 - \tfrac{3}{8}\text{ in.}(5\text{ bolts})(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\\ &= 9.89\text{ in.}^2\end{aligned}$$

From AISC *Specification* Equation J4-4:

LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_u A_{nv}$ $= 0.75(0.60)(58\text{ ksi})(9.89\text{ in.}^2)$ $= 258\text{ kips} > 232\text{ kips}/2 = 116\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58\text{ ksi})(9.89\text{ in.}^2)}{2.00}$ $= 172\text{ kips} > 166\text{ kips}/2 = 83.0\text{ kips}$ **o.k.**

Bolt Bearing on Gusset Plate and Horizontal Flange

Return to Table of Contents

By comparison to the preceding calculations for the diagonal connection, bolt bearing does not control.

Vertical Connection

LRFD	ASD
Required axial strength: P_u = 286 kips As determined previously, the design bolt shear strength is 17.3 kips/bolt. 286 kips / 17.3 kips/bolt = 16.5 bolts 2 lines both sides = 16.5 bolts / 4 = 4.13 Use 5 bolts per line.	Required axial strength: P_a = 204 kips As determined previously, the allowable bolt shear strength is 11.5 kips/bolt. 204 kips / 11.5 kips/bolt = 17.7 bolts 2 lines both sides = 17.7 bolts / 4 = 4.43 Use 5 bolts per line.

Shear Yielding of Plate

Try plate with, t_p = ⅜ in.

$$\begin{aligned} A_{gv} &= \tfrac{3}{8}\text{ in.}(31.75\text{ in.}) \\ &= 11.9\text{ in.}^2 \end{aligned}$$

From AISC *Specification* Equation J4-3:

LRFD	ASD
$\phi = 1.00$ $\phi R_n = \phi 0.60 F_y A_{gv}$ $= 1.00(0.60)(36\text{ ksi})(11.9\text{ in.}^2)$ $= 257\text{ kips} > 286\text{ kips}/2 = 143\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_y A_{gv}}{\Omega}$ $= \dfrac{0.60(36\text{ ksi})(11.9\text{ in.}^2)}{1.50}$ $= 171\text{ kips} > 204\text{ kips}/2 = 102\text{ kips}$ **o.k.**

Shear Rupture of Plate

$$\begin{aligned} A_{nv} &= 11.9\text{ in.}^2 - \tfrac{3}{8}\text{ in.}(7\text{ bolts})(1\tfrac{1}{16}\text{ in.} + \tfrac{1}{16}\text{ in.}) \\ &= 8.95\text{ in.}^2 \end{aligned}$$

From AISC *Specification* Equation J4-4:

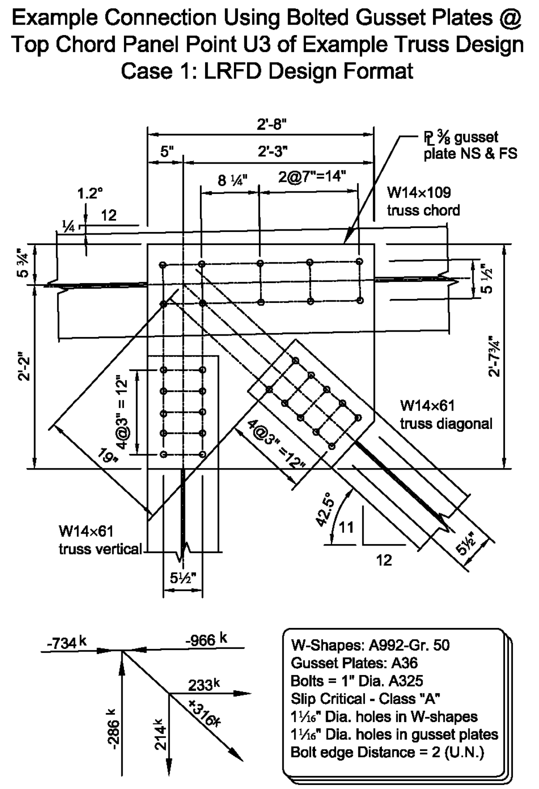
LRFD	ASD
$\phi = 0.75$ $\phi R_n = \phi 0.60 F_y A_{nv}$ $= 0.75(0.60)(58\text{ ksi})(8.95\text{ in.}^2)$ $= 234\text{ kips} > 286\text{ kips}/2 = 143\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{0.60 F_u A_{nv}}{\Omega}$ $= \dfrac{0.60(58\text{ ksi})(8.95\text{ in.}^2)}{2.00}$ $= 156\text{ kips} > 204\text{ kips}/2 = 102\text{ kips}$ **o.k.**

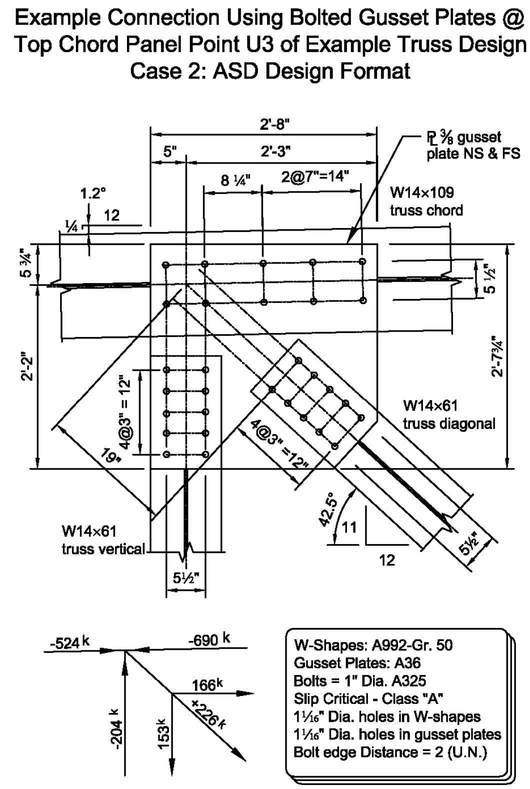
Return to Table of Contents

Bolt Bearing on Gusset Plate and Vertical Flange

By comparison to the preceding calculations for the diagonal connection, bolt bearing does not control.

The final layout for the connection is as follows:





Note that because of the difference in depths between the top chord and the vertical and diagonal members, $\frac{3}{16}$-in. loose shims are required on each side of the shallower members.

Chapter IID
Miscellaneous Connections

This section contains design examples on connections in the AISC *Steel Construction Manual* that are not covered in other sections of the AISC *Design Examples*.

Return to Table of Contents

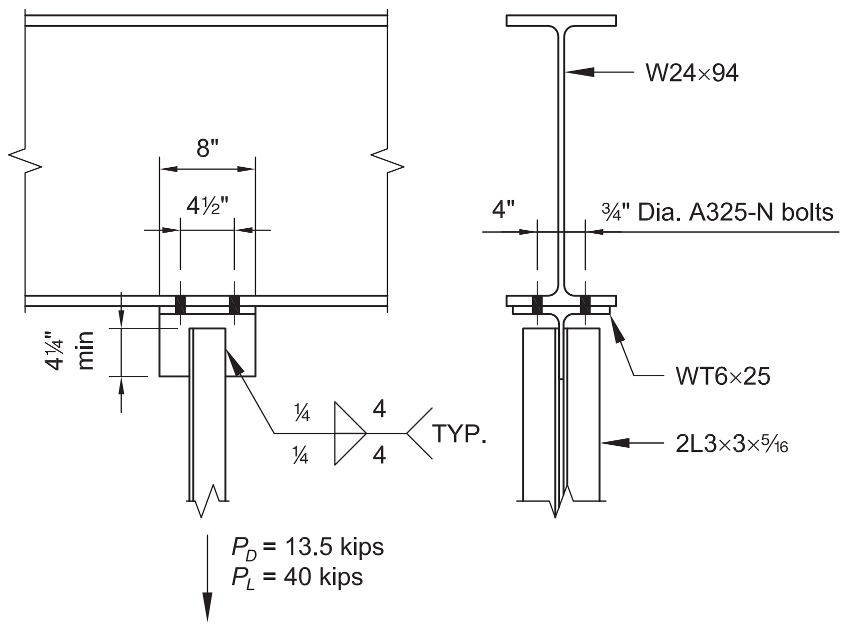
EXAMPLE II.D-1 PRYING ACTION IN TEES AND IN SINGLE ANGLES

Given:

Design an ASTM A992 WT hanger connection between an ASTM A36 2L3×3×5⁄16 tension member and an ASTM A992 W24×94 beam to support the following loads:

P_D = 13.5 kips
P_L = 40 kips

Use ¾-in.-diameter ASTM A325-N or F1852-N bolts and 70-ksi electrodes.



Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

Hanger
WT
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Beam
W24×94
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Angles
2L3×3×5⁄16
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1, 1-7 and 1-15, the geometric properties are as follows:

Beam
W24×94
d = 24.3 in.
t_w = 0.515 in.
b_f = 9.07 in.
t_f = 0.875 in.

Angles
2L3×3×5⁄16
A = 3.56 in.^{2}
$\bar{x}$ = 0.860 in. for single angle

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD	ASD
$P_u = 1.2(13.5 \text{ kips}) + 1.6(40 \text{ kips})$ $= 80.2 \text{ kips}$	$P_a = 13.5 \text{ kips} + 40 \text{ kips}$ $= 53.5 \text{ kips}$

Tensile Yielding of Angles

$$P_n = F_y A_g \qquad \text{(Spec. Eq. D2-1)}$$
$$= 36 \text{ ksi}\left(3.56 \text{ in.}^2\right)$$
$$= 128 \text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi P_n = 0.90(128 \text{ kips})$ $= 115 \text{ kips} > 80.2 \text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{P_n}{\Omega} = \dfrac{128 \text{ kips}}{1.67}$ $= 76.6 \text{ kips} > 53.5 \text{ kips}$ **o.k.**

From AISC *Specification* Table J2.4, the minimum size of fillet weld based on a material thickness of 5⁄16 in. is 3⁄16 in.

From AISC *Specification* Section J2.2b, the maximum size of fillet weld is:

$$w_{max} = \text{thickness} - 1/16 \text{ in.}$$
$$= 5/16 \text{ in.} - 1/16 \text{ in.}$$
$$= 1/4 \text{ in.}$$

Try ¼-in. fillet welds.

From AISC *Manual* Part 8, Equations 8-2:

LRFD	ASD
$l_{min} = \dfrac{P_u}{1.392D}$ $= \dfrac{80.2 \text{ kips}}{1.392(4 \text{ sixteenths})}$ $= 14.4 \text{ in.}$	$l_{min} = \dfrac{P_a}{0.928D}$ $= \dfrac{53.5 \text{ kips}}{0.928(4 \text{ sixteenths})}$ $= 14.4 \text{ in.}$

Use four 4-in. welds (16 in. total), one at the toe and heel of each angle.

Tensile Rupture Strength of Angles

$$U = 1 - \frac{\bar{x}}{L} \text{ from AISC } \textit{Specification} \text{ Table D3.1 case 2}$$
$$= 1 - \frac{0.860 \text{ in.}}{4.00 \text{ in.}}$$
$$= 0.785$$

$$A_e = A_n U \qquad (\textit{Spec}. \text{ Eq. D3-1})$$
$$= 3.56 \text{ in.}^2 (0.785)$$
$$= 2.79 \text{ in.}^2$$

$$P_n = F_u A_e \qquad (\textit{Spec}. \text{ Eq. D2-2})$$
$$= 58 \text{ ksi}\left(2.79 \text{ in.}^2\right)$$
$$= 162 \text{ kips}$$

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(162 \text{ kips})$ $= 122 \text{ kips} > 80.2 \text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{162 \text{ kips}}{2.00}$ $= 81.0 \text{ kips} > 53.5 \text{ kips}$ **o.k.**

Preliminary WT Selection Using Beam Gage

$g = 4$ in.

Try four ¾-in.-diameter ASTM A325-N bolts.

From AISC *Manual* Table 7-2:

LRFD	ASD
$T = r_{ut} = \dfrac{P_u}{n}$ $= \dfrac{80.2 \text{ kips}}{4}$ $= 20.1 \text{ kips/bolt}$ $B = \phi r_n = 29.8 \text{ kips} > 20.1 \text{ kips}$ **o.k.**	$T = r_{at} = \dfrac{P_a}{n}$ $= \dfrac{53.5 \text{ kips}}{4}$ $= 13.4 \text{ kips/bolt}$ $B = r_n / \Omega = 19.9 \text{ kips} > 13.4 \text{ kips}$ **o.k.**

Determine tributary length per pair of bolts, p, using AISC *Manual* Figure 9-4 and assuming a ½-in. web thickness.

$$p = \frac{4.00\text{ in.} - \frac{1}{2}\text{ in.}}{2} + \frac{8.00\text{ in.} - 4\frac{1}{2}\text{ in.}}{2}$$
$$= 3.50\text{ in.} \leq 4\frac{1}{2}\text{ in.}$$

LRFD	ASD
$\dfrac{2\text{ bolts}(20.1\text{ kips/bolt})}{3.50\text{ in.}} = 11.5\text{ kips/in.}$	$\dfrac{2\text{ bolts}(13.4\text{ kips/bolt})}{3.50\text{ in.}} = 7.66\text{ kips/in.}$

From AISC *Manual* Table 15-2b, with an assumed b = (4.00 in. – ½ in.)/2 = 1.75 in., the flange thickness, $t = t_f$, of the WT hanger should be approximately ⅝ in.

The minimum depth WT that can be used is equal to the sum of the weld length plus the weld size plus the k-dimension for the selected section. From AISC *Manual* Table 1-8 with an assumed b = 1.75 in., $t_f \approx \frac{5}{8}\text{ in.}$, and $d_{min} = 4\text{ in.} + \frac{1}{4}\text{ in.} + k \approx 6\text{ in.}$, appropriate selections include:

WT6×25
WT7×26.5
WT8×25
WT9×27.5

Try a WT6×25.

From AISC *Manual* Table 1-8, the geometric properties are as follows:

b_f = 8.08 in.
t_f = 0.640 in.
t_w = 0.370 in.

Prying Action Using AISC Manual Part 9

The beam flange is thicker than the WT flange; therefore, prying in the tee flange will control over prying in the beam flange.

$$b = \frac{g - t_w}{2}$$
$$= \frac{4.00\text{ in.} - 0.370\text{ in.}}{2}$$
$$= 1.82\text{ in.} > 1\frac{1}{4}\text{-in. entering and tightening clearance, and the fillet toe is cleared}$$

$$a = \frac{b_f - g}{2}$$
$$= \frac{8.08\text{ in.} - 4.00\text{ in.}}{2}$$
$$= 2.04\text{ in.}$$

$$b' = b - \frac{d_b}{2} \quad \text{(\textit{Manual} Eq. 9-21)}$$

$$= 1.82 \text{ in.} - \left(\frac{\text{¾ in.}}{2}\right)$$
$$= 1.45 \text{ in.}$$

$$a' = \left(a + \frac{d_b}{2}\right) \le \left(1.25b + \frac{d_b}{2}\right)$$ (*Manual* Eq. 9-27)

$$= 2.04 \text{ in.} + \left(\frac{\text{¾ in.}}{2}\right) \le 1.25(1.82 \text{ in.}) + \frac{\text{¾ in.}}{2}$$
$$= 2.42 \text{ in.} \le 2.65 \text{ in.}$$

$$\rho = \frac{b'}{a'}$$ (*Manual* Eq. 9-26)

$$= \frac{1.45 \text{ in.}}{2.42 \text{ in.}}$$
$$= 0.599$$

LRFD	ASD
$\beta = \frac{1}{\rho}\left(\frac{B}{T} - 1\right)$ (*Manual* Eq. 9-25) $= \frac{1}{0.599}\left(\frac{29.8 \text{ kips/bolt}}{20.1 \text{ kips/bolt}} - 1\right)$ $= 0.806$	$\beta = \frac{1}{\rho}\left(\frac{B}{T} - 1\right)$ (*Manual* Eq. 9-25) $= \frac{1}{0.599}\left(\frac{19.9 \text{ kips/bolt}}{13.4 \text{ kips/bolt}} - 1\right)$ $= 0.810$

$$\delta = 1 - \frac{d'}{p}$$ (*Manual* Eq. 9-24)

$$= 1 - \frac{\text{¾ in.} + \text{1/16 in.}}{3.50 \text{ in.}}$$
$$= 0.768$$

Since $\beta < 1.0$,

LRFD	ASD
$\alpha' = \frac{1}{\delta}\left(\frac{\beta}{1-\beta}\right) \le 1.0$ $= \frac{1}{0.768}\left(\frac{0.806}{1-0.806}\right)$ $= 5.41$, therefore, $\alpha' = 1.0$ $\phi = 0.90$ $t_{min} = \sqrt{\frac{4Tb'}{\phi p F_u (1+\delta\alpha')}}$ (*Manual* Eq. 9-23a) $= \sqrt{\frac{4(20.1 \text{ kips/bolt})(1.45 \text{ in.})}{0.90(3.50 \text{ in.})(65 \text{ ksi})\left[1+(0.768)(1.0)\right]}}$ $= 0.567 \text{ in.} < t_f = 0.640 \text{ in.}$ **o.k.**	$\alpha' = \frac{1}{\delta}\left(\frac{\beta}{1-\beta}\right) \le 1.0$ $= \frac{1}{0.768}\left(\frac{0.810}{1-0.810}\right)$ $= 5.55$, therefore, $\alpha' = 1.0$ $\Omega = 1.67$ $t_{min} = \sqrt{\frac{\Omega 4Tb'}{p F_u (1+\delta\alpha')}}$ (*Manual* Eq. 9-23b) $= \sqrt{\frac{1.67(4)(13.4 \text{ kips/bolt})(1.45 \text{ in.})}{3.50 \text{ in.}(65 \text{ ksi})\left[1+(0.768)(1.0)\right]}}$ $= 0.568 \text{ in.} < t_f = 0.640 \text{ in.}$ **o.k.**

Tensile Yielding of the WT Stem on the Whitmore Section Using AISC Manual Part 9

The effective width of the WT stem (which cannot exceed the actual width of 8 in.) is:

$$l_w = 3.00\text{ in.} + 2(4.00\text{ in.})(\tan 30^\circ) \le 8.00\text{ in.}$$
$$= 7.62\text{ in.}$$

The nominal strength is determined as:

$$R_n = F_y A_g \qquad \text{(}\textit{Spec}\text{. Eq. J4-1)}$$
$$= 50\text{ ksi}(7.62\text{ in.})(0.370\text{ in.})$$
$$= 141\text{ kips}$$

LRFD	ASD
$\phi = 0.90$ $\phi R_n = 0.90(141\text{ kips})$ $= 127\text{ kips} > 80.2\text{ kips}$ **o.k.**	$\Omega = 1.67$ $\dfrac{R_n}{\Omega} = \dfrac{141\text{ kips}}{1.67}$ $= 84.4\text{ kips} > 53.5\text{ kips}$ **o.k.**

Shear Rupture of the WT Stem Base Metal

$$t_{min} = \frac{6.19D}{F_u} \qquad \text{(}\textit{Manual}\text{ Eq. 9-3)}$$
$$= 6.19\left(\frac{4\text{ sixteenths}}{65\text{ ksi}}\right)$$
$$= 0.381\text{ in.} > 0.370\text{ in.}$$ **shear rupture strength of WT stem controls over weld rupture strength**

Block Shear Rupture of the WT Stem

$$A_{gv} = (2\text{ shear planes})(4.00\text{ in.})(0.370\text{ in.})$$
$$= 2.96\text{ in.}^2$$

Tension stress is uniform, therefore U_{bs} = 1.0.

$$A_{nt} = A_{gt} = 3.00\text{ in.}(0.370\text{ in.})$$
$$= 1.11\text{ in.}^2$$

$$R_n = 0.60F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.60F_yA_{gv} + U_{bs}F_uA_{nt} \qquad \text{(}\textit{Spec}\text{. Eq. J4-5)}$$

Because the angles are welded to the WT-hanger, shear yielding on the gross area will control (that is, the portion of the block shear rupture equation that addresses shear rupture on the net area does not control).

$$R_n = 0.60F_y A_{gv} + U_{bs} F_u A_{nt}$$
$$= 0.60(50\text{ ksi})(2.96\text{ in.}^2) + 1.0(65\text{ ksi})(1.11\text{ in.}^2)$$
$$= 161\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(161\text{ kips})$ $= 121\text{ kips} > 80.2\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{161\text{ kips}}{2.00}$ $= 80.5\text{ kips} > 53.5\text{ kips}$ **o.k.**

Note: As an alternative to the preceding calculations, the designer can use a simplified procedure to select a WT hanger with a flange thick enough to reduce the effect of prying action to an insignificant amount, i.e., $q \approx 0$. Assuming $b' = 1.45$ in.

From AISC *Manual* Part 9:

LRFD	ASD
$\phi = 0.90$ $t_{min} = \sqrt{\dfrac{4Tb'}{\phi pF_u}}$ (*Manual* Eq. 9-20a) $= \sqrt{\dfrac{4(20.1\text{ kips/bolt})(1.45\text{ in.})}{0.90(3.50\text{ in./bolt})(65\text{ ksi})}}$ $= 0.755$ in.	$\Omega = 1.67$ $t_{min} = \sqrt{\dfrac{\Omega 4Tb'}{pF_u}}$ (*Manual* Eq. 9-20b) $= \sqrt{\dfrac{1.67(4)(13.4\text{ kips/bolt})(1.45\text{ in.})}{(3.50\text{ in./bolt})(65\text{ ksi})}}$ $= 0.755$ in.

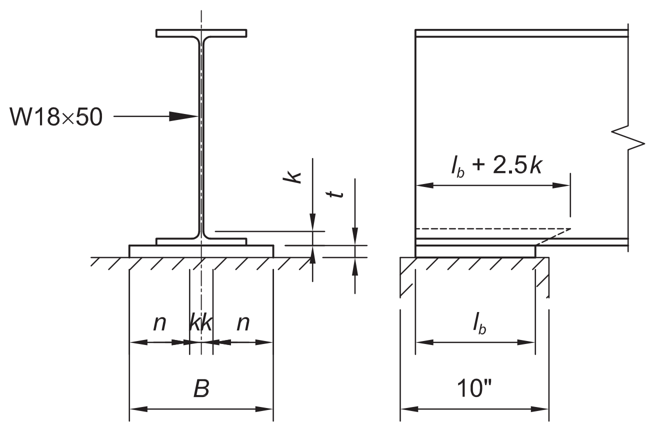
A WT6×25, with t_f = 0.640 in. < 0.755 in., does not have a sufficient flange thickness to reduce the effect of prying action to an insignificant amount. In this case, the simplified approach requires a WT section with a thicker flange.

EXAMPLE II.D-2 BEAM BEARING PLATE

Given:

An ASTM A992 W18×50 beam with a dead load end reaction of 15 kips and a live load end reaction of 45 kips is supported by a 10-in.-thick concrete wall. Assuming the concrete has f_c' = 3 ksi, and the bearing plate is ASTM A36 material determine the following:

a. If a bearing plate is required if the beam is supported by the full wall thickness
b. The bearing plate required if l_b = 10 in. (the full wall thickness)
c. The bearing plate required if l_b = 6½ in. and the bearing plate is centered on the thickness of the wall



Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W18×50
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Bearing Plate (if required)
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Concrete Wall
f_c' = 3 ksi

From AISC *Manual* Table 1-1, the geometric properties are as follows:

Beam
W18×50
d = 18.0 in.
t_w = 0.355 in.
b_f = 7.50 in.
t_f = 0.570 in.
k_{des} = 0.972 in.
k_1 = 13⁄16 in.

Concrete Wall
h = 10.0 in.

Solution a:

LRFD	ASD
Calculate required strength. R_u = 1.2(15 kips) + 1.6(45 kips) = 90.0 kips Check web local yielding using AISC *Manual* Table 9-4 and *Manual* Equation 9-45a. $l_{b\,req} = \dfrac{R_u - \phi R_1}{\phi R_2} \geq k_{des}$ $= \dfrac{90.0\text{ kips} - 43.1\text{ kips}}{17.8\text{ kips/in.}} \geq 0.972\text{ in.}$ $= 2.63\text{ in.} < 10.0\text{ in.}$ **o.k.** Check web local crippling using AISC *Manual* Table 9-4. $\dfrac{l_b}{d} = \dfrac{10.0\text{ in.}}{18.0\text{ in.}}$ $= 0.556$ Since $\dfrac{l_b}{d} > 0.2$, use *Manual* Equation 9-48a. $l_{b\,req} = \dfrac{R_u - \phi R_5}{\phi R_6}$ $= \dfrac{90.0\text{ kips} - 52.0\text{ kips}}{6.30\text{ kips/in.}}$ $= 6.03\text{ in.} < 10.0\text{ in.}$ **o.k.** Verify $\dfrac{l_b}{d} > 0.2$, $\dfrac{l_b}{d} = \dfrac{6.03\text{ in.}}{18.0\text{ in.}}$ $= 0.335 > 0.2$ **o.k.** Check the bearing strength of concrete. Note that AISC *Specification* Equation J8-1 is used because A_2 is not larger than A_1 in this case. $P_p = 0.85 f_c{}' A_1$ (*Spec*. Eq. J8-1)	Calculate required strength. R_a = 15 kips + 45 kips = 60.0 kips Check web local yielding using AISC *Manual* Table 9-4 and *Manual* Equation 9-45b. $l_{b\,req} = \dfrac{R_a - R_1 / \Omega}{R_2 / \Omega} \geq k_{des}$ $= \dfrac{60.0\text{ kips} - 28.8\text{ kips}}{11.8\text{ kips/in.}} \geq 0.972\text{ in.}$ $= 2.64\text{ in.} < 10.0\text{ in.}$ **o.k.** Check web local crippling using AISC *Manual* Table 9-4. $\dfrac{l_b}{d} = \dfrac{10.0\text{ in.}}{18.0\text{ in.}}$ $= 0.556$ Since $\dfrac{l_b}{d} > 0.2$, use *Manual* Equation 9-48b. $l_{b\,req} = \dfrac{R_a - R_5 / \Omega}{R_6 / \Omega}$ $= \dfrac{60.0\text{ kips} - 34.7\text{ kips}}{4.20\text{ kips/in.}}$ $= 6.02\text{ in.} < 10.0\text{ in.}$ **o.k.** Verify $\dfrac{l_b}{d} > 0.2$, $\dfrac{l_b}{d} = \dfrac{6.02\text{ in.}}{18.0\text{ in.}}$ $= 0.334 > 0.2$ **o.k.** Check the bearing strength of concrete. Note that AISC *Specification* Equation J8-1 is used because A_2 is not larger than A_1 in this case. $P_p = 0.85 f_c{}' A_1$ (*Spec*. Eq. J8-1)

LRFD	ASD
$\phi_c = 0.65$ $\phi_c P_p = \phi_c 0.85 f_c{}' A_1$ $= 0.65(0.85)(3\text{ ksi})(7.50\text{ in.})(10.0\text{ in.})$ $= 124\text{ kips} > 90.0\text{ kips}$ **o.k.**	$\Omega_c = 2.31$ $\dfrac{P_p}{\Omega_c} = \dfrac{0.85 f_c{}' A_1}{\Omega_c}$ $= \dfrac{0.85(3\text{ ksi})(7.50\text{ in.})(10.0\text{ in.})}{2.31}$ $= 82.8\text{ kips} > 60.0\text{ kips}$ **o.k.**

Beam Flange Thickness Check Using AISC Manual Part 14

LRFD	ASD
Determine the cantilever length from *Manual* Equation 14-1. $n = \dfrac{b_f}{2} - k_{des}$ $= \dfrac{7.50\text{ in.}}{2} - 0.972\text{ in.}$ $= 2.78\text{ in.}$ Determine bearing pressure. $f_p = \dfrac{R_u}{A_1}$ Determine the minimum beam flange thickness required if no bearing plate is provided. The beam flanges along the length, *n*, are assumed to be fixed end cantilevers with a minimum thickness determined using the limit state of flexural yielding. $M_u = \dfrac{f_p n^2}{2} = \dfrac{R_u n^2}{2A_1}$ $Z = \frac{1}{4} t^2$ $M_u \le \phi F_y Z = \phi F_y \left(\dfrac{t^2}{4}\right)$ $t_{min} = \sqrt{\dfrac{4M_u}{\phi F_y}} = \sqrt{\dfrac{2R_u n^2}{\phi A_1 F_y}}$ $\phi = 0.90$ $t_{min} = \sqrt{\dfrac{2R_u n^2}{\phi A_1 F_y}}$	Determine the cantilever length from *Manual* Equation 14-1. $n = \dfrac{b_f}{2} - k_{des}$ $= \dfrac{7.50\text{ in.}}{2} - 0.972\text{ in.}$ $= 2.78\text{ in.}$ Determine bearing pressure. $f_p = \dfrac{R_a}{A_1}$ Determine the minimum beam flange thickness required if no bearing plate is provided. The beam flanges along the length, *n*, are assumed to be fixed end cantilevers with a minimum thickness determined using the limit state of flexural yielding. $M_a = \dfrac{f_p n^2}{2} = \dfrac{R_a n^2}{2A_1}$ $Z = \frac{1}{4} t^2$ $M_a \le \dfrac{F_y Z}{\Omega} = \dfrac{F_y}{\Omega}\left(\dfrac{t^2}{4}\right)$ $t_{min} = \sqrt{\dfrac{\Omega 4 M_a}{F_y}} = \sqrt{\dfrac{\Omega 2 R_a n^2}{A_1 F_y}}$ $\Omega = 1.67$ $t_{min} = \sqrt{\dfrac{\Omega 2 R_a n^2}{A_1 F_y}}$

LRFD	ASD
$= \sqrt{\dfrac{2(90.0\text{ kips})(2.78\text{ in.})^2}{0.90(7.50\text{ in.})(10.0\text{ in.})(50\text{ ksi})}}$ $= 0.642\text{ in.} > 0.570\text{ in.}$ **n.g.** **A bearing plate is required. See note following.**	$= \sqrt{\dfrac{1.67(2)(60.0\text{ kips})(2.78\text{ in.})^2}{(7.50\text{ in.})(10.0\text{ in.})(50\text{ ksi})}}$ $= 0.643\text{ in.} > 0.570\text{ in.}$ **n.g.** **A bearing plate is required. See note following.**

Note: The designer may assume a bearing width narrower than the beam flange in order to justify a thinner flange. In this case, if 5.44 in. ≤ bearing width ≤ 6.56 in., a 0.570 in. flange thickness is ok and the concrete has adequate bearing strength.

Solution b:

$l_b = 10$ in.

From Solution a, web local yielding and web local crippling are o.k.

LRFD	ASD
Calculate the required bearing-plate width using AISC *Specification* Equation J8-1. $\phi_c = 0.65$ $A_{1\,req} = \dfrac{R_u}{\phi_c 0.85 f_c'}$ $= \dfrac{90.0\text{ kips}}{0.65(0.85)(3\text{ ksi})}$ $= 54.3\text{ in.}^2$ $B_{req} = \dfrac{A_{1\text{ req}}}{N}$ $= \dfrac{54.3\text{ in.}^2}{10.0\text{ in.}}$ $= 5.43\text{ in.}$ Use B = 8 in. (selected as the least whole-inch dimension that exceeds b_f). Calculate the required bearing-plate thickness using AISC *Manual* Part 14. $n = \dfrac{B}{2} - k_{des}$ (*Manual* Eq. 14-1) $= \dfrac{8.00\text{ in.}}{2} - 0.972\text{ in.}$ $= 3.03\text{ in.}$ $t_{min} = \sqrt{\dfrac{2R_u n^2}{\phi A_1 F_y}}$	Calculate the required bearing-plate width using AISC *Specification* Equation J8-1. $\Omega_c = 2.31$ $A_{1\,req} = \dfrac{R_a \Omega_c}{0.85 f_c'}$ $= \dfrac{60.0\text{ kips}(2.31)}{(0.85)(3\text{ ksi})}$ $= 54.4\text{ in.}^2$ $B_{req} = \dfrac{A_{1\text{ req}}}{N}$ $= \dfrac{54.4\text{ in.}^2}{10.0\text{ in.}}$ $= 5.44\text{ in.}$ Use B = 8 in. (selected as the least whole-inch dimension that exceeds b_f). Calculate the required bearing-plate thickness using AISC *Manual* Part 14. $n = \dfrac{B}{2} - k_{des}$ (*Manual* Eq. 14-1) $= \dfrac{8.00\text{ in.}}{2} - 0.972\text{ in.}$ $= 3.03\text{ in.}$ $t_{min} = \sqrt{\dfrac{\Omega 2 R_a n^2}{A_1 F_y}}$

$= \sqrt{\dfrac{2(90.0 \text{ kips})(3.03 \text{ in.})^2}{0.90(10.0 \text{ in.})(8.00 \text{ in.})(36 \text{ ksi})}}$ $= 0.798$ in. Use PL ⅞ in.×10 in.×0 ft 8 in.	$= \sqrt{\dfrac{1.67(2)(60.0 \text{ kips})(3.03 \text{ in.})^2}{(10.0 \text{ in.})(8.00 \text{ in.})(36 \text{ ksi})}}$ $= 0.799$ in. Use PL ⅞ in.×10 in.×0 ft 8 in.

Note: The calculations for t_{min} are conservative. Taking the strength of the beam flange into consideration results in a thinner required bearing plate or no bearing plate at all.

Solution c:

$l_b = N = 6.50$ in.

From Solution a, web local yielding and web local crippling are o.k.

Try $B = 8$ in.

$$\begin{aligned} A_1 &= BN \\ &= 8.00 \text{ in.}(6.50 \text{ in.}) \\ &= 52.0 \text{ in.}^2 \end{aligned}$$

To determine the dimensions of the area A_2, the load is spread into the concrete until an edge or the maximum condition $\sqrt{A_2/A_1} = 2$ is met. There is also a requirement that the area, A_2, be geometrically similar to A_1 or, in other words, have the same aspect ratio as A_1.

$$\begin{aligned} N_1 &= 6.50 \text{ in.} + 2(1.75 \text{ in.}) \\ &= 10.0 \text{ in.} \end{aligned}$$

$$\begin{aligned} \frac{B}{N} &= \frac{8.00 \text{ in.}}{6.50 \text{ in.}} \\ &= 1.23 \end{aligned}$$

$$\begin{aligned} B_1 &= 1.23(10.0 \text{ in.}) \\ &= 12.3 \text{ in.} \end{aligned}$$

$$\begin{aligned} A_2 &= B_1 N_1 \\ &= 12.3 \text{ in.}\ (10.0 \text{ in.}) \\ &= 123 \text{ in.}^2 \end{aligned}$$

$$\text{Check } \sqrt{\frac{A_2}{A_1}} = \sqrt{\frac{123 \text{ in.}^2}{52.0 \text{ in.}^2}} = 1.54 \le 2 \quad \textbf{o.k.}$$

$$\begin{aligned} P_p &= 0.85 f_c{}' A_1 \sqrt{A_2/A_1} \le 1.7 f_c{}' A_1 \\ &= 0.85(3 \text{ ksi})(52.0 \text{ in.}^2)(1.54) \le 1.7(3 \text{ ksi})(52.0 \text{ in.}^2) \\ &= 204 \text{ kips} \le 265 \text{ kips} \end{aligned}$$

(*Spec*. Eq. J8-2)

LRFD	ASD
$\phi_c = 0.65$ $\phi_c P_p = 0.65(204 \text{ kips})$ $= 133 \text{ kips}$ 133 kips > 90.0 kips **o.k.** Calculate the required bearing-plate thickness using AISC *Manual* Part 14. $n = \dfrac{B}{2} - k$ (*Manual* Eq. 14-1) $= \dfrac{8.00 \text{ in.}}{2} - 0.972 \text{ in.}$ $= 3.03 \text{ in.}$ $t_{min} = \sqrt{\dfrac{2R_u n^2}{\phi A_1 F_y}}$ $= \sqrt{\dfrac{2(90.0 \text{ kips})(3.03 \text{ in.})^2}{0.90(6.50 \text{ in.})(8.00 \text{ in.})(36 \text{ ksi})}}$ $= 0.990 \text{ in.}$ Use PL 1 in.× 6½ in.× 0 ft 8 in.	$\Omega_c = 2.31$ $\dfrac{P_p}{\Omega} = \dfrac{204 \text{ kips}}{2.31}$ $= 88.3 \text{ kips}$ 88.3 kips > 60.0 kips **o.k.** Calculate the required bearing-plate thickness using AISC *Manual* Part 14. $n = \dfrac{B}{2} - k \; \dfrac{B}{2} - k$ (*Manual* Eq. 14-1) $= \dfrac{8.00 \text{ in.}}{2} - 0.972 \text{ in.}$ $= 3.03 \text{ in.}$ $t_{min} = \sqrt{\dfrac{\Omega 2 R_a n^2}{A_1 F_y}}$ $= \sqrt{\dfrac{1.67(2)(60.0 \text{ kips})(3.03 \text{ in.})^2}{(6.50 \text{ in.})(8.00 \text{ in.})(36 \text{ ksi})}}$ $= 0.991 \text{ in.}$ Use PL 1 in.× 6½ in.× 0 ft 8 in.

Note: The calculations for t_{min} are conservative. Taking the strength of the beam flange into consideration results in a thinner required bearing plate or no bearing plate at all.

EXAMPLE II.D-3 SLIP-CRITICAL CONNECTION WITH OVERSIZED HOLES

Given:

Design the connection of an ASTM A36 2L3×3×5⁄16 tension member to an ASTM A36 plate welded to an ASTM A992 beam as shown in Figure II.D-3-1 for a dead load of 15 kips and a live load of 45 kips. The angles have standard holes and the plate has oversized holes per AISC *Specification* Table J3.3. Use ¾-in.-diameter ASTM A325-SC bolts with Class A surfaces.

P_D = 15 kips
P_L = 45 kips

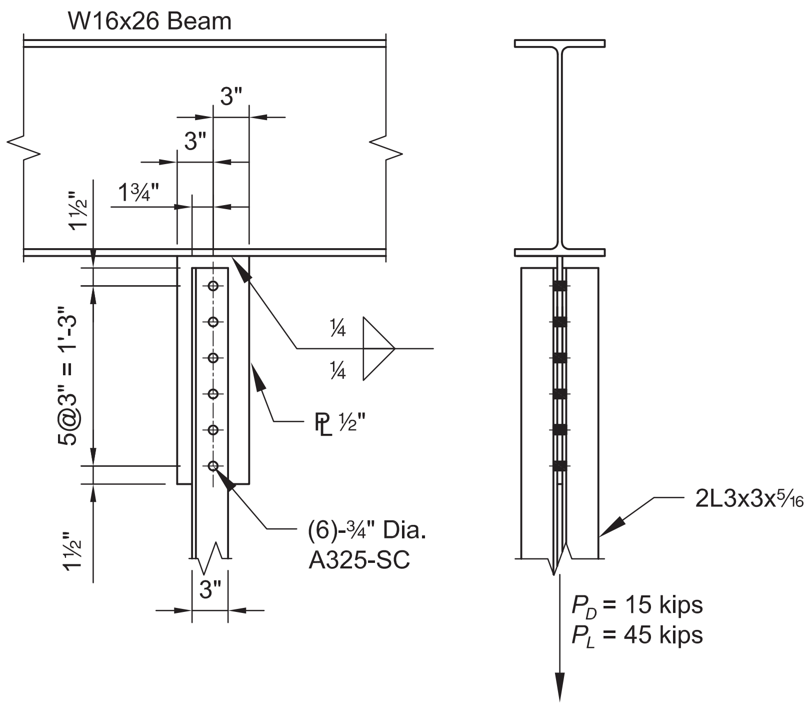


Fig. II.D-3-1. Connection Configuration for Example II.D-3.

Solution:

From AISC *Manual* Tables 2-4 and 2-5, the material properties are as follows:

Beam
W16×26
ASTM A992
F_y = 50 ksi
F_u = 65 ksi

Hanger
2L3×3×5⁄16
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

Plate
ASTM A36
F_y = 36 ksi
F_u = 58 ksi

From AISC *Manual* Tables 1-1, 1-7 and 1-15, the geometric properties are as follows:

Beam
W16×26
t_f = 0.345 in.
t_w = 0.250 in.
k_{des} = 0.747 in.

Hanger
2L3×3×5⁄16
A = 3.56 in.^{2}
$\bar{x}$ = 0.860 in. for single angle

Plate
t_p = 0.500 in.

LRFD	ASD
Calculate required strength.	Calculate required strength.
$R_u = (1.2)(15 \text{ kips}) + (1.6)(45 \text{ kips})$ $= 90.0$ kips	$R_a = 15 \text{ kips} + 45 \text{ kips}$ $= 60.0$ kips
Check the available slip resistance of the bolts using AISC *Manual* Table 7-3.	Check the available slip resistance of the bolts using AISC *Manual* Table 7-3.
For ¾-in.-diameter ASTM A325-SC bolts with Class A faying surfaces in oversized holes and double shear:	For ¾-in.-diameter ASTM A325-SC bolts with Class A faying surfaces in oversized holes and double shear:
$\phi r_n = 16.1$ kips/bolt	$\dfrac{r_n}{\Omega} = 10.8$ kips/bolt
$n = \dfrac{R_u}{\phi r_n} = \dfrac{90.0 \text{ kips}}{16.1 \text{ kips/bolt}}$ $= 5.59 \rightarrow$ 6 bolts	$n = \dfrac{R_a}{(r_n/\Omega)} = \dfrac{60.0 \text{ kips}}{10.8 \text{ kips/bolt}}$ $= 5.56 \rightarrow$ 6 bolts
Slip-critical connections must also be designed for the limit states of bearing-type connections. Check bolt shear strength using AISC *Manual* Table 7-1.	Slip-critical connections must also be designed for the limit states of bearing-type connections. Check bolt shear strength using AISC *Manual* Table 7-1.
$\phi r_n = \phi F_v A_b = 35.8$ kips/bolt	$\dfrac{r_n}{\Omega} = \dfrac{F_v A_b}{\Omega} = 23.9$ kips/bolt
$\phi R_n = \phi r_n n$ $= (35.8 \text{ kips/bolt})(6 \text{ bolts})$ $= 215 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\dfrac{R_n}{\Omega} = \dfrac{r_n}{\Omega} n$ $= (23.9 \text{ kips/bolt})(6 \text{ bolts})$ $= 143 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Tensile Yielding Strength of the Angles

$$P_n = F_y A_g \quad \text{(Spec. Eq. D2-1)}$$
$$= 36\text{ ksi}\left(3.56\text{ in.}^2\right)$$
$$= 128\text{ kips}$$

LRFD	ASD
$\phi_t = 0.90$ $\phi_t P_n = 0.90(128\text{ kips})$ $= 115\text{ kips} > 90.0\text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $\dfrac{P_n}{\Omega} = \dfrac{128\text{ kips}}{1.67}$ $= 76.6\text{ kips} > 60.0\text{ kips}$ **o.k.**

Tensile Rupture Strength of the Angles

$$U = 1 - \frac{\bar{x}}{l} \text{ from AISC } \textit{Specification} \text{ Table D3.1 Case 2}$$
$$= 1 - \frac{0.860\text{ in.}}{15.0\text{ in.}}$$
$$= 0.943$$

$$A_e = A_n U \quad \text{(Spec. Eq. D3-1)}$$
$$= \left[3.56\text{ in.}^2 - 2(\tfrac{5}{16}\text{ in.})(\tfrac{13}{16}\text{ in.} + \tfrac{1}{16}\text{ in.})\right](0.943)$$
$$= 2.84\text{ in.}^2$$

$$P_n = F_u A_e \quad \text{(Spec. Eq. D2-2)}$$
$$= 58\text{ ksi}(2.84\text{ in.}^2)$$
$$= 165\text{ kips}$$

LRFD	ASD
$\phi_t = 0.75$ $\phi_t P_n = 0.75(165\text{ kips})$ $= 124\text{ kips} > 90.0\text{ kips}$ **o.k.**	$\Omega_t = 2.00$ $\dfrac{P_n}{\Omega_t} = \dfrac{165\text{ kips}}{2.00}$ $= 82.5\text{ kips} > 60.0\text{ kips}$ **o.k.**

Block Shear Rupture Strength of the Angles

Use a single vertical row of bolts.

$U_{bs} = 1,\ n = 6,\ L_{ev} = 1\tfrac{1}{2}\text{ in.}$, and $L_{eh} = 1\tfrac{1}{4}\text{ in.}$

$$R_n = 0.60 F_u A_{nv} + U_{bs} F_u A_{nt} \le 0.60 F_y A_{gv} + U_{bs} F_u A_{nt} \quad \text{(Spec. Eq. J4-5)}$$

Shear Yielding Component

$$A_{gv} = \left[5(3.00\text{ in.}) + 1.50\text{ in.}\right](\tfrac{5}{16}\text{ in.})$$
$$= 5.16\text{ in.}^2 \text{ per angle}$$

$$0.60F_yA_{gv} = 0.60(36\text{ ksi})(5.16\text{ in.}^2)$$
$$= 111\text{ kips per angle}$$

Shear Rupture Component

$$A_{nv} = 5.16\text{ in.}^2 - 5.5(^{13}\!/_{16}\text{ in.} + ^1\!/_{16}\text{ in.})(^5\!/_{16}\text{ in.})$$
$$= 3.66\text{ in.}^2\text{ per angle}$$

$$0.60F_uA_{nv} = 0.60(58\text{ ksi})(3.66\text{ in.}^2)$$
$$= 127\text{ kips per angle}$$

Shear yielding controls over shear rupture.

Tension Rupture Component

$$A_{nt} = [1.25\text{ in.} - 0.5(^{13}\!/_{16}\text{ in.} + ^1\!/_{16}\text{ in.})](^5\!/_{16}\text{ in.})$$
$$= 0.254\text{ in.}^2\text{ per angle}$$

$$U_{bs}F_uA_{nt} = 1.0(58\text{ ksi})(0.254\text{ in.}^2)$$
$$= 14.7\text{ kips per angle}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(2)(111\text{ kips} + 14.7\text{ kips})$ $= 189\text{ kips} > 90.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{2(111\text{kips} + 14.7\text{ kips})}{2.00}$ $= 126\text{ kips} > 60.0\text{ kips}$ **o.k.**

Bearing / Tear Out Strength of the Angles

Holes are standard 13⁄16-in. diameter.

Check strength for edge bolt.

$$l_c = 1.50\text{ in.} - \frac{^3\!/_4\text{ in.} + ^1\!/_{16}\text{ in.}}{2}$$
$$= 1.09\text{ in.}$$

$$r_n = 1.2l_ctF_u \le 2.4dtF_u \qquad (\textit{Spec}.\text{ Eq. J3-6a})$$
$$= 1.2(1.09\text{ in.})(^5\!/_{16}\text{ in.})(2)(58\text{ ksi}) \le 2.4(^3\!/_4\text{ in.})(^5\!/_{16}\text{ in.})(2)(58\text{ ksi})$$
$$= 47.4\text{ kips} \le 65.3\text{ kips}$$

Check strength for interior bolts.

$$l_c = 3.00\text{ in.} - (^3\!/_4\text{ in.} + ^1\!/_{16}\text{ in.})$$
$$= 2.19\text{ in.}$$

$$r_n = 1.2l_ctF_u \le 2.4dtF_u \qquad (\textit{Spec}.\text{ Eq. J3-6a})$$
$$= 1.2(2.19\text{ in.})(^5\!/_{16}\text{ in.})(2)(58\text{ ksi}) \le 2.4(^3\!/_4\text{ in.})(^5\!/_{16}\text{ in.})(2)(58\text{ ksi})$$

$= 95.3 \text{ kips} \leq 65.3 \text{ kips}$

Total strength for all bolts.

$$r_n = 1(47.4 \text{ kips}) + 5(65.3 \text{ kips})$$
$$= 374 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(374 \text{ kips})$ $= 281 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{r_n}{\Omega} = \dfrac{374 \text{ kips}}{2.00}$ $= 187 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Tensile Yielding Strength of the ½-in. Plate

By inspection, the Whitmore section includes the entire width of the ½-in. plate.

$$R_n = F_y A_g \qquad (\textit{Spec}. \text{ Eq. J4-1})$$
$$= 36 \text{ ksi}(\tfrac{1}{2} \text{ in.})(6.00 \text{ in.})$$
$$= 108 \text{ kips}$$

LRFD	ASD
$\phi_t = 0.90$ $\phi R_n = 0.90(108 \text{ kips})$ $= 97.2 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega_t = 1.67$ $\dfrac{R_n}{\Omega_t} = \dfrac{108 \text{ kips}}{1.67}$ $= 64.7 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Tensile Rupture Strength of the ½-in. Plate

Holes are oversized $^{15}/_{16}$-in. diameter.

Calculate the effective net area.

$A_e = A_n \leq 0.85 A_g$ from AISC *Specification* Section J4.1

$$\leq 0.85\left(3.00 \text{ in.}^2\right)$$
$$\leq 2.55 \text{ in.}^2$$

$$A_n = 3.00 \text{ in.}^2 - (\tfrac{1}{2} \text{ in.})(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})$$
$$= 2.50 \text{ in.}^2 \leq 2.55 \text{ in.}^2$$

$$A_e = A_n U \qquad (\textit{Spec}. \text{ Eq. D3-1})$$
$$= 2.50 \text{ in.}^2 (1.0)$$
$$= 2.50 \text{ in.}^2$$

$$R_n = F_u A_e \qquad (\textit{Spec}. \text{ Eq. J4-2})$$
$$= 58 \text{ ksi}(2.50 \text{ in.}^2)$$
$$= 145 \text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(145 \text{ kips})$ $= 109 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{145 \text{ kips}}{2.00}$ $= 72.5 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Block Shear Rupture Strength of the ½-in. Plate

Use a single vertical row of bolts.

$U_{bs} = 1.0,\ n = 6,\ L_{ev} = 1\tfrac{1}{2} \text{ in., and } L_{eh} = 3 \text{ in.}$

$$R_n = 0.60F_u A_{nv} + U_{bs} F_u A_{nt} \leq 0.60F_y A_{gv} + U_{bs} F_u A_{nt} \qquad (Spec. \text{ Eq. J4-5})$$

Shear Yielding Component

$$\begin{aligned} A_{gv} &= \left[5(3.00 \text{ in.}) + 1.50 \text{ in.}\right](\tfrac{1}{2} \text{ in.}) \\ &= 8.25 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_y A_{gv} &= 0.60(36 \text{ ksi})(8.25 \text{ in.}^2) \\ &= 178 \text{ kips} \end{aligned}$$

Shear Rupture Component

$$\begin{aligned} A_{nv} &= 8.25 \text{ in.}^2 - 5.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})(\tfrac{1}{2} \text{ in.}) \\ &= 5.50 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} 0.60F_u A_{nv} &= 0.60(58 \text{ ksi})(5.50 \text{ in.}^2) \\ &= 191 \text{ kips} \end{aligned}$$

Shear yielding controls over shear rupture.

Tension Rupture Component

$$\begin{aligned} A_{nt} &= \left[3.00 \text{ in.} - 0.5(\tfrac{15}{16} \text{ in.} + \tfrac{1}{16} \text{ in.})\right](\tfrac{1}{2} \text{ in.}) \\ &= 1.25 \text{ in.}^2 \end{aligned}$$

$$\begin{aligned} U_{bs} F_u A_{nt} &= 1.0(58 \text{ ksi})(1.25 \text{ in.}^2) \\ &= 72.5 \text{ kips} \end{aligned}$$

LRFD	ASD
$\phi = 0.75$ $\phi R_n = 0.75(178 \text{ kips} + 72.5 \text{ kips})$ $= 188 \text{ kips} > 90.0 \text{ kips}$ **o.k.**	$\Omega = 2.00$ $\dfrac{R_n}{\Omega} = \dfrac{(178 \text{ kips} + 72.5 \text{ kips})}{2.00}$ $= 125 \text{ kips} > 60.0 \text{ kips}$ **o.k.**

Bearing/Tear Out Strength of the ½-in. Plate

Holes are oversized 15⁄16-in. diameter.

Check strength for edge bolt.

$$l_c = 1.50\text{ in.} - \frac{15/16\text{ in.}}{2}$$
$$= 1.03\text{ in.}$$

$$r_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad (\textit{Spec}.\text{ Eq. J3-6a})$$
$$= 1.2(1.03\text{ in.})(½\text{ in.})(58\text{ ksi}) \leq 2.4(¾)(½\text{ in.})(58\text{ ksi})$$
$$= 35.8\text{ kips} \leq 52.2\text{ kips}$$

Check strength for interior bolts.

$$l_c = 3.00\text{ in.} - 15/16\text{ in.}$$
$$= 2.06\text{ in.}$$

$$r_n = 1.2 l_c t F_u \leq 2.4 d t F_u \qquad (\textit{Spec}.\text{ Eq. J3-6a})$$
$$= 1.2(2.06\text{ in.})(½\text{ in.})(58\text{ ksi}) \leq 2.4(¾\text{ in.})(½\text{ in.})(58\text{ ksi})$$
$$= 71.7\text{ kips} \leq 52.2\text{ kips}$$

Total strength for all bolts.

$$r_n = 1(35.8\text{ kips}) + 5(52.2\text{ kips})$$
$$= 297\text{ kips}$$

LRFD	ASD
$\phi = 0.75$ $\phi r_n = 0.75(297\text{ kips})$ $= 223\text{ kips} > 90.0\text{ kips}$ **o.k.**	$\Omega = 2.00$ $\frac{r_n}{\Omega} = \frac{297\text{ kips}}{2.00}$ $= 149\text{ kips} > 60.0\text{ kips}$ **o.k.**

Fillet Weld Required for the ½-in. Plate to the W-Shape Beam

Because the angle of the force relative to the axis of the weld is 90°, the strength of the weld can be increased by the following factor from AISC *Specification* Section J2.4.

$$(1.0 + 0.50\sin^{1.5}\theta) = (1.0 + 0.50\sin^{1.5} 90°)$$
$$= 1.50$$

From AISC *Manual* Equations 8-2,

LRFD	ASD
$D_{req} = \frac{R_u}{1.50(1.392l)}$ $= \frac{90.0\text{ kips}}{1.50(1.392)(2)(6.00\text{ in.})}$ $= 3.59\text{ sixteenths}$	$D_{req} = \frac{P_a}{1.50(0.928l)}$ $= \frac{60.0\text{ kips}}{1.50(0.928)(2)(6.00\text{ in.})}$ $= 3.59\text{ sixteenths}$

From AISC *Manual* Table J2.4, the minimum fillet weld size is 3⁄16 in.

Use a ¼-in. fillet weld on both sides of the plate.

Beam Flange Base Metal Check

$$t_{min} = \frac{3.09D}{F_u} \qquad \text{(}Manual\text{ Eq. 9-2)}$$

$$= \frac{3.09(3.59\text{ sixteenths})}{65\text{ ksi}}$$

$$= 0.171\text{ in.} < 0.345\text{ in.} \quad \textbf{o.k.}$$

Concentrated Forces Check for W16x26 Beam

Check web local yielding. (Assume the connection is at a distance from the member end greater than the depth of the member, *d*.)

$$R_n = F_{yw}t_w(5k_{des} + l_b) \qquad \text{(}Spec\text{. Eq. J10-2)}$$

$$= 50\text{ ksi}(\text{¼ in.})\left[5(0.747\text{ in.}) + 6.00\text{ in.}\right]$$

$$= 122\text{ kips}$$

LRFD	ASD
$\phi = 1.00$ $\phi R_n = 1.00(122\text{ kips})$ $= 122\text{ kips} > 90.0\text{ kips}$ **o.k.**	$\Omega = 1.50$ $\frac{R_n}{\Omega} = \frac{122\text{ kips}}{1.50}$ $= 81.3\text{ kips} > 60.0\text{ kips}$ **o.k.**

Part III
System Design Examples

EXAMPLE III-1 Design of Selected Members and Lateral Analysis of a Four-Story Building

INTRODUCTION

This section illustrates the load determination and selection of representative members that are part of the gravity and lateral frame of a typical four-story building. The design is completed in accordance with the 2010 AISC *Specification for Structural Steel Buildings* and the 14th Edition AISC *Steel Construction Manual*. Loading criteria are based on ASCE/SEI 7-10 (ASCE, 2010).

This section includes:

- Analysis and design of a typical steel frame for gravity loads
- Analysis and design of a typical steel frame for lateral loads
- Examples illustrating three methods for satisfying the stability provisions of AISC *Specification* Chapter C

The building being analyzed in this design example is located in a Midwestern city with moderate wind and seismic loads. The loads are given in the description of the design example. All members are ASTM A992 steel.

CONVENTIONS

The following conventions are used throughout this example:

1. Beams or columns that have similar, but not necessarily identical, loads are grouped together. This is done because such grouping is generally a more economical practice for design, fabrication and erection.

2. Certain calculations, such as design loads for snow drift, which might typically be determined using a spreadsheet or structural analysis program, are summarized and then incorporated into the analysis. This simplifying feature allows the design example to illustrate concepts relevant to the member selection process.

3. Two commonly used deflection calculations, for uniform loads, have been rearranged so that the conventional units in the problem can be directly inserted into the equation for steel design. They are as follows:

Simple Beam:

$$\Delta = \frac{5\,w\text{ kip/in.}\left(L\text{ in.}\right)^4}{384\left(29{,}000\text{ ksi}\right)\left(I\text{ in.}^4\right)} = \frac{w\text{ kip/ft}\left(L\text{ ft}\right)^4}{1{,}290\left(I\text{ in.}^4\right)}$$

Beam Fixed at both Ends:

$$\Delta = \frac{w\text{ kip/in.}\left(L\text{ in.}\right)^4}{384\left(29{,}000\text{ ksi}\right)\left(I\text{ in.}^4\right)} = \frac{w\text{ kip/ft}\left(L\text{ ft}\right)^4}{6{,}440\left(I\text{ in.}^4\right)}$$

DESIGN SEQUENCE

The design sequence is presented as follows:

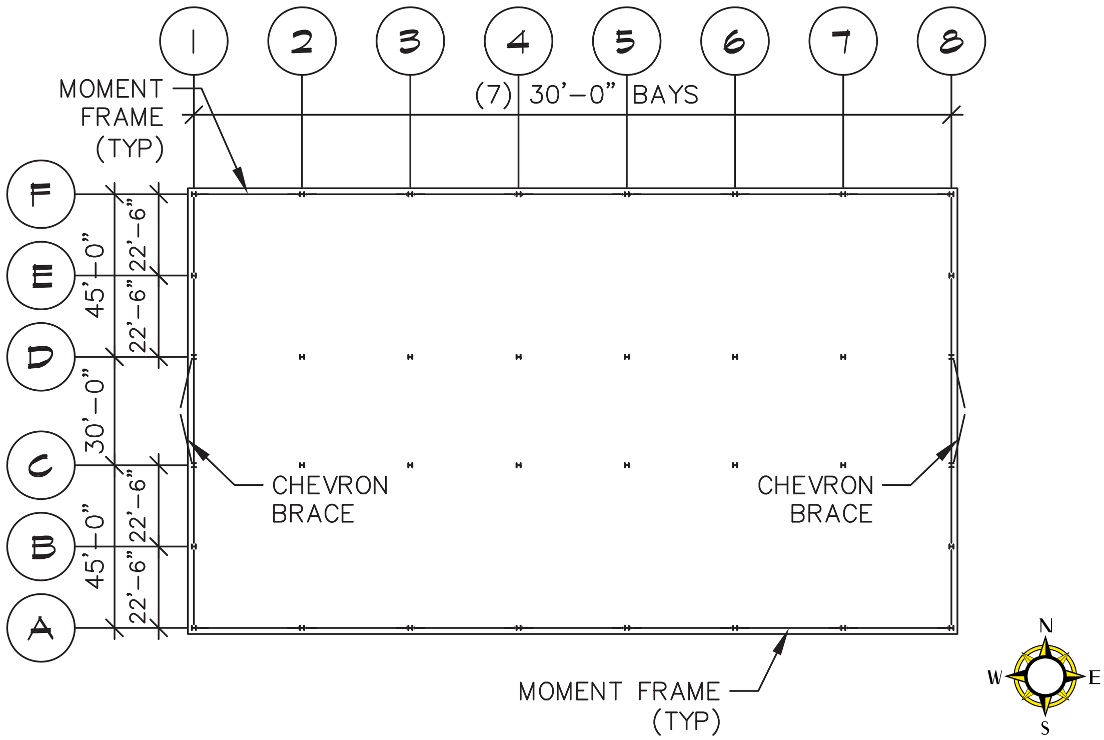
1. General description of the building including geometry, gravity loads and lateral loads
2. Roof member design and selection
3. Floor member design and selection
4. Column design and selection for gravity loads
5. Wind load determination
6. Seismic load determination
7. Horizontal force distribution to the lateral frames
8. Preliminary column selection for the moment frames and braced frames
9. Seismic load application to lateral systems
10. Stability (P-Δ) analysis

Return to Table of Contents

GENERAL DESCRIPTION OF THE BUILDING

Geometry

The design example is a four-story building, comprised of seven bays at 30 ft in the East-West (numbered grids) direction and bays of 45 ft, 30 ft and 45 ft in the North-South (lettered grids) direction. The floor-to-floor height for the four floors is 13 ft 6 in. and the height from the fourth floor to the roof (at the edge of the building) is 14 ft 6 in. Based on discussions with fabricators, the same column size will b e used for the whole height of the building.



Basic Building Layout

The plans of these floors and the roof are shown on Sheets S2.1 thru S2.3, found at the end of this Chapter. The exterior of the building is a ribbon window system with brick spandrels supported and back-braced with steel and infilled with metal studs. The spandrel wall extends 2 ft above the elevation of the edge of the roof. The window and spandrel system is shown on design drawing Sheet S4.1.

The roof system is 1½-in. metal deck on bar joists. These bar joists are supported on steel beams as shown on Sheet S2.3. The roof slopes to interior drains. The middle 3 bays have a 6 ft tall screen wall around them and house the mechanical equipment and the elevator over run. This area has steel beams, in place of steel bar joists, to support the mechanical equipment.

The three elevated floors have 3 in. of normal weight concrete over 3-in. composite deck for a total slab thickness of 6 in. The supporting beams are spaced at 10 ft on center. These beams are carried by composite girders in the East-West direction to the columns. There is a 30 ft by 29 ft opening in the second floor, to create a two-story atrium at the entrance. These floor layouts are shown on Sheets S2.1 and S2.2. The first floor is a slab on grade and the foundation consists of conventional spread footings.

The building includes both moment frames and braced frames for lateral resistance. The lateral system in the North-South direction consists of chevron braces at the end of the building, located adjacent to the stairways. In the East-West direction there are no locations in which chevron braces can be concealed; consequently, the lateral system in the East-West direction is composed of moment frames at the North and South faces of the building.

This building is sprinklered and has large open spaces around it, and consequently does not require fireproofing for the floors.

Wind Forces

The Basic Wind Speed is 90 miles per hour (3 second gust). Because it is sited in an open, rural area, it will be analyzed as Wind Exposure Category C. Because it is an ordinary (Risk Category II) office occupancy, the wind importance factor is 1.0.

Seismic Forces

The sub-soil has been evaluated and the site class has been determined to be Category D. The area has a short period $S_s = 0.121g$ and a one-second period $S_1 = 0.060g$. The seismic importance factor is 1.0, that of an ordinary office occupancy (Risk Category II).

Roof and Floor Loads

Roof loads:

The ground snow load (p_g) is 20 psf. The slope of the roof is ¼ in./ft or more at all locations, but not exceeding ½ in./ft; consequently, 5 psf rain-on-snow surcharge is to be considered, but ponding instability design calculations are not required. This roof can be designed as a fully exposed roof, but, per ASCE/SEI 7 Section 7.3, cannot be designed for less than $p_f = (I)p_g = 20$ psf uniform snow load. Snow drift will be applied at the edges of the roof and at the screen wall around the mechanical area. The roof live load for this building is 20 psf, but may be reduced per ASCE/SEI 7 Section 4.8 where applicable.

Floor Loads:

The basic live load for the floor is 50 psf. An additional partition live load of 20 psf is specified. Because the locations of partitions and, consequently, corridors are not known, and will be subject to change, the entire floor will be designed for a live load of 80 psf. This live load will be reduced, based on type of member and area per the ASCE provisions for live-load reduction.

Wall Loads:

A wall load of 55 psf will be used for the brick spandrels, supporting steel, and metal stud back-up. A wall load of 15 psf will be used for the ribbon window glazing system.

ROOF MEMBER DESIGN AND SELECTION

Calculate dead load and snow load.

Dead Load

Roofing	=	5 psf
Insulation	=	2 psf
Deck	=	2 psf
Beams	=	3 psf
Joists	=	3 psf
Misc.	=	5 psf
Total	=	20 psf

Return to Table of Contents

Snow Load from ASCE/SEI 7 Section 7.3 and 7.10

Snow	= 20 psf
Rain on Snow	= 5 psf
Total	= 25 psf

Note: In this design, the rain and snow load is greater than the roof live load

The deck is 1½ in., wide rib, 22 gage, painted roof deck, placed in a pattern of three continuous spans minimum. The typical joist spacing is 6 ft on center. At 6 ft on center, this deck has an allowable total load capacity of 89 psf. The roof diaphragm and roof loads extend 6 in. past the centerline of grid as shown on Sheet S4.1.

From Section 7.7 of ASCE/SEI 7, the following drift loads are calculated:

Flat roof snow load = 20 psf, Density γ = 16.6 lbs/ft^3, h_b = 1.20 ft

Summary of Drifts

	Upwind Roof Length (l_u)	**Proj. Height**	**Max. Drift Load**	**Max Drift Width (W)**
Side Parapet	121 ft	2 ft	13.2 psf	6.36 ft
End Parapet	211 ft	2 ft	13.2 psf	6.36 ft
Screen Wall	60.5 ft	6 ft	30.5 psf	7.35 ft

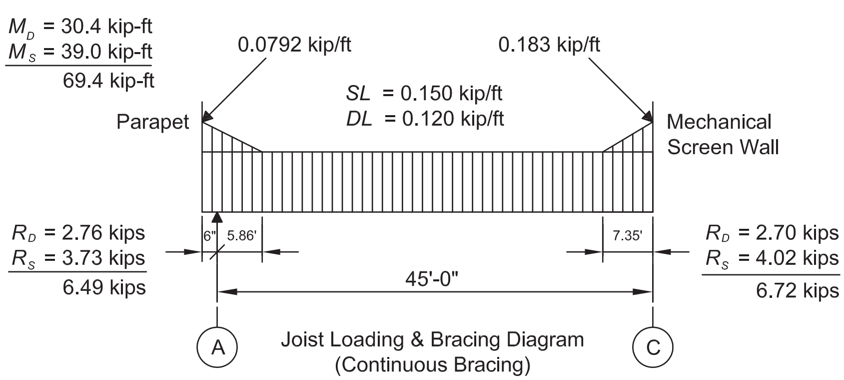
The snow drift at the penthouse was calculated for the maximum effect, using the East-West wind and an upwind fetch from the parapet to the centerline of the columns at the penthouse. This same drift is conservatively used for wind in the North-South direction. The precise location of the drift will depend upon the details of the penthouse construction, but will not affect the final design in this case.

SELECT ROOF JOISTS

Layout loads and size joists.

User Note: Joists may be specified using ASD or LRFD but are most commonly specified by ASD as shown here.

The 45-ft side joist with the heaviest loads is shown below along with its end reactions and maximum moment:



Joist Loading & Bracing Diagram
(Continuous Bracing)

Because the load is not uniform, select a 24KCS4 joist from the Steel Joist Institute load tables (SJI, 2005). This joist has an allowable moment of 92.3 kip-ft, an allowable shear of 8.40 kips, a gross moment of inertia of 453 in.^{4} and weighs 16.6 plf.

The first joist away from the end of the building is loaded with snow drift along the length of the member. Based on analysis, a 24KCS4 joist is also acceptable for this uniform load case.

As an alternative to directly specifying the joist sizes on the design document, as done in this example, loading diagrams can be included on the design documents to allow the joist manufacturer to economically design the joists.

The typical 30-ft joist in the middle bay will have a uniform load of

w = (20 psf + 25 psf)(6 ft) = 270 plf

w_{SL} = (25 psf)(6 ft) = 150 plf

From the Steel Joist Institute load tables, select an 18K5 joist which weighs approximately 7.7 plf and satisfies both strength and deflection requirements.

Note: the first joist away from the screen wall and the first joist away from the end of the building carry snow drift. Based on analysis, an 18K7 joist will be used in these locations.

SELECT ROOF BEAMS

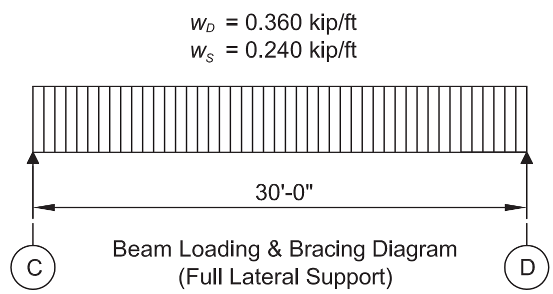
Calculate loads and select beams in the mechanical area.

For the beams in the mechanical area, the mechanical units could weigh as much as 60 psf. Use 40 psf additional dead load, which will account for the mechanical units and the screen wall around the mechanical area. Use 15 psf additional snow load, which will account for any snow drift which could occur in the mechanical area. The beams in the mechanical area are spaced at 6 ft on center.

Per AISC Design Guide 3 (West et al., 2003), calculate the minimum I_x to limit deflection to l/360 = 1.00 in. because a plaster ceiling will be used in the lobby area. Use 40 psf as an estimate of the snow load, including some drifting that could occur in this area, for deflection calculations.

Note: The beams and supporting girders in this area should be rechecked when the final weights and locations for the mechanical units have been determined.

$$I_{req}\text{ (Live Load)} = \frac{0.240\text{ kip/ft}(30.0\text{ ft})^4}{1{,}290(1.00\text{ in.})}$$
$$= 151\text{ in.}^4$$



Beam Loading & Bracing Diagram
(Full Lateral Support)

Calculate the required strengths from Chapter 2 of ASCE/SEI 7 and select the beams in the mechanical area.

LRFD	ASD
$w_u = 6.00\text{ ft}[1.2\,(0.020\text{ kip/ft}^2 + 0.040\text{ kip/ft}^2) + 1.6(0.025\text{ kip/ft}^2 + 0.015\text{ kip/ft}^2)]$ $= 0.816\text{ kip/ft}$	$w_a = 6.00\text{ ft}[0.020\text{ kip/ft}^2 + 0.040\text{ kip/ft}^2 + 0.025\text{ kip/ft}^2 + 0.015\text{ kip/ft}^2]$ $= 0.600\text{ kip/ft}$
$R_u = \frac{30.0\text{ ft}}{2}(0.816\text{ kip/ft})$ $= 12.2\text{ kips}$	$R_a = \frac{30.0\text{ ft}}{2}(0.600\text{ kip/ft})$ $= 9.00\text{ kips}$
$M_u = \frac{0.816\text{ kip/ft}(30.0\text{ ft})^2}{8}$ $= 91.8\text{ kip-ft}$	$M_a = \frac{0.600\text{ kip/ft}(30.0\text{ ft})^2}{8}$ $= 67.5\text{ kip-ft}$
Assuming the beam has full lateral support, use *Manual* Table 3-2, select an ASTM A992 W14×22, which has a design flexural strength of 125 kip-ft, a design shear strength of 94.5 kips, and an I_x of 199 in.^{4}	Assuming the beam has full lateral support, use *Manual* Table 3-2, select an ASTM A992 W14×22, which has an allowable flexural strength of 82.8 kip-ft, an allowable shear strength of 63.0 kips and an I_x of 199 in.^{4}

SELECT ROOF BEAMS AT THE END (EAST & WEST) OF THE BUILDING

The beams at the ends of the building carry the brick spandrel panel and a small portion of roof load. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. Therefore, per AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to L/600 or ⅜ in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to L/600 or 0.3 in. max to accommodate the brick and L/360 or ¼ in. max to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to L/600 or ¼ in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. In calculating the wall loads, the spandrel panel weight is taken as 55 psf. The spandrel panel weight is approximately:

$$w_D = 7.50\text{ ft}(0.055\text{ kip/ft}^2) = 0.413\text{ kip/ft}$$

The dead load from the roof is equal to:

$$w_D = 3.50\text{ ft}(0.020\text{ kip/ft}^2) = 0.070\text{ kip/ft}$$

Use 8 psf for the initial dead load.

$$w_{D(initial)} = 3.50\text{ ft}(0.008\text{ kip/ft}^2) = 0.0280\text{ kip/ft}$$

Use 12 psf for the superimposed dead load.

$$w_{D(super)} = 3.50\text{ ft}(0.012\text{ kip/ft}^2) = 0.0420\text{ kip/ft}$$

The snow load from the roof can be conservatively taken as:

$$w_S = 3.50\text{ ft}(0.025\text{ kip/ft}^2 + 0.0132\text{ kip/ft}^2) = 0.134\text{ kip/ft}$$

to account for the maximum snow drift as a uniform load.

Assume the beams are simple spans of 22.5 ft.

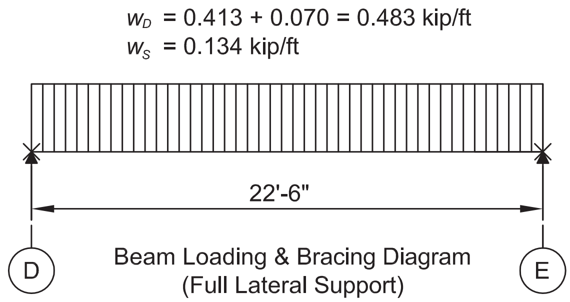
Calculate minimum I_x to limit the superimposed dead and live load deflection to ¼ in.

$$I_{req} = \frac{0.176\text{ kip/ft}(22.5\text{ ft})^4}{1{,}290(\tfrac{1}{4}\text{ in.})} = 140\text{ in.}^4$$

Calculate minimum I_x to limit the cladding and initial dead load deflection to ⅜ in.

$$I_{req} = \frac{0.441\text{ kip/ft}(22.5\text{ ft})^4}{1{,}290(\tfrac{3}{8}\text{ in.})} = 234\text{ in.}^4$$

The beams are full supported by the deck as shown in Detail 4 on Sheet S4.1. The loading diagram is as follows:



Calculate the required strengths from Chapter 2 of ASCE/SEI 7 and select the beams for the roof ends.

LRFD	ASD
w_u =1.2(0.070 kip/ft + 0.413 kip/ft) + 1.6(0.134 kip/ft) = 0.794 kip/ft $R_u = \dfrac{22.5\text{ ft}}{2}(0.794\text{ kip/ft})$ = 8.93 kips $M_u = \dfrac{0.794\text{ kip/ft}(22.5\text{ ft})^2}{8}$ = 50.2 kip-ft Assuming the beam has full lateral support, use *Manual* Table 3-2, select an ASTM A992 W16×26, which has a design flexural strength of 166 kip-ft, a design shear strength of 106 kips, and an I_x of 301 in.^{4}	w_a = (0.070 kip/ft + 0.413 kip/ft) + 0.134 kip/ft = 0.617 kip/ft $R_a = \dfrac{22.5\text{ ft}}{2}(0.617\text{ kip/ft})$ = 6.94 kips $M_a = \dfrac{0.617\text{ kip/ft}(22.5\text{ ft})^2}{8}$ = 39.0 kip-ft Assuming the beam has full lateral support, use *Manual* Table 3-2, select an ASTM A992 W16×26, which has an allowable flexural strength of 110 kip-ft, an allowable shear strength of 70.5 kips, and an I_x of 301 in.^{4}

SELECT ROOF BEAMS ALONG THE SIDE (NORTH & SOUTH) OF THE BUILDING

The beams along the side of the building carry the spandrel panel and a substantial roof dead load and live load. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. Therefore, per AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to L/600 or ⅜ in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to L/600 or 0.3 in. max to accommodate the brick and L/360 or ¼ in. max to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to L/600 or ¼ in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. These beams will be part of the moment frames on the side of the building and therefore will be designed as fixed at both ends. The roof dead load and snow load on this edge beam is equal to the joist end dead load and snow load reaction. Treating this as a uniform load, divide this by the joist spacing.

$$w_D = 2.76 \text{ kips/6.00 ft} = 0.460 \text{ kip/ft}$$

$$w_S = 3.73 \text{ kips/6.00 ft} = 0.622 \text{ kip/ft}$$

$$w_{D(initial)} = 23.0 \text{ ft } (0.008 \text{ kip/ft}^2) = 0.184 \text{ kip/ft}$$

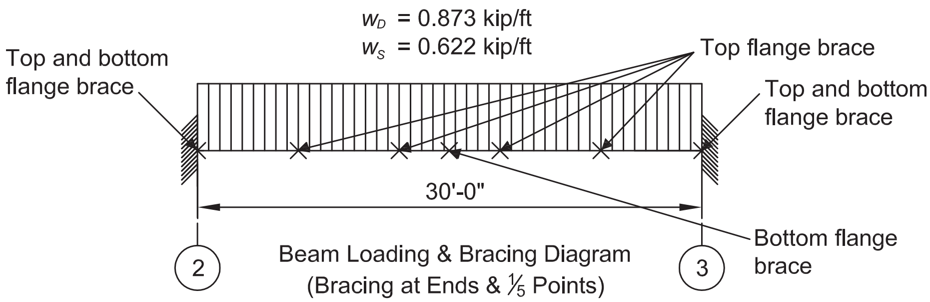
$$w_{D(super)} = 23.0 \text{ ft } (0.012 \text{ kip/ft}^2) = 0.276 \text{ kip/ft}$$

Calculate the minimum I_x to limit the superimposed dead and live load deflection to ¼ in.

$$I_{req} = \frac{(0.898 \text{ kip/ft})(30.0 \text{ ft})^4}{6{,}440(\tfrac{1}{4} \text{ in.})} = 452 \text{ in.}^4$$

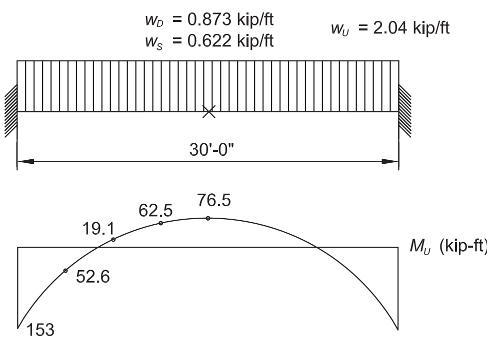
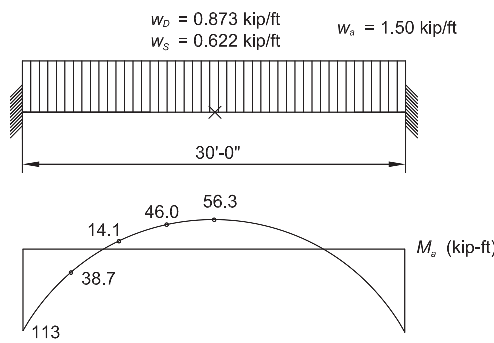
Calculate the minimum I_x to limit the cladding and initial dead load deflection to ⅜ in.

$$I_{req} = \frac{(0.597 \text{ kip/ft})(30.0 \text{ ft})^4}{6{,}440(\tfrac{3}{8} \text{ in.})} = 200 \text{ in.}^4$$



Return to Table of Contents

Calculate the required strengths from Chapter 2 of ASCE/SEI 7 and select the beams for the roof sides.

LRFD	ASD
$w_u = 1.2(0.460\text{ kip/ft} + 0.413\text{ kip/ft}) + 1.6(0.622\text{ kip/ft})$ $= 2.04\text{ kip/ft}$	$w_a = (0.460\text{ kip/ft} + 0.413\text{ kip/ft}) + 0.622\text{ kip/ft}$ $= 1.50\text{ kip/ft}$
$R_u = \dfrac{30.0\text{ ft}}{2}(2.04\text{ kip/ft})$ $= 30.6\text{ kips}$	$R_a = \dfrac{30.0\text{ ft}}{2}(1.50\text{ kip/ft})$ $= 22.5\text{ kips}$
Calculate C_b for compression in the bottom flange braced at the midpoint and supports using AISC *Specification* Equation F1-1.	Calculate C_b for compression in the bottom flange braced at the midpoint and supports using AISC *Specification* Equation F1-1.
	
$M_{uMax} = \dfrac{2.04\text{ kip/ft}(30.0\text{ ft})^2}{12}$ $= 153\text{ kip-ft at supports}$	$M_{aMax} = \dfrac{1.50\text{ kip/ft}(30.0\text{ ft})^2}{12}$ $= 113\text{ kip-ft at supports}$
$M_u = \dfrac{2.04\text{ kip/ft}(30.0\text{ ft})^2}{24}$ $= 76.5\text{ kip-ft at midpoint}$	$M_a = \dfrac{1.50\text{ kip/ft}(30.0\text{ ft})^2}{24}$ $= 56.3\text{ kip-ft at midpoint}$
From AISC *Manual* Table 3-23,	From AISC *Manual* Table 3-23,
$M_{uA} = \dfrac{2.04\text{ kip/ft}}{12}\left(\begin{array}{l}6(30.0\text{ ft})(3.75\text{ ft})\\-(30.0\text{ ft})^2-6(3.75\text{ ft})^2\end{array}\right)$ $= 52.6\text{ kip-ft}$ at quarter point of unbraced segment	$M_{aA} = \dfrac{1.50\text{ kip/ft}}{12}\left(\begin{array}{l}6(30.0\text{ ft})(3.75\text{ ft})\\-(30.0\text{ ft})^2-6(3.75\text{ ft})^2\end{array}\right)$ $= 38.7\text{ kip-ft}$ at quarter point of unbraced segment
$M_{uB} = \dfrac{2.04\text{ kip/ft}}{12}\left(\begin{array}{l}6(30.0\text{ ft})(7.50\text{ ft})\\-(30.0\text{ ft})^2-6(7.50\text{ ft})^2\end{array}\right)$ $= 19.1\text{ kip-ft at midpoint of unbraced segment}$	$M_{aB} = \dfrac{1.50\text{ kip/ft}}{12}\left(\begin{array}{l}6(30.0\text{ ft})(7.50\text{ ft})\\-(30.0\text{ ft})^2-6(7.50\text{ ft})^2\end{array}\right)$ $= 14.1\text{ kip-ft at midpoint of unbraced segment}$
$M_{uC} = \dfrac{2.04\text{ kip/ft}}{12}\left(\begin{array}{l}6(30.0\text{ ft})(11.3\text{ ft})\\-(30.0\text{ ft})^2-6(11.3\text{ ft})^2\end{array}\right)$ $= 62.5\text{ kip-ft at three quarter point of unbraced segment}$	$M_{aC} = \dfrac{1.50\text{ kip/ft}}{12}\left(\begin{array}{l}6(30.0\text{ ft})(11.3\text{ ft})\\-(30.0\text{ ft})^2-6(11.3\text{ ft})^2\end{array}\right)$ $= 46.0\text{ kip-ft at three quarter point of unbraced segment}$

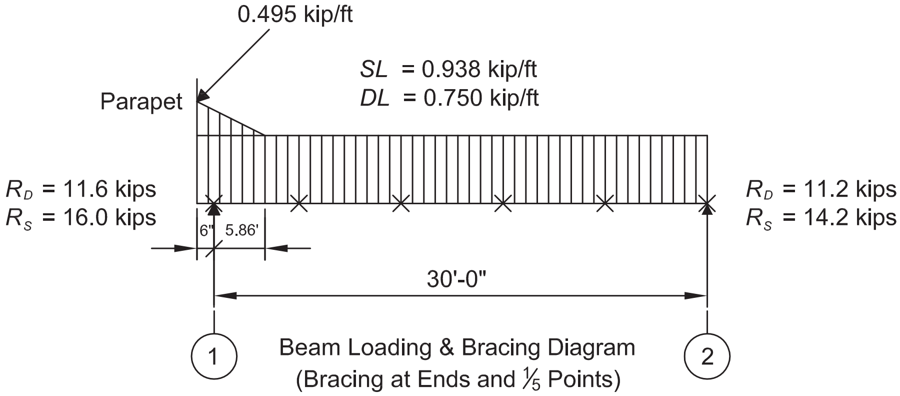
LRFD	ASD
Using AISC *Specification* Equation F1-1, $C_b = \dfrac{12.5M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C}$ $= \dfrac{12.5(153\text{ kip-ft})}{\begin{bmatrix}2.5(153\text{ kip-ft})+3(52.6\text{ kip-ft})\\+4(19.1\text{ kip-ft})+3(62.5\text{ kip-ft})\end{bmatrix}}$ $= 2.38$ From AISC *Manual* Table 3-10, select W18×35. For L_b = 6 ft and C_b = 1.0 $\phi_b M_n$ = 229 kip-ft > 76.5 kip-ft **o.k.** For L_b = 15 ft and C_b = 2.38, $\phi_b M_n$ = (109 kip-ft)2.38 = 259 kip-ft ≤ $\phi_b M_p$ $\phi_b M_p$ = 249 kip-ft > 153 kip-ft **o.k.** From AISC *Manual* Table 3-2, a W18×35 has a design shear strength of 159 kips and an I_x of 510 in.^{4} **o.k.**	Using AISC *Specification* Equation F1-1, $C_b = \dfrac{12.5M_{max}}{2.5M_{max}+3M_A+4M_B+3M_C}$ $= \dfrac{12.5(113\text{ kip-ft})}{\begin{bmatrix}2.5(113\text{ kip-ft})+3(38.7\text{ kip-ft})\\+4(14.1\text{ kip-ft})+3(46.0\text{ kip-ft})\end{bmatrix}}$ $= 2.38$ From AISC *Manual* Table 3-10, select W18×35. For L_b = 6 ft and C_b = 1.0 M_n / Ω_b = 152 kip-ft > 56.3 kip-ft **o.k.** For L_b = 15 ft and C_b = 2.38, M_n / Ω_b = (72.7 kip-ft)2.38 = 173 kip-ft ≤ M_p / Ω_b Ω_b / M_p = 166 kip-ft > 113 kip-ft **o.k.** From AISC *Manual* Table 3-2, a W18×35 has an allowable shear strength of 106 kips and an I_x of 510 in.^{4} **o.k.**

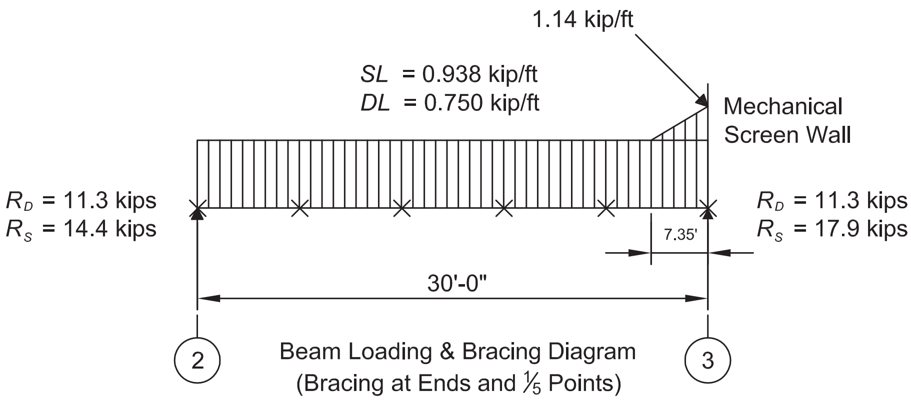
Note: This roof beam may need to be upsized during the lateral load analysis to increase the stiffness and strength of the member and improve lateral frame drift performance.

Return to Table of Contents

SELECT THE ROOF BEAMS ALONG THE INTERIOR LINES OF THE BUILDING

There are three individual beam loadings that occur along grids C and D. The beams from 1 to 2 and 7 to 8 have a uniform snow load except for the snow drift at the end at the parapet. The snow drift from the far ends of the 45-ft joists is negligible. The beams from 2 to 3 and 6 to 7 are the same as the first group, except they have snow drift at the screen wall. The loading diagrams are shown below. A summary of the moments, left and right reactions, and required I_x to keep the live load deflection to equal or less than the span divided by 240 (or 1.50 in.) is given below.





Calculate required strengths from Chapter 2 of ASCE/SEI 7 and required moment of inertia.

LRFD	ASD
Grids 1 to 2 and 7 to 8 (opposite hand) R_u (left) = 1.2(11.6 kips) + 1.6(16.0 kips) = 39.5 kips R_u (right) = 1.2(11.2 kips) + 1.6(14.2 kips) = 36.2 kips M_u = 1.2(84.3 kip-ft) + 1.6(107 kip-ft) = 272 kip-ft	Grids 1 to 2 and 7 to 8 (opposite hand) R_a (left) = 11.6 kips + 16.0 kips = 27.6 kips R_a (right) = 11.2 kips + 14.2 kips = 25.4 kips M_a = 84.3 kip-ft + 107 kip-ft = 191 kip-ft

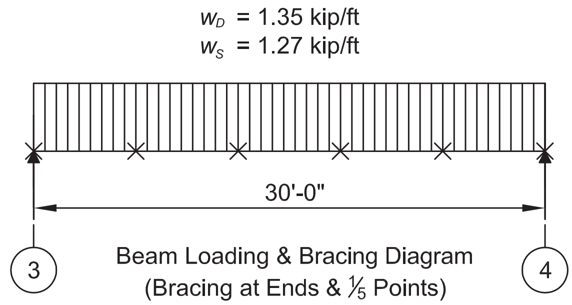
LRFD	ASD
$I_{x\,req'd} = \dfrac{(0.938 \text{ kip/ft})(30.0 \text{ ft})^4}{1{,}290(1.50 \text{ in.})}$ $= 393 \text{ in.}^4$	$I_{x\,req'd} = \dfrac{(0.938 \text{ kip/ft})(30.0 \text{ ft})^4}{1{,}290(1.50 \text{ in.})}$ $= 393 \text{ in.}^4$
From AISC *Manual* Table 3-10, for L_b = 6 ft and C_b = 1.0, select W21×44 which has a design flexural strength of 332 kip-ft, a design shear strength of 217 kips, and I_x = 843 in.^{4}	From AISC *Manual* Table 3-10, for L_b = 6 ft and C_b = 1.0, select W21×44 which has an allowable flexural strength of 221 kip-ft, an allowable shear strength of 145 kips, and I_x = 843 in.^{4}
Grids 2 to 3 and 6 to 7(opposite hand)	Grids 2 to 3 and 6 to 7(opposite hand)
R_u (left) = 1.2(11.3 kips) + 1.6(14.4 kips) = 36.6 kips	R_a (left) = 11.3 kips + 14.4 kips = 25.7 kips
R_u (right) = 1.2(11.3 kips) + 1.6(17.9) kips) = 42.2 kips	R_a (right) = 11.3 kips + 17.9 kips = 29.2 kips
M_u = 1.2(84.4 kip-ft) + 1.6(111 kip-ft) = 279 kip-ft	M_a = 84.4 kip-ft + 111 kip-ft = 195 kip-ft
$I_{x\,req'd} = \dfrac{(0.938 \text{ kip/ft})(30.0 \text{ ft})^4}{1{,}290(1.50 \text{ in.})}$ $= 393 \text{ in.}^4$	$I_{x\,req'd} = \dfrac{(0.938 \text{ kip/ft})(30.0 \text{ ft})^4}{1{,}290(1.50 \text{ in.})}$ $= 393 \text{ in.}^4$
From AISC *Manual* Table 3-10, for L_b = 6 ft and C_b = 1.0, select W21×44 which has a design flexural strength of 332 kip-ft, a design shear strength of 217 kips and I_x = 843 in.^{4}	From AISC *Manual* Table 3-10, for L_b = 6 ft and C_b = 1.0, select W21×44 which has an allowable flexural strength of 221 kip-ft, an allowable shear strength of 145 kips, and I_x = 843 in.^{4}

The third individual beam loading occurs at the beams from 3 to 4, 4 to 5, and 5 to 6. This is the heaviest load.

Return to Table of Contents

SELECT THE ROOF BEAMS ALONG THE SIDES OF THE MECHANICAL AREA

The beams from 3 to 4, 4 to 5, and 5 to 6 have a uniform snow load outside the screen walled area, except for the snow drift at the parapet ends and the screen wall ends of the 45-ft joists. Inside the screen walled area the beams support the mechanical equipment. A summary of the moments, left and right reactions, and required I_x to keep the live load deflection to equal or less than the span divided by 240 (or 1.50 in.) is given below.



LRFD	ASD
w_u =1.2 (1.35 kip/ft) +1.6(1.27 kip/ft) = 3.65 kip/ft	w_a = 1.35 kip/ft + 1.27 kip/ft[2] = 2.62 kip/ft
$M_u = \dfrac{3.65\text{ kip/ft}\left(30.0\text{ ft}\right)^2}{8}$ = 411 kip-ft	$M_a = \dfrac{2.62\text{ kip/ft}\left(30.0\text{ ft}\right)^2}{8}$ = 295 kip-ft
$R_u = \dfrac{30.0\text{ ft}}{2}\left(3.65\text{ kip/ft}\right)$ = 54.8 kips	$R_a = \dfrac{30.0\text{ ft}}{2}\left(2.62\text{ kip/ft}\right)$ = 39.3 kips
$I_{x\ req'd} = \dfrac{1.27\text{ kip/ft}\left(30.0\text{ ft}\right)^4}{1{,}290\left(1.50\text{ in.}\right)}$ = 532 in.^{4}	$I_{x\ req'd} = \dfrac{1.27\text{ kip/ft}\left(30.0\text{ ft}\right)^4}{1{,}290\left(1.50\text{ in.}\right)}$ = 532 in.^{4}
From AISC *Manual* Table 3-2, for L_b = 6 ft and C_b = 1.0, select W21×55, which has a design flexural strength of 473 kip-ft, a design shear strength of 234 kips, and an I_x of 1,140 in.^{4}	From AISC *Manual* Table 3-2, for L_b = 6 ft and C_b = 1.0, select W21×55, which has an allowable flexural strength of 314 kip-ft, an allowable shear strength of 156 kips, and an I_x of 1,140 in.^{4}

FLOOR MEMBER DESIGN AND SELECTION

Calculate dead load and live load.

Dead Load

Slab and Deck	= 57 psf
Beams (est.)	= 8 psf
Misc. (ceiling, mechanical, etc.)	= 10 psf
Total	= 75 psf

Note: The weight of the floor slab and deck was obtained from the manufacturer's literature.

Live Load

Total (can be reduced for area per ASCE/SEI 7) = 80 psf

The floor and deck will be 3 in. of normal weight concrete, $f_c' = 4$ ksi, on 3-in. 20 gage, galvanized, composite deck, laid in a pattern of three or more continuous spans. The total depth of the slab is 6 in. The Steel Deck Institute maximum unshored span for construction with this deck and a three-span condition is 10 ft 11 in. The general layout for the floor beams is 10 ft on center; therefore, the deck does not need to be shored during construction. At 10 ft on center, this deck has an allowable superimposed live load capacity of 143 psf. In addition, it can be shown that this deck can carry a 2,000 pound load over an area of 2.5 ft by 2.5 ft as required by Section 4.4 of ASCE/SEI 7. The floor diaphragm and the floor loads extend 6 in. past the centerline of grid as shown on Sheet S4.1.

SELECT FLOOR BEAMS (composite and noncomposite)

Note: There are two early and important checks in the design of composite beams. First, select a beam that either does not require camber, or establish a target camber and moment of inertia at the start of the design process. A reasonable approximation of the camber is between $L/300$ minimum and $L/180$ maximum (or a maximum of 1½ to 2 in.).

Second, check that the beam is strong enough to safely carry the wet concrete and a 20 psf construction live load (per ASCE 37-05), when designed by the ASCE/SEI 7 load combinations and the provisions of Chapter F of the AISC *Specification*.

SELECT TYPICAL 45-FT INTERIOR COMPOSITE BEAM (10 FT ON CENTER)

Find a target moment of inertia for an unshored beam.

Hold deflection to around 2 in. maximum to facilitate concrete placement.

$$w_D = (0.057\text{ kip/ft}^2 + 0.008\text{ kip/ft}^2)(10.0\text{ ft})$$
$$= 0.650\text{ kip/ft}$$

$$I_{req} \approx \frac{0.650\text{ kip/ft}(45.0\text{ ft})^4}{1{,}290(2.00\text{ in.})}$$
$$= 1{,}030\text{ in.}^4$$

Determine the required strength to carry wet concrete and construction live load.

$$w_{DL} = 0.065\text{ kip/ft}^2(10.0\text{ ft})$$
$$= 0.650\text{ kip/ft}$$
$$w_{LL} = 0.020\text{ kip/ft}^2(10.0\text{ ft})$$
$$= 0.200\text{ kip/ft}$$

Determine the required flexural strength due to wet concrete only.

LRFD	ASD
$w_u = 1.4(0.650\text{ kip/ft})$ $= 0.910\text{ kip/ft}$ $M_u = \frac{0.910\text{ kip/ft}(45.0\text{ ft})^2}{8}$ $= 230\text{ kip-ft}$	$w_a = 0.650\text{ kip/ft}$ $M_a = \frac{0.650\text{ kip/ft}(45.0\text{ ft})^2}{8}$ $= 165\text{ kip-ft}$

Determine the required flexural strength due to wet concrete and construction live load.

LRFD	ASD
$w_u = 1.2(0.650\text{ kip/ft}) + 1.6(0.200\text{ kip/ft})$ $= 1.10\text{ kip/ft}$ $M_u = \frac{1.10\text{ kip/ft}(45.0\text{ ft})^2}{8}$ $= 278\text{ kip-ft}$ **controls**	$w_a = 0.650\text{ kip/ft} + 0.200\text{ kip/ft}$ $= 0.850\text{ kip/ft}$ $M_a = \frac{0.850\text{ kip/ft}(45.0\text{ ft})^2}{8}$ $= 215\text{ kip-ft}$ **controls**

Use AISC *Manual* Table 3-2 to select a beam with $I_x \geq 1{,}030\text{ in.}^4$. Select W21×50, which has $I_x = 984\text{ in.}^4$, close to our target value, and has available flexural strengths of 413 kip-ft (LRFD) and 274 kip-ft (ASD).

Check for possible live load reduction due to area in accordance with Section 4.7.2 of ASCE/SEI 7.

For interior beams, $K_{LL} = 2$

The beams are at 10.0 ft on center, therefore the area $A_T = (45.0\text{ ft})(10.0\text{ ft}) = 450\text{ ft}^2$.

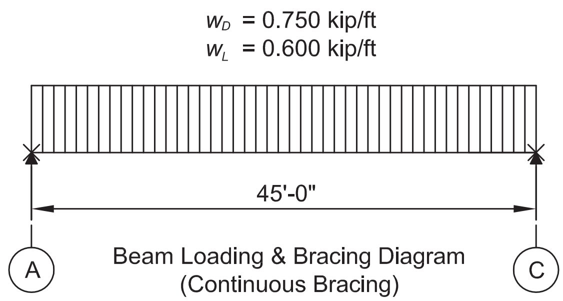
Since $K_{LL}A_T = 2(450\text{ ft}^2) = 900\text{ ft}^2 > 400\text{ ft}^2$, a reduced live load can be used.

From ASCE/SEI 7, Equation 4.7-1:

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right)$$
$$= 80.0\text{ psf}\left(0.25 + \frac{15}{\sqrt{900\text{ ft}^2}}\right)$$
$$= 60.0\text{ psf} \geq 0.50L_o = 40.0\text{ psf}$$

Therefore, use 60.0 psf.
The beam is continuously braced by the deck.

The beams are at 10 ft on center, therefore the loading diagram is as shown below.



Beam Loading & Bracing Diagram
(Continuous Bracing)

Calculate the required flexural strength from Chapter 2 of ASCE/SEI 7.

LRFD	ASD
$w_u = 1.2(0.750\text{ kip/ft}) + 1.6(0.600\text{ kip/ft})$ $= 1.86\text{ kip/ft}$ $M_u = \dfrac{1.86\text{ kip/ft}(45.0\text{ ft})^2}{8}$ $= 471\text{ kip-ft}$	$w_a = 0.750\text{ kip/ft} + 0.600\text{ kip/ft}$ $= 1.35\text{ kip/ft}$ $M_a = \dfrac{1.35\text{ kip/ft}(45.0\text{ ft})^2}{8}$ $= 342\text{ kip-ft}$

Assume initially $a = 1.00$ in.

$$Y2 = Y_{con} - a/2$$
$$= 6.00\text{ in.} - 1.00\text{ in.}/2$$
$$= 5.50\text{ in.}$$

Use AISC *Manual* Table 3-19 to check W21×50 selected above. Using required strengths of 471 kip-ft (LRFD) or 342 kip-ft (ASD) and a $Y2$ value of 5.50 in.

LRFD	ASD
Select W21×50 beam, where PNA = Location 7 and $\Sigma Q_n = 184$ kips $\phi_b M_n = 598$ kip-ft > 471 kip-ft **o.k.**	Select W21×50 beam, where PNA = Location 7 and $\Sigma Q_n = 184$ kips $M_p / \Omega_n = 398$ kip-ft > 342 kip-ft **o.k.**

Determine the effective width, b_{eff}.

Per *Specification* AISC Section I3.1a, the effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline, which shall not exceed:

(1) one-eighth of the span of the beam, center-to-center of supports

$$\frac{45.0\text{ ft}}{8}(2\text{ sides}) = 11.3\text{ ft}$$

(2) one-half the distance to the centerline of the adjacent beam

$$\frac{10.0\text{ ft}}{2}(2\text{ sides}) = 10.0\text{ ft} \quad \textbf{controls}$$

(3) the distance to the edge of the slab

Not applicable

Determine the height of the compression block, a.

$$a = \frac{\Sigma Q_n}{0.85 f_c' b} \qquad (\textit{Manual}\text{ Eq. 3-7})$$

$$= \frac{184\text{ kips}}{0.85(4\text{ ksi})(10.0\text{ ft})(12\text{ in./ft})}$$

$$= 0.451\text{ in.} < 1.00\text{ in.} \quad \textbf{o.k.}$$

Check the W21×50 end shear strength.

LRFD	ASD
$R_u = \frac{45.0\text{ ft}}{2}(1.86\text{ kip/ft})$ $= 41.9$ kips From AISC *Manual* Table 3-2, $\phi_v V_n = 237$ kips > 41.9 kips **o.k.**	$R_a = \frac{45.0\text{ ft}}{2}(1.35\text{ kip/ft})$ $= 30.4$ kips From AISC *Manual* Table 3-2, $V_n / \Omega_v = 158$ kips > 30.4 kips **o.k.**

Check live load deflection.

$$\Delta_{LL} = l/360 = (45.0\text{ ft})(12\text{ in./ft})/360 = 1.50\text{ in.}$$

For a W21×50, from AISC *Manual* Table 3-20,

$Y2 = 5.50$ in.

PNA Location 7
I_{LB} = 1,730 in.^{4}

$$\Delta_{LL} = \frac{w_{LL} l^4}{1,290 I_{LB}}$$
$$= \frac{0.600 \text{ kip/ft}\left(45.0 \text{ ft}\right)^4}{1,290\left(1,730 \text{ in.}^4\right)}$$
$$= 1.10 \text{ in.} < 1.50 \text{ in.} \quad \textbf{o.k.}$$

Based on AISC Design Guide 3 (West, Fisher and Griffis, 2003) limit the live load deflection, using 50% of the (unreduced) design live load, to L / 360 with a maximum absolute value of 1.0 in. across the bay.

$$\Delta_{LL} = \frac{0.400 \text{ kip/ft}\left(45.0 \text{ ft}\right)^4}{1,290\left(1,730 \text{ in.}^4\right)}$$
$$= 0.735 \text{ in.} < 1.00 \text{ in.} \quad \textbf{o.k.}$$

1.00 in. – 0.735 in. = 0.265 in.

Note: Limit the supporting girders to 0.265 in. deflection under the same load case at the connection point of the beam.

Determine the required number of shear stud connectors.

From AISC *Manual* Table 3-21, using perpendicular deck with one ¾-in.-diameter stud per rib in normal weight, 4 ksi concrete, in weak position; Q_n = 17.2 kips/stud.

$$\frac{\Sigma Q_n}{Q_n} = \frac{184 \text{ kips}}{17.2 \text{ kips/stud}}$$
$$= 10.7 \text{ studs / side}$$

Therefore use 22 studs.

Based on AISC Design Guide 3, limit the wet concrete deflection in a bay to L / 360, not to exceed 1.00 in.

Camber the beam for 80% of the calculated wet deflection.

$$\Delta_{DL(wet\ conc)} = \frac{0.650 \text{ kip/ft}\left(45.0 \text{ ft}\right)^4}{1,290\left(984 \text{ in.}^4\right)}$$
$$= 2.10 \text{ in.}$$

$$\text{Camber} = 0.80(2.10 \text{ in.})$$
$$= 1.68 \text{ in.}$$

Round the calculated value down to the nearest ¼ in; therefore, specify 1.50 in. of camber.

2.10 in. – 1.50 in. = 0.600 in.

1.00 in. – 0.600 in. = 0.400 in.

Note: Limit the supporting girders to 0.400 in. deflection under the same load combination at the connection point of the beam.

SELECT TYPICAL 30-FT INTERIOR COMPOSITE (OR NONCOMPOSITE) BEAM (10 FT ON CENTER)

Find a target moment of inertia for an unshored beam.

Hold deflection to around 1.50 in. maximum to facilitate concrete placement.

$$I_{req} \approx \frac{0.650 \text{ kip/ft}\left(30.0 \text{ ft}\right)^4}{1{,}290\left(1.50 \text{ in.}\right)}$$
$$= 272 \text{ in.}^4$$

Determine the required strength to carry wet concrete and construction live load.

$$w_{DL} = 0.065 \text{ kip/ft}^2(10.0 \text{ ft})$$
$$= 0.650 \text{ kip/ft}$$

$$w_{LL} = 0.020 \text{ kip/ft}^2(10.0 \text{ ft})$$
$$= 0.200 \text{ kip/ft}$$

Determine the required flexural strength due to wet concrete only.

LRFD	ASD
$w_u = 1.4(0.650 \text{ kip/ft})$ $= 0.910 \text{ kip/ft}$ $M_u = \dfrac{0.910 \text{ kip/ft}\left(30.0 \text{ ft}\right)^2}{8}$ $= 102 \text{ kip-ft}$	$w_a = 0.650 \text{ kip/ft}$ $M_a = \dfrac{0.650 \text{ kip/ft}\left(30.0 \text{ ft}\right)^2}{8}$ $= 73.1 \text{ kip-ft}$

Determine the required flexural strength due to wet concrete and construction live load.

LRFD	ASD
$w_u = 1.2(0.650 \text{ kip/ft}) + 1.6(0.200 \text{ kip/ft})$ $= 1.10 \text{ kip/ft}$ $M_u = \dfrac{1.10 \text{ kip/ft}\left(30.0 \text{ ft}\right)^2}{8}$ $= 124 \text{ kip-ft}$ **controls**	$w_a = 0.650 \text{ kip/ft} + 0.200 \text{ kip/ft}$ $= 0.850 \text{ kip/ft}$ $M_a = \dfrac{0.850 \text{ kip/ft}\left(30.0 \text{ ft}\right)^2}{8}$ $= 95.6 \text{ kip-ft}$ **controls**

Use AISC *Manual* Table 3-2 to find a beam with an $I_x \geq 272$ in.^{4} Select W16×26, which has an $I_x = 301$ in.^{4} which exceeds our target value, and has available flexural strengths of 166 kip-ft (LRFD) and 110 kip-ft (ASD).

Check for possible live load reduction due to area in accordance with Section 4.7.2 of ASCE/SEI 7.

For interior beams, $K_{LL} = 2$.

The beams are at 10 ft on center, therefore the area $A_T = 30.0 \text{ ft} \times 10.0 \text{ ft} = 300 \text{ ft}^2$.

Since $K_{LL}A_T = 2(300 \text{ ft}^2) = 600 \text{ ft}^2 > 400 \text{ ft}^2$, a reduced live load can be used.

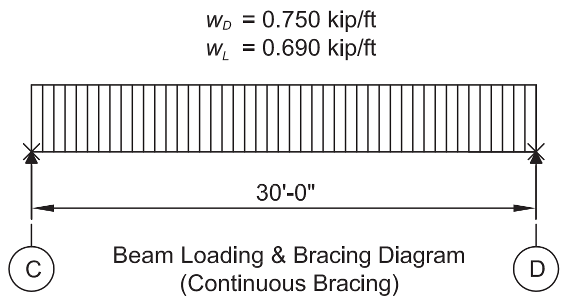
From ASCE/SEI 7, Equation 4.7-1:

Return to Table of Contents

$$L = L_o\left(0.25+\frac{15}{\sqrt{K_{LL}A_T}}\right)$$
$$= 80.0\text{ psf}\left(0.25+\frac{15}{\sqrt{600\text{ ft}^2}}\right)$$
$$= 69.0\text{ psf} \geq 0.50\,L_o = 40.0\text{ psf}$$

Therefore, use 69.0 psf.

The beams are at 10 ft on center, therefore the loading diagram is as shown below.



Beam Loading & Bracing Diagram
(Continuous Bracing)

From Chapter 2 of ASCE/SEI 7, calculate the required strength.

LRFD	ASD
$w_u = 1.2(0.750\text{ kip/ft}) + 1.6\,(0.690\text{ kip/ft})$ $= 2.00\text{ kip/ft}$ $M_u = \dfrac{2.00\text{ kip/ft}(30.0\text{ ft})^2}{8}$ $= 225\text{ kip-ft}$	$w_a = 0.750\text{ kip/ft} + 0.690\text{ kip/ft}$ $= 1.44\text{ kip/ft}$ $M_a = \dfrac{1.44\text{ kip/ft}(30.0\text{ ft})^2}{8}$ $= 162\text{ kip-ft}$

Assume initially a = 1.00

$$Y2 = 6.00\text{ in.} - \frac{1.00\text{ in.}}{2}$$
$$= 5.50\text{ in.}$$

Use AISC *Manual* Table 3-19 to check the W16×26 selected above. Using required strengths of 225 kip-ft (LRFD) or 162 kip-ft (ASD) and a $Y2$ value of 5.50 in.

LRFD	ASD
Select W16×26 beam, where PNA Location 7 and $\sum Q_n = 96.0$ kips $\phi_b M_n = 248\text{ kip-ft} > 225\text{ kip-ft}$ **o.k.**	Select W16×26 beam, where PNA Location 7 and $\sum Q_n = 96.0$ kips $M_n / \Omega_n = 165\text{ kip-ft} > 162\text{ kip-ft}$ **o.k.**

Determine the effective width, b_{eff}.

From AISC *Specification* Section I3.1a, the effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline, which shall not exceed:

(1) one-eighth of the span of the beam, center-to-center of supports

$$\frac{30.0\text{ ft}}{8}(2\text{ sides}) = 7.50\text{ ft} \quad \textbf{controls}$$

(2) one-half the distance to the centerline of the adjacent beam

$$\frac{10.0\text{ ft}}{2}(2\text{ sides}) = 10.0\text{ ft}$$

(3) the distance to the edge of the slab

Not applicable

Determine the height of the compression block, *a*.

$$a = \frac{\Sigma Q_n}{0.85 f_c{}' b} \qquad (\textit{Manual}\text{ Eq. 3-7})$$

$$= \frac{96.0\text{ kips}}{0.85(4\text{ ksi})(7.50\text{ ft})(12\text{ in./ft})}$$

$$= 0.314\text{ in.} < 1.00\text{ in.} \qquad \textbf{o.k.}$$

Check the W16×26 end shear strength.

LRFD	ASD
$R_u = \frac{30.0\text{ ft}}{2}(2.00\text{ kip/ft})$ $= 30.0\text{ kips}$ From AISC *Manual* Table 3-2, $\phi_v V_n = 106\text{ kips} > 30.0\text{ kips}$ **o.k.**	$R_a = \frac{30.0\text{ ft}}{2}(1.44\text{ kip/ft})$ $= 21.6\text{ kips}$ From AISC *Manual* Table 3-2, $V_n / \Omega_v = 70.5\text{ kips} > 21.6\text{ kips}$ **o.k.**

Check live load deflection.

$$\Delta_{LL} = l/360$$
$$= (30.0\text{ ft})(12\text{ in./ft})/360$$
$$= 1.00\text{ in.}$$

For a W16×26, from AISC *Manual* Table 3-20,

$Y2 = 5.50$ in.
PNA Location 7
$I_{LB} = 575$ in.^{4}

$$\Delta_{LL} = \frac{w_{LL} l^4}{1{,}290 I_{LB}}$$

$$= \frac{0.690\text{ kip/ft}(30.0\text{ ft})^4}{1{,}290(575\text{ in.}^4)}$$

$$= 0.753\text{ in.} < 1.00\text{ in.} \qquad \textbf{o.k.}$$

Based on AISC Design Guide 3, limit the live load deflection, using 50% of the (unreduced) design live load, to L/360 with a maximum absolute value of 1.0 in. across the bay.

$$\Delta_{LL} = \frac{0.400 \text{ kip/ft}\left(30.0 \text{ ft}\right)^4}{1,290\left(575 \text{ in.}^4\right)}$$
$$= 0.437 \text{ in.} < 1.00 \text{ in.} \quad \textbf{o.k.}$$

1.00 in. – 0.437 in. = 0.563 in.

Note: Limit the supporting girders to 0.563 in. deflection under the same load combination at the connection point of the beam.

Determine the required number of shear stud connectors.

From AISC *Manual* Table 3-21, using perpendicular deck with one ¾-in.-diameter stud per rib in normal weight, 4 ksi concrete, in the weak position; Q_n = 17.2 kips/stud

$$\frac{\sum Q_n}{Q_n} = \frac{96.0 \text{ kips}}{17.2 \text{ kips/stud}}$$
$$= 5.58 \text{ studs/side}$$

Use 12 studs

Note: Per AISC *Specification* Section I8.2d, there is a maximum spacing limit of 8(6 in.) = 4 ft not to exceed 36 in. between studs.

Therefore use 12 studs, uniformly spaced at no more than 36 in. on center.

Note: Although the studs may be placed up to 36 in. o.c. the steel deck must still be anchored to the supporting member at a spacing not to exceed 18 in. per AISC *Specification* Section I3.2c.

Based on AISC Design Guide 3, limit the wet concrete deflection in a bay to L/360, not to exceed 1.00 in.

Camber the beam for 80% of the calculated wet dead load deflection.

$$\Delta_{DL(wet\ conc)} = \frac{0.650 \text{ kip/ft}\left(30.0 \text{ ft}\right)^4}{1,290\left(301 \text{ in.}^4\right)}$$
$$= 1.36 \text{ in.}$$

Camber = 0.800(1.36 in.)
= 1.09 in.

Round the calculated value down to the nearest ¼ in. Therefore, specify 1.00 in. of camber.

1.36 in. – 1.00 in. = 0.360 in.

1.00 in. – 0.360 in. = 0.640 in.

Note: Limit the supporting girders to 0.640 in. deflection under the same load combination at the connection point of the beam.

This beam could also be designed as a noncomposite beam. Use AISC *Manual* Table 3-2 with previous moments and shears:

LRFD	ASD
Select W18×35 From AISC *Manual* Table 3-2, $\phi_b M_n = \phi_b M_p$ $= 249$ kip-ft > 225 kip-ft **o.k.** $\phi_v V_n = 159 > 30.0$ kips **o.k.**	Select W18×35 From AISC *Manual* Table 3-2, $M_n/\Omega_b = M_p/\Omega_b$ $= 166$ kip-ft > 162 kip-ft **o.k.** $V_n / \Omega_v = 106$ kips > 21.6 kips **o.k.**

Check beam deflections.

Check live load deflection of the W18×35 with an $I_x = 510$ in.^{4}, from AISC *Manual* Table 3-2.

$$\Delta_{LL} = \frac{0.690 \text{ kip/ft}\left(30.0 \text{ ft}\right)^4}{1{,}290\left(510 \text{ in.}^4\right)}$$

$$= 0.850 \text{ in.} < 1.00 \text{ in.} \qquad \textbf{o.k.}$$

Based on AISC Design Guide 3, limit the live load deflection, using 50% of the (unreduced) design live load, to L/360 with a maximum absolute value of 1.0 in. across the bay.

$$\Delta_{LL} = \frac{0.400 \text{ kip/ft}\left(30.0 \text{ ft}\right)^4}{1{,}290\left(510 \text{ in.}^4\right)}$$

$$= 0.492 \text{ in.} < 1.00 \text{ in.} \qquad \textbf{o.k.}$$

1.00 in. – 0.492 in. = 0.508 in.

Note: Limit the supporting girders to 0.508 in. deflection under the same load combination at the connection point of the beam.

Note: Because this beam is stronger than the W16×26 composite beam, no wet concrete strength checks are required in this example.

Based on AISC Design Guide 3, limit the wet concrete deflection in a bay to L/360, not to exceed 1.00 in.

Camber the beam for 80% of the calculated wet deflection.

$$\Delta_{DL(wet\ conc)} = \frac{0.650 \text{ kip/ft}\left(30.0 \text{ ft}\right)^4}{1{,}290\left(510 \text{ in.}^4\right)}$$

$$= 0.800 \text{ in.} < 1.50 \text{ in.} \qquad \textbf{o.k.}$$

Camber = 0.800(0.800 in.) = 0.640 in. < 0.750 in.

A good break point to eliminate camber is ¾ in.; therefore, do not specify a camber for this beam.

1.00 in. – 0.800 in. = 0.200 in.

Note: Limit the supporting girders to 0.200 in. deflection under the same load case at the connection point of the beam.

Therefore, selecting a W18×35 will eliminate both shear studs and cambering. The cost of the extra steel weight may be offset by the elimination of studs and cambering. Local labor and material costs should be checked to make this determination.

SELECT TYPICAL NORTH-SOUTH EDGE BEAM

The influence area ($K_{LL}A_T$) for these beams is less than 400 ft^2, therefore no live load reduction can be taken.

These beams carry 5.50 ft of dead load and live load as well as a wall load.

The floor dead load is:

$$w = 5.50 \text{ ft}(0.075 \text{ kips/ft}^2)$$
$$= 0.413 \text{ kip/ft}$$

Use 65 psf for the initial dead load.

$$w_{D(initial)} = 5.50 \text{ ft}(0.065 \text{ kips/ft}^2)$$
$$= 0.358 \text{ kips/ft}$$

Use 10 psf for the superimposed dead load.

$$w_{D(super)} = 5.50 \text{ ft}(0.010 \text{ kips/ft}^2)$$
$$= 0.055 \text{ kips/ft}$$

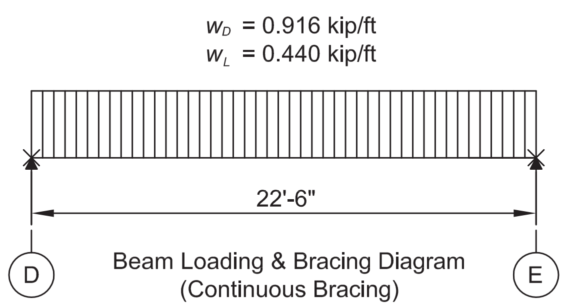
The dead load of the wall system at the floor is:

$$w = 7.50 \text{ ft}\left(0.055 \text{ kip / ft}^2\right) + 6.00 \text{ ft}\left(0.015 \text{ kip / ft}^2\right)$$
$$= 0.413 \text{ kip/ft} + 0.090 \text{ kip/ft}$$
$$= 0.503 \text{ kip/ft}$$

The total dead load is w_{DL} $= 0.413 \text{ kip/ft} + 0.503 \text{ kip/ft}$
$= 0.916 \text{ kip/ft}$

The live load is $w_{LL} = 5.5 \text{ ft}(0.080 \text{ kip/ft}^2)$
$= 0.440 \text{ kip/ft}$

The loading diagram is as follows.



Beam Loading & Bracing Diagram
(Continuous Bracing)

Calculate the required strengths from Chapter 2 of ASCE/SEI 7.

LRFD	ASD
$w_u = 1.2(0.916 \text{ kip/ft}) + 1.6\,(0.440 \text{ kip/ft})$ $= 1.80 \text{ kip/ft}$ $M_u = \dfrac{1.80 \text{ kip/ft}\left(22.5 \text{ ft}\right)^2}{8}$	$w_a = 0.916 \text{ kip/ft} + 0.440 \text{ kip/ft}$ $= 1.36 \text{ kip/ft}$ $M_a = \dfrac{1.36 \text{ kip/ft}\left(22.5 \text{ ft}\right)^2}{8}$

$= 114$ kip-ft	$= 86.1$ kip-ft
$R_u = \dfrac{22.5\text{ ft}}{2}(1.80\text{ kip/ft})$	$R_a = \dfrac{22.5\text{ ft}}{2}(1.36\text{ kip/ft})$
$= 20.3$ kips	$= 15.3$ kips

Because these beams are less than 25 ft long, they will be most efficient as noncomposite beams. The beams at the edges of the building carry a brick spandrel panel. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. Therefore, per AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to L/600 or ⅜ in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to L/600 or 0.3 in. max to accommodate the brick and L/360 or 0.25 in. max to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to L/600 or 0.25 in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. Note that it is typically not recommended to camber beams supporting spandrel panels.

Calculate minimum I_x to limit the superimposed dead and live load deflection to ¼ in.

$$I_{req} = \frac{0.495\text{ kip/ft}(22.5\text{ ft})^4}{1{,}290(\text{¼ in.})}$$
$$= 393\text{ in.}^4 \quad \textbf{controls}$$

Calculate minimum I_x to limit the cladding and initial dead load deflection to ⅜ in.

$$I_{req} = \frac{0.861\text{ kip/ft}(22.5\text{ ft})^4}{1{,}290(\text{⅜ in.})}$$
$$= 456\text{ in.}^4$$

Select beam from AISC *Manual* Table 3-2.

LRFD	ASD
Select W18×35 with $I_x = 510$ in.^{4} $\phi_b M_n = \phi_b M_p$ $= 249$ kip-ft > 114 kip-ft **o.k.** $\phi_v V_n = 159 > 20.3$ kips **o.k.**	Select W18×35 with $I_x = 510$ in.^{4} $M_n / \Omega_b = M_p / \Omega_b$ $= 166$ kip-ft > 86.1 kip-ft **o.k.** $V_n / \Omega_v = 106$ kips > 15.3 kips **o.k.**

SELECT TYPICAL EAST-WEST SIDE GIRDER

The beams along the sides of the building carry the spandrel panel and glass, and dead load and live load from the intermediate floor beams. For these beams, the cladding weight exceeds 25% of the total dead load on the beam. Therefore, per AISC Design Guide 3, limit the vertical deflection due to cladding and initial dead load to L/600 or ⅜ in. maximum. In addition, because these beams are supporting brick above and there is continuous glass below, limit the superimposed dead and live load deflection to L/600 or 0.3 in. max to accommodate the brick and L/360 or 0.25 in. max to accommodate the glass. Therefore, combining the two limitations, limit the superimposed dead and live load deflection to L/600 or 0.25 in. The superimposed dead load includes all of the dead load that is applied after the cladding has been installed. These beams will be part of the moment frames on the North and South sides of the building and therefore will be designed as fixed at both ends.

Establish the loading.

The dead load reaction from the floor beams is:

$$\begin{aligned}P_D &= 0.750\text{ kip/ft}(45.0\text{ ft} / 2)\\ &= 16.9\text{ kips}\end{aligned}$$

$$\begin{aligned}P_{D(initial)} &= 0.650\text{ kip/ft}(45.0\text{ ft} / 2)\\ &= 14.6\text{ kips}\end{aligned}$$

$$\begin{aligned}P_{D(super)} &= 0.100\text{ kip/ft}(45.0\text{ ft} / 2)\\ &= 2.25\text{ kips}\end{aligned}$$

The uniform dead load along the beam is:

$$\begin{aligned}w_D &= 0.500\text{ ft}(0.075\text{ kip/ft}^2) + 0.503\text{ kip/ft}\\ &= 0.541\text{ kip/ft}\end{aligned}$$

$$\begin{aligned}w_{D(initial)} &= 0.500\text{ ft}(0.065\text{ kip/ft}^2)\\ &= 0.033\text{ kip/ft}\end{aligned}$$

$$\begin{aligned}w_{D(super)} &= 0.500\text{ ft}(0.010\text{ kip/ft}^2)\\ &= 0.005\text{ kip/ft}\end{aligned}$$

Select typical 30-ft composite (or noncomposite) girders.

Check for possible live load reduction in accordance with Section 4.7.2 of ASCE/SEI 7.

For edge beams with cantilevered slabs, K_{LL} = 1, per ASCE/SEI 7, Table 4-2. However, it is also permissible to calculate the value of K_{LL} based upon influence area. Because the cantilever dimension is small, K_{LL} will be closer to 2 than 1. The calculated value of K_{LL} based upon the influence area is

$$\begin{aligned}K_{LL} &= \frac{(45.5\text{ ft})(30.0\text{ ft})}{\left(\dfrac{45.0\text{ ft}}{2} + 0.500\text{ ft}\right)(30.0\text{ ft})}\\ &= 1.98\end{aligned}$$

The area A_T = (30.0 ft)(22.5 ft + 0.500 ft) = 690 ft^2

Using Equation 4.7-1 of ASCE/SEI 7

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right)$$

$$= 80.0 \text{ psf}\left(0.25 + \frac{15}{\sqrt{(1.98)(690 \text{ ft}^2)}}\right)$$

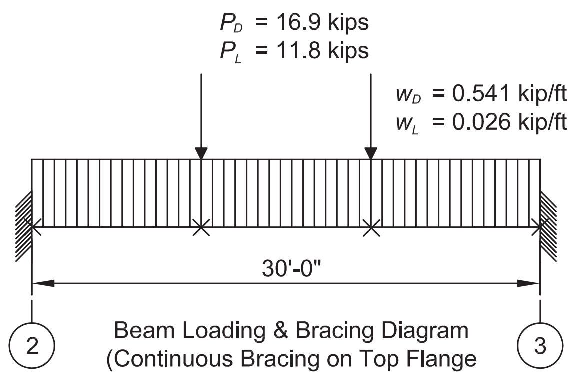
$$= 52.5 \text{ psf} \geq 0.50L_o = 40.0 \text{ psf}$$

Therefore, use 52.5 psf.

The live load from the floor beams is P_{LL} = 0.525 kip/ft(45.0 ft / 2)
= 11.8 kips

The uniform live load along the beam is w_{LL} = 0.500 ft(0.0525 kip/ft^2)
= 0.026 kip/ft

The loading diagram is shown below.



Beam Loading & Bracing Diagram
(Continuous Bracing on Top Flange
At Ends and ⅓ Points on Bottom Flange)

A summary of the moments, reactions and required moments of inertia, determined from a structural analysis of a fixed-end beam, is as follows:

Calculate the required strengths and select the beams for the floor side beams.

LRFD	ASD
Typical side beam R_u = 49.5 kips M_u at ends =313 kip-ft M_u at ctr. =156 kip-ft	Typical side beam R_a = 37.2 kips M_a at ends = 234 kip-ft M_a at ctr. = 117 kip-ft

The maximum moment occurs at the support with compression in the bottom flange. The bottom laterally braced at 10 ft o.c. by the intermediate beams.

Note: During concrete placement, because the deck is parallel to the beam, the beam will not have continuous lateral support. It will be braced at 10 ft o.c. by the intermediate beams. By inspection, this condition will not control because the maximum moment under full loading causes compression in the bottom flange, which is braced at 10 ft o.c.

LRFD	ASD
Calculate C_b = for compression in the bottom flange braced at 10 ft o.c.	Calculate C_b = for compression in the bottom flange braced at 10 ft o.c.

LRFD	ASD
C_b = 2.21 (from computer output) Select W21×44 With continuous bracing, from AISC *Manual* Table 3-2, $\phi_b M_n = \phi_b M_p$ $= 358 \text{ kip-ft} > 156 \text{ kip-ft}$ **o.k.** For L_b = 10 ft and C_b = 2.21, from AISC *Manual* Table 3-10, $\phi M_n = (265 \text{ kip-ft})(2.21)$ $= 586 \text{ kip-ft}$ According to AISC *Specification* Section F2.2, the nominal flexural strength is limited M_p, therefore $\phi_b M_n = \phi_b M_p = 358$ kip-ft. 358 kip-ft > 313 kip-ft **o.k.** From AISC *Manual* Table 3-2, a W21×44 has a design shear strength of 217 kips. From Table 1-1, I_x = 843 in.^{4} Check deflection due to cladding and initial dead load. $\Delta = 0.295 \text{ in.} < 3/8 \text{ in.}$ **o.k.** Check deflection due to superimposed dead and live loads. $\Delta = 0.212 \text{ in.} < 0.250 \text{ in.}$ **o.k.**	C_b = 2.22 (from computer output) Select W21×44 With continuous bracing, from AISC *Manual* Table 3-2, $M_n / \Omega_b = M_p / \Omega_b$ $= 238 \text{ kip-ft} > 117 \text{ kip-ft}$ **o.k.** For L_b = 10 ft and C_b = 2.22, from AISC *Manual* Table 3-10, $M_n / \Omega = (176 \text{ kip-ft})(2.22)$ $= 391 \text{ kip-ft}$ According to AISC *Specification* Section F2.2, the nominal flexural strength is limited M_p, therefore $M_n/\Omega_b = M_p/\Omega_b = 238$ kip-ft. 238 kip-ft > 234 kip-ft **o.k.** From AISC *Manual* Table 3-2, a W21×44 has an allowable shear strength of 145 kips. From Table 1-1, I_x = 843 in.^{4} Check deflection due to cladding and initial dead load. $\Delta = 0.295 \text{ in.} < 3/8 \text{ in.}$ **o.k.** Check deflection due to superimposed dead and live loads. $\Delta = 0.212 \text{ in.} < 0.250 \text{ in.}$ **o.k.**

Note that both of the deflection criteria stated previously for the girder and for the locations on the girder where the floor beams are supported have also been met.

Also noted previously, it is not typically recommended to camber beams supporting spandrel panels. The W21×44 is adequate for strength and deflection, but may be increased in size to help with moment frame strength or drift control.

Return to Table of Contents

SELECT TYPICAL EAST-WEST INTERIOR GIRDER

Establish loads

The dead load reaction from the floor beams is

$$P_{DL} = 0.750 \text{ kip/ft}(45.0 \text{ ft} + 30.0 \text{ ft})/2$$
$$= 28.1 \text{ kips}$$

Check for live load reduction due to area in accordance with Section 4.7.2 of ASCE/SEI 7.

For interior beams, $K_{LL} = 2$

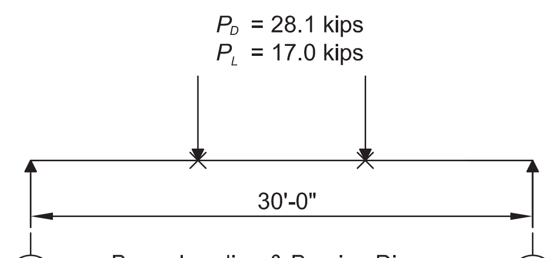
The area $A_T = (30.0 \text{ ft})(37.5 \text{ ft}) = 1{,}130 \text{ ft}^2$

Using ASCE/SEI 7, Equation 4.7-1:

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL}A_T}}\right)$$
$$= 80.0 \text{ psf}\left(0.25 + \frac{15}{\sqrt{(2)(1{,}130 \text{ ft}^2)}}\right)$$
$$= 45.2 \text{ psf} \geq 0.50\,L_o = 40.0 \text{ psf}$$

Therefore, use 45.2 psf.

The live load from the floor beams is $P_{LL} = 0.0452 \text{ kip/ft}^2(10.0 \text{ ft})(37.5 \text{ ft})$
$= 17.0 \text{ kips}$



Beam Loading & Bracing Diagram
(Continuous Bracing)

Note: The dead load for this beam is included in the assumed overall dead load.

A summary of the simple moments and reactions is shown below:

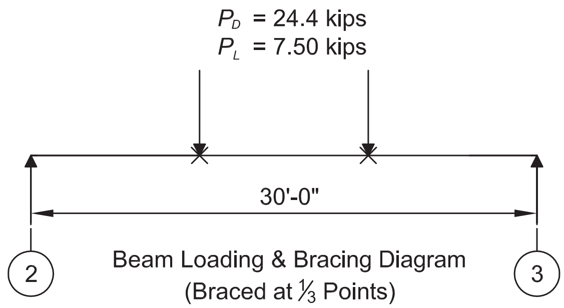
Calculate the required strengths and select the size for the interior beams.

LRFD	ASD
Typical interior beam R_u = 60.9 kips M_u = 609 kip-ft	Typical interior beam R_a = 45.1 kips M_a = 451 kip-ft

Check for beam requirements when carrying wet concrete.

Return to Table of Contents

Note: During concrete placement, because the deck is parallel to the beam, the beam will not have continuous lateral support. It will be braced at 10 ft on center by the intermediate beams. Also, during concrete placement, a construction live load of 20 psf will be present. This load pattern and a summary of the moments, reactions, and deflection requirements is shown below. Limit wet concrete deflection to 1.5 in.



LRFD	ASD
Typical interior beam with wet concrete only R_u = 34.2 kips M_u = 342 kip-ft	Typical interior beam with wet concrete only R_a = 24.4 kips M_a = 244 kip-ft

Assume $I_x \geq$ 935 in.^{4}, where 935 in.^{4} is determined based on a wet concrete deflection of 1.5 in.

LRFD	ASD
Typical interior beam with wet concrete and construction load R_u = 41.3 kips M_u (midspan) = 413 kip-ft Select a beam with an unbraced length of 10.0 ft and a conservative C_b = 1.0. From AISC *Manual* Tables 3-2 and 3-10, select a W21×68, which has a design flexural strength of 532 kip-ft, a design shear strength of 272 kips, and from Table 1-1, an I_x of 1,480 in.^{4} $\phi_b M_p$ = 532 kip-ft > 413 kip-ft **o.k.**	Typical interior beam with wet concrete and construction load R_a = 31.9 kips M_a (midspan) = 319 kip-ft Select a beam with an unbraced length of 10.0 ft and a conservative C_b = 1.0. From AISC *Manual* Tables 3-2 and 3-10, select a W21×68, which has an allowable flexural strength of 354 kip-ft, an allowable shear strength of 181 kips, and from Table 1-1 an I_x of 1,480 in.^{4} M_p / Ω_b = 354 kip-ft > 319 kip-ft **o.k.**

Check W21×68 as a composite beam.

From previous calculations:

LRFD	ASD
Typical interior Beam R_u = 60.9 kips M_u (midspan) = 609 kip-ft	Typical interior beam R_a = 45.1 kips M_a (midspan) = 451 kip-ft

*Y*2 (from previous calculations, assuming an initial *a* = 1.00 in.) = 5.50 in.

Using AISC *Manual* Table 3-19, check a W21×68, using required flexural strengths of 609 kip-ft (LRFD) and 451 kip-ft (ASD) and *Y*2 value of 5.5 in.

LRFD		ASD	
Select a W21×68 At PNA Location 7, $\sum Q_n = 250$ kips $\phi_b M_n = 844$ kip-ft > 609 kip-ft	**o.k.**	Select a W21×68 At PNA Location 7, $\sum Q_n = 250$ kips $M_n / \Omega_b = 561$ kip-ft > 461 kip-ft	**o.k.**

Based on AISC Design Guide 3, limit the wet concrete deflection in a bay to L/360, not to exceed 1.00 in.

Camber the beam for 80% of the calculated wet deflection.

$$\Delta_{DL(wet\ conc)} = \frac{24.4 \text{ kips}(30.0 \text{ ft})^3 (12 \text{ in./ft})^3}{28(29{,}000 \text{ ksi})(1{,}480 \text{ in.}^4)}$$
$$= 0.947 \text{ in.}$$

$$\text{Camber} = 0.80(0.947 \text{ in.})$$
$$= 0.758 \text{ in.}$$

Round the calculated value down to the nearest ¼ in. Therefore, specify ¾ in. of camber.

$$0.947 \text{ in.} - \tfrac{3}{4} \text{ in.} = 0.197 \text{ in.} < 0.200 \text{ in.}$$

Therefore, the total deflection limit of 1.00 in. for the bay has been met.

Determine the effective width, b_{eff}.

Per AISC *Specification* Section I3.1a, the effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline, which shall not exceed:

(1) one-eighth of the span of the beam, center-to-center of supports

$$\frac{30.0 \text{ ft}}{8}(2 \text{ sides}) = 7.50 \text{ ft} \quad \textbf{controls}$$

(2) one-half the distance to the centerline of the adjacent beam

$$\left(\frac{45.0 \text{ ft}}{2} + \frac{30.0 \text{ ft}}{2}\right) = 37.5 \text{ ft}$$

(3) the distance to the edge of the slab

Not applicable.

Determine the height of the compression block.

$$a = \frac{\sum Q_n}{0.85 f_c' b}$$
$$= \frac{250 \text{ kips}}{0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})}$$
$$= 0.817 \text{ in.} < 1.00 \text{ in.} \qquad \textbf{o.k.}$$

Check end shear strength.

LRFD	ASD
R_u = 60.9 kips From AISC *Manual* Table 3-2, $\phi_v V_n$ = 272 kips > 60.9 kips **o.k.**	R_a = 45.1 kips From AISC *Manual* Table 3-2, V_n / Ω_v = 181 kips > 45.1 kips **o.k.**

Check live load deflection.

$$\Delta_{LL} = l/360 = (30.0\text{ ft})(12\text{ in./ft})/360 = 1.00\text{ in.}$$

From AISC *Manual* Table 3-20,

W21×68: $Y2$ = 5.50 in., PNA Location 7

$$I_{LB} = 2{,}510\text{ in.}^4$$

$$\begin{aligned}\Delta_{LL} &= \frac{Pl^3}{28EI_{LB}} \\ &= \frac{17.0\text{ kips}(30.0\text{ ft})^3(12\text{ in./ft})^3}{28(29{,}000\text{ ksi})(2{,}510\text{ in.}^4)} \\ &= 0.389\text{ in.} < 1.00\text{ in.} \quad \textbf{o.k.}\end{aligned}$$

Based on AISC Design Guide 3, limit the live load deflection, using 50% of the (unreduced) design live load, to L/360 with a maximum absolute value of 1.00 in. across the bay.

The maximum deflection is,

$$\begin{aligned}\Delta_{LL} &= \frac{15.0\text{ kips}(30.0\text{ ft})^3(12\text{ in./ft})^3}{28(29{,}000\text{ ksi})(2{,}510\text{ in.}^4)} \\ &= 0.343\text{ in.} < 1.00\text{ in.} \quad \textbf{o.k.}\end{aligned}$$

Check the deflection at the location where the floor beams are supported.

$$\begin{aligned}\Delta_{LL} &= \frac{15.0\text{ kips}(120\text{ in.})}{6(29{,}000\text{ ksi})(2{,}510\text{ in.}^4)}\left[3(360\text{ in.})(120\text{ in.}) - 4(120\text{ in.})^2\right] \\ &= 0.297\text{ in.} > 0.265\text{ in.} \quad \textbf{o.k.}\end{aligned}$$

Therefore, the total deflection in the bay is 0.297 in. + 0.735 in. = 1.03 in., which is acceptably close to the limit of 1.00 in, where Δ_{LL} = 0.735 in. is from the 45 ft interior composite beam running north-south.

Determine the required shear stud connectors.

Using *Manual* Table 3-21, for parallel deck with, w_r / h_r > 1.5, one ¾-in.-diameter stud in normal weight, 4-ksi concrete and Q_n = 21.5 kips/stud.

$$\begin{aligned}\frac{\Sigma Q_n}{Q_n} &= \frac{250\text{ kips}}{21.5\text{ kips/stud}} \\ &= 11.6\text{ studs/side}\end{aligned}$$

Therefore, use a minimum 24 studs for horizontal shear.

Per AISC *Specification* Section I8.2d, the maximum stud spacing is 36 in.

Since the load is concentrated at ⅓ points, the studs are to be arranged as follows:

Use 12 studs between supports and supported beams at ⅓ points. Between supported beams (middle ⅓ of span), use 4 studs to satisfy minimum spacing requirements.

Thus, 28 studs are required in a 12:4:12 arrangement.

Notes: Although the studs may be placed up to 3'-0" o.c. the steel deck must still be anchored to be the supporting member at a spacing not to exceed 18 in. in accordance with AISC *Specification* Section I3.2c.

This W21×68 beam, with full lateral support, is very close to having sufficient available strength to support the imposed loads without composite action. A larger noncomposite beam might be a better solution.

COLUMN DESIGN AND SELECTION FOR GRAVITY COLUMNS

Estimate column loads

Roof	(from previous calculations)	
	Dead Load	20 psf
	Live (Snow)	25 psf
	Total	45 psf

Snow drift loads at the perimeter of the roof and at the mechanical screen wall from previous calculations

Reaction to column (side parapet):

$$w = (3.73 \text{ kips} / 6.00 \text{ ft}) - (0.025 \text{ ksf})(23.0 \text{ ft}) = 0.0467 \text{ kip/ft}$$

Reaction to column (end parapet):

$$w = (16.0 \text{ kips} / 37.5 \text{ ft}) - (0.025 \text{ ksf})(15.5 \text{ ft}) = 0.0392 \text{ kip/ft}$$

Reaction to column (screen wall along lines C & D):

$$w = (4.02 \text{ kips} / 6.00 \text{ ft}) - (0.025 \text{ ksf})(22.5 \text{ ft}) = 0.108 \text{ kip/ft}$$

Mechanical equipment and screen wall (average):

$$w = 40 \text{ psf}$$

Return to Table of Contents

Column	**Loading Width ft**	**Loading Length ft**	**Area ft^2**	**DL kip/ft^2**	P_D **kips**	**SL kip/ft^2**	P_S **kips**
2A, 2F, 3A, 3F, 4A, 4F	23.0	30.0	690	0.020	13.8	0.025	17.3
5A, 5F, 6A, 6F, 7A, 7F							
snow drifting side		30.0				0.0467 klf	1.40
exterior wall		30.0		0.413 klf	12.4		
					26.2		**18.7**
1B, 1E, 8B, 8E	3.50	22.5	78.8	0.020	1.58	0.025	1.97
snow drifting end		22.5				0.0418 klf	0.941
exterior wall		22.5		0.413 klf	9.29		
					10.9		**2.91**
1A, 1F, 8A, 8F	23.0	15.5	357 $-\dfrac{(78.8\text{ ft}^2)}{2}$	0.020	6.36	0.025	7.95
		=	318				
snow drifting end		11.8				0.0418 klf	0.493
snow drifting side		15.5				0.0467 klf	0.724
exterior wall		27.3		0.413 klf	11.3		
					17.7		**9.17**
1C, 1D, 8C, 8D	37.5	15.5	581 $-\dfrac{(78.8\text{ ft}^2)}{2}$	0.020	10.8	0.025	13.6
		=	542				
snow-drifting end		26.3				0.0418 klf	1.10
exterior wall		26.3		0.413 klf	10.9		
					21.7		**14.7**
2C, 2D, 7C, 7D	37.5	30.0	1,125	0.020	**22.5**	0.025	**28.1**
3C, 3D, 4C, 4D	22.5	30.0	675	0.020	13.5	0.025	16.9
5C, 5D, 6C, 6D							
snow-drifting		30.0				0.108 klf	3.24
mechanical area	15.0	30.0	450	0.060	27.0	0.040 klf	18.0
					40.5		**38.1**

Floor Loads (from previous calculations)

Dead load	75 psf
Live load	80 psf
Total load	155 psf

Calculate reduction in live loads, analyzed at the base of three floors using Section 4.7.2 of ASCE/SEI 7.

Note: The 6-in. cantilever of the floor slab has been ignored for the calculation of K_{LL} for columns in this building because it has a negligible effect.

Columns: 2A, 2F, 3A, 3F, 4A, 4F, 5A, 5F, 6A, 6F, 7A, 7F
Exterior column without cantilever slabs
$K_{LL} = 4$
$L_o = 80.0$ psf
$n = 3$

$$A_T = (23.0\text{ ft})(30.0\text{ ft})$$
$$= 690\text{ ft}^2$$

Using ASCE/SEI 7 Equation 4.7-1

$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL} n A_T}}\right)$$
$$= 80.0\text{ psf}\left(0.25 + \frac{15}{\sqrt{(4)(3)(690\text{ ft}^2)}}\right)$$
$$= 33.2\text{ psf} \geq 0.4L_o = 32.0\text{ psf}$$

Use $L = 33.2$ psf.

Columns: 1B, 1E, 8B, 8E
Exterior column without cantilever slabs
$K_{LL} = 4$
$L_o = 80.0$ psf
$n = 3$

$$A_T = (5.50\text{ ft})(22.5\text{ ft})$$
$$= 124\text{ ft}^2$$
$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL} n A_T}}\right)$$
$$= 80.0\text{ psf}\left(0.25 + \frac{15}{\sqrt{(4)(3)(124\text{ ft}^2)}}\right)$$
$$= 51.1\text{ psf} \geq 0.4L_o = 32.0\text{ psf}$$

Use $L = 51.1$ psf

Columns: 1A, 1F, 8A, 8F
Corner column without cantilever slabs

$K_{LL} = 4$ $L_o = 80.0$ psf $n = 3$

$$A_T = (15.5 \text{ ft})(23.0 \text{ ft}) - (124 \text{ ft}^2 / 2)$$
$$= 295 \text{ ft}^2$$
$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL} n A_T}}\right) \geq 0.4L_o$$
$$= 80.0 \text{ psf}\left(0.25 + \frac{15}{\sqrt{(4)(3)(295 \text{ ft}^2)}}\right) \geq 0.4(80.0 \text{ psf})$$
$$= 40.2 \text{ psf} \geq 32.0 \text{ psf}$$

Use L = 40.2 psf.

Columns: 1C, 1D, 8C, 8D
Exterior column without cantilever slabs
$K_{LL} = 4$
$L_o = 80.0$ psf
$n = 3$

$$A_T = (15.5 \text{ ft})(37.5 \text{ ft}) - (124 \text{ ft}^2 / 2)$$
$$= 519 \text{ ft}^2$$
$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL} n A_T}}\right) \geq 0.4L_o$$
$$= 80.0 \text{ psf}\left(0.25 + \frac{15}{\sqrt{(4)(3)(519 \text{ ft}^2)}}\right) \geq 0.4(80.0 \text{ psf})$$
$$= 35.2 \text{ psf} \geq 32.0 \text{ psf}$$

Use L = 35.2 psf.

Columns: 2C, 2D, 3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D, 7C, 7D
Interior column
$K_{LL} = 4$
$L_o = 80.0$ psf
$n = 3$

$$A_T = (37.5 \text{ ft})(30.0 \text{ ft})$$
$$= 1{,}125 \text{ ft}^2$$
$$L = L_o\left(0.25 + \frac{15}{\sqrt{K_{LL} n A_T}}\right) \geq 0.4L_o$$
$$= 80.0 \text{ psf}\left(0.25 + \frac{15}{\sqrt{(4)(3)(1{,}125 \text{ ft}^2)}}\right) \geq 0.4(80.0 \text{ psf})$$
$$= 30.3 \text{ psf} \leq 32.0 \text{ psf}$$

Use L= 32.0 psf.

Column	**Loading Width ft**	**Loading Length ft**	**Tributary Area ft²**	**DL kip/ft²**	P_D **kips**	**LL kip/ft²**	P_L **kips**
2A, 2F, 3A, 3F, 4A, 4F 5A, 5F, 6A, 6F, 7A, 7F	23.0	30.0	690	0.075	51.8	0.0332	22.9
exterior wall		30.0		0.503 klf	15.1		
					66.9		**22.9**
1B, 1E, 8B, 8E	5.50	22.5	124	0.075	9.30	0.0511	6.34
exterior wall		22.5		0.503 klf	11.3		
					20.6		**6.34**
1A, 1F, 8A, 8F	23.0	15.5	$357 - \frac{(124\text{ in.}^2)}{2} = 295$	0.075	22.1	0.0402	11.9
exterior wall		27.3		0.503 klf	13.7		
					35.8		**11.9**
1C, 1D, 8C, 8D	37.5	15.5	$581 - \frac{(124\text{ in.}^2)}{2} = 519$	0.075	38.9	0.0352	18.3
exterior wall		26.3		0.503 klf	13.2		
					52.1		**18.3**
2C, 2D, 3C, 3D, 4C, 4D 5C, 5D, 6C, 6D, 7C, 7D	37.5	30.0	1,125	0.075	**84.4**	0.0320	**36.0**

Column load summary

Column	Floor		P_D kips	P_L kips
2A, 2F, 3A, 3F, 4A, 4F	Roof		26.2	18.7
5A, 5F, 6A, 6F, 7A, 7F	4th	66	.9	22.9
	3rd		66.9	22.9
	2nd	66	.9	22.9
	Total		**227**	**87.4**
1B, 1E, 8B, 8E	Roof	10	.9	2.91
	4th	20	.6	6.34
	3rd		20.6	6.34
	2nd	20	.6	6.34
	Total		**72.7**	**21.9**
1A, 1F, 8A, 8F	Roof	17	.7	9.14
	4th	35	.8	11.9
	3rd		35.8	11.9
	2nd	35	.8	11.9
	Total		**125**	**44.8**
1C, 1D, 8C, 8D	Roof		21.7	14.6
	4th	52	.1	18.3
	3rd		52.1	18.3
	2nd	52	.1	18.3
	Total		**178**	**69.5**
2C, 2D, 7C, 7D	Roof		22.5	28.1
	4th	84	.4	36.0
	3rd		84.4	36.0
	2nd	84	.4	36.0
	Total		**276**	**136**
3C, 3D, 4C, 4D	Roof		40.5	38.1
5C, 5D, 6C, 6D	4th	84	.4	36.0
	3rd		84.4	36.0
	2nd	84	.4	36.0
	Total		**294**	**146**

SELECT TYPICAL INTERIOR LEANING COLUMNS

Columns: 3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D

Elevation of second floor slab: 113.5 ft
Elevation of first floor slab: 100 ft
Column unbraced length: $K_xL_x = K_yL_y = 13.5$ ft

From ASCE/SEI 7, determine the required strength,

LRFD	ASD
$P_u = 1.2(294 \text{ kips}) + 1.6(3)(36.0 \text{ kips}) + 0.5(38.1 \text{ kips}) = 545 \text{ kips}$	$P_a = 294 \text{ kips} + 0.75(3)(36.0 \text{ kips}) + 0.75(38.1 \text{ kips}) = 404 \text{ kips}$

Using AISC *Manual* Table 4-1, enter with the effective length of 13.5 ft, and proceed across the table until reaching the lightest size that has sufficient available strength at the required unbraced length.

LRFD	ASD
W12×65	W12×65
$\phi_cP_n = 696 \text{ kips} > 545 \text{ kips}$ **o.k.**	$P_n / \Omega_c = 463 \text{ kips} > 404 \text{ kips}$ **o.k.**
W14×68	W14×68
$\phi_cP_n = 656 \text{ kips} > 545 \text{ kips}$ **o.k.**	$P_n / \Omega_c = 436 \text{ kips} > 404 \text{ kips}$ **o.k.**

Columns: 2C, 2D, 7C, 7D

Elevation of second floor slab: 113.5 ft
Elevation of first floor slab: 100.0 ft
Column unbraced length: $K_xL_x = K_yL_y = 13.5$ ft

LRFD	ASD
$P_u = 1.2(276 \text{ kips}) + 1.6(3)(36.0 \text{ kips}) + 0.5(28.1 \text{ kips}) = 518 \text{ kips}$	$P_a = 276 \text{ kips} + 0.75(3)(36.0 \text{ kips}) + 0.75(28.1 \text{ kips}) = 378 \text{ kips}$

Using AISC *Manual* Table 4-1, enter with the effective length of 13.5 ft, and proceed across the table until reaching the lightest size that has sufficient available strength at the required unbraced length.

LRFD	ASD
W12×65	W12×65
$\phi_cP_n = 696 \text{ kips} > 518 \text{ kips}$ **o.k.**	$P_n / \Omega_c = 463 \text{ kips} > 378 \text{ kips}$ **o.k.**
W14×61	W14×61
$\phi_cP_n = 585 \text{ kips} > 518 \text{ kips}$ **o.k.**	$P_n / \Omega_c = 389 \text{ kips} > 378 \text{ kips}$ **o.k.**

SELECT TYPICAL EXTERIOR LEANING COLUMNS

Columns: 1B, 1E, 8B, 8E

Elevation of second floor slab: 113.5 ft
Elevation of first floor slab: 100.0 ft
Column unbraced length: $K_xL_x = K_yL_y = 13.5$ ft

LRFD	ASD
$P_u = 1.2(72.7 \text{ kips}) + 1.6(3)(6.34 \text{ kips}) + 0.5(2.91 \text{ kips}) = 119 \text{ kips}$	$P_a = 72.7 \text{ kips} + 0.75(3)(6.34 \text{ kips}) + 0.75(2.91 \text{ kips}) = 89.1 \text{ kips}$

Using AISC *Manual* Table 4-1, enter with the effective length of 13.5 ft, and proceed across the table until reaching the lightest size that has sufficient available strength at the required unbraced length.

LRFD	ASD
W12×40 $\phi_cP_n = 316 \text{ kips} > 119 \text{ kips}$ **o.k.**	W12×40 $P_n / \Omega_c = 210 \text{ kips} > 89.1 \text{ kips}$ **o.k.**

Note: A 12 in. column was selected above for ease of erection of framing beams.
(Bolted double-angle connections can be used without bolt staggering.)

WIND LOAD DETERMINATION

Use the Envelope Procedure for simple diaphragm buildings from ASCE/SEI 7, Chapter 28, Part 2.

To qualify for the simplified wind load method for low-rise buildings, per ASCE/SEI 7, Section 26.2, the following must be true.

1. Simple diaphragm building **o.k.**
2. Low-rise building <= 60 ft **o.k.**
3. Enclosed **o.k.**
4. Regular-shaped **o.k.**
5. Not a flexible building **o.k.**
6. Does not have response characteristics requiring special considerations **o.k.**
7. Symmetrical shape **o.k.**
8. Torsional load cases from ASCE/SEI 7, Figure 28.4-1 do not control design of MWFRS **o.k.**

Define input parameters

1. Risk category: II from ASCE/SEI 7, Table 1.5-1

2. Basic wind speed V: 115 mph (3-s) from ASCE/SEI 7, Figure 26.5-1A

3. Exposure category: C from ASCE/SEI 7, Section 26.7.3

4. Topographic factor, K_{zt} : 1.0 from ASCE/SEI 7, Section 26.8.2

5. Mean roof height: 55' - 0"

6. Height and exposure adjustment, λ: 1.59 from ASCE/SEI 7, Figure 28.6-1

7. Roof angle: 0°

$p_s = \lambda K_{zt} p_{s30}$ (ASCE 7 Eq. 28.6-7)

$= (1.59)(1.0)(21.0 \text{ psf}) = 33.4 \text{ psf}$ Horizontal pressure zone A
$= (1.59)(1.0)(13.9 \text{ psf}) = 22.1 \text{ psf}$ Horizontal pressure zone C
$= (1.59)(1.0)(-25.2 \text{ psf}) = -40.1 \text{ psf}$ Vertical pressure zone E
$= (1.59)(1.0)(-14.3 \text{ psf}) = -22.7 \text{ psf}$ Vertical pressure zone F
$= (1.59)(1.0)(-17.5 \text{ psf}) = -27.8 \text{ psf}$ Vertical pressure zone G
$= (1.59)(1.0)(-11.1 \text{ psf}) = -17.6 \text{ psf}$ Vertical pressure zone H

a = 10% of least horizontal dimension or 0.4h, whichever is smaller, but not less than either 4% of least horizontal dimension or 3 ft

a = the lesser of:	10% of least horizontal dimension	12.3 ft
	40% of eave height	22 .0 ft
but not less than	4% of least horizontal dimension or 3 ft	4.92 ft

a = 12.3 ft
$2a$ = 24.6 ft

Zone A – End zone of wall (width = 2a)
Zone C – Interior zone of wall
Zone E – End zone of windward roof (width = 2a)
Zone F – End zone of leeward roof (width = 2a)

Zone G – Interior zone of windward roof
Zone H – Interior zone of leeward roof

Calculate load to roof diaphragm

Mechanical screen wall height: 6 ft

Wall height: $\frac{1}{2}\left[55.0 \text{ ft} - 3(13.5 \text{ ft})\right] = 7.25 \text{ ft}$

Parapet wall height: 2 ft

Total wall height at roof at screen wall: 6 ft + 7.25 ft = 13.3 ft

Total wall height at roof at parapet: 2 ft + 7.25 ft = 9.25 ft

Calculate load to fourth floor diaphragm

Wall height: ½ (55.0 ft – 40.5 ft) = 7.25 ft

½ (40.5 ft – 27.0 ft) = 6.75 ft

Total wall height at floor: 6.75 ft + 7.25 ft = 14.0 ft

Calculate load to third floor diaphragm

Wall height: ½ (40.5 ft – 27.0 ft) = 6.75 ft

½ (27.0 ft – 13.5 ft) = 6.75 ft

Total wall height at floor: 6.75 ft + 6.75 ft = 13.5 ft

Calculate load to second floor diaphragm

Wall height: ½ (27.0 ft – 13.5 ft) = 6.75 ft

½ (13.5 ft – 0.0 ft) = 6.75 ft

Total wall height at floor: 6.75 ft + 6.75 ft = 13.5 ft

Total load to diaphragm:

Load to diaphragm at roof: $w_{s(A)}$ = (33.4 psf)(9.25 ft) = 309 plf

$w_{s(C)}$ = (22.1 psf)(9.25 ft) = 204 plf at parapet

$w_{s(C)}$ = (22.1 psf)(13.3 ft) = 294 plf at screenwall

Load to diaphragm at fourth floor: $w_{s(A)}$ = (33.4 psf)(14.0 ft) = 468 plf

$w_{s(C)}$ = (22.1 psf)(14.0 ft) = 309 plf

Load to diaphragm at second and third floors: $w_{s(A)}$ = (33.4 psf)(13.5 ft) = 451 plf

$w_{s(C)}$ = (22.1 psf)(13.5 ft) = 298 plf

l = length of structure, ft

b = width of structure, ft

h = height of wall at building element, ft

Determine the wind load to each frame at each level. Conservatively apply the end zone pressures on both ends of the building simultaneously.

Wind from a north or south direction:

Total load to each frame: $P_{W(n-s)} = w_{s(A)}\left(2a\right) + w_{s(C)}\left(l/2 - 2a\right)$

Shear in diaphragm: $v_{(n-s)} = P_{W(n-s)}/120$ ft for roof

$v_{(n-s)} = P_{W(n-s)}/90$ ft for floors (deduction for stair openings)

Wind from an east or west direction:

Total load to each frame: $P_{W(e-w)} = w_{s(A)}\left(2a\right) + w_{s(C)}\left(b/2 - 2a\right)$

Shear in diaphragm: $v_{(e-w)} = P_{W(e-w)}/210$ ft for roof and floors

	l	***b***	***2a***	***h***	$p_{s(A)}$	$p_{s(C)}$	$w_{s(A)}$	$w_{s(C)}$	$P_{W(n-s)}$	$P_{W(e-w)}$	$v_{(n-s)}$	$v_{(e-w)}$
	ft	**ft**	**ft**	**ft**	**psf**	**psf**	**plf**	**plf**	**kips**	**kips**	**plf**	**plf**
Screen	93.0	33.0	0	13.3	0	22.1	0	294	13.7	4.85	–	–
Roof	120	90.0	24.6	9.25	33.4	22.1	309	204	14.8	11.8	238	79
4th	213	123	24.6	14.0	33.4	22.1	468	309	36.8	22.9	409	109
3rd	213	123	24.6	13.5	33.4	22.1	451	298	35.5	22.1	394	105
2nd	213	123	24.6	13.5	33.4	22.1	451	298	35.5	22.1	394	105
Base of Frame									136	83.8		

Note: The table above indicates the total wind load in each direction acting on a steel frame at each level. The wind load at the ground level has not been included in the chart because it does not affect the steel frame.

Return to Table of Contents

SEISMIC LOAD DETERMINATION

The floor plan area: 120 ft, column center line to column center line, by 210 ft, column centerline to column center line, with the edge of floor slab or roof deck 6 in. beyond the column center line.

Area = (121 ft)(211 ft)
= 25,500 ft^2

The perimeter cladding system length:

Length = (2)(123 ft) + (2)(213 ft)
= 672 ft

The perimeter cladding weight at floors:

Brick spandrel panel with metal stud backup	(7.50 ft)(0.055 ksf) = 0.413 klf
Window wall system	(6.00 ft)(0.015 ksf) = 0.090 klf
Total	**0.503 klf**

Typical roof dead load (from previous calculations):

Although 40 psf was used to account for the mechanical units and screen wall for the beam and column design, the entire mechanical area will not be uniformly loaded. Use 30% of the uniform 40 psf mechanical area load to determine the total weight of all of the mechanical equipment and screen wall for the seismic load determination.

Roof Area = (25,500 ft^2)(0.020 ksf) =	510 kips
Wall perimeter = (672 ft)(0.413 klf) =	278 kips
Mechanical Area = (2,700 ft^2)(0.300)(0.040 ksf) =	32.4 kips
Total	**820 kips**

Typical third and fourth floor dead load:

Note: An additional 10 psf has been added to the floor dead load to account for partitions per Section 12.7.2.2 of ASCE/SEI 7.

Floor Area = (25,500 ft^2)(0.085 ksf) =	2,170 kips
Wall perimeter = (672 ft)(0.503 klf) =	338 kips
Total	**2,510 kips**

Second floor dead load: the floor area is reduced because of the open atrium

Floor Area = (24,700 ft^2)(0.085 ksf) =	2,100 kips
Wall perimeter = (672 ft)(0.503 klf) =	338 kips
Total	**2,440 kips**

Total dead load of the building:

Roof	820 kips
Fourth floor	2,510 kips
Third floor	2,510 kips
Second floor	2,440 kips
Total	**8,280 kips**

Calculate the seismic forces.

Determine the seismic risk category and importance factors.

Office Building: Risk Category II from ASCE/SEI 7 Table 1.5-1

Seismic Importance Factor: I_e = 1.00 from ASCE/SEI 7 Table 1.5-2

The site coefficients are given in this example. S_S and S_1 can also be determined from ASCE/SEI 7, Figures 22-1 and 22-2, respectively.

$S_S = 0.121g$

$S_1 = 0.060g$

Soil, site class D (given)

F_a @ $S_S \leq 0.25$ = 1.6 from ASCE/SEI 7, Table 11.4-1

F_v @ $S_1 \leq 0.1$ = 2.4 from ASCE/SEI 7, Table 11.4-2

Determine the maximum considered earthquake accelerations.

$S_{MS} = F_a S_S = (1.6)(0.121g) = 0.194g$ from ASCE/SEI 7, Equation 11.4-1

$S_{M1} = F_v S_1 = (2.4)(0.060g) = 0.144g$ from ASCE/SEI 7, Equation 11.4-2

Determine the design earthquake accelerations.

$S_{DS} = ⅔ S_{MS} = ⅔ (0.194g) = 0.129g$ from ASCE/SEI 7, Equation 11.4-3

$S_{D1} = ⅔ S_{M1} = ⅔ (0.144g) = 0.096g$ from ASCE/SEI 7, Equation 11.4-4

Determine the seismic design category.

$S_{DS} < 0.167$g, Seismic Risk Category II: Seismic Design Category: A from ASCE/SEI 7, Table 11.6-1

$0.067\text{g} \leq S_{D1} < 0.133$g, Seismic Risk Category II: Seismic Design Category: B from ASCE/SEI 7, Table 11.6-2

Select the seismic force resisting system.

Seismic Design Category B may be used and it is therefore permissible to select a structural steel system not specifically detailed for seismic resistance, for which the seismic response modification coefficient, R = 3

Determine the approximate fundamental period.

Building Height, h_n = 55.0 ft

C_t = 0.02: x = 0.75 from ASCE/SEI 7, Table 12.8-2

$T_a = C_t (h_n)^x = (0.02)(55.0 \text{ ft})^{0.75} = 0.404$ sec from ASCE/SEI 7, Equation 12.8-7

Determine the redundancy factor from ASCE/SEI 7, Section 12.3.4.1.

ρ = 1.0 because the Seismic Design Category = B

Determine the vertical seismic effect term.

$E_v = 0.2S_{DS}D$ (ASCE 7 Eq. 12.4-4)
$= 0.2(0.129g)D$
$= 0.0258D$

The following seismic load combinations are as specified in ASCE/SEI 7, Section 12.4.2.3.

LRFD	ASD
$(1.2+0.2S_{DS})D+\rho Q_E+L+0.2S$ $=[1.2+0.2(0.129)]D+1.0Q_E+L+0.2S$ $=1.23D+1.0Q_E+L+0.2S$ $(0.9-0.2S_{DS})D+\rho Q_E+1.6H$ $=[0.9-0.2(0.129)]D+1.0Q_E+0.0$ $=0.874D+1.0Q_E$	$(1.0+0.14S_{DS})D+H+F+0.7\rho Q_E$ $=[1.0+0.14(0.129)]D+0.0+0.0+0.7(1.0)Q_E$ $=1.02D+0.7Q_E$ $(1.0+0.10S_{DS})D+H+F+0.525\rho Q_E+0.75L+0.75(L_r \text{ or } S \text{ or } R)$ $=[1.0+0.10(0.129)]D+0.0+0.0+0.525(1.0)Q_E+0.75L+0.75S$ $=1.01D+0.525Q_E+0.75L+0.75S$ $(0.6-0.14S_{DS})D+0.7\rho Q_E+H$ $=[0.6-0.14(0.129)]D+0.7(1.0)Q_E+0$ $=0.582D+0.7Q_E$

Note: ρQ_E = effect of horizontal seismic (earthquake induced) forces

Overstrength Factor: Ω_o = 3 for steel systems not specifically detailed for seismic resistance, excluding cantilever column systems, per ASCE/SEI 7, Table 12.2-1.

Calculate the seismic base shear using ASCE/SEI 7, Section 12.8.1.

Determine the seismic response coefficient from ASCE/SEI 7, Equation 12.8-2

$$C_s = \frac{S_{DS}}{\left(\dfrac{R}{I_e}\right)} = \frac{0.129}{\left(\dfrac{3}{1}\right)} = 0.0430 \quad \textbf{controls}$$

Let $T_a = T$. From ASCE/SEI 7 Figure 22-12, T_L = 12 > T (midwestern city); therefore use ASCE/SEI 7, Equation 12.8-3 to determine the upper limit of C_s.

$$C_s = \frac{S_{D1}}{T\left(\dfrac{R}{I_e}\right)} = \frac{0.096}{0.404\left(\dfrac{3}{1}\right)} = 0.0792$$

From ASCE/SEI 7, Equation 12.8-5, C_s shall not be taken less than:

$$\begin{aligned} C_s &= 0.044 S_{DS} I_e \geq 0.01 \\ &= 0.044(0.129)(1.0) \\ &= 0.00568 \end{aligned}$$

Therefore, $C_s = 0.0430$.

Calculate the seismic base shear from ASCE/SEI 7 Equation 12.8-1

$$\begin{aligned} V &= C_s W \\ &= 0.0430(8{,}280 \text{ kips}) \\ &= 356 \text{ kips} \end{aligned}$$

Calculate vertical distribution of seismic forces from ASCE/SEI 7, Section 12.8.3.

$$\begin{aligned} F_x &= C_{vx} V \\ &= C_{vx}(356 \text{ kips}) \end{aligned} \qquad \text{(ASCE Eq. 12.8-11)}$$

$$C_{vx} = \frac{w_x h_x^{\,k}}{\sum_{i=1}^{n} w_i h_i^{\,k}} \quad F_x = C_{vx} V \qquad \text{(ASCE Eq. 12.8-12)}$$

For structures having a period of 0.5 s or less, $k = 1$.

Calculate horizontal shear distribution at each level per ASCE/SEI 7, Section 12.8.4.

$$V_x = \sum_{i=x}^{n} F_i \qquad \text{(ASCE Eq. 12.8-13)}$$

Calculate the overturning moment at each level per ASCE/SEI 7, Section 12.8.5.

$$M_x = \sum_{i=x}^{n} F_i (h_i - h_x)$$

	w_x kips	h_x^k ft	$w_x h_x^k$ kip-ft	C_{vx} kips	F_x kips	V_x kips	M_x k-ft
Roof	820	55.0	45,100	0.182	64.8	64.8	
Fourth	2,510	40.5	102,000	0.411	146	211	940
Third	2,510	27.0	67,800	0.273	97.2	308	3,790
Second	2,440	13.5	32,900	0.133	47.3	355	7,940
Base	**8,280**		**248,000**		**355**		**12,700**

Calculate strength and determine rigidity of diaphragms.

Determine the diaphragm design forces from Section 12.10.1.1 of ASCE/SEI 7.

F_{px} is the largest of:

1. The force F_x at each level determined by the vertical distribution above

2. $F_{px} = \dfrac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px} \leq 0.4 S_{DS} I_e w_{px}$ from ASCE/SEI 7, Equation 12.10-1 and 12.10-3

$$\leq 0.4(0.129)(1.0)\,w_{px}$$
$$\leq 0.0516 w_{px}$$

3. $F_{px} = 0.2 S_{DS} I_e w_{px}$ from ASCE/SEI 7, Equation 12.10-2

$$= 0.2(0.129)(1.0)\,w_{px}$$
$$= 0.0258 w_{px}$$

	w_{px} **kips**	*A* **kips**	*B* **kips**	*C* **kips**	F_{px} **kips**	$v_{(n\text{-}s)}$ **plf**	$v_{(e\text{-}w)}$ **plf**
Roof	820	64.8	42.3	21.2	64.8	297	170
Fourth	2,510	146	130	64.8	146	892	382
Third	2,510	97.2	130	64.8	130	791	339
Second	2,440	47.3	105	63.0	105	641	275

where

A = force at a level based on the vertical distribution of seismic forces

$$B = F_{px} = \frac{\sum_{i=x}^{n} F_i}{\sum_{i=x}^{n} w_i} w_{px} \leq 0.4 S_{DS} I_e w_{px}$$

$$C = 0.2 S_{DS} I_e w_{px}$$

F_{px} = max(*A*, *B*, *C*)

Note: The diaphragm shear loads include the effects of openings in the diaphragm and a 10% increase to account for accidental torsion.

Roof

Roof deck: 1½ in. deep, 22 gage, wide rib
Support fasteners: ⅝ in. puddle welds in 36 / 5 pattern
Sidelap fasteners: 3 #10 TEK screws
Joist spacing = s = 6.0 ft
Diaphragm length = 210 ft
Diaphragm width = l_v =120 ft

By inspection, the critical condition for the diaphragm is loading from the north or south directions.

Calculate the required diaphragm strength, including a 10% increase for accidental torsion.

LRFD	ASD
From the ASCE/SEI 7 load combinations for strength design, the earthquake load is, $v_r = 1.0\dfrac{Q_E}{l_v}$ $= 1.0(0.55)\dfrac{F_{px}}{l_v}$ $= 1.0(0.55)\dfrac{(64.8 \text{ kips})}{(120 \text{ ft})}$ $= 0.297 \text{ klf}$ The wind load is, $v_r = 1.0W$ $= 1.0(0.238 \text{ klf})$ $= 0.238 \text{ klf}$	From the ASCE/SEI 7 load combinations for allowable stress design, the earthquake load is, $v_r = 0.7\dfrac{Q_E}{l_v}$ $= 0.7(0.55)\dfrac{F_{px}}{l_v}$ $= 0.7(0.55)\dfrac{(64.8 \text{ kips})}{(120 \text{ ft})}$ $= 0.208 \text{ klf}$ The wind load is, $v_r = 0.6W$ $= 0.6(0.238 \text{ klf})$ $= 0.143 \text{ klf}$

Note: The 0.55 factor in the earthquake load accounts for half the shear to each braced frame plus the 10% increase for accidental torsion.

From the SDI *Diaphragm Design Manual* (SDI, 2004)*,* the nominal shear strengths are:

1. For panel buckling strength, v_n = 1.425 klf
2. For connection strength, v_n = 0.820 klf

Calculate the available strengths.

LRFD	ASD
Panel Buckling Strength (SDI, 2004) $v_a = \phi v_n$ $= 0.80(1.425 \text{ klf})$ $= 1.14 \text{ klf} > 0.297 \text{ klf}$ **o.k.** Connection Strength (SDI, 2004) Earthquake	Panel Buckling Strength (SDI, 2004) $v_a = \dfrac{v_n}{\Omega}$ $= \dfrac{1.425 \text{ klf}}{2.00}$ $= 0.713 \text{ klf} > 0.208 \text{ klf}$ **o.k.** Connection Strength (SDI, 2004) Earthquake

Return to Table of Contents

LRFD	ASD
$v_a = \phi v_n$ $= 0.55(0.820 \text{ klf})$ $= 0.451 \text{ klf} > 0.297 \text{ klf}$ **o.k.**	$v_a = \dfrac{v_n}{\Omega}$ $= \dfrac{0.820 \text{ klf}}{3.00}$ $= 0.273 \text{ klf} > 0.208 \text{ klf}$ **o.k.**
Wind (SDI, 2004) $v_a = \phi v_n$ $= 0.70(0.820 \text{ klf})$ $= 0.574 \text{ klf} > 0.238 \text{ klf}$ **o.k.**	Wind (SDI, 2004) $v_a = \dfrac{v_n}{\Omega}$ $= \dfrac{0.820 \text{ klf}}{2.35}$ $= 0.349 \text{ klf} > 0.143 \text{ klf}$ **o.k.**

Check diaphragm flexibility.

From the Steel Deck Institute *Diaphragm Design Manual,*

$D_{xx} = 758$ ft $\qquad K_1 = 0.286$ ft^{-1} $\qquad K_2 = 870$ kip/in. $\qquad K_4 = 3.78$

$$G' = \frac{K_2}{K_4 + \dfrac{0.3D_{xx}}{s} + 3K_1 s}$$
$$= \frac{870 \text{ kips/in.}}{3.78 + \dfrac{0.3(758 \text{ ft})}{6.00 \text{ ft}} + 3\left(\dfrac{0.286}{\text{ft}}\right)(6.00 \text{ ft})}$$
$$= 18.6 \text{ kips/in.}$$

Seismic loading to diaphragm.

$$w = (64.8 \text{ kips})/(210 \text{ ft})$$
$$= 0.309 \text{ klf}$$

Calculate the maximum diaphragm deflection.

$$\Delta = \frac{wL^2}{8l_v G'}$$
$$= \frac{(0.309 \text{ klf})(210 \text{ ft})^2}{8(120 \text{ ft})(18.6 \text{ kips/in.})}$$
$$= 0.763 \text{ in.}$$

Story drift = 0.141 in. (from computer output)

The diaphragm deflection exceeds two times the story drift; therefore, the diaphragm may be considered to be flexible in accordance with ASCE/SEI 7, Section 12.3.1.3

The roof diaphragm is flexible in the N-S direction, but using a rigid diaphragm distribution is more conservative for the analysis of this building. By similar reasoning, the roof diaphragm will also be treated as a rigid diaphragm in the E-W direction.

Third and Fourth floors

Floor deck: 3 in. deep, 22 gage, composite deck with normal weight concrete,
Support fasteners; ⅝ in. puddle welds in a 36 / 4 pattern
Sidelap fasteners: 1 button punched fastener
Beam spacing = s = 10.0 ft
Diaphragm length = 210 ft
Diaphragm width = 120 ft
l_v = 120 ft – 30 ft = 90 ft to account for the stairwell

By inspection, the critical condition for the diaphragm is loading from the north or south directions

Calculate the required diaphragm strength, including a 10% increase for accidental torsion.

LRFD	ASD
From the ASCE/SEI 7 load combinations for strength design, the earthquake load for the fourth floor is, $v_r = 1.0\dfrac{Q_E}{l_v}$ $= 1.0(0.55)\dfrac{F_{px}}{l_v}$ $= 1.0(0.55)\dfrac{(146\text{ kips})}{(90\text{ ft})}$ $= 0.892\text{ klf}$	From the ASCE/SEI 7 load combinations for strength design, the earthquake load, $v_r = 0.7\dfrac{Q_E}{l_v}$ $= 0.7(0.55)\dfrac{F_{px}}{l_v}$ $= 0.7(0.55)\dfrac{(146\text{ kips})}{(90\text{ ft})}$ $= 0.625\text{ klf}$
For the fourth floor, the wind load is, $v_r = 1.0W$ $= 1.0(0.409\text{ klf})$ $= 0.409\text{ klf}$	For the fourth floor, the wind load is, $v_r = 0.6W$ $= 0.6(0.409\text{ klf})$ $= 0.245\text{ klf}$
From the ASCI/SEI 7 load combinations for strength design, the earthquake load for the third floor is, $v_r = 1.0\dfrac{Q_E}{l_v}$ $= 1.0(0.55)\dfrac{F_{px}}{l_v}$ $= 1.0(0.55)\dfrac{(130\text{ kips})}{(90\text{ ft})}$ $= 0.794\text{ klf}$	From the ASCI/SEI 7 load combinations for strength design, the earthquake load for the third floor is, $v_r = 0.7\dfrac{Q_E}{l_v}$ $= 0.7(0.55)\dfrac{F_{px}}{l_v}$ $= 0.7(0.55)\dfrac{(130\text{ kips})}{(90\text{ ft})}$ $= 0.556\text{ klf}$
For the third floor, the wind load is, $v_r = 1.0W$ $= 1.0(0.394\text{ klf})$ $= 0.394\text{ klf}$	For the third floor, the wind load is, $v_r = 0.6W$ $= 0.6(0.394\text{ klf})$ $= 0.236\text{ klf}$

From the SDI *Diaphragm Design Manual,* the nominal shear strengths are:

For connection strength, v_n = 5.16 klf

Calculate the available strengths.

LRFD	ASD
Connection Strength (same for earthquake or wind) (SDI, 2004) $v_a = \phi v_n$ $= 0.5(5.16 \text{ klf})$ $= 2.58 \text{ klf} > 0.892 \text{ klf}$ **o.k.**	Connection Strength (same for earthquake or wind) (SDI, 2004) $v_a = \dfrac{v_n}{\Omega}$ $= \dfrac{5.16 \text{ klf}}{3.25}$ $= 1.59 \text{ klf} > 0.625 \text{ klf}$ **o.k.**

Check diaphragm flexibility.

From the Steel Deck Institute *Diaphragm Design Manual,*

$K_1 = 0.729 \text{ ft}^{-1}$ $K_2 = 870$ kip/in. $K_3 = 2{,}380$ kip/in. $K_4 = 3.78$

$$
\begin{aligned}
G' &= \left(\frac{K_2}{K_4 + 3K_1 s}\right) + K_3 \\
&= \left(\frac{870 \text{ kip/in.}}{3.78 + 3\left(\dfrac{0.729}{\text{ft}}\right)(10.0 \text{ ft})}\right) + 2{,}380 \text{ kip/in.} \\
&= 2{,}410 \text{ kips/in.}
\end{aligned}
$$

Fourth Floor

Calculate seismic loading to diaphragm based on the fourth floor seismic load.

$$
\begin{aligned}
w &= (146 \text{ kips})/(210 \text{ ft}) \\
&= 0.695 \text{ klf}
\end{aligned}
$$

Calculate the maximum diaphragm deflection on the fourth floor.

$$
\begin{aligned}
\Delta &= \frac{wL^2}{8l_v G'} \\
&= \frac{(0.695 \text{ klf})(210 \text{ ft})^2}{8(90 \text{ ft})(2{,}410 \text{ kips/in.})} \\
&= 0.0177 \text{ in.}
\end{aligned}
$$

Third Floor

Calculate seismic loading to diaphragm based on the third floor seismic load.

$$
\begin{aligned}
w &= (130 \text{ kips})/(210 \text{ ft}) \\
&= 0.619 \text{ klf}
\end{aligned}
$$

Calculate the maximum diaphragm deflection on the third floor.

$$\Delta = \frac{wL^2}{8l_v G'}$$
$$= \frac{(0.619 \text{ klf})(210 \text{ ft})^2}{8(90 \text{ ft})(2{,}410 \text{ kips/in.})}$$
$$= 0.0157 \text{ in.}$$

The diaphragm deflection at the third and fourth floors is less than two times the story drift (story drift = 0.245 in. from computer output); therefore, the diaphragm is considered rigid in accordance with ASCE/SEI 7, Section 12.3.1.3. By inspection, the floor diaphragm will also be rigid in the E-W direction.

Second floor

Floor deck: 3 in. deep, 22 gage, composite deck with normal weight concrete,
Support fasteners: ⅝ in. puddle welds in a 36 / 4 pattern
Sidelap fasteners: 1 button punched fasteners
Beam spacing = s = 10.0 ft
Diaphragm length = 210 ft
Diaphragm width = 120 ft
Because of the atrium opening in the floor diaphragm, an effective diaphragm depth of 75 ft will be used for the deflection calculations.

By inspection, the critical condition for the diaphragm is loading from the north or south directions.

Calculate the required diaphragm strength, including a 10% increase for accidental torsion.

LRFD	ASD
From the ASCE/SEI 7 load combinations for strength design, the earthquake load is, $v_r = 1.0\frac{Q_E}{l_v}$ $= 1.0(0.55)\frac{F_{px}}{l_v}$ $= 1.0(0.55)\frac{(105 \text{ kips})}{(90 \text{ ft})}$ $= 0.642 \text{ klf}$ The wind load is, $v_r = 1.0W$ $= 1.0(0.395 \text{ klf})$ $= 0.395 \text{ klf}$	From the ASCE/SEI 7 load combinations for strength design, the earthquake load is, $v_r = 0.7\frac{Q_E}{l_v}$ $= 0.7(0.55)\frac{F_{px}}{l_v}$ $= 0.7(0.55)\frac{(105 \text{ kips})}{(90 \text{ ft})}$ $= 0.449 \text{ klf}$ The wind load is, $v_r = 0.6W$ $= 0.6(0.395 \text{ klf})$ $= 0.237 \text{ klf}$

From the SDI *Diaphragm Design Manual,* the nominal shear strengths are:

For connection strength, v_n = 5.16 klf

Calculate the available strengths.

LRFD	ASD
Connection Strength (same for earthquake or wind) (SDI, 2004) $v_a = \phi v_n$ $= 0.50(5.16 \text{ klf})$ $= 2.58 \text{ klf} > 0.642 \text{ klf}$ **o.k.**	Connection Strength (same for earthquake or wind) (SDI, 2004) $v_a = \dfrac{v_n}{\Omega}$ $= \dfrac{5.16 \text{ klf}}{3.25}$ $= 1.59 \text{ klf} > 0.449 \text{ klf}$ **o.k.**

Check diaphragm flexibility.

From the Steel Deck Institute *Diaphragm Design Manual,*

$K_1 = 0.729 \text{ ft}^{-1}$ $K_2 = 870$ kip/in. $K_3 = 2{,}380$ kip/in. $K_4 = 3.78$

$$G' = \left(\frac{K_2}{K_4 + 3K_1 s}\right) + K_3$$
$$= \left(\frac{870 \text{ kip/in.}}{3.78 + 3\left(\dfrac{0.729}{\text{ft}}\right)(10.0 \text{ ft})}\right) + 2{,}380 \text{ kip/in.}$$
$$= 2{,}410 \text{ kip/in.}$$

Calculate seismic loading to diaphragm.

$$w = (105 \text{ kips})/(210 \text{ ft})$$
$$= 0.500 \text{ klf}$$

Calculate the maximum diaphragm deflection.

$$\Delta = \frac{wL^2}{8bG'}$$
$$= \frac{(0.500 \text{ klf})(210 \text{ ft})^2}{8(75 \text{ ft})(2{,}410 \text{ kip/in.})}$$
$$= 0.0152 \text{ in.}$$

Story drift = 0.228 in. (from computer output)

The diaphragm deflection is less than two times the story drift; therefore, the diaphragm is considered rigid in accordance with ASCE/SEI 7, Section 12.3.1.3. By inspection, the floor diaphragm will also be rigid in the E-W direction.

Horizontal shear distribution and torsion:

Calculate the seismic forces to be applied in the frame analysis in each direction, including the effect of accidental torsion, in accordance with ASCE/SEI 7, Section 12.8.4.

	F_y kips	Load to Frame %	Load to Frame kips	Load to Grids 1 and 8 Accidental Torsion %	Load to Grids 1 and 8 Accidental Torsion kips	Total kips
Roof	**64.8**	**50**	**32.4**	**5**	**3.24**	**35.6**
Fourth	**146**	**50**	**73.0**	**5**	**7.30**	**80.3**
Third	**97.2**	**50**	**48.6**	**5**	**4.86**	**53.5**
Second	**47.3**	**50**	**23.7**	**5**	**2.37**	**26.1**
Base						**196**

	F_x kips	Load to Frame %	Load to Frame kips	Load to Grids A and F Accidental Torsion %	Load to Grids A and F Accidental Torsion kips	Total kips
Roof	**64.8**	**50**	**32.4**	**5**	**3.24**	**35.6**
Fourth	**146**	**50**	**73.0**	**5**	**7.30**	**80.3**
Third	**97.2**	**50**	**48.6**	**5**	**4.86**	**53.5**
Second	**47.3**	**50.8**(1)	**24.0**	**5**	**2.37**	**26.4**
Base						**196**

(1) Note: In this example, Grids A and F have both been conservatively designed for the slightly higher load on Grid A due to the atrium opening. The increase in load is calculated as follows

	Area ft^2	Mass kips	*y*-dist ft	M_y k-ft
I	25,500	2,170	60.5	131,000
II	841	71.5	90.5	6,470
	24,700	**2,100**		**125,000**

$$y = 125{,}000\text{ kip-ft}/2{,}100\text{ kips} = 59.5\text{ ft}$$

$$(100\%)(121\text{ ft} - 59.5\text{ ft})/121\text{ ft} = 50.8\%$$

MOMENT FRAME MODEL

Grids 1 and 8 were modeled in conventional structural analysis software as two-dimensional models. The second-order option in the structural analysis program was not used. Rather, for illustration purposes, second-order effects are calculated separately, using the "Approximate Second-Order Analysis" method described in AISC *Specification* Appendix 8.

The column and beam layouts for the moment frames follow. Although the frames on Grids A and F are the same, slightly heavier seismic loads accumulate on grid F after accounting for the atrium area and accidental torsion. The models are half-building models. The frame was originally modeled with W14×82 interior columns and W21×44 non-composite beams, but was revised because the beams and columns did not meet the strength requirements. The W14×82 column size was increased to a W14×90 and the W21×44 beams were upsized to W24×55 beams. Minimum composite studs are specified for the beams (corresponding to $\sum Q_n = 0.25 F_y A_s$), but the beams were modeled with a stiffness of $I_{eq} = I_s$.

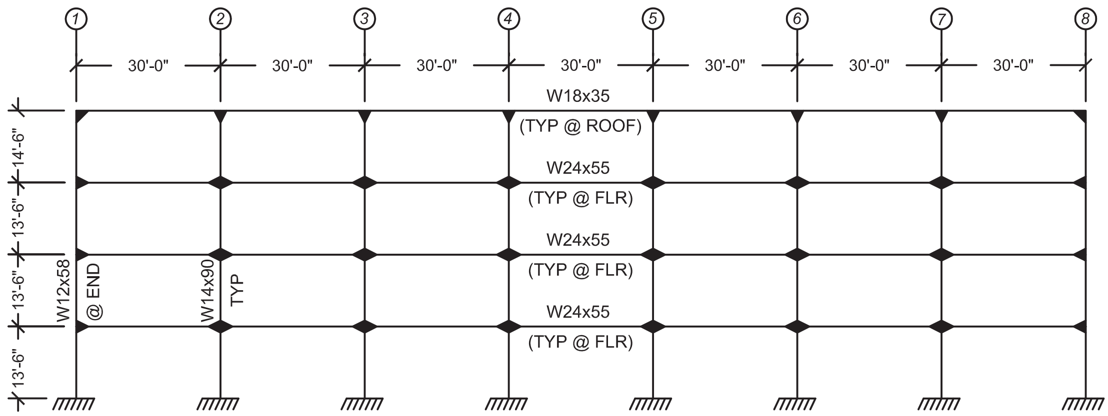
The frame was checked for both wind and seismic story drift limits. Based on the results on the computer analysis, the frame meets the L/400 drift criterion for a 10 year wind (0.7W) indicated in Commentary Section CC.1.2 of ASCE/SEI 7. In addition, the frame meets the 0.025h_{sx} allowable story drift limit given in ASCE/SEI 7 Table 12.12-1 for Seismic Risk Category II.

All of the vertical loads on the frame were modeled as point loads on the frame. The dead load and live load are shown in the load cases that follow. The wind, seismic, and notional loads from leaning columns are modeled and distributed 1/14 to exterior columns and 1/7 to the interior columns. This approach minimizes the tendency to accumulate too much load in the lateral system nearest an externally applied load.

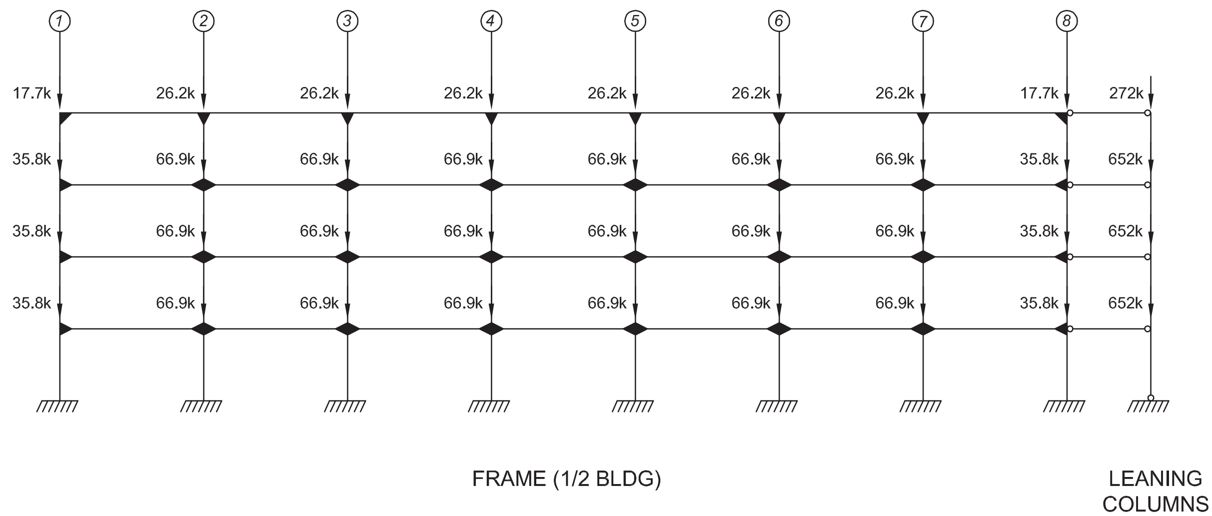
Also shown in the models below are the remainder of the half-building model gravity loads from the interior leaning columns accumulated in a single leaning column which was connected to the frame portion of the model with pinned ended links. Because the second-order analyses that follow will use the "Approximate Second-Order Analysis (amplified first-order) approach given in the AISC *Specification* Appendix 8, the inclusion of the leaning column is unnecessary, but serves to summarize the leaning column loads and illustrate how these might be handled in a full second-order analysis. See Geschwindner (1994), "A Practical Approach to the 'Leaning' Column."

There are five lateral load cases. Two are the wind load and seismic load, per the previous discussion. In addition, notional loads of $N_i = 0.002Y_i$ were established. The model layout, nominal dead, live, and snow loads with associated notional loads, wind loads and seismic loads are shown in the figures below.

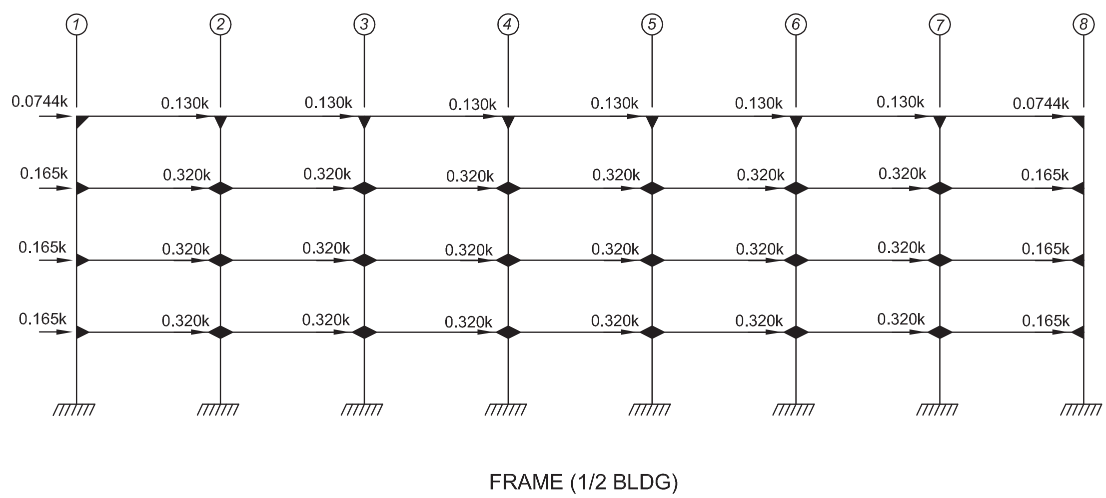
The same modeling procedures were used in the braced frame analysis. If column bases are not fixed in construction, they should not be fixed in the analysis.



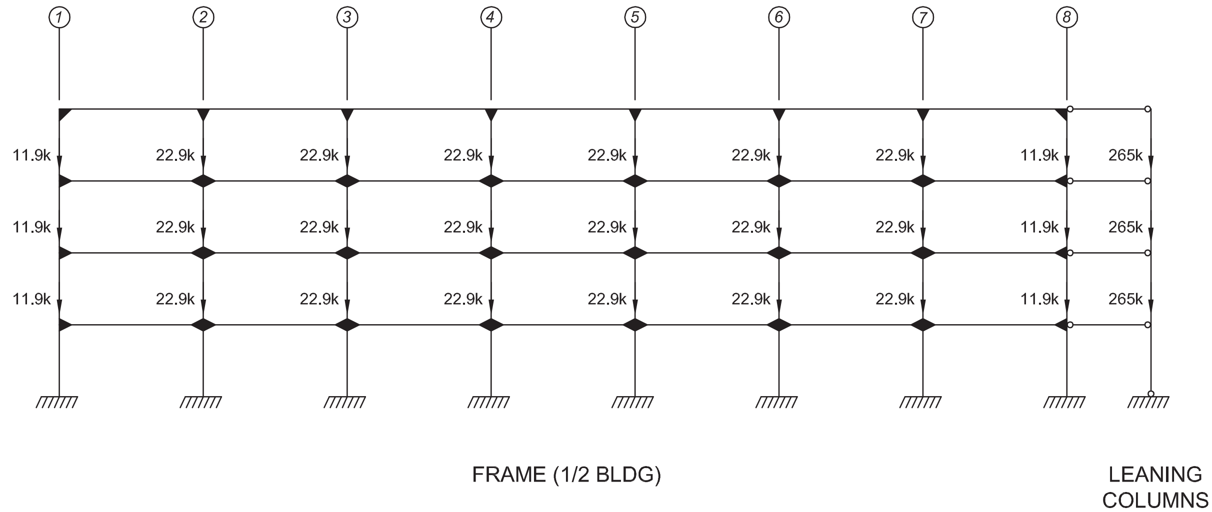
FRAME LAYOUT (GRID A & F)



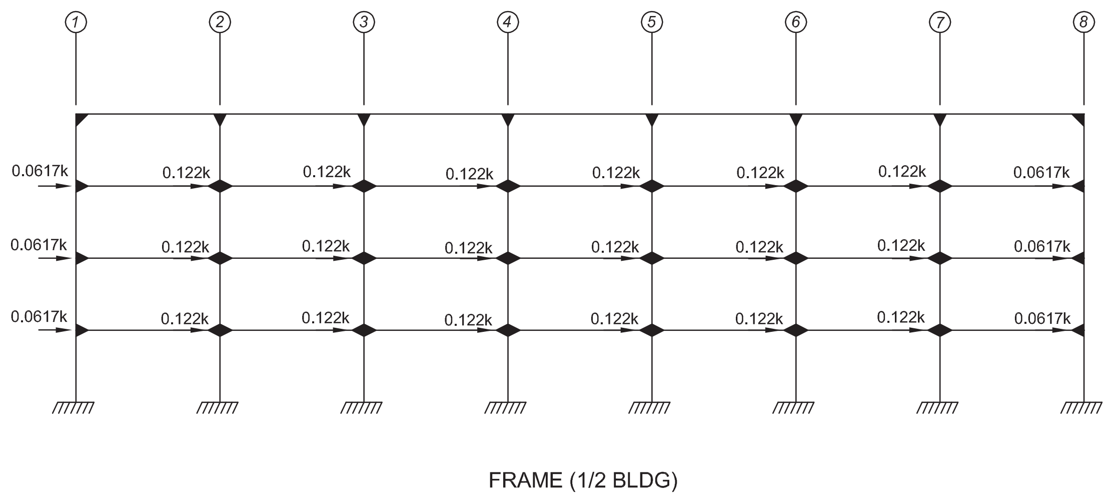
NOMINAL DEAD LOADS



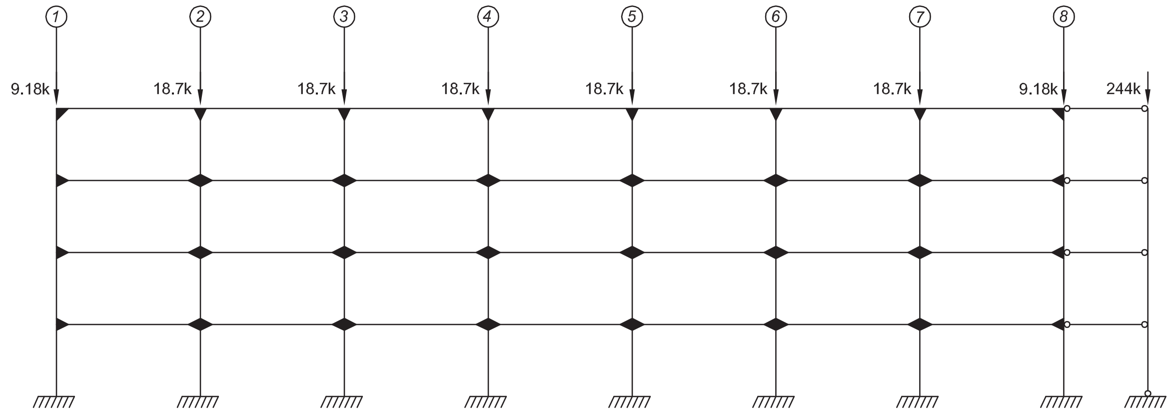
NOTIONAL DEAD LOADS



NOMINAL LIVE LOADS



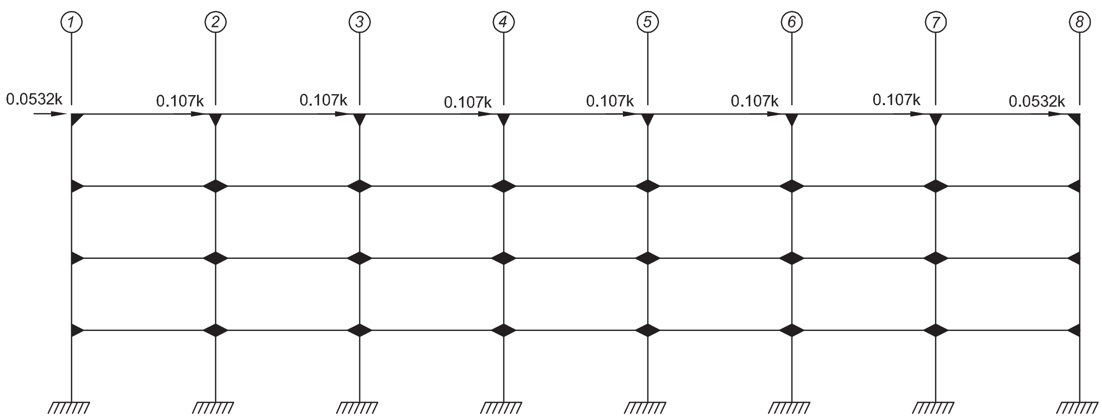
NOTIONAL LIVE LOADS



FRAME (1/2 BLDG)

LEANING
COLUMNS

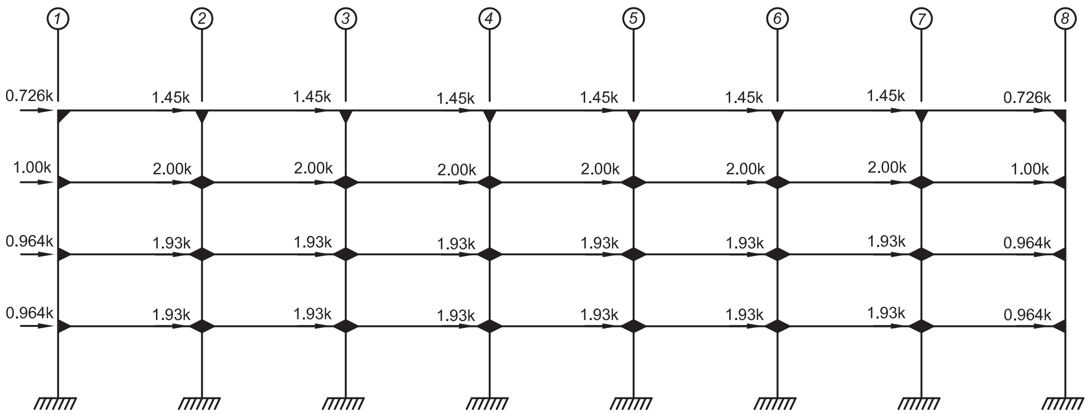
NOMINAL SNOW LOADS



FRAME (1/2 BLDG)

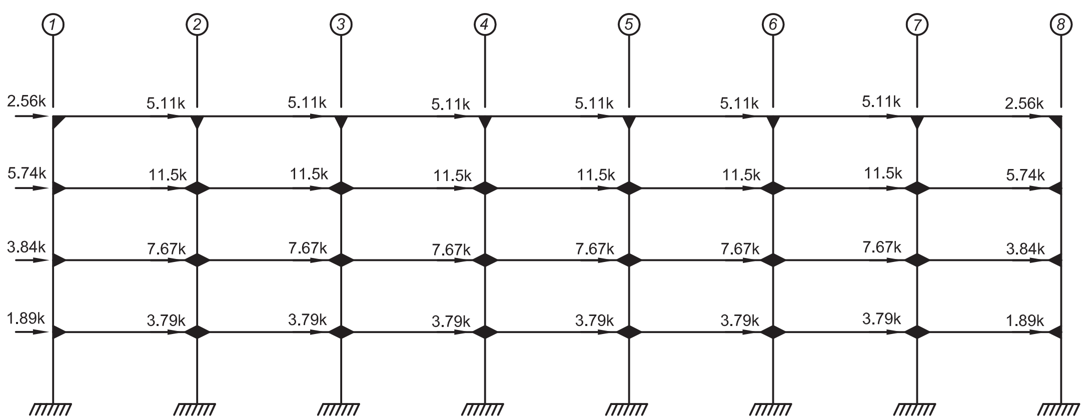
NOTIONAL SNOW LOADS

Return to Table of Contents



FRAME (1/2 BLDG)

<u>NOMINAL WIND LOADS</u>



FRAME (1/2 BLDG)

<u>SEISMIC LOADS ($1.0Q_E$)</u>

CALCULATION OF REQUIRED STRENGTH—THREE METHODS

Three methods for checking one of the typical interior column designs at the base of the building are presented below. All three of presented methods require a second-order analysis (either direct via computer analysis techniques or by amplifying a first-order analysis). A fourth method called the "First-Order Analysis Method" is also an option. This method does not require a second-order analysis; however, this method is not presented below. For additional guidance on applying any of these methods, see the discussion in AISC *Manual* Part 2 titled Required Strength, Stability, Effective Length, and Second-Order Effects.

GENERAL INFORMATION FOR ALL THREE METHODS

Seismic load combinations controlled over wind load combinations in the direction of the moment frames in the example building. The frame analysis was run for all LRFD and ASD load combinations; however, only the controlling combinations have been illustrated in the following examples. A lateral load of 0.2% of gravity load was included for all gravity-only load combinations.

The second-order analysis for all the examples below was carried out by doing a first-order analysis and then amplifying the results to achieve a set of second-order design forces using the approximate second-order analysis procedure from AISC *Specification* Appendix 8.

METHOD 1. DIRECT ANALYSIS METHOD

Design for stability by the direct analysis method is found in Chapter C of the AISC *Specification*. This method requires that both the flexural and axial stiffness are reduced and that 0.2% notional lateral loads are applied in the analysis to account for geometric imperfections and inelasticity. Any general second-order analysis method that considers both $P-\delta$ and $P-\Delta$ effects is permitted. The amplified first-order analysis method of AISC *Specification* Appendix 8 is also permitted provided that the B_1 and B_2 factors are based on the reduced flexural and axial stiffnesses. A summary of the axial loads, moments and 1st floor drifts from first-order analysis is shown below. The floor diaphragm deflection in the east-west direction was previously determined to be very small and will thus be neglected in these calculations. Second-order member forces are determined using the amplified first-order procedure of AISC *Specification* Appendix 8.

It was assumed, subject to verification, that B_2 is less than 1.7 for each load combination; therefore, per AISC *Specification* Section C2.2b(4), the notional loads were applied to the gravity-only load combinations . The required seismic load combinations are given in ASCE/SEI 7, Section 12.4.2.3.

LRFD	ASD
$1.23D \pm 1.0Q_E + 0.5L + 0.2S$ (Controls columns and beams) From a first-order analysis with notional loads where appropriate and reduced stiffnesses: For Interior Column Design: P_u = 317 kips M_{1u} = 148 kip-ft (from first-order analysis) M_{2u} = 233 kip-ft (from first-order analysis) First story drift with reduced stiffnesses = 0.718 in.	$1.01D + 0.75L + 0.75(0.7Q_E) + 0.75S$ (Controls columns and beams) From a first-order analysis with notional loads where appropriate and reduced stiffnesses: For Interior Column Design: P_a = 295 kips M_{1a} = 77.9 kip-ft M_{2a} = 122 kip-ft First story drift with reduced stiffnesses = 0.377 in.

Note: For ASD, ordinarily the second-order analysis must be carried out under 1.6 times the ASD load combinations and the results must be divided by 1.6 to obtain the required strengths. For this example, second-order analysis by the amplified first-order analysis method is used. The amplified first-order analysis method incorporates the 1.6 multiplier directly in the B_1 and B_2 amplifiers, such that no other modification is needed.

The required second-order flexural strength, M_r, and axial strength, P_r, are determined as follows. For typical interior columns, the gravity-load moments are approximately balanced, therefore, M_{nt} = 0.0 kip-ft

Calculate the amplified forces and moments in accordance with AISC *Specification* Appendix 8.

LRFD	ASD
$M_r = B_1M_{nt} + B_2M_{lt}$ (*Spec*. Eq. A-8-1) *Determine* B_1 P_r = required second-order axial strength using LRFD or ASD load combinations, kips. Note that for members subject to axial compression, B_1 may be calculated based on the first-order estimate $P_r = P_{nt} + P_{lt}$. Therefore, P_r = 317 kips (from the first-order computer analysis) I_x = 999 in.^{4} (W14×90) τ_b = 1.0 $P_{e1} = \dfrac{\pi^2 EI^*}{(K_1L)^2}$ (*Spec*. Eq. A-8-5) $= \dfrac{\pi^2(0.8)(29{,}000\text{ ksi})(999\text{ in.}^4)}{\left[(1.0)(13.5\text{ ft})(12\text{ in./ft})\right]^2}$ $= 8{,}720$ kips $C_m = 0.6 - 0.4(M_1 / M_2)$ (*Spec*. Eq. A-8-4) $= 0.6 - 0.4$ (148 kip-ft / 233 kip-ft) $= 0.346$ α = 1.0 $B_1 = \dfrac{C_m}{1-\dfrac{\alpha P_r}{P_{e1}}} \geq 1$ (*Spec*. Eq. A-8-3) $= \dfrac{0.346}{1-\dfrac{1.0(317\text{ kips})}{8{,}720\text{ kips}}} \geq 1$ $= 0.359 \geq 1;$ Use 1.0 *Determine* B_2 $B_2 = \dfrac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) where α = 1.0 P_{story} = 5,440 kips (from computer output)	$M_r = B_1M_{nt} + B_2M_{lt}$ (*Spec*. Eq. A-8-1) *Determine* B$_1$ P_r = required second-order axial strength using LRFD or ASD load combinations, kips. Note that for members subject to axial compression, B_1 may be calculated based on the first-order estimate $P_r = P_{nt} + P_{lt}$. Therefore, P_r = 295 kips (from the first-order computer analysis) I_x = 999 in.^{4} (W14×90) τ_b = 1.0 $P_{e1} = \dfrac{\pi^2 EI^*}{(K_1L)^2}$ (*Spec*. Eq. A-8-5) $= \dfrac{\pi^2(0.8)(29{,}000\text{ ksi})(999\text{ in.}^4)}{\left[(1.0)(13.5\text{ ft})(12\text{ in./ft})\right]^2}$ $= 8{,}720$ kips $C_m = 0.6 - 0.4(M_1 / M_2)$ (*Spec*. Eq. A-8-4) $= 0.6 - 0.4$ (77.9 kip-ft / 122 kip-ft) $= 0.345$ α = 1.6 $B_1 = \dfrac{C_m}{1-\dfrac{\alpha P_r}{P_{e1}}} \geq 1$ (*Spec*. Eq. A-8-3) $= \dfrac{0.345}{1-\dfrac{1.6(295\text{ kips})}{8{,}720\text{ kips}}} \geq 1$ $= 0.365 \geq 1;$ Use 1.0 *Determine* B_2 $B_2 = \dfrac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) where α= 1.6 P_{story} = 5,120 kips (from computer output)

LRFD	ASD
$P_{e\,story}$ may be taken as: $P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec.* Eq. A-8-7) where $R_M = 1 - 0.15\dfrac{P_{mf}}{P_{story}}$ (*Spec.* Eq. A-8-8) where $P_{mf} = 2{,}250\text{ kips}$ (gravity load in moment frame) $R_M = 1 - 0.15\dfrac{2{,}250\text{ kips}}{5{,}440\text{ kips}}$ $= 0.938$ $H = 1.0Q_E$ $= 1.0(196\text{ kips})$ (Lateral) $= 196\text{ kips}$ (previous seismic force distribution calculations) $\Delta_H = 0.718\text{ in.}$ (from computer output) $P_{e\,story} = 0.938\dfrac{(196\text{ kips})(13.5\text{ ft})(12\text{ in./ft})}{0.718\text{ in.}}$ $= 41{,}500\text{ kips}$ $B_2 = \dfrac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec.* Eq. A-8-6) $= \dfrac{1}{1 - \dfrac{1.0(5{,}440\text{ kips})}{41{,}500\text{ kips}}} \geq 1$ $= 1.15 \geq 1$ Because $B_2 < 1.7$, it is verified that it was unnecessary to add the notional loads to the lateral loads for this load combination. *Calculate amplified moment* From AISC *Specification* Equation A-8-1, $M_r = (1.0)(0.0\text{ kip-ft}) + (1.15)(233\text{ kip-ft})$ $= 268\text{ kip-ft}$ *Calculate amplified axial load* $P_{nt} = 317\text{ kips}$ (from computer analysis)	$P_{e\,story}$ may be taken as: $P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec.* Eq. A-8-7) where $R_M = 1 - 0.15\dfrac{P_{mf}}{P_{story}}$ (*Spec.* Eq. A-8-8) where $P_{mf} = 2{,}090\text{ kips}$ (gravity load in moment frame) $R_M = 1 - 0.15\dfrac{2{,}090\text{ kips}}{5{,}120\text{ kips}}$ $= 0.939$ $H = 0.75(0.7Q_E)$ $= 0.75(0.7)(196\text{ kips})$ (Lateral) $= 103\text{ kips}$ (previous seismic force distribution calculations) $\Delta_H = 0.377\text{ in.}$ (from computer output) $P_{e\,story} = 0.939\dfrac{(103\text{ kips})(13.5\text{ ft})(12\text{ in./ft})}{0.377\text{ in.}}$ $= 41{,}600\text{ kips}$ $B_2 = \dfrac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec.* Eq. A-8-6) $= \dfrac{1}{1 - \dfrac{1.6(5{,}120\text{ kips})}{41{,}600\text{ kips}}} \geq 1$ $= 1.25 \geq 1$ Because $B_2 < 1.7$, it is verified that it was unnecessary to add the notional loads to the lateral loads for this load combination. *Calculate amplified moment* From AISC *Specification* Equation A-8-1, $M_r = (1.0)(0.0\text{ kip-ft}) + (1.25)(122\text{ kip-ft})$ $= 153\text{ kip-ft}$ *Calculate amplified axial load* $P_{nt} = 295\text{ kips}$ (from computer analysis) For a long frame, such as this one, the change in load

LRFD	ASD
For a long frame, such as this one, the change in load to the interior columns associated with lateral load is negligible. $P_r = P_{nt} + B_2P_{lt}$ (*Spec.* Eq. A-8-2) $= 317 \text{ kips} + (1.15)(0.0 \text{ kips})$ $= 317 \text{ kips}$ The flexural and axial stiffness of all members in the moment frame were reduced using 0.8E in the computer analysis. Check that the flexural stiffness was adequately reduced for the analysis per AISC *Specification* Section C2.3(2). $\alpha = 1.0$ $P_r = 317 \text{ kips}$ $P_y = AF_y = 26.5 \text{ in.}^2 (50.0 \text{ ksi}) = 1{,}330 \text{ kips}$ (W14×90 column) $\dfrac{\alpha P_r}{P_y} = \dfrac{1.0(317 \text{ kips})}{1{,}330 \text{ kips}} = 0.238 \le 0.5$ Therefore, $\tau_b = 1.0$ **o.k.** Note: By inspection $\tau_b = 1.0$ for all of the beams in the moment frame. For the direct analysis method, K = 1.0. From AISC *Manual* Table 4-1, P_c = 1,040 kips (W14×90 @ KL = 13.5 ft) From AISC *Manual* Table 3-2, $M_{cx} = \phi_b M_{px}$ = 574 kip-ft (W14×90 with L_b = 13.5 ft) $\dfrac{P_r}{P_c} = \dfrac{317 \text{ kips}}{1{,}040 \text{ kips}} = 0.305 \ge 0.2$ Because $\dfrac{P_r}{P_c} \ge 0.2$, use AISC *Specification* interaction Equation H1-1a. $\dfrac{P_r}{P_c} + \left(\dfrac{8}{9}\right)\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$ (*Spec.* Eq. H1-1a)	to the interior columns associated with lateral load is negligible. $P_r = P_{nt} + B_2P_{lt}$ (*Spec.* Eq. A-8-2) $= 295 \text{ kips} + (1.25)(0.0 \text{ kips})$ $= 295 \text{ kips}$ The flexural and axial stiffness of all members in the moment frame were reduced using 0.8E in the computer analysis. Check that the flexural stiffness was adequately reduced for the analysis per AISC *Specification* Section C2.3(2). $\alpha = 1.6$ $P_r = 295 \text{ kips}$ $P_y = AF_y = 26.5 \text{ in.}^2 (50.0 \text{ ksi}) = 1{,}330 \text{ kips}$ (W14×90 column) $\dfrac{\alpha P_r}{P_y} = \dfrac{1.6(295 \text{ kips})}{1{,}330 \text{ kips}} = 0.355 \le 0.5$ Therefore, $\tau_b = 1.0$ **o.k.** Note: By inspection $\tau_b = 1.0$ for all of the beams in the moment frame. For the direct analysis method, K = 1.0. From AISC *Manual* Table 4-1, P_c = 690 kips (W14×90 @ KL = 13.5 ft) From AISC *Manual* Table 3-2, $M_{cx} = \dfrac{M_{px}}{\Omega_b}$ = 382 kip-ft (W14×90 with L_b = 13.5 ft) $\dfrac{P_r}{P_c} = \dfrac{295 \text{ kips}}{690 \text{ kips}} = 0.428 \ge 0.2$ Because $\dfrac{P_r}{P_c} \ge 0.2$, use AISC *Specification* interaction Equation H1-1a. $\dfrac{P_r}{P_c} + \left(\dfrac{8}{9}\right)\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) \le 1.0$ (*Spec.* Eq. H1-1a)

Return to Table of Contents

LRFD	ASD
$0.305+\left(\frac{8}{9}\right)\left(\frac{268\text{ kip-ft}}{574\text{ kip-ft}}\right)\leq 1.0$ $0.720\leq 1.0$ **o.k.**	$0.428+\left(\frac{8}{9}\right)\left(\frac{153\text{ kip-ft}}{382\text{ kip-ft}}\right)\leq 1.0$ $0.784\leq 1.0$ **o.k.**

METHOD 2. EFFECTIVE LENGTH METHOD

Required strengths of frame members must be determined from a second-order a nalysis. In t his example the second-order analysis is performed by amplifying the axial forces and moments in members and connections from a first-order analysis using the provisions of AISC *Specification* Appendix 8. The av ailable strengths of compression members are calculated using effective length factors computed from a sidesway stability analysis.

A first-order frame analysis is conducted using the load combinations for LRFD or ASD. A minimum lateral load (notional load) equal to 0.2% of the gravity loads is included for any gravity-only load combination. The required load combinations are given in ASCE/SEI 7 and are summarized in Part 2 of the AISC *Manual*.

A summary of the axial loads, moments and 1st floor drifts from the first-order computer analysis is shown below. The floor diaphragm deflection in t he east-west direction was previously determined to be very sm all and will thus be neglected in these calculations.

LRFD	ASD
$1.23D \pm 1.0Q_E + 0.5L + 0.2S$ (Controls columns and beams) For Interior Column Design: $P_u = 317$ kips $M_{1u} = 148$ kip-ft (from first-order analysis) $M_{2u} = 233$ kip-ft (from first-order analysis) First-order first story drift = 0.575 in.	$1.01D + 0.75L + 0.75(0.7Q_E) + 0.75S$ (Controls columns and beams) For Interior Column Design: $P_a = 295$ kips $M_{1a} = 77.9$ kip-ft (from first-order analysis) $M_{2a} = 122$ kip-ft (from first-order analysis) First-order first story drift = 0.302 in.

The required second-order flexural strength, M_r, and axial strength, P_r, are calculated as follows:

For typical interior columns, the gravity load moments are approximately balanced; therefore, M_{nt} = 0.0 kips.

LRFD	ASD
$M_r = B_1M_{nt} + B_2M_{lt}$ (*Spec*. Eq. A-8-1) Determine B_1. P_r = required second-order axial strengt h using LRFD or ASD load combinations, kips Note that for members subject to axial compression, B_1 may be calculated based on the first-order estimate $P_r = P_{nt} + P_{lt}$. Therefore, $P_r = 317$ kips (from first-order computer analysis) $I = 999$ in.^{4} (W14×90) $P_{e1} = \frac{\pi^2 EI^*}{(K_1L)^2}$ (*Spec*. Eq. A-8-5)	$M_r = B_1M_{nt} + B_2M_{lt}$ (*Spec*. Eq. A-8-1) Determine B_1. P_r = required second-order axial strength using LRFD or ASD load combinations, kips Note that for members subject to axial com pression, B_1 may be calculated based on the first-order estimate $P_r = P_{nt} + P_{lt}$. Therefore, $P_r = 295$ kips (from first-order computer analysis) $I = 999$ in.^{4} (W14×90) $P_{e1} = \frac{\pi^2 EI^*}{(K_1L)^2}$ (*Spec*. Eq. A-8-5)

LRFD	ASD
$= \dfrac{\pi^2 (29{,}000 \text{ ksi})(999 \text{ in.}^4)}{\left[(1.0)(13.5 \text{ ft})(12 \text{ in./ft})\right]^2}$ $= 10{,}900$ kips	$= \dfrac{\pi^2 (29{,}000 \text{ ksi})(999 \text{ in.}^4)}{\left[(1.0)(13.5 \text{ ft})(12 \text{ in./ft})\right]^2}$ $= 10{,}900$ kips
$C_m = 0.6 - 0.4(M_1 / M_2)$ (*Spec*. Eq. A-8-4) $= 0.6 - 0.4$ (148 kip-ft / 233 kip-ft) $= 0.346$	$C_m = 0.6 - 0.4(M_1 / M_2)$ (*Spec*. Eq. A-8-4) $= 0.6 - 0.4$ (77.9 kip-ft / 122 kip-ft) $= 0.345$
$\alpha = 1.0$	$\alpha = 1.6$
$B_1 = \dfrac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} \geq 1$ (*Spec*. Eq. A-8-3) $= \dfrac{0.346}{1 - \dfrac{1.0(317 \text{ kips})}{10{,}900 \text{ kips}}} \geq 1$ $= 0.356 \geq 1$; Use 1.00	$B_1 = \dfrac{C_m}{1 - \dfrac{\alpha P_r}{P_{e1}}} \geq 1$ (*Spec*. Eq. A-8-3) $= \dfrac{0.345}{1 - \dfrac{1.6(295 \text{ kips})}{10{,}900 \text{ kips}}} \geq 1$ $= 0.361 \geq 1$; Use 1.00
Determine B_2.	Determine B_2.
$B_2 = \dfrac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6)	$B_2 = \dfrac{1}{1 - \dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6)
where $\alpha = 1.0$ $P_{story} = 5{,}440$ kips (from computer output)	where $\alpha = 1.6$ $P_{story} = 5{,}120$ kips (from computer output)
$P_{e\,story}$ may be taken as $P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7)	$P_{e\,story}$ may be taken as $P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7)
where $R_M = 1 - 0.15 \dfrac{P_{mf}}{P_{story}}$ (*Spec*. Eq. A-8-8)	where $R_M = 1 - 0.15 \dfrac{P_{mf}}{P_{story}}$ (*Spec*. Eq. A-8-8)
where $P_{mf} = 2{,}250$ kips (gravity load in moment frame)	where $P_{mf} = 2{,}090$ kips (gravity load in moment frame)
$R_M = 1 - 0.15 \dfrac{2{,}250 \text{ kips}}{5{,}440 \text{ kips}}$ $= 0.938$	$R_M = 1 - 0.15 \dfrac{2{,}090 \text{ kips}}{5{,}120 \text{ kips}}$ $= 0.939$
$H = 196$ kips (Lateral) (from previous seismic force distribution calculations)	$H = 103$ kips (Lateral) (from previous seismic force distribution calculations)
$\Delta_H = 0.575$ in. (from computer output)	$\Delta_H = 0.302$ in. (from computer output)

<table>
<tr><th>LRFD</th><th>ASD</th></tr>
<tr><td>

$$P_{e\,story} = 0.938\frac{(196 \text{ kips})(13.5 \text{ ft})(12 \text{ in./ft})}{0.575 \text{ in.}}$$
$$= 51{,}800 \text{ kips}$$

$$B_2 = \frac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1 \quad (\textit{Spec.}\text{ Eq. A-8-6})$$
$$= \frac{1}{1-\dfrac{1.0(5{,}440 \text{ kips})}{51{,}800 \text{ kips}}} \geq 1$$
$$= 1.12 \geq 1$$

Note: B_2 < 1.5, therefore use of the effective length method is acceptable.

Calculate amplified moment

From AISC *Specification* Equation A-8-1,

$$M_r = (1.00)(0.0 \text{ kip-ft}) + (1.12)(233 \text{ kip-ft})$$
$$= 261 \text{ kip-ft}$$

Calculate amplified axial load.

P_{nt} = 317 kips (from computer analysis)

For a long frame, such as this one, the change in load to the interior columns associated with lateral load is negligible.

Therefore, P_{lt} = 0

$$P_r = P_{nt} + B_2P_{lt} \quad (\textit{Spec.}\text{ Eq. A-8-2})$$
$$= 317 \text{ kips} + (1.12)(0.0 \text{ kips})$$
$$= 317 \text{ kips}$$

Determine the controlling effective length.

For out-of-plane buckling in the moment frame

K_y = 1.0

$$K_yL_y = 1.0(13.5 \text{ ft}) = 13.5 \text{ ft}$$

For in-plane buckling in the moment frame, use the story stiffness procedure from the AISC *Specification* Commentary for Appendix 7 to determine K_x with *Specification* Commentary Equation C-A-7-5.

</td><td>

$$P_{e\,story} = 0.939\frac{(103 \text{ kips})(13.5 \text{ ft})(12 \text{ in./ft})}{0.302 \text{ in.}}$$
$$= 51{,}900 \text{ kips}$$

$$B_2 = \frac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1 \quad (\textit{Spec.}\text{ Eq. A-8-6})$$
$$= \frac{1}{1-\dfrac{1.6(5{,}120 \text{ kips})}{51{,}900 \text{ kips}}} \geq 1$$
$$= 1.19 \geq 1$$

Note: B_2 < 1.5, therefore use of the effective length method is acceptable.

Calculate amplified moment

From AISC *Specification* Equation A-8-1,

$$M_r = (1.00)(0.0 \text{ kip-ft}) + (1.19)(122 \text{ kip-ft})$$
$$= 145 \text{ kip-ft}$$

Calculate amplified axial load.

P_{nt} = 295 kips (from computer analysis)

For a long frame, such as this one, the change in load to the interior columns associated with lateral load is negligible.

Therefore, P_{lt} = 0

$$P_r = P_{nt} + B_2P_{lt} \quad (\textit{Spec.}\text{ Eq. A-8-2})$$
$$= 295 \text{ kips} + (1.19)(0.0 \text{ kips})$$
$$= 295 \text{ kips}$$

Determine the controlling effective length.

For out-of-plane buckling in the moment frame

K_y = 1.0

$$K_yL_y = 1.0(13.5 \text{ ft}) = 13.5 \text{ ft}$$

For in-plane buckling in the moment frame, use the story stiffness procedure from the AISC *Specification* Commentary for Appendix 7 to determine K_x with *Specification* Commentary Equation C-A-7-5.

</td></tr>
</table>

LRFD	ASD
$K_2 = \sqrt{\dfrac{\Sigma P_r}{(0.85+0.15R_L)P_r}\left(\dfrac{\pi^2 EI}{L^2}\right)\left(\dfrac{\Delta_H}{\Sigma HL}\right)} \geq \sqrt{\dfrac{\pi^2 EI}{L^2}\left(\dfrac{\Delta_H}{1.7HL}\right)}$	$K_2 = \sqrt{\dfrac{\Sigma P_r}{(0.85+0.15R_L)P_r}\left(\dfrac{\pi^2 EI}{L^2}\right)\left(\dfrac{\Delta_H}{\Sigma HL}\right)} \geq \sqrt{\dfrac{\pi^2 EI}{L^2}\left(\dfrac{\Delta_H}{1.7HL}\right)}$
Simplifying and substituting terms previously calculated results in:	Simplifying and substituting terms previously calculated results in:
$K_x = \sqrt{\dfrac{P_{story}}{P_{e\,story}}\left(\dfrac{P_e}{P_r}\right)} \geq \sqrt{P_e\left(\dfrac{\Delta_H}{1.7HL}\right)}$	$K_x = \sqrt{\dfrac{P_{story}}{P_{e\,story}}\left(\dfrac{P_e}{P_r}\right)} \geq \sqrt{P_e\left(\dfrac{\Delta_H}{1.7HL}\right)}$
where	where
$P_e = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2(29{,}000\text{ ksi})(999\text{ in.}^4)}{[12\text{ in./ft}(13.5\text{ ft})]^2} = 10{,}900\text{ kips}$	$P_e = \dfrac{\pi^2 EI}{L^2} = \dfrac{\pi^2(29{,}000\text{ ksi})(999\text{ in.}^4)}{[12\text{ in./ft}(13.5\text{ ft})]^2} = 10{,}900\text{ kips}$
$K_x = \sqrt{\dfrac{5{,}440\text{ kips}}{51{,}800\text{ kips}}\left(\dfrac{10{,}900\text{ kips}}{317\text{ kips}}\right)} \geq \sqrt{10{,}900\text{ kips}\left(\dfrac{0.575\text{ in.}}{1.7(196\text{ kips})\left(\dfrac{12\text{ in.}}{\text{ft}}\right)(13.5\text{ ft})}\right)} = 1.90 \geq 0.341$	$K_x = \sqrt{\dfrac{5{,}120\text{ kips}}{51{,}900\text{ kips}}\left(\dfrac{10{,}900\text{ kips}}{295\text{ kips}}\right)} \geq \sqrt{10{,}900\text{ kips}\left(\dfrac{0.302\text{ in.}}{1.7(103\text{ kips})\left(\dfrac{12\text{ in.}}{\text{ft}}\right)(13.5\text{ ft})}\right)} = 1.91 \geq 0.341$
Use $K_x = 1.90$	Use $K_x = 1.91$
$\dfrac{KL_x}{r_x / r_y} = \dfrac{1.90(13.5\text{ ft})}{1.66} = 15.5\text{ ft}$	$\dfrac{KL_x}{r_x / r_y} = \dfrac{1.91(13.5\text{ ft})}{1.66} = 15.5\text{ ft}$
Because $\dfrac{K_x L_x}{r_x/r_y} > K_y L_y$, use KL = 15.5 ft	Because $\dfrac{K_x L_x}{r_x/r_y} > K_y L_y$, use KL = 15.5 ft
From AISC *Manual* Table 4-1,	From AISC *Manual* Table 4-1,
P_c = 990 kips (W14×90 @ KL = 15.5 ft)	P_c = 660 kips (W14×90 @ KL = 15.5 ft)
From AISC *Manual* Table 3-2,	From AISC *Manual* Table 3-2,
M_{cx} = 574 kip-ft (W14×90 with L_b =13.5 ft)	M_{cx} = 382 kip-ft (W14×90 with L_b = 13.5 ft)
$\dfrac{P_r}{P_c} = \dfrac{317\text{ kips}}{990\text{ kips}} = 0.320 \geq 0.2$	$\dfrac{P_r}{P_c} = \dfrac{295\text{ kips}}{660\text{ kips}} = 0.447 \geq 0.2$

LRFD	ASD
Because $\frac{P_r}{P_c} \geq 0.2$, use interaction Equation H1-1a. $\frac{P_r}{P_c}+\left(\frac{8}{9}\right)\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\leq 1.0$ (*Spec*. Eq. H1.1-a) $0.320+\left(\frac{8}{9}\right)\left(\frac{261\text{ kip-ft}}{574\text{ kip-ft}}\right)\leq 1.0$ $0.724 \leq 1.0$ **o.k.**	Because $\frac{P_r}{P_c} \geq 0.2$, use interaction Equation H1-1a. $\frac{P_r}{P_c}+\left(\frac{8}{9}\right)\left(\frac{M_{rx}}{M_{cx}}+\frac{M_{ry}}{M_{cy}}\right)\leq 1.0$ (*Spec*. Eq. H1.1-a) $0.447+\left(\frac{8}{9}\right)\left(\frac{145\text{ kip-ft}}{382\text{ kip-ft}}\right)\leq 1.0$ $0.784 \leq 1.0$ **o.k.**

METHOD 3. SIMPLIFIED EFFECTIVE LENGTH METHOD

A simplification of the effective length method using a method of second-order analysis based upon drift limits and other assumptions is described in Chapter 2 of the AISC *Manual*. A first-order frame analysis is conducted using the load combinations for LRFD or ASD. A minimum lateral load (notional load) equal to 0.2% of the gravity loads is included for all gravity-only load combinations. The floor diaphragm deflection in the east-west direction was previously determined to be very small and will thus be neglected in these calculations.

LRFD	ASD
$1.23D \pm 1.0Q_E + 0.5L + 0.2S$ (Controls columns and beams) From a first-order analysis For interior column design: P_u = 317 kips M_{1u} = 148 kip-ft (from first-order analysis) M_{2u} = 233 kip-ft (from first-order analysis) First story first-order drift = 0.575 in.	$1.01D + 0.75L + 0.75(0.7Q_E) + 0.75S$ (Controls columns and beams) From a first-order analysis For interior column design: P_a = 295 kips M_{1a} = 77.9 kip-ft (from first-order analysis) M_{2a} = 122 kip-ft (from first-order analysis) First story first-order drift = 0.302 in.

Then the following steps are executed.

LRFD	ASD
Step 1: Lateral load = 196 kips Deflection due to first-order elastic analysis Δ = 0.575 in. between first and second floor Floor height = 13.5 ft Drift ratio = (13.5 ft)(12 in./ft) / 0.575 in. = 282 *Step 2:* Design story drift limit = H/400 Adjusted Lateral load = (282/ 400)(196 kips) = 138 kips	*Step 1:* Lateral load = 103 kips Deflection due to first-order elastic analysis Δ = 0.302 in. between first and second floor Floor height = 13.5 ft Drift ratio = (13.5 ft)(12 in./ft) / 0.302 in. = 536 *Step 2:* Design story drift limit = H/400 Adjusted Lateral load = (536 / 400)(103 kips) = 138 kips

LRFD	ASD
Step 3: $\text{Load ratio} = (1.0)\dfrac{\text{total story load}}{\text{lateral load}}$ $= (1.0)\dfrac{5{,}440 \text{ kips}}{138 \text{ kips}}$ $= 39.4$ From AISC *Manual* Table 2-1: $B_2 = 1.1$ Which matches the value obtained in Method 2 to the two significant figures of the table	*Step 3:* (for an ASD design the ratio must be multiplied by 1.6) $\text{Load ratio} = (1.6)\dfrac{\text{total story load}}{\text{lateral load}}$ $= (1.6)\dfrac{5{,}120 \text{ kips}}{138 \text{ kips}}$ $= 59.4$ From AISC *Manual* Table 2-1: $B_2 = 1.2$ Which matches the value obtained in Method 2 to the two significant figures of the table

Note: Intermediate values are not interpolated from the table because the precision of the table is two significant digits. Additionally, the design story drift limit used in Step 2 need not be the same as other strength or serviceability drift limits used during the analysis and design of the structure.

Step 4. Multiply all the forces and moment from the first-order analysis by the value of B_2 obtained from the table. This presumes that B_1 is less than or equal to B_2, which is usually the case for members without transverse loading between their ends.

LRFD	ASD
Step 5. Since the selection is in the shaded area of the chart, ($B_2 \leq 1.1$). For LRFD design, use $K = 1.0$. Multiply both sway and non-sway moments by B_2. $M_r = B_2(M_{nt} + M_{lt})$ $= 1.1(0 \text{ kip-ft} + 233 \text{ kip-ft}) = 256 \text{ kip-ft}$ $P_r = B_2(P_{nt} + P_{lt})$ $= 1.1(317 \text{ kips} + 0.0 \text{ kips}) = 349 \text{ kips}$ From AISC *Manual* Table 4-1, $P_c = 1{,}040$ kips (W14×90 @ $KL = 13.5$ ft) From AISC *Manual* Table 3-2, $M_{cx} = \phi_b M_{px} = 574$ kip-ft (W14×90 with $L_b = 13.5$ ft) $\dfrac{P_r}{P_c} = \dfrac{349 \text{ kips}}{1{,}040 \text{ kips}} = 0.336 \geq 0.2$ Because $\dfrac{P_r}{P_c} \geq 0.2$, use interaction Equation H1-1a.	Step 5. Since the selection is in the unshaded area of the chart ($B_2 > 1.1$), For ASD design, the effective length factor, K, must be determined through analysis. From previous analysis, use an effective length of 15.5 ft. Multiply both sway and non-sway moments by B_2 $M_r = B_2(M_{nt} + M_{lt})$ $= 1.2(0 \text{ kip-ft} + 122 \text{ kip-ft}) = 146 \text{ kip-ft}$ $P_r = B_2(P_{nt} + P_{lt})$ $= 1.2(295 \text{ kips} + 0.0 \text{ kips}) = 354 \text{ kips}$ From AISC *Manual* Table 4-1, $P_c = 675$ kips (W14×90 @ $KL = 13.5$ ft) From AISC *Manual* Table 3-2, $M_{cx} = \dfrac{M_{px}}{\Omega_b} = 382$ kip-ft (W14×90 with $L_b = 13.5$ ft) $\dfrac{P_r}{P_c} = \dfrac{354 \text{ kips}}{675 \text{ kips}} = 0.524 \geq 0.2$ Because $\dfrac{P_r}{P_c} \geq 0.2$, use interaction Equation H1-1a.

LRFD	ASD
$\dfrac{P_r}{P_c}+\left(\dfrac{8}{9}\right)\left(\dfrac{M_{rx}}{M_{cx}}+\dfrac{M_{ry}}{M_{cy}}\right)\le 1.0$	$\dfrac{P_r}{P_c}+\left(\dfrac{8}{9}\right)\left(\dfrac{M_{rx}}{M_{cx}}+\dfrac{M_{ry}}{M_{cy}}\right)\le 1.0$
$0.336+\left(\dfrac{8}{9}\right)\left(\dfrac{256\text{ kip-ft}}{574\text{ kip-ft}}\right)\le 1.0$	$0.524+\left(\dfrac{8}{9}\right)\left(\dfrac{146\text{ kip-ft}}{382\text{ kip-ft}}\right)\le 1.0$
$0.732\le 1.0$ **o.k.**	$0.864\le 1.0$ **o.k.**

BEAM ANALYSIS IN THE MOMENT FRAME

The controlling load combinations for the beams in the moment frames are shown below and evaluated for the second floor beam. The dead load, live load and seismic moments were taken from a computer analysis. The table summarizes the calculation of B_2 for the stories above and below the second floor.

1st – 2nd	LRFD Combination	ASD Combination 1	ASD Combination 2
	$1.23D + 1.0Q_E + 0.5L + 0.2S$	$1.02D + 0.7Q_E$	$1.01D + 0.75L + 0.75(0.7Q_E) + 0.75S$
H	196 kips	137 kips	103 kips
L	13.5 ft	13.5 ft	13.5 ft
Δ_H	0.575 in.	0.402 in.	0.302 in.
P_{mf}	2,250 kips	1,640 kips	2,090 kips
R_M	0.938	0.937	0.939
$P_{e\,story}$	51,800 kips	51,700 kips	51,900 kips
P_{story}	5,440 kips	3,920 kips	5,120 kips
B_2	1.12	1.14	1.19

2nd – 3rd	LRFD Combination	ASD Combination 1	ASD Combination 2
	$1.23D + 1.0Q_E + 0.5L +0.2S$	$1.02D + 0.7Q_E$	$1.01D + 0.75L + 0.75(0.7Q_E) + 0.75S$
H	170 kips	119 kips	89.3 kips
L	13.5 ft	13.5 ft	13.5 ft
Δ_H	0.728 in.	0.509 in.	0.382 in.
P_{mf}	1,590	1,160	1,490
R_M	0.938	0.937	0.939
$P_{e\,story}$	35,500 kips	35,500 kips	35,600 kips
P_{story}	3,840 kips	2,770 kips	3,660 kips
B_2	1.12	1.14	1.20

For beam members, the larger of the B_2 values from the story above or below is used.

From computer output at the controlling beam:

M_{dead} = 153 kip-ft M_{live} = 80.6 kip-ft M_{snow} = 0.0 kip-ft $M_{earthquake}$ = 154 kip-ft

LRFD Combination	ASD Combination 1	ASD Combination 2
$B_2M_{lt} = 1.12(154\text{ kip-ft})$ $= 172\text{ kip-ft}$	$B_2M_{lt} = 1.14(154\text{ kip-ft})$ $= 176\text{ kip-ft}$	$B_2M_{lt} = 1.20(154\text{ kip-ft})$ $= 185\text{ kip-ft}$

$M_u = \begin{bmatrix} 1.23(153\text{ kip-ft}) \\ +1.0(172\text{ kip-ft}) \\ +0.5(80.6\text{ kip-ft}) \end{bmatrix}$ $= 400\text{ kip-ft}$	$M_a = \begin{bmatrix} 1.02(153\text{ kip-ft}) \\ +0.7(176\text{ kip-ft}) \end{bmatrix}$ $= 279\text{ kip-ft}$	$M_a = \begin{bmatrix} 1.01(153\text{ kip-ft}) \\ +0.525(185\text{ kip-ft}) \\ +0.75(80.6\text{ kip-ft}) \end{bmatrix}$ $= 312\text{ kip-ft}$

Calculate C_b for compression in the bottom flange braced at 10.0 ft o.c.

LRFD	ASD
$C_b = 1.86$ (from computer output) Check W24×55 From AISC *Manual* Table 3-2, with continuous bracing $\phi M_n = 503\text{ kip-ft}$ From AISC *Manual* Table 3-10, for $L_b = 10.0$ ft and $C_b = 1.86$ $\phi M_n = (386\text{ kip-ft})1.86 \leq 503\text{ kip-ft}$ $= 718\text{ kip-ft} \leq 503\text{ kip-ft}$ Use $\phi M_n = 503\text{ kip-ft} > 400\text{ kip-ft}$ **o.k.** From AISC *Manual* Table 3-2, a W24×55 has a design shear strength of 252 kips and an I_x of 1350 in.^{4}	$1.02D + 0.7Q_E$ $C_b = 1.86$ (from computer output) $1.01D + 0.75(0.7Q_E) + 0.75L$ $C_b = 2.01$ (from computer output) Check W24×55 $1.02D + 0.7E$ From AISC *Manual* Table 3-2, with continuous bracing $\dfrac{M_n}{\Omega} = 334\text{ kip-ft}$ From AISC *Manual* Table 3-10, for $L_b = 10.0$ ft and $C_b = 1.86$ $\dfrac{M_n}{\Omega} = (256\text{ kip-ft})1.86 \leq 334\text{ kip-ft}$ $= 476\text{ kip-ft} \leq 334\text{ kip-ft}$ Use $\dfrac{M_n}{\Omega} = 334\text{ kip-ft} > 279\text{ kip-ft}$ **o.k.** $1.01D + 0.75(0.7Q_E) + 0.75L$ With continuous bracing $\dfrac{M_n}{\Omega} = 334\text{ kip-ft}$ From AISC *Manual* Table 3-10, for $L_b = 10$ ft and $C_b = 2.01$ $\dfrac{M_n}{\Omega} = (256\text{ kip-ft})2.01$ $= 515\text{ kip-ft} \leq 334\text{ kip-ft}$ Use $\dfrac{M_n}{\Omega} = 334\text{ kip-ft} > 312\text{ kip-ft}$ **o.k.**

LRFD	ASD
	From AISC *Manual* Table 3-2, a W24×55 has an allowable shear strength of 167 kips and an I_x of 1,350 in.^{4}

The moments and shears on the roof beams due to the lateral loads were also checked but do not control the design.

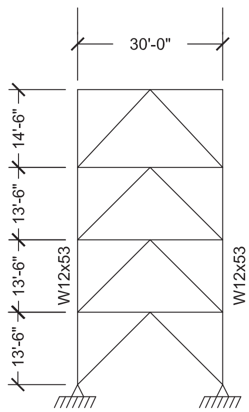
The connections of these beams can be designed by one of the techniques illustrated in the Chapter IIB of the design examples.

Return to Table of Contents

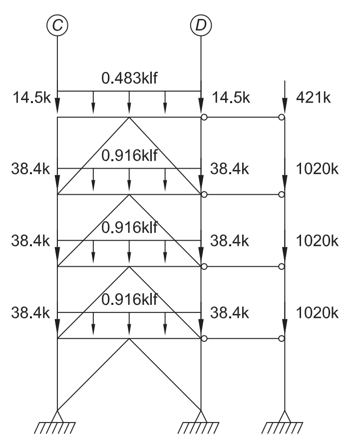
BRACED FRAME ANALYSIS

The braced frames at Grids 1 and 8 were analyzed for the required load combinations. The stability design requirements from Chapter C were applied to this system.

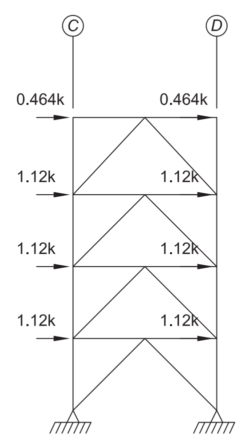
The model layout, nominal dead, live, and snow loads with associated notional loads, wind loads and seismic loads are shown in the figures below:



BRACED FRAME LAYOUT (GRID 1 & 8)

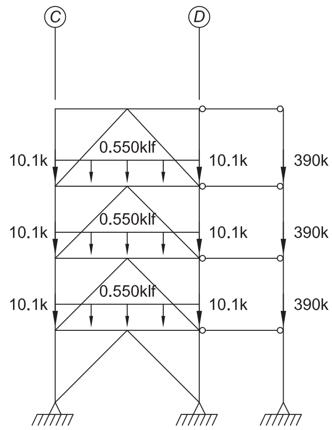


NOMINAL DEAD LOADS LEANING COLUMNS

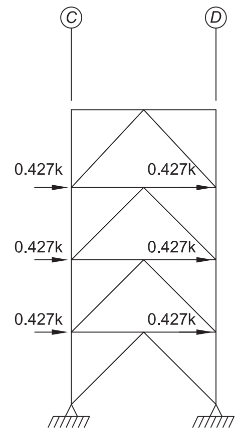


NOTIONAL DEAD LOADS

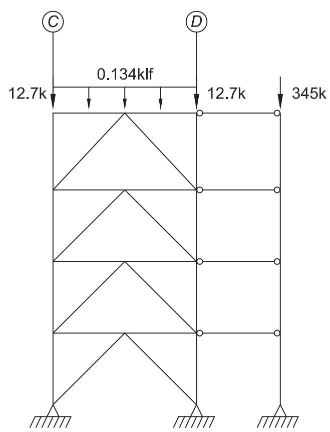
Return to Table of Contents



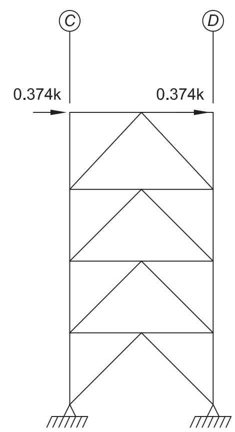
NOMINAL LIVE LOADS LEANING COLUMNS



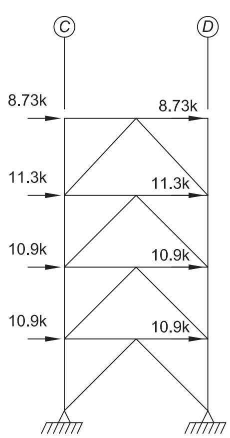
NOTIONAL LIVE LOADS



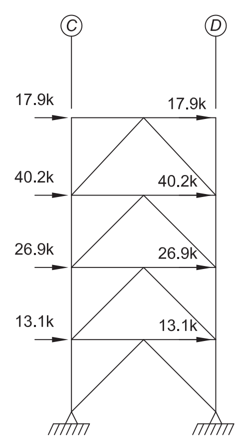
NOMINAL SNOW LOADS LEANING COLUMNS



NOTIONAL SNOW LOADS



NOMINAL WIND LOADS



SEISMIC LOADS (1.0 Q_E)

Second-order analysis by amplified first-order analysis

In the following, the approximate second-order analysis method from AISC *Specification* Appendix 8 is used to account for second-order effects in the braced frames by amplifying the axial forces in members and connections from a first-order analysis.

A first-order frame analysis is conducted using the load combinations for LRFD and ASD. From this analysis the critical axial loads, moments and deflections are obtained.

A summary of the axial loads and 1st floor drifts from the first-order computer analysis is shown below. The floor diaphragm deflection in the north-south direction was previously determined to be very small and will thus be neglected in these calculations.

LRFD	ASD
$1.23D \pm 1.0Q_E + 0.5L + 0.2S$ (Controls columns and beams) From a first-order analysis For interior column design: $P_{nt} = 236$ kips $P_{lt} = 146$ kips The moments are negligible First story first-order drift = 0.211 in.	$1.01D + 0.75L + 0.75(0.7Q_E) + 0.75S$ (Controls columns and beams) From a first-order analysis For interior column design: $P_{nt} = 219$ kips $P_{lt} = 76.6$ kips The moments are negligible First story first-order drift = 0.111 in.

The required second-order axial strength, P_r, is computed as follows:

LRFD	ASD
$P_r = P_{nt} + B_2 P_{lt}$ (*Spec*. Eq. A-8-2) Determine B_2. $B_2 = \dfrac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) $P_{story} = 5{,}440$ kips (previously calculated) $P_{e\,story}$ may be calculated as: $P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7) where H = 196 kips (from previous calculations) $\Delta_H = 0.211$ in. (from computer output) $R_M = 1.0$ for braced frames $P_{e\,story} = 1.0\dfrac{(196\text{ kips})(13.5\text{ ft})(12\text{ in./ft})}{0.211\text{ in.}}$ $= 150{,}000$ kips	$P_r = P_{nt} + B_2 P_{lt}$ (*Spec*. Eq. A-8-2) Determine B_2. $B_2 = \dfrac{1}{1-\dfrac{\alpha P_{story}}{P_{e\,story}}} \geq 1$ (*Spec*. Eq. A-8-6) $P_{story} = 5{,}120$ kips (previously calculated) $P_{e\,story}$ may be calculated as: $P_{e\,story} = R_M \dfrac{HL}{\Delta_H}$ (*Spec*. Eq. A-8-7) where H = 103 kips (from previous calculations) $\Delta_H = 0.111$ in. (from computer output) $R_M = 1.0$ for braced frames $P_{e\,story} = 1.0\dfrac{(103\text{ kips})(13.5\text{ ft})(12\text{ in./ft})}{0.111\text{ in.}}$ $= 150{,}000$ kips

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,story}}} \geq 1$$

$$= \frac{1}{1 - \frac{1.0(5{,}440 \text{ kips})}{150{,}000 \text{ kips}}} \geq 1$$

$$= 1.04 \geq 1$$

$P_r = P_{nt} + B_2 P_{lt}$ (*Spec*. Eq. A-8-2)
= 236 kips + (1.04)(146 kips)
= 388 kips

From AISC *Manual* Table 4-1,

P_c = 514 kips (W12×53 @ KL = 13.5 ft)

From AISC *Specification* Equation H1-1a,

$$\frac{P_r}{P_c} = \frac{388 \text{ kips}}{514 \text{ kips}} = 0.755 \leq 1.0$$ **o.k.**

$$B_2 = \frac{1}{1 - \frac{\alpha P_{story}}{P_{e\,story}}} \geq 1$$

$$= \frac{1}{1 - \frac{1.6(5{,}120 \text{ kips})}{150{,}000 \text{ kips}}} \geq 1$$

$$= 1.06 \geq 1$$

$P_r = P_{nt} + B_2 P_{lt}$ (*Spec*. Eq. A-8-2)
= 219 kips + (1.06)(76.6 kips)
= 300 kips

From AISC *Manual* Table 4-1,

P_c = 342 kips (W12×53 @ KL = 13.5 ft)

From AISC *Specification* Equation H1-1a,

$$\frac{P_r}{P_c} = \frac{300 \text{ kips}}{342 \text{ kips}} = 0.877 \leq 1.0$$ **o.k.**

Note: Notice that the lower sidesway displacements of the braced frame produce much lower values of B_2 than those of the moment frame. Similar results could be expected for the other two methods of analysis.

Although not presented here, second-order effects should be accounted for in the design of the beams and diagonal braces in the braced frames at Grids 1 and 8.

ANALYSIS OF DRAG STRUTS

The fourth floor delivers the highest diaphragm force to the braced frames at the ends of the building: E = 80.3 kips (from previous calculations). This force is transferred to the braced frame through axial loading of the W18×35 beams at the end of the building.

The gravity dead loads for the edge beams are the floor loading of 75.0 psf (5.50 ft) plus the exterior wall loading of 0.503 kip/ft, giving a total dead load of 0.916 kip/ft. The gravity live load for these beams is the floor loading of 80.0 psf (5.50 ft) = 0.440 kip/ft. The resulting midspan moments are M_{Dead} = 58.0 kip-ft and M_{Live} = 27.8 kip-ft.

The controlling load combination for LRFD is 1.23D + 1.0Q_E + 0.50L. The controlling load combinations for ASD are 1.01D + 0.75L + 0.75(0.7Q_E) or 1.02D + 0.7Q_E

LRFD	ASD
M_u = 1.23(58.0 kip-ft) + 0.50(27.8 kip-ft) = 85.2 kip-ft	M_a = 1.01(58.0 kip-ft) + 0.75(27.8 kip-ft) = 79.4 kip-ft or M_a = 1.02(58.0 kip-ft) = 59.2 kip-ft
Load from the diaphragm shear due to earthquake loading	Load from the diaphragm shear due to earthquake loading
F_p = 80.3 kips	F_p = 0.75(0.70)(80.3 kips) = 42.2 kips or F_p = 0.70(80.3 kips) = 56.2 kips

Only the two 45 ft long segments on either side of the brace can transfer load into the brace, because the stair opening is in front of the brace.

Use AISC *Specification* Section H2 to check the combined bending and axial stresses.

LRFD	ASD
$V = \dfrac{80.3 \text{ kips}}{2(45.0 \text{ ft})} = 0.892 \text{ kip/ft}$	$V = \dfrac{42.2 \text{ kips}}{2(45.0 \text{ ft})} = 0.469 \text{ kip/ft}$ or $V = \dfrac{56.2 \text{ kips}}{2(45.0 \text{ ft})} = 0.624 \text{ kip/ft}$
The top flange bending stress is	The top flange stress due to bending
$f_b = \dfrac{M_u}{S_x} = \dfrac{85.2 \text{ kip-ft}(12 \text{ in./ft})}{57.6 \text{ in.}^3} = 17.8 \text{ ksi}$	$f_b = \dfrac{M_a}{S_x} = \dfrac{79.4 \text{ kip-ft}(12 \text{ in./ft})}{57.6 \text{ in.}^3} = 16.5 \text{ ksi}$ or

	$f_b = \frac{M_a}{S_x}$ $= \frac{59.2 \text{ kip-ft}(12 \text{ in./ft})}{57.6 \text{ in.}^3}$ $= 12.3 \text{ ksi}$

Note: It is often possible to resist the drag strut force using the slab directly. For illustration purposes, this solution will instead use the beam to resist the force independently of the slab. The full cross section can be used to resist the force if the member is designed as a column braced at one flange only (plus any other intermediate bracing present, such as from filler beams). Alternatively, a reduced cross section consisting of the top flange plus a portion of the web can be used. Arbitrarily use the top flange and 8 times an area of the web equal to its thickness times a depth equal to its thickness, as an area to carry the drag strut component.

$$\text{Area} = 6.00 \text{ in.}(0.425 \text{ in.}) + 8(0.300 \text{ in.})^2 = 2.55 \text{ in.}^2 + 0.720 \text{ in.}^2 = 3.27 \text{ in.}^2$$

Ignoring the small segment of the beam between Grid C and D, the axial stress due to the drag strut force is:

LRFD	ASD
$f_a = \frac{80.3 \text{ kips}}{2(3.27 \text{ in.}^2)}$ $= 12.3 \text{ ksi}$	$f_a = \frac{42.2 \text{ kips}}{2(3.27 \text{ in.}^2)}$ $= 6.45 \text{ ksi}$ or $f_a = \frac{90.0 \text{ ft}(0.624 \text{ kip/ft})}{2(3.27 \text{ in.}^2)}$ $= 8.59 \text{ ksi}$
Using AISC *Specification* Section H2, assuming the top flange is continuously braced: $F_a = \phi_c F_y$ $= 0.90(50 \text{ ksi})$ $= 45.0 \text{ ksi}$ $F_{bw} = \phi_b F_y$ $= 0.90(50 \text{ ksi})$ $= 45.0 \text{ ksi}$ $\frac{f_a}{F_a} + \frac{f_{bw}}{F_{bw}} \leq 1.0$ (from *Spec*. Eq. H2-1) $\frac{12.3 \text{ ksi}}{45.0 \text{ ksi}} + \frac{17.8 \text{ ksi}}{45.0 \text{ ksi}} = 0.669$ **o.k.**	From AISC *Specification* Section H2, assuming the top flange is continuously braced: $F_a = F_y / \Omega_c$ $= 50 \text{ ksi}/1.67$ $= 29.9 \text{ ksi}$ $F_{bw} = F_y / \Omega_b$ $= 50 \text{ ksi}/1.67$ $= 29.9 \text{ ksi}$ $\frac{f_a}{F_a} + \frac{f_{bw}}{F_{bw}} \leq 1.0$ (from *Spec*. Eq. H2-1) Load Combination 1: $\frac{6.45 \text{ ksi}}{29.9 \text{ ksi}} + \frac{16.5 \text{ ksi}}{29.9 \text{ ksi}} = 0.768$ **o.k.** Load Combination 2:

Return to Table of Contents

	$\dfrac{8.59\text{ ksi}}{29.9\text{ ksi}}+\dfrac{12.3\text{ ksi}}{29.9\text{ ksi}}=0.699$ **o.k.**

Note: Because the drag strut load is a horizontal load, the method of transfer into the strut, and the extra horizontal load which must be accommodated by the beam end connections should be indicated on the drawings.

Return to Table of Contents

PART III EXAMPLE REFERENCES

ASCE (2010), *Minimum Design Loads for Buildings and Other Structures*, ASCE/SEI 7-10, Reston, VA.

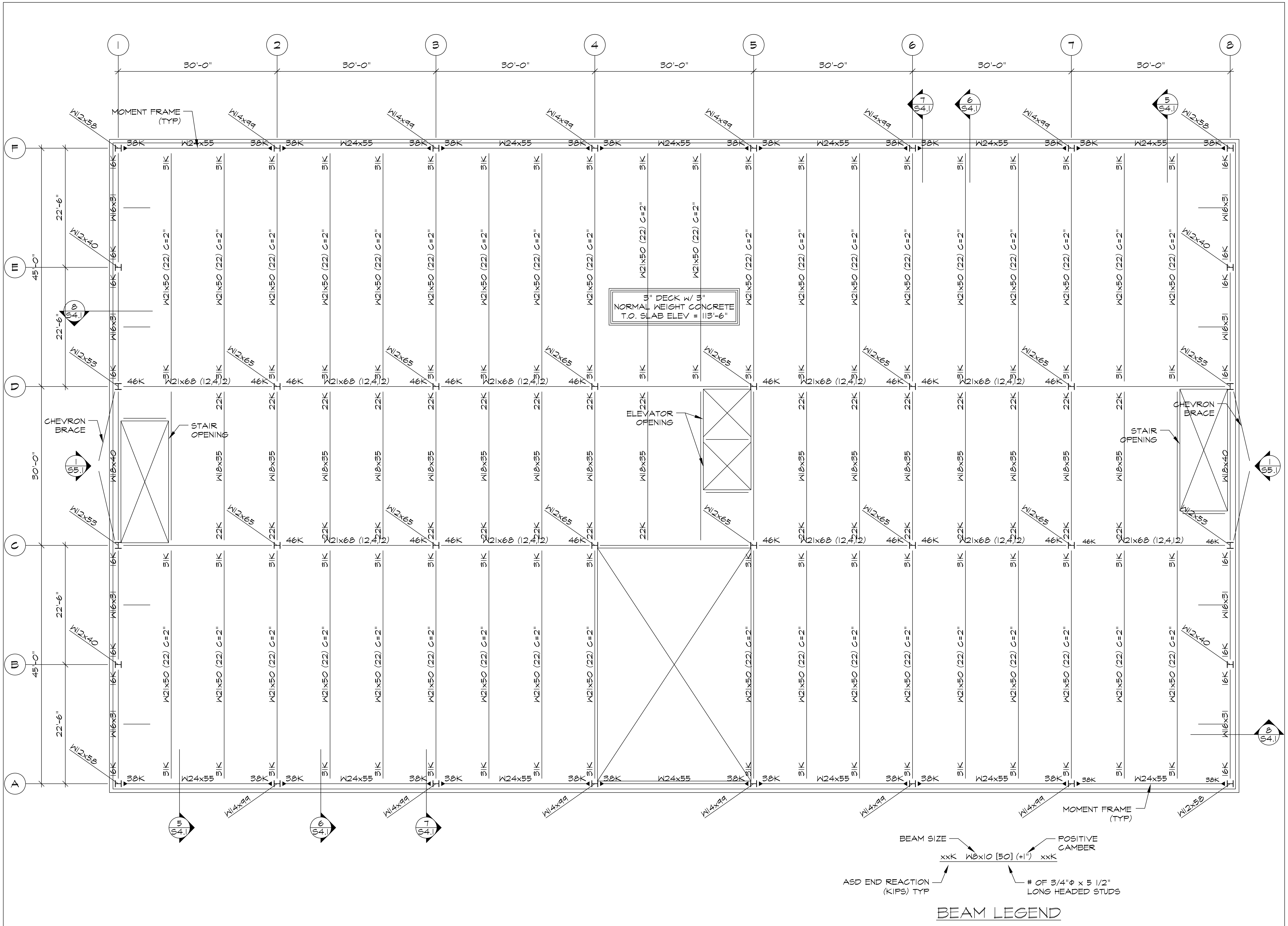
Geschwindner, L.F. (1994), "A Practical Approach to the Leaning Column," *Engineering Journal*, AISC, Vol. 31, No. 4, 4th Quarter, pp. 141-149.

SDI (2004), *Diaphragm Design Manual,* 3rd Ed., Steel Deck Institute, Fox River Grove, IL.

SJI (2005), *Load Tables and Weight Tables for Steel Joists and Joist Girders*, 42nd Ed., Steel Joist Institute, Forest, VA.

West, M., Fisher, J. and Griffis, L.A. (2003), *Serviceability Design Considerations for Steel Buildings,* Design Guide 3, 2nd Ed., AISC, Chicago, IL.

Return to Table of Contents



2ND FLOOR FRAMING PLAN

1/8" = 1'-0"

1 / S2.1

NORTH

ABC ARCHITECTS ASSOCIATES
123456 STREET ADDRESS
SUITE · CITY, STATE ZIP
PHONE NUMBER · FAX NUMBER
ON THE WEB

- ABLE -
STRUCTURAL ENGINEERS
654321 ADDRESS STREET
SUITE · CITY, STATE ZIP
PHONE NUMBER · FAX NUMBER
ON THE WEB

BUILDING 3
ANYWHERE

SEAL

REVISIONS

DESCRIPTION	DATE
△	
△	
△	
△	
△	

DATE: | PROJECT #: 123456

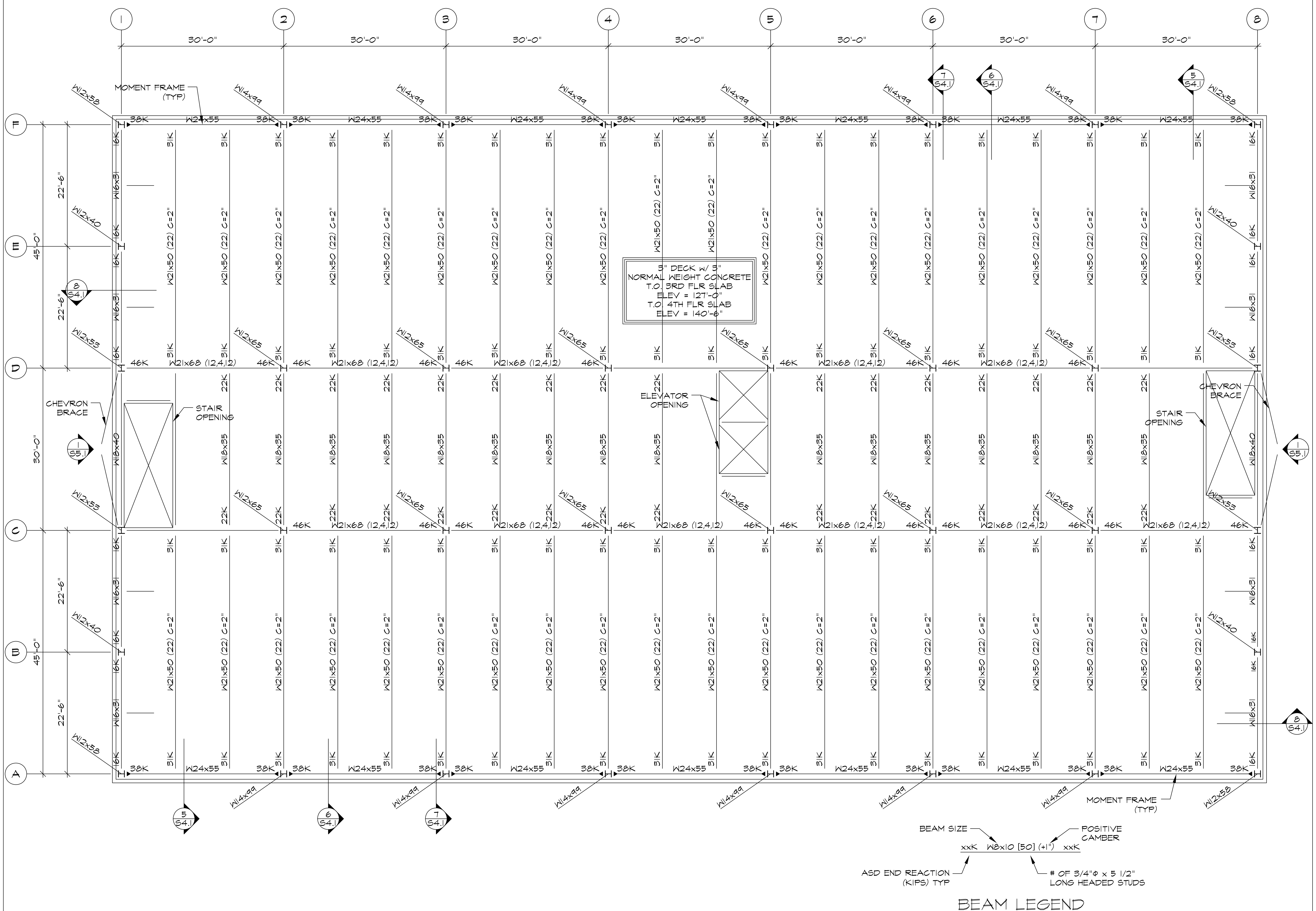
DRAWN BY: ME | CHECKED BY: YOU

ISSUE:

2ND FLOOR FRAMING PLAN

S2.1

Return to Table of Contents



3RD & 4TH FLOOR FRAMING PLAN 1 S2.2

1/8" = 1'-0"

ABC ARCHITECTS ASSOCIATES

123456 STREET ADDRESS
SUITE · CITY, STATE ZIP
PHONE NUMBER · FAX NUMBER
ON THE WEB

- ABLE -
STRUCTURAL ENGINEERS

654321 ADDRESS STREET
SUITE · CITY, STATE ZIP
PHONE NUMBER · FAX NUMBER
ON THE WEB

BUILDING 3

ANYWHERE

SEAL

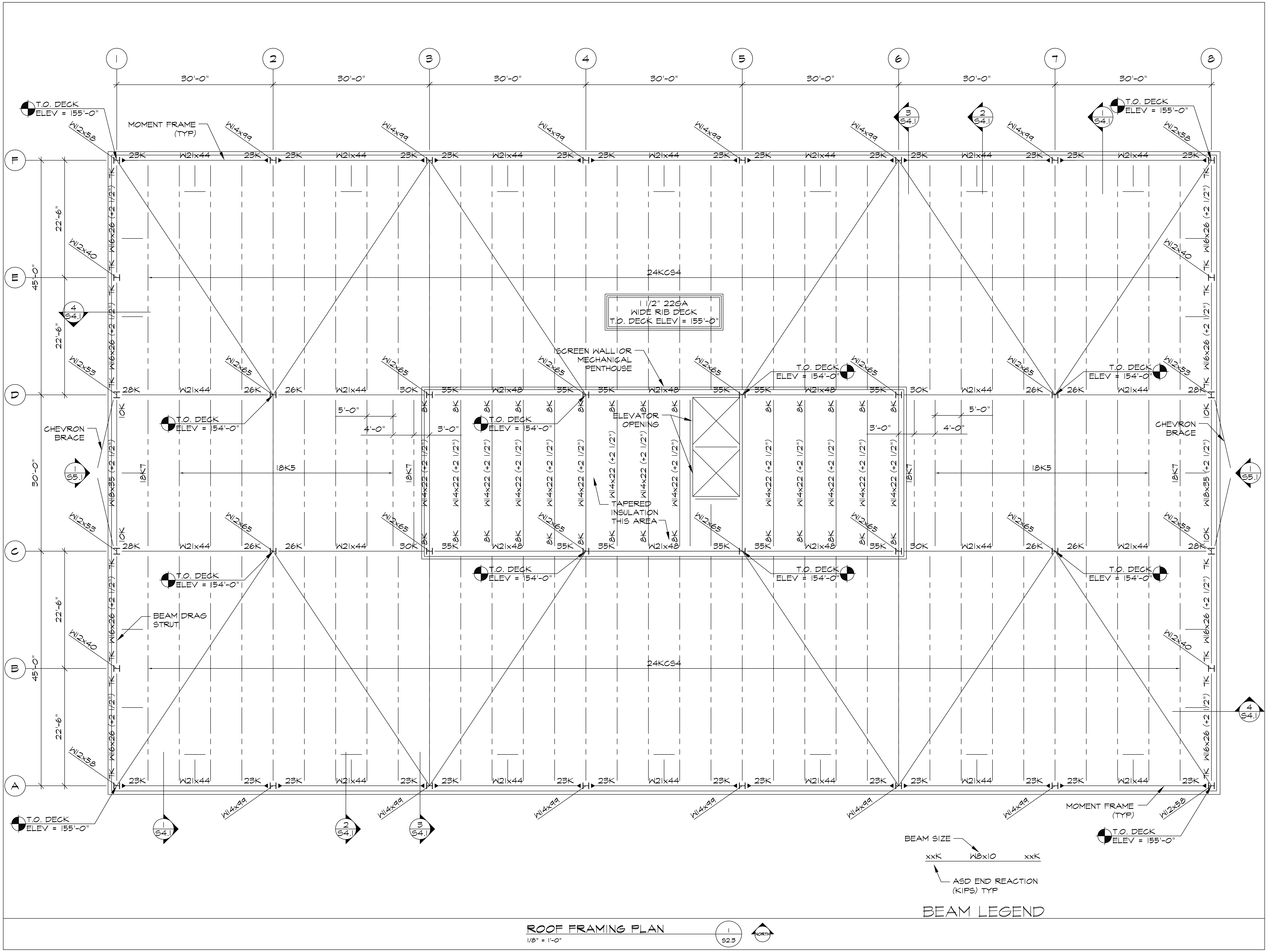
REVISIONS	
DESCRIPTION	DATE
△	
△	
△	
△	
△	

DATE:	PROJECT #: 123456
DRAWN BY: ME	CHECKED BY: YOU
ISSUE:	

3RD FLOOR FRAMING PLAN

S2.2

Return to Table of Contents



ABC ARCHITECTS ASSOCIATES
123456 STREET ADDRESS
SUITE · CITY, STATE ZIP
PHONE NUMBER · FAX NUMBER
ON THE WEB

- ABLE -
STRUCTURAL
ENGINEERS
654321 ADDRESS STREET
SUITE · CITY, STATE ZIP
PHONE NUMBER · FAX NUMBER
ON THE WEB

BUILDING 3
ANYWHERE

SEAL

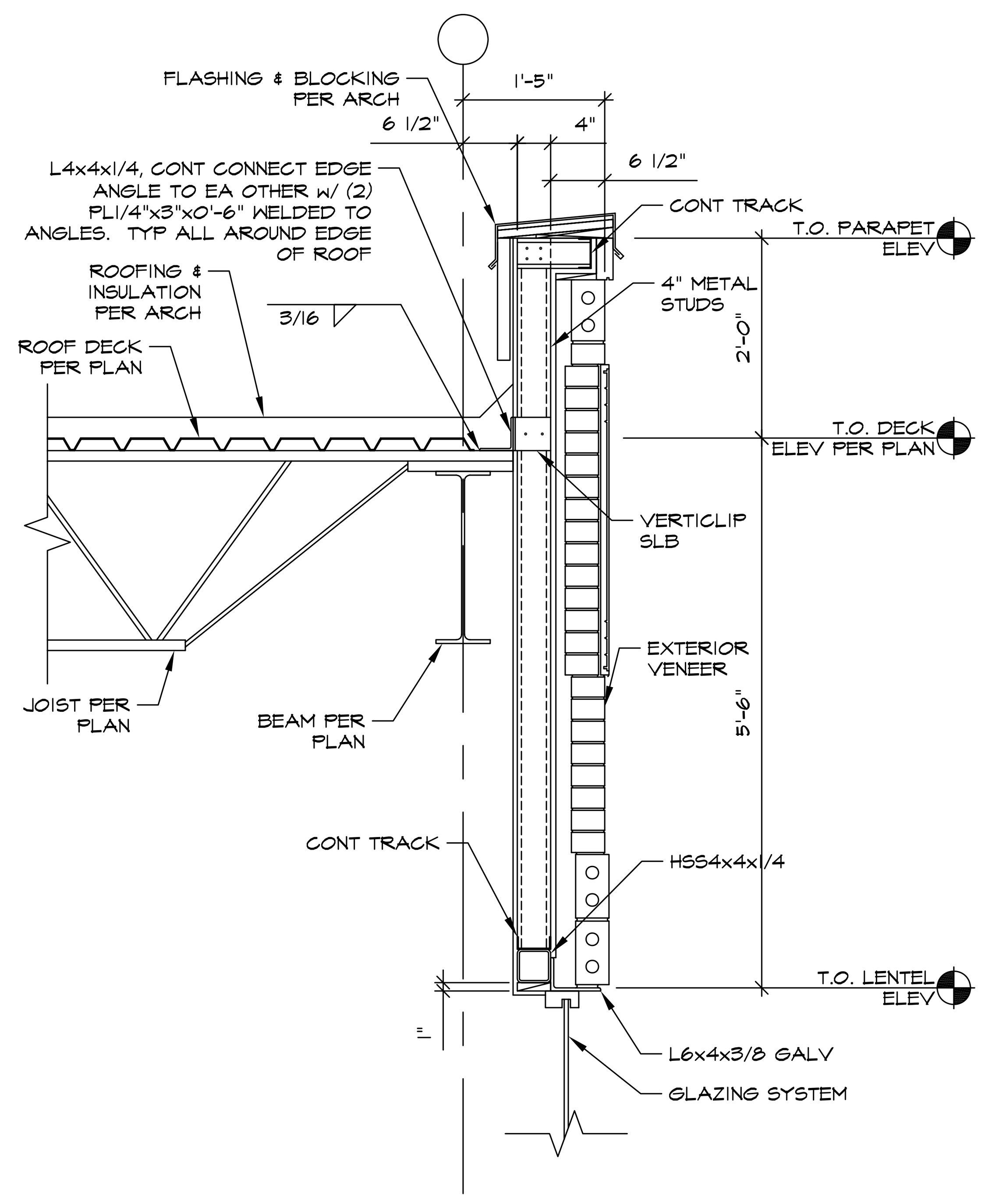
REVISIONS	
DESCRIPTION	DATE
△	
△	
△	
△	
△	

DATE:	PROJECT #: 123456
DRAWN BY: ME	CHECKED BY: YOU
ISSUE:	

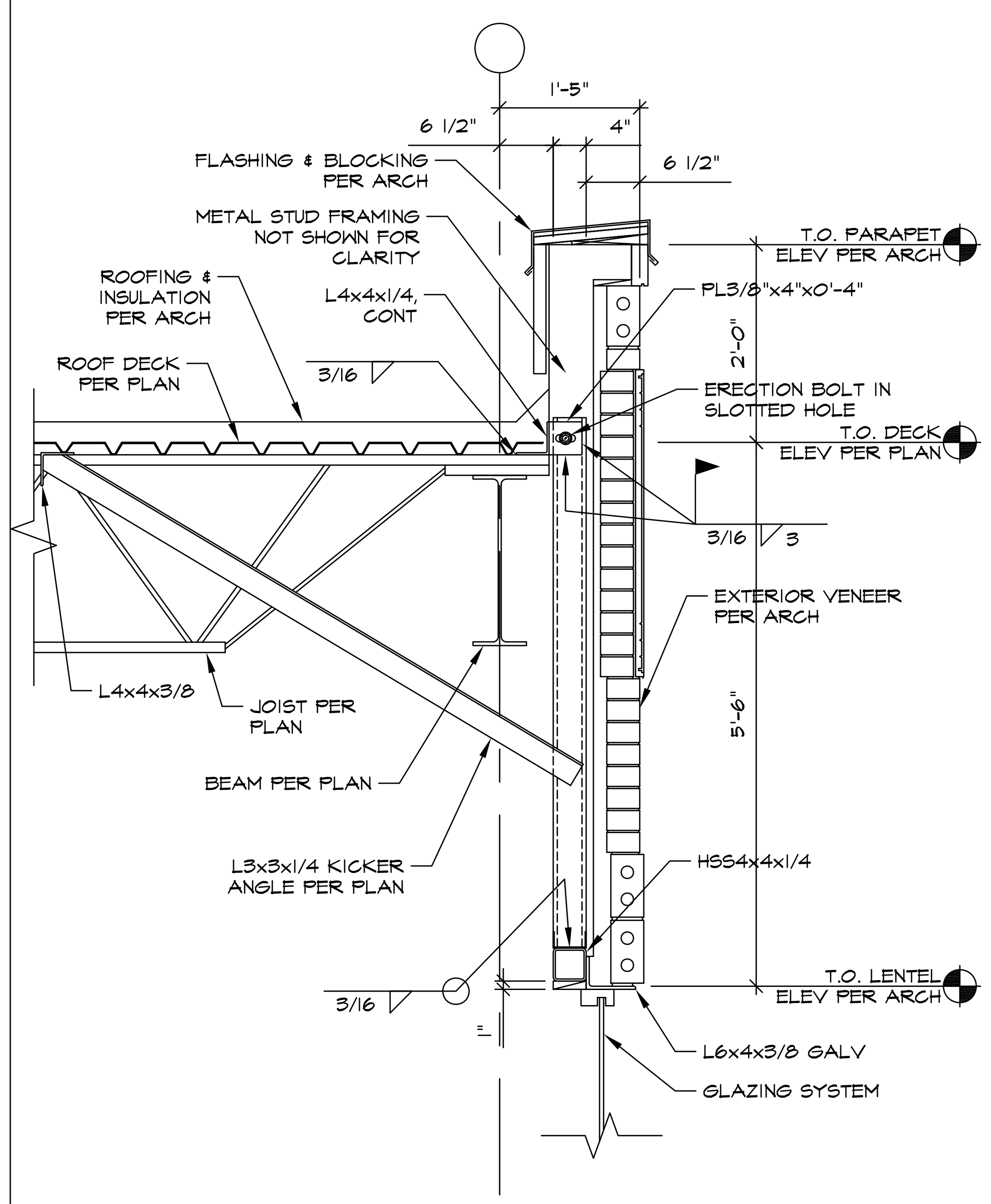
ROOF FRAMING PLAN

S2.3

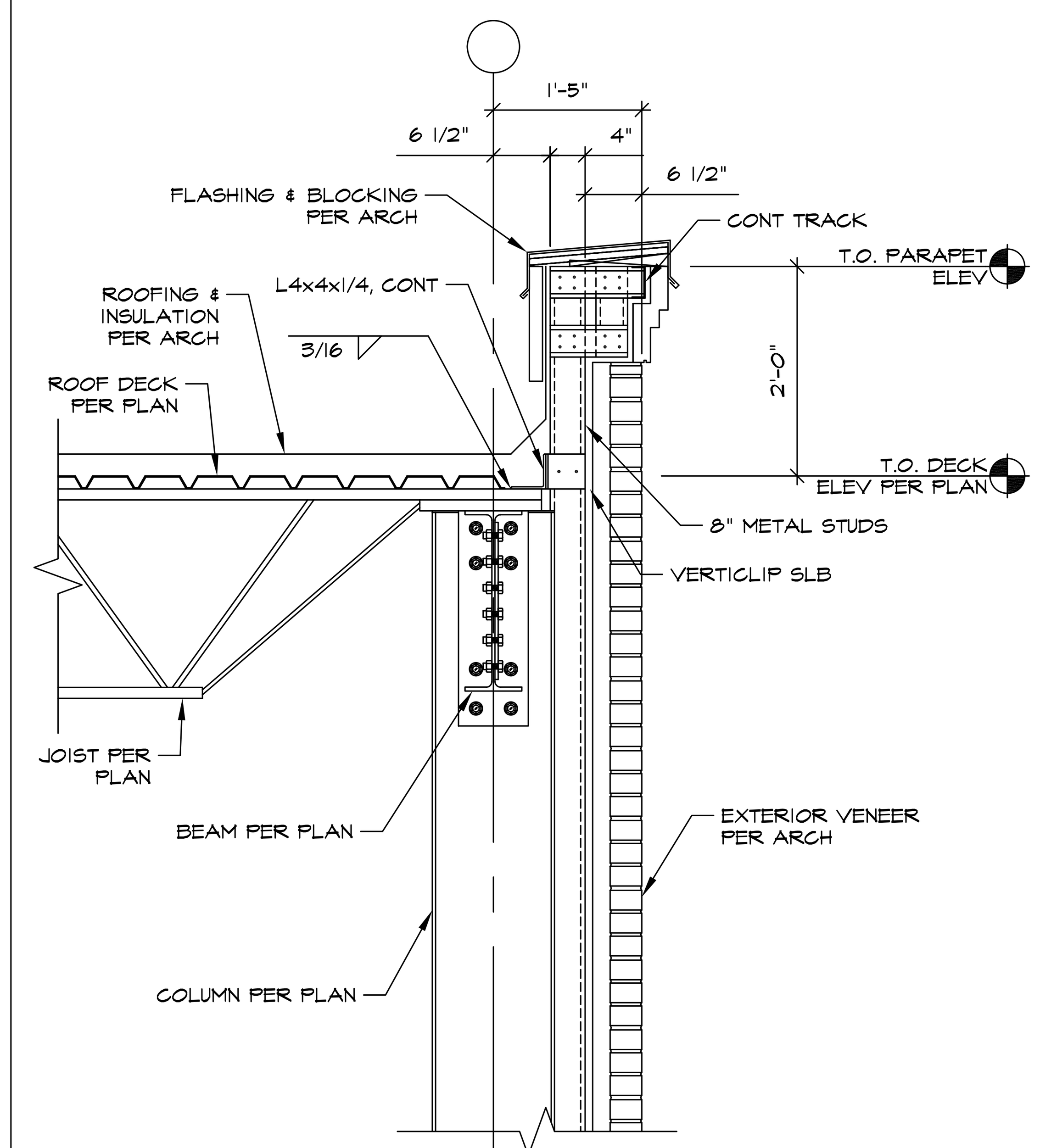
Return to Table of Contents



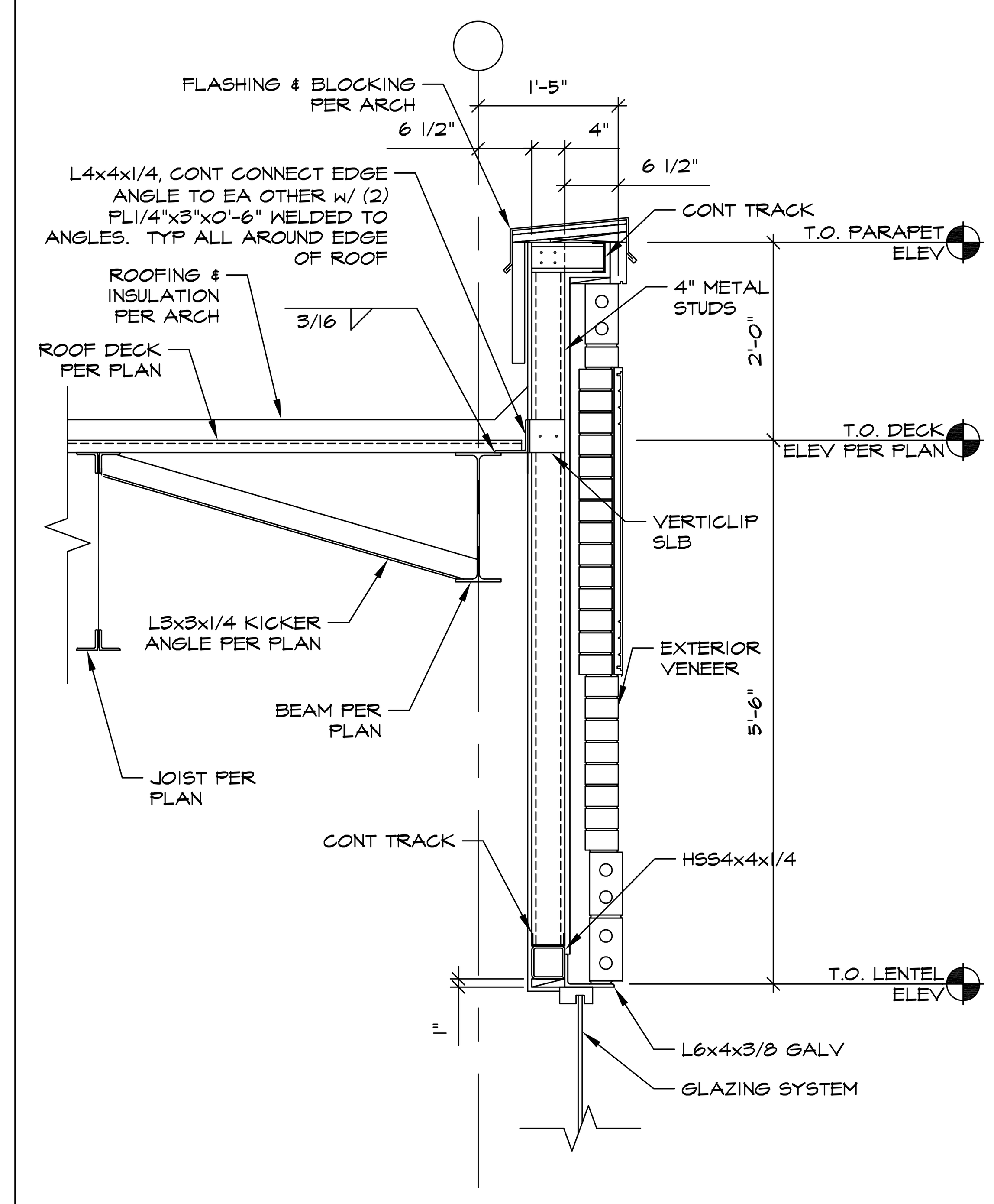
SECTION 1
3/4" = 1'-0"
S4.1



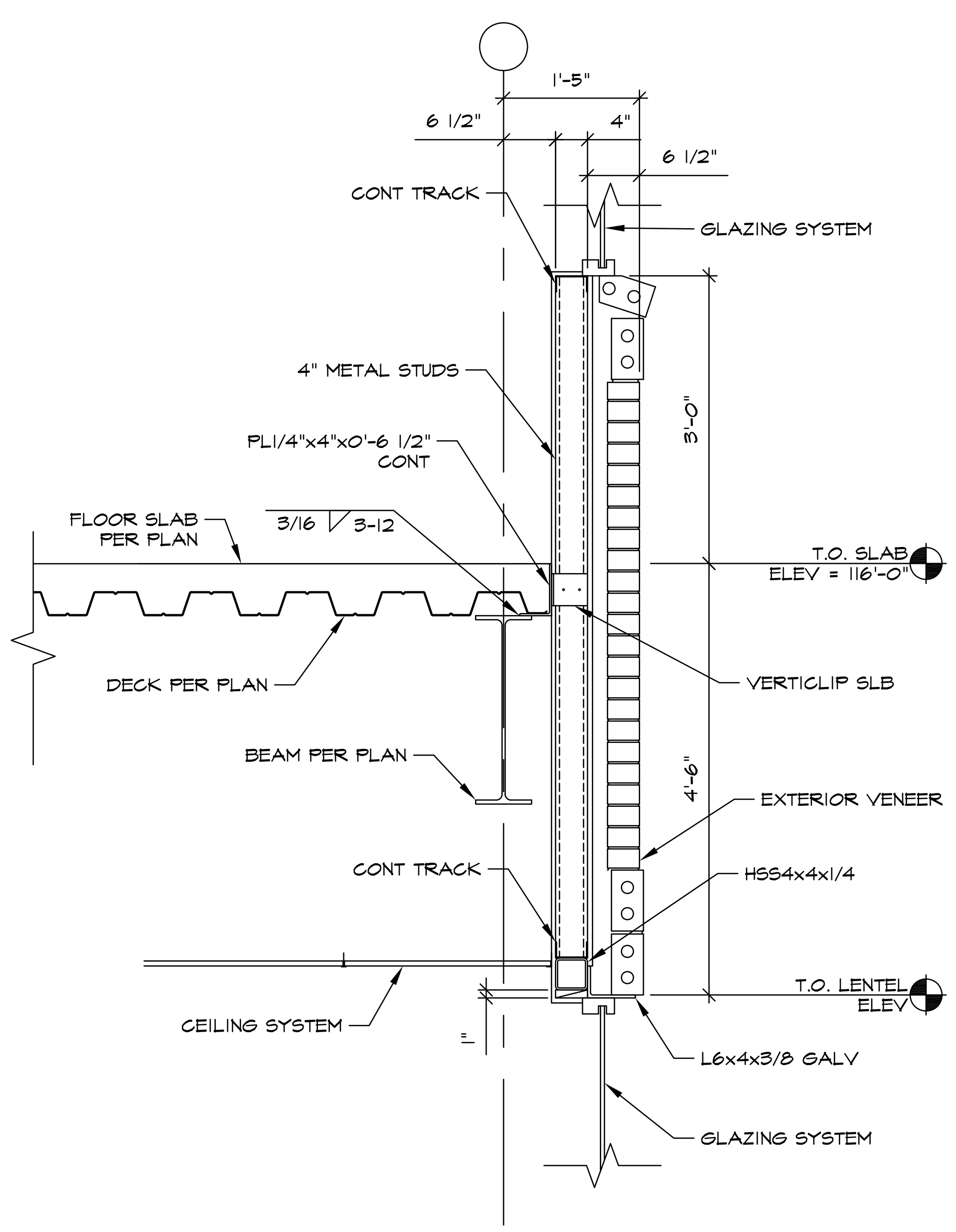
SECTION 2
3/4" = 1'-0"
S4.1



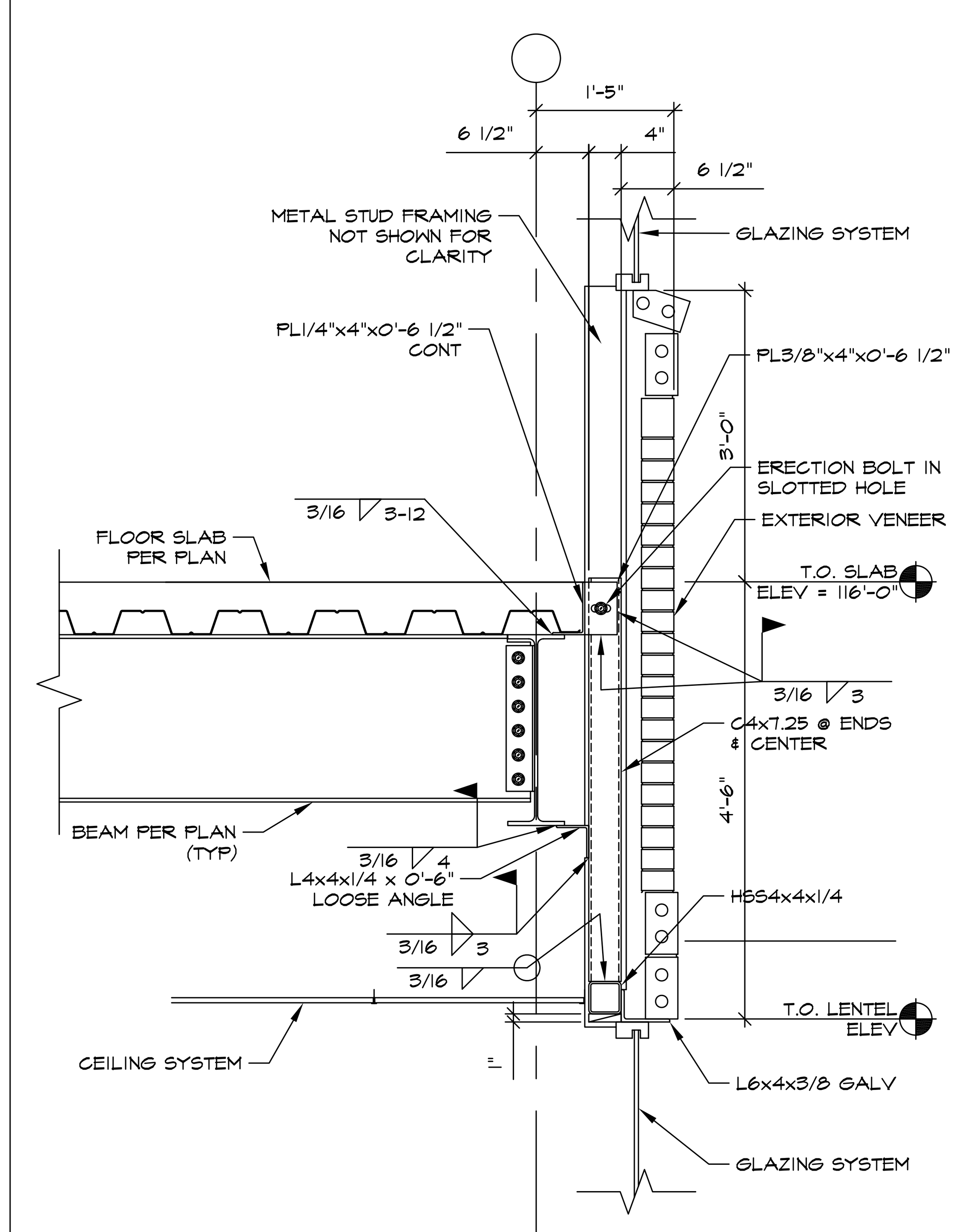
SECTION 3
3/4" = 1'-0"
S4.1



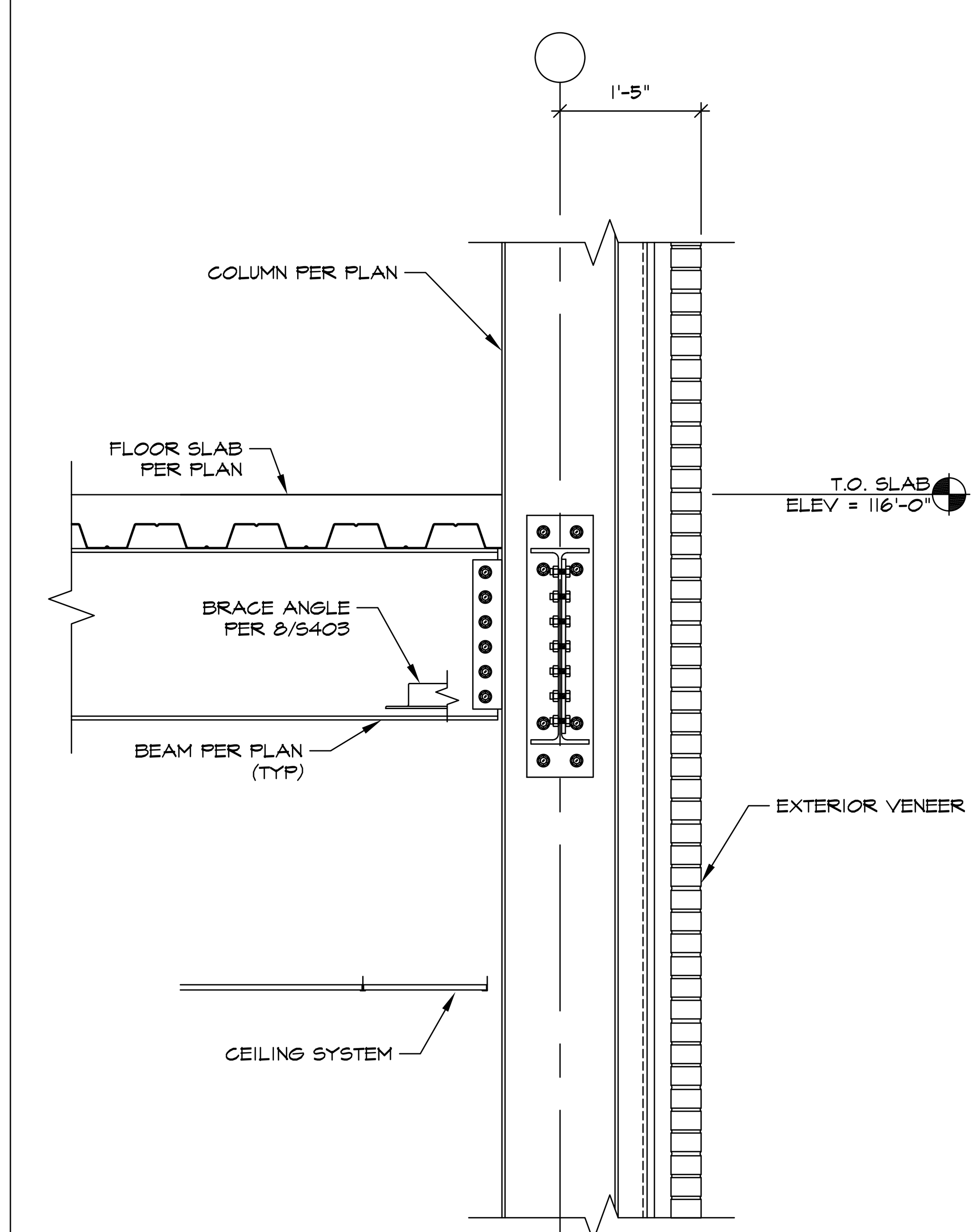
SECTION 4
3/4" = 1'-0"
S4.1



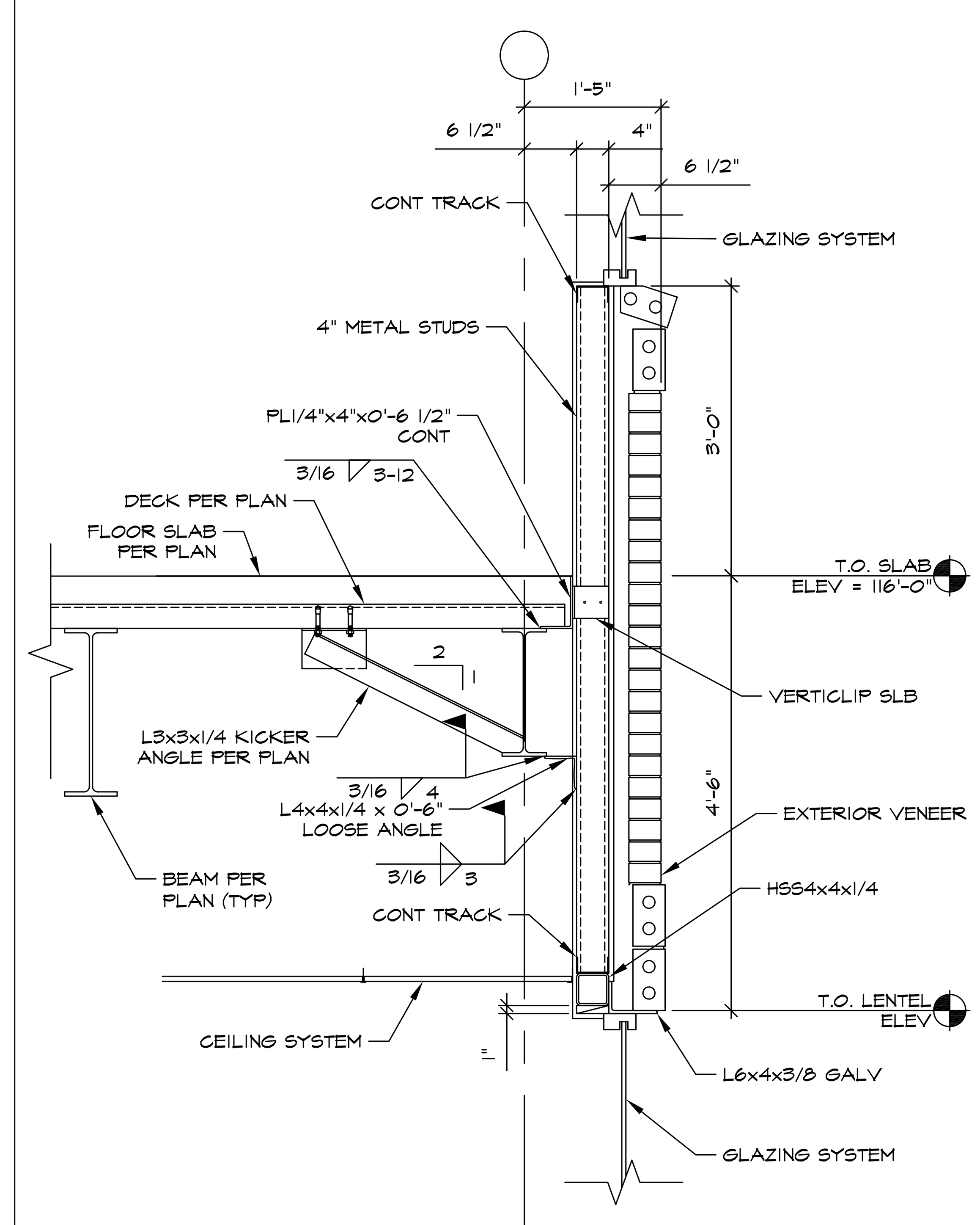
SECTION 5
3/4" = 1'-0"
S4.1



SECTION 6
3/4" = 1'-0"
S4.1



SECTION 7
3/4" = 1'-0"
S4.1



SECTION 8
3/4" = 1'-0"
S4.1

ABC ARCHITECTS ASSOCIATES

123456 STREET ADDRESS
SUITE · CITY, STATE ZIP
PHONE NUMBER · FAX NUMBER
ON THE WEB

- ABLE -
STRUCTURAL
ENGINEERS
654321 ADDRESS STREET
SUITE · CITY, STATE ZIP
PHONE NUMBER · FAX NUMBER
ON THE WEB

BUILDING 3
ANYWHERE

SEAL

REVISIONS

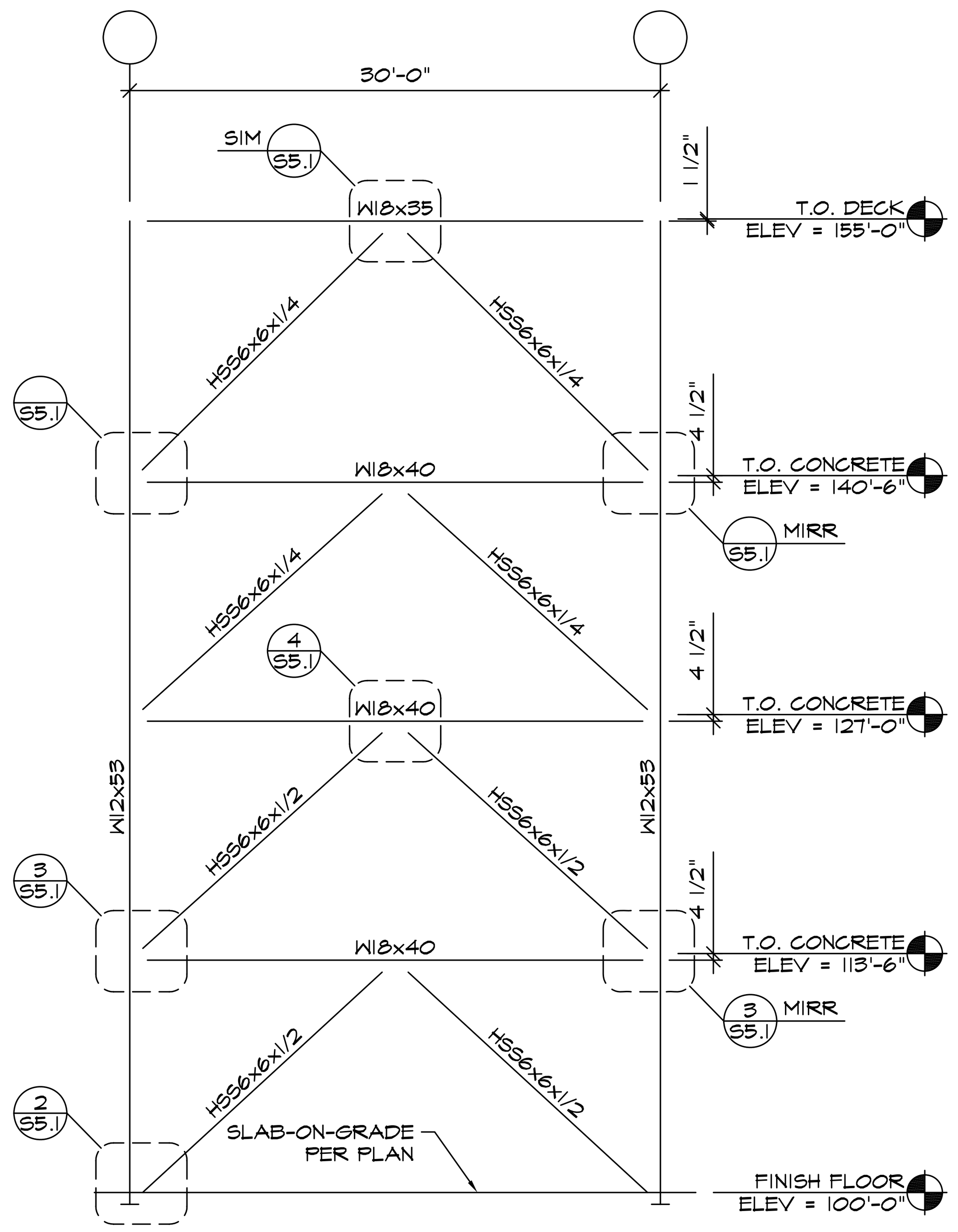
DESCRIPTION	DATE
△	
△	
△	
△	
△	

DATE:	PROJECT #: 123456
DRAWN BY: ME	CHECKED BY: YOU
ISSUE:	

ROOF FRAMING SECTIONS

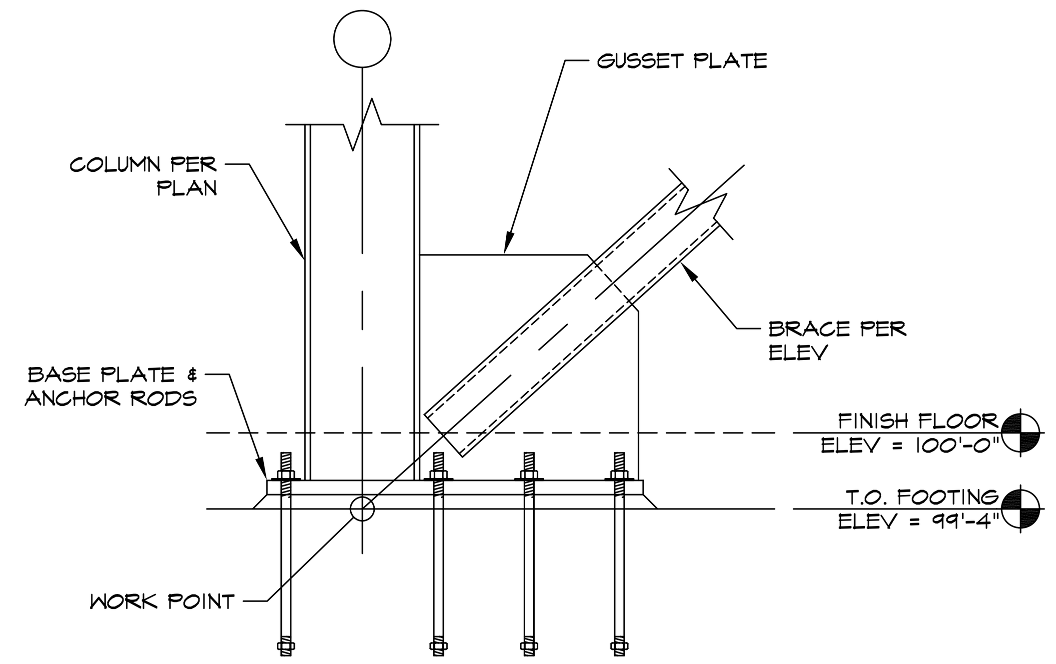
S4.1

Return to Table of Contents



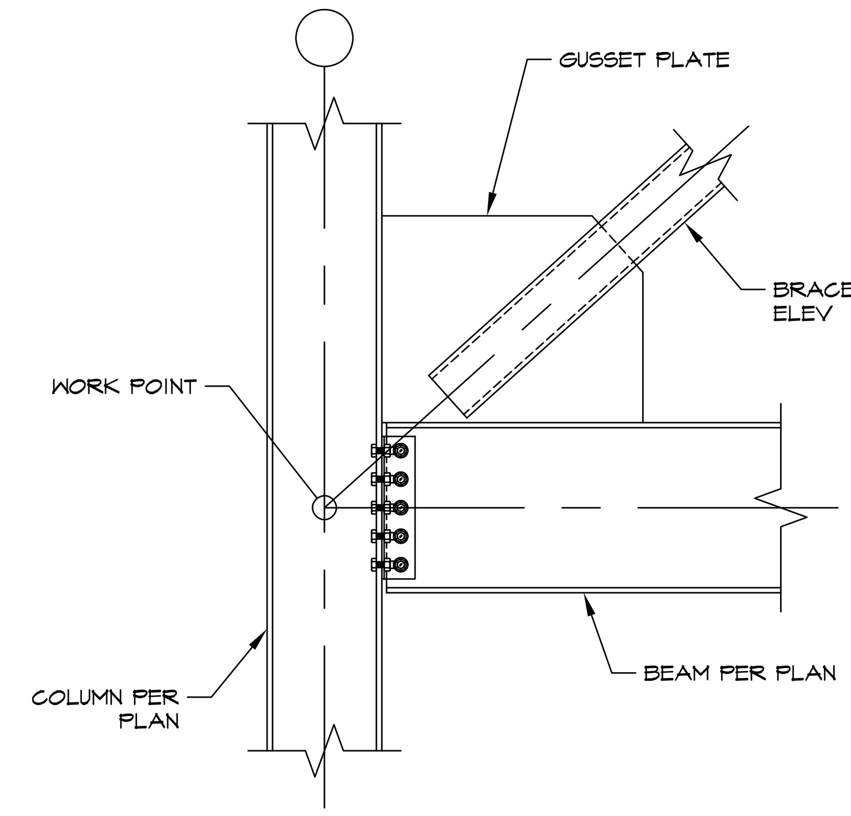
CHEVRON BRACE ELEVATION 1 S5.1

N.T.S.



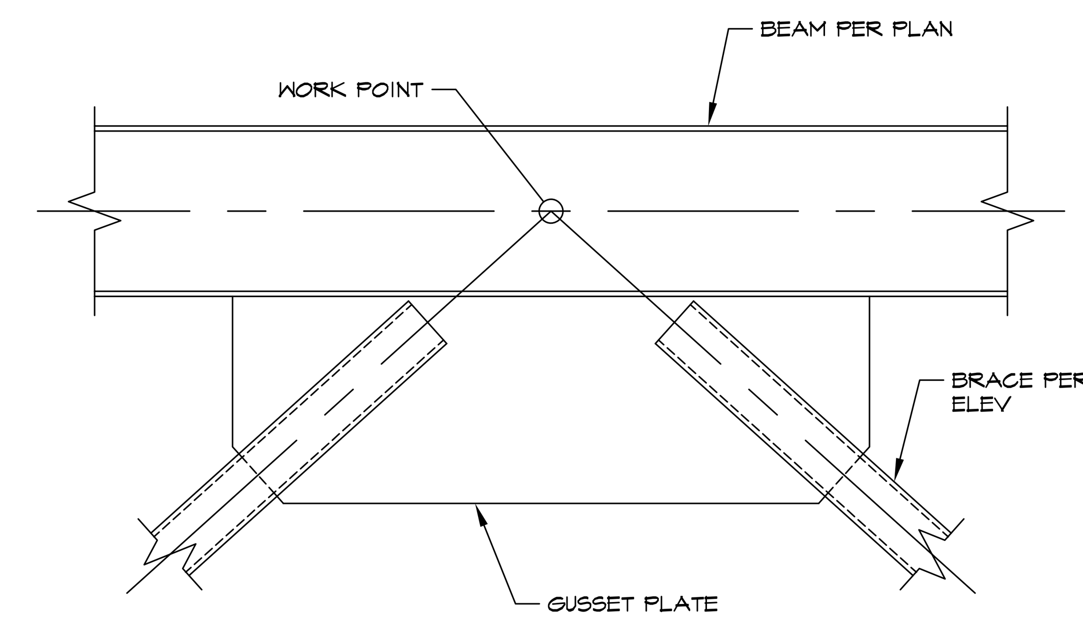
DETAIL 2 S5.1

1 1/2" = 1'-0"



DETAIL 3 S5.1

1 1/2" = 1'-0"



DETAIL 4 S5.1

1 1/2" = 1'-0"

NOT USED 5 S5.1

NOT USED 6 S5.1

NOT USED 7 S5.1

NOT USED 8 S5.1

ABC ARCHITECTS ASSOCIATES
123456 STREET ADDRESS
SUITE · CITY, STATE ZIP
PHONE NUMBER · FAX NUMBER
ON THE WEB

- ABLE -
STRUCTURAL
ENGINEERS
654321 ADDRESS STREET
SUITE · CITY, STATE ZIP
PHONE NUMBER · FAX NUMBER
ON THE WEB

BUILDING 3
ANYWHERE

SEAL

REVISIONS

DESCRIPTION	DATE
△	
△	
△	
△	
△	

DATE:	PROJECT #: 123456
DRAWN BY: ME	CHECKED BY: YOU
ISSUE:	

CHEVRON BRACE ELEVATION & DETAILS

S5.1

Part IV
Additional Resources

This part contains additional design aids that are not available in the AISC *Steel Construction Manual*.

DESIGN TABLE DISCUSSION

Table 4-1. W-Shapes in Axial Compression, 65 ksi steel

Available strengths in axial compression are given for W-shapes with F_y = 65 ksi (ASTM A913 Grade 65). The tabulated values are given for the effective length with respect to the y-axis $(KL)_y$. However, the effective length with respect to the x-axis $(KL)_x$ must also be investigated. To determine the available strength in axial compression, the table should be entered at the larger of $(KL)_y$ and $(KL)_{y\,eq}$, where

$$(KL)_{y\,eq} = \frac{(KL)_x}{\dfrac{r_x}{r_y}} \quad (4\text{-}1)$$

The available strength is based on the limit states of flexural buckling, torsional buckling, and flexural-torsional buckling. The limit between elastic and inelastic buckling is $\dfrac{KL}{r} = 99.5$ with F_y = 65 ksi.

The slenderness limit between a nonslender web and a slender web is $\lambda_{rw} = 31.5$ with $F_y = 65$ ksi. All current ASTM A6 W-shapes have nonslender flanges with F_y = 65 ksi.

Values of the ratio r_x/r_y and other properties useful in the design of W-shape compression members are listed at the bottom of Table 4-1.

Variables P_{wo}, P_{wi}, P_{wb} and P_{fb} shown in Table 4-1 can be used to determine the strength of W-shapes without stiffeners to resist concentrated forces applied normal to the face(s) of the flange(s). In these tables it is assumed that the concentrated forces act far enough away from the member ends that end effects are not considered (end effects are addressed in Chapter 9). When $P_r \le \phi R_n$ or R_n/Ω, column web stiffeners are not required. Figures 4-1, 4-2 and 4-3 illustrate the limit states and the applicable variables for each.

Web Local Yielding: The variables P_{wo} and P_{wi} can be used in the calculation of the available web local yielding strength for the column as follows:

LRFD	ASD
$\phi R_n = P_{wo} + P_{wi} l_b$ (4-2a)	$R_n/\Omega = P_{wo} + P_{wi} l_b$ (4-2b)

where

R_n = $F_{yw} t_w (5k + l_b) = 5F_{yw} t_w k + F_{yw} t_w l_b$, kips (AISC *Specification* Equation J10-2)

P_{wo} = $\phi 5F_{yw} t_w k$ for LRFD and $5F_{yw} t_w k/\Omega$ for ASD, kips

P_{wi} = $\phi F_{yw} t_w$ for LRFD and $F_{yw} t_w/\Omega$ for ASD, kips/in.

k = distance from outer face of flange to the web toe of fillet, in.
l_b = length of bearing, in.
t_w = thickness of web, in.
ϕ = 1.00
Ω = 1.50

Web Compression Buckling: The variable P_{wb} is the available web compression buckling strength for the column as follows:

LRFD	ASD
$\phi R_n = P_{wb}$ (4-3a)	$R_n/\Omega = P_{wb}$ (4-3b)

where

R_n = $\dfrac{24 t_w^3 \sqrt{E F_{yw}}}{h}$ (AISC *Specification* Equation J10-8)

P_{wb} = $\dfrac{\phi 24 t_w^3 \sqrt{EF_{yw}}}{h}$ for LRFD and $\dfrac{24 t_w^3 \sqrt{EF_{yw}}}{\Omega h}$ for ASD, kips
F_{yw} = specified minimum yield stress of the web, ksi
h = clear distance between flanges less the fillet or corner radius for rolled shapes, in.
ϕ = 0.90
Ω = 1.67

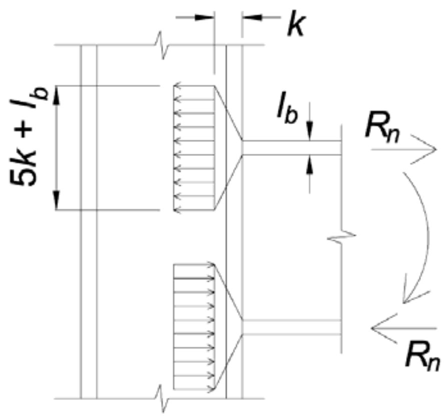


Fig. 4-1. Illustration of web local yielding limit state (AISC Specification *Section J10.2).*

Flange Local Buckling: The variable P_{fb} is the available flange local bending strength for the column as follows:

LRFD	ASD
$\phi R_n = P_{fb}$ (4-4a)	$R_n / \Omega = P_{fb}$ (4-4a)

where

R_n = $6.25 F_{yf} t_f^2$, kips (AISC *Specification* Equation J10-1)
P_{fb} = $\phi 6.25 F_{yf} t_f^2$ for LRFD and $6.25 F_{yf} t_f^2 / \Omega$ for ASD, kips
ϕ = 0.90
Ω = 1.67

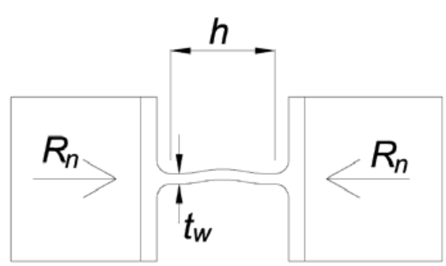


Fig. 4-2. Illustration of web compression buckling limit state (AISC Specification *Section J10.5).*

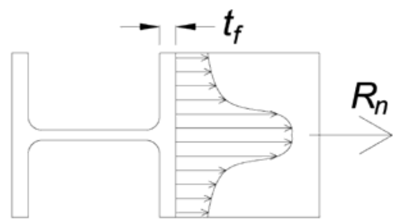


Fig. 4-3. Illustration of flange local bending limit state (AISC Specification *Section J10.1).*

Return to Table of Contents

F_y = 65 ksi

Table 4-1
Available Strength in Axial Compression, kips
W-Shapes

W14

Shape	W14×											
lb/ft	730[h]		665[h]		605[h]		550[h]		500[h]		455[h]	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y												
0	8370	12600	7630	11500	6930	10400	6310	9480	5720	8600	5220	7840
6	8180	12300	7450	11200	6770	10200	6150	9250	5580	8390	5080	7640
7	8120	12200	7390	11100	6710	10100	6100	9170	5530	8310	5040	7570
8	8040	12100	7320	11000	6640	9980	6040	9070	5470	8220	4980	7490
9	7960	12000	7240	10900	6570	9870	5970	8970	5410	8130	4920	7400
10	7860	11800	7150	10800	6480	9750	5890	8850	5340	8020	4860	7300
11	7760	11700	7060	10600	6400	9610	5810	8730	5260	7900	4780	7190
12	7650	11500	6960	10500	6300	9470	5720	8590	5170	7780	4710	7070
13	7530	11300	6850	10300	6200	9310	5620	8450	5090	7640	4620	6950
14	7410	11100	6730	10100	6090	9150	5520	8300	4990	7500	4530	6820
15	7270	10900	6600	9930	5970	8970	5410	8130	4890	7350	4440	6680
16	7140	10700	6470	9730	5850	8790	5300	7970	4790	7190	4340	6530
17	6990	10500	6340	9530	5720	8600	5180	7790	4680	7030	4240	6380
18	6840	10300	6200	9310	5590	8410	5060	7610	4560	6860	4140	6220
19	6680	10000	6050	9100	5460	8200	4930	7420	4450	6690	4030	6060
20	6520	9810	5900	8870	5320	7990	4810	7220	4330	6510	3920	5890
22	6190	9310	5590	8410	5030	7560	4540	6820	4080	6140	3690	5550
24	5850	8790	5270	7920	4730	7120	4260	6410	3830	5750	3460	5200
26	5490	8260	4950	7430	4430	6660	3980	5990	3570	5370	3220	4840
28	5140	7720	4610	6940	4130	6200	3700	5570	3310	4980	2980	4480
30	4780	7180	4280	6440	3820	5740	3420	5140	3050	4590	2740	4120
32	4420	6650	3960	5950	3520	5290	3150	4730	2800	4210	2510	3780
34	4080	6130	3640	5460	3230	4850	2880	4320	2550	3840	2290	3440
36	3740	5620	3320	4990	2940	4420	2620	3930	2320	3480	2070	3110
38	3410	5120	3020	4540	2660	4000	2360	3550	2090	3130	1860	2790
40	3090	4640	2730	4100	2400	3610	2130	3200	1880	2830	1680	2520
Properties												
P_{wo}, kips	3670	5500	3140	4710	2680	4020	2280	3420	1950	2920	1670	2500
P_{wi}, kips/in.	133	200	123	184	113	169	103	155	94.9	142	87.5	131
P_{wb}, kips	50100	75300	39200	58900	30400	45700	23300	35100	18200	27300	14200	21400
P_{fb}, kips	5860	8810	4970	7470	4210	6330	3550	5340	2980	4480	2510	3770
L_p, ft	14.5		14.3		14.1		13.9		13.7		13.6	
L_r, ft	212		195		178		164		151		138	
A_g, in.^{2}	215		196		178		162		147		134	
I_x, in.^{4}	14300		12400		10800		9430		8210		7190	
I_y, in.^{4}	4720		4170		3680		3250		2880		2560	
r_y, in.	4.69		4.62		4.55		4.49		4.43		4.38	
r_x/r_y	1.74		1.73		1.71		1.70		1.69		1.67	
$P_{ex}(KL)^2/10^4$, k-in.^{2}	409000		355000		309000		270000		235000		206000	
$P_{ey}(KL)^2/10^4$, k-in.^{2}	135000		119000		105000		93000		82400		73300	

ASD	LRFD
Ω_c = 1.67	ϕ_c = 0.90

[h] Flange thickness is greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

W14

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 65 ksi

Shape		W14×											
lb/ft		426[h]		398[h]		370[h]		342[h]		311[h]		283[h]	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y	0	4870	7310	4550	6840	4240	6380	3930	5910	3560	5350	3240	4870
	11	4460	6700	4170	6260	3870	5820	3590	5390	3240	4870	2950	4430
	12	4380	6590	4100	6160	3810	5720	3520	5290	3180	4780	2890	4350
	13	4300	6470	4020	6040	3740	5620	3460	5200	3120	4690	2840	4270
	14	4220	6340	3940	5920	3660	5500	3390	5090	3060	4590	2780	4180
	15	4130	6210	3860	5800	3580	5390	3310	4980	2990	4490	2720	4080
	16	4040	6070	3770	5670	3500	5260	3230	4860	2920	4380	2650	3980
	17	3940	5930	3680	5530	3420	5130	3150	4740	2840	4270	2580	3880
	18	3840	5780	3590	5390	3330	5000	3070	4620	2770	4160	2510	3780
	19	3740	5630	3490	5250	3240	4860	2990	4490	2690	4040	2440	3670
	20	3640	5470	3390	5100	3140	4720	2900	4360	2610	3920	2370	3560
	22	3420	5140	3190	4790	2950	4430	2720	4090	2440	3670	2220	3330
	24	3200	4810	2980	4480	2750	4140	2540	3810	2280	3420	2060	3100
	26	2980	4470	2770	4160	2550	3840	2350	3530	2110	3160	1900	2860
	28	2750	4140	2560	3840	2360	3540	2160	3250	1940	2910	1750	2630
	30	2530	3800	2350	3530	2160	3240	1980	2980	1770	2660	1600	2400
	32	2310	3470	2140	3220	1970	2960	1800	2710	1610	2420	1450	2180
	34	2100	3160	1940	2920	1780	2680	1630	2450	1450	2180	1310	1960
	36	1900	2850	1750	2630	1600	2410	1460	2200	1300	1950	1170	1750
	38	1700	2560	1570	2360	1440	2160	1310	1970	1170	1750	1050	1570
	40	1540	2310	1420	2130	1300	1950	1180	1780	1050	1580	945	1420
	42	1390	2090	1290	1930	1180	1770	1070	1610	954	1430	857	1290
	44	1270	1910	1170	1760	1070	1610	979	1470	869	1310	781	1170
	46	1160	1750	1070	1610	980	1470	896	1350	795	1200	715	1070
	48	1070	1600	985	1480	900	1350	823	1240	730	1100	656	986
	50	983	1480	907	1360	830	1250	758	1140	673	1010	605	909
Properties													
P_{wo}, kips		1480	2220	1320	1980	1170	1760	1020	1540	874	1310	746	1120
P_{wi}, kips/in.		81.5	122	76.7	115	71.9	108	66.7	100	61.1	91.7	55.9	83.9
P_{wb}, kips		11500	17200	9600	14400	7890	11900	6320	9490	4850	7290	3710	5580
P_{fb}, kips		2250	3380	1980	2970	1720	2590	1480	2230	1240	1870	1040	1570
L_p, ft		13.4		13.4		13.2		13.1		13.0		12.9	
L_r, ft		130		122		114		106		96.7		88.3	
A_g, in.^{2}		125		117		109		101		91.4		83.3	
I_x, in.^{4}		6600		6000		5440		4900		4330		3840	
I_y, in.^{4}		2360		2170		1990		1810		1610		1440	
r_y, in.		4.34		4.31		4.27		4.24		4.20		4.17	
r_x/r_y		1.67		1.66		1.66		1.65		1.64		1.63	
$P_{ex}(KL)^2/10^4$, k-in.^{2}		189000		172000		156000		140000		124000		110000	
$P_{ey}(KL)^2/10^4$, k-in.^{2}		67500		62100		57000		51800		46100		41200	

ASD	LRFD
Ω_c = 1.67	ϕ_c = 0.90

[h] Flange thickness is greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

F_y = 65 ksi

Table 4-1 (continued)

Available Strength in Axial Compression, kips

W-Shapes

W14

Shape		W14×											
lb/ft		257		233		211		193		176		159	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y	0	2940	4420	2670	4010	2410	3630	2210	3320	2020	3030	1820	2730
	6	2860	4300	2590	3890	2340	3520	2150	3220	1960	2940	1760	2650
	7	2830	4250	2560	3850	2320	3480	2120	3190	1930	2910	1740	2620
	8	2800	4200	2530	3800	2290	3440	2100	3150	1910	2870	1720	2590
	9	2760	4140	2500	3750	2260	3390	2070	3110	1880	2830	1700	2550
	10	2720	4080	2460	3690	2220	3340	2030	3060	1850	2780	1670	2510
	11	2670	4010	2420	3630	2180	3280	2000	3000	1820	2740	1640	2460
	12	2620	3940	2370	3560	2140	3220	1960	2950	1780	2680	1610	2420
	13	2570	3860	2320	3490	2100	3150	1920	2890	1750	2630	1570	2360
	14	2510	3780	2270	3420	2050	3080	1880	2820	1710	2570	1540	2310
	15	2460	3690	2220	3340	2000	3010	1830	2750	1670	2500	1500	2250
	16	2400	3600	2160	3250	1950	2940	1790	2680	1620	2440	1460	2190
	17	2330	3510	2110	3170	1900	2860	1740	2610	1580	2370	1420	2130
	18	2270	3410	2050	3080	1850	2780	1690	2540	1530	2300	1380	2070
	19	2200	3310	1990	2990	1790	2690	1640	2460	1490	2230	1330	2010
	20	2130	3210	1930	2890	1730	2610	1580	2380	1440	2160	1290	1940
	22	2000	3000	1800	2700	1620	2430	1480	2220	1340	2010	1200	1810
	24	1850	2790	1670	2510	1500	2250	1370	2050	1240	1860	1110	1670
	26	1710	2570	1540	2310	1380	2070	1260	1890	1140	1710	1020	1530
	28	1570	2360	1410	2120	1260	1900	1150	1730	1040	1560	929	1400
	30	1430	2150	1280	1930	1150	1720	1040	1570	941	1410	842	1270
	32	1290	1940	1160	1740	1040	1560	941	1410	847	1270	757	1140
	34	1160	1750	1040	1560	927	1390	841	1260	756	1140	675	1010
	36	1040	1560	927	1390	827	1240	750	1130	674	1010	602	905
	38	932	1400	832	1250	742	1120	673	1010	605	909	540	812
	40	841	1260	751	1130	670	1010	608	914	546	821	487	733
Properties													
P_{wo}, kips		637	955	538	807	459	688	393	590	343	515	289	433
P_{wi}, kips/in.		51.1	76.7	46.4	69.6	42.5	63.7	38.6	57.9	36.0	54.0	32.3	48.4
P_{wb}, kips		2830	4250	2110	3170	1630	2460	1220	1840	992	1490	716	1080
P_{fb}, kips		869	1310	720	1080	592	890	504	758	417	627	344	518
L_p, ft		12.8		12.7		12.6		12.5		12.5		12.4	
L_r, ft		80.7		73.5		67.2		61.8		57.1		52.4	
A_g, in.^{2}		75.6		68.5		62.0		56.8		51.8		46.7	
I_x, in.^{4}		3400		3010		2660		2400		2140		1900	
I_y, in.^{4}		1290		1150		1030		931		838		748	
r_y, in.		4.13		4.10		4.07		4.05		4.02		4.00	
r_x/r_y		1.62		1.62		1.61		1.60		1.60		1.60	
$P_{ex}(KL)^2/10^4$, k-in.^{2}		97300		86200		76100		68700		61300		54400	
$P_{ey}(KL)^2/10^4$, k-in.^{2}		36900		32900		29500		26600		24000		21400	

ASD	LRFD
Ω_c = 1.67	ϕ_c = 0.90

Return to Table of Contents

W14

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 65 ksi

Shape		W14×											
lb/ft		145		132		120		109		99		90	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y	0	1660	2500	1510	2270	1370	2070	1250	1870	1130	1700	1030	1550
	6	1610	2420	1460	2190	1330	1990	1200	1810	1090	1640	995	1500
	7	1590	2390	1440	2160	1310	1970	1190	1780	1080	1620	982	1480
	8	1570	2360	1420	2130	1290	1940	1170	1760	1060	1600	968	1450
	9	1550	2330	1400	2100	1270	1910	1150	1730	1040	1570	951	1430
	10	1520	2290	1370	2060	1250	1870	1130	1700	1030	1540	933	1400
	11	1500	2250	1340	2020	1220	1830	1110	1660	1000	1510	914	1370
	12	1470	2210	1310	1970	1190	1790	1080	1620	982	1480	893	1340
	13	1440	2160	1280	1930	1160	1750	1050	1590	957	1440	871	1310
	14	1400	2110	1250	1880	1130	1700	1030	1540	932	1400	848	1270
	15	1370	2060	1210	1830	1100	1660	998	1500	906	1360	824	1240
	16	1330	2000	1180	1770	1070	1610	968	1460	878	1320	799	1200
	17	1290	1950	1140	1720	1040	1560	937	1410	850	1280	773	1160
	18	1260	1890	1100	1660	1000	1500	906	1360	821	1230	746	1120
	19	1220	1830	1060	1600	965	1450	873	1310	791	1190	719	1080
	20	1180	1770	1030	1540	929	1400	840	1260	761	1140	691	1040
	22	1090	1640	945	1420	856	1290	774	1160	700	1050	636	956
	24	1010	1520	865	1300	782	1180	707	1060	639	960	580	872
	26	927	1390	785	1180	709	1070	640	963	578	869	525	789
	28	844	1270	707	1060	638	959	576	866	519	781	471	708
	30	764	1150	632	950	569	856	514	772	463	696	419	630
	32	686	1030	559	840	503	756	454	682	408	614	370	556
	34	611	918	495	744	446	670	402	604	362	544	328	492
	36	545	819	442	664	398	598	359	539	323	485	292	439
	38	489	735	397	596	357	536	322	484	290	435	262	394
	40	441	663	358	538	322	484	290	437	261	393	237	356
Properties													
P_{wo}, kips		249	373	228	342	197	295	166	249	145	218	125	187
P_{wi}, kips/in.		29.5	44.2	28.0	41.9	25.6	38.4	22.8	34.1	21.0	31.5	19.1	28.6
P_{wb}, kips		543	816	464	697	356	535	251	377	197	297	147	222
P_{fb}, kips		289	434	258	388	215	323	180	270	148	222	123	184
L_p, ft		12.3		11.6		11.6		11.6		11.5		11.5	
L_r, ft		48.7		44.3		41.5		39.1		36.8		34.9	
A_g, in.^{2}		42.7		38.8		35.3		32.0		29.1		26.5	
I_x, in.^{4}		1710		1530		1380		1240		1110		999	
I_y, in.^{4}		677		548		495		447		402		362	
r_y, in.		3.98		3.76		3.74		3.73		3.71		3.70	
r_x/r_y		1.59		1.67		1.67		1.67		1.66		1.66	
$P_{ex}(KL)^2/10^4$, k-in.^{2}		48900		43800		39500		35500		31800		28600	
$P_{ey}(KL)^2/10^4$, k-in.^{2}		19400		15700		14200		12800		11500		10400	

ASD	LRFD
Ω_c = 1.67	ϕ_c = 0.90

Return to Table of Contents

F_y = 65 ksi

Table 4-1 (continued)

Available Strength in Axial Compression, kips

W-Shapes

W14

Shape	W14×													
lb/ft	82		74		68		61		53		48[c]		43[c]	
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y														
0	934	1400	849	1280	778	1170	697	1050	607	913	541	813	474	712
6	862	1300	783	1180	718	1080	642	965	531	798	479	720	419	630
7	838	1260	761	1140	697	1050	623	936	506	761	457	686	401	603
8	810	1220	736	1110	674	1010	602	905	479	720	432	649	381	572
9	780	1170	709	1060	648	974	579	871	449	676	405	609	358	539
10	748	1120	679	1020	621	933	555	834	419	630	377	567	334	502
11	714	1070	648	974	592	890	529	795	387	582	349	524	308	464
12	678	1020	616	926	562	845	502	754	356	535	320	481	282	425
13	641	964	583	876	531	798	474	712	324	487	291	438	257	386
14	604	908	549	824	500	751	446	670	293	441	263	395	231	348
15	566	851	514	773	468	703	417	627	263	396	236	355	207	311
16	528	794	480	721	436	656	389	584	234	352	210	315	184	276
17	491	738	446	670	405	609	360	542	208	312	186	279	163	244
18	454	683	413	620	374	562	333	500	185	278	166	249	145	218
19	418	629	380	571	344	517	306	460	166	250	149	224	130	196
20	384	576	348	524	315	473	280	421	150	226	134	202	117	177
22	318	478	289	435	261	392	232	348	124	186	111	167	97.1	146
24	267	402	243	365	219	330	195	293	104	157	93.2	140	81.6	123
26	228	343	207	311	187	281	166	249	88.8	133	79.4	119	69.5	104
28	197	295	179	268	161	242	143	215	76.6	115	68.5	103	59.9	90.1
30	171	257	156	234	140	211	125	187	66.7	100	59.7	89.7	52.2	78.5
32	150	226	137	205	123	185	110	165	58.6	88.1				
34	133	200	121	182	109	164	97.0	146						
36	119	179	108	162	97.5	147	86.5	130						
38	107	160	96.9	146	87.5	131	77.7	117						
40	96.3	145	87.5	131	79.0	119	70.1	105						
Properties														
P_{wo}, kips	160	240	135	202	118	177	101	151	100	150	87.7	131	74.0	111
P_{wi}, kips/in.	22.1	33.2	19.5	29.3	18.0	27.0	16.3	24.4	16.0	24.1	14.7	22.1	13.2	19.8
P_{wb}, kips	229	344	157	236	124	186	91.3	137	87.4	131	67.9	102	49.1	73.8
P_{fb}, kips	178	267	150	225	126	190	101	152	106	159	86.1	129	68.3	103
L_p, ft	7.68		7.68		7.62		7.59		5.95		5.92		5.86	
L_r, ft	26.7		25.2		24.0		22.7		18.4		17.6		16.8	
A_g, in.^{2}	24.0		21.8		20.0		17.9		15.6		14.1		12.6	
I_x, in.^{4}	881		795		722		640		541		484		428	
I_y, in.^{4}	148		134		121		107		57.7		51.4		45.2	
r_y, in.	2.48		2.48		2.46		2.45		1.92		1.91		1.89	
r_x/r_y	2.44		2.44		2.44		2.44		3.07		3.06		3.08	
$P_{ex}(KL)^2/10^4$, k-in.^{2}	25200		22800		20700		18300		15500		13900		12300	
$P_{ey}(KL)^2/10^4$, k-in.^{2}	4240		3840		3460		3060		1650		1470		1290	

ASD	LRFD
Ω_c = 1.67	ϕ_c = 0.90

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W12

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 65 ksi

Shape	W12×											
lb/ft	336[h]		305[h]		279[h]		252[h]		230[h]		210	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y												
0	3850	5790	3480	5240	3190	4790	2880	4330	2640	3960	2410	3620
6	3700	5550	3340	5020	3050	4590	2760	4150	2520	3790	2300	3450
7	3640	5470	3290	4940	3010	4520	2720	4080	2480	3730	2260	3400
8	3580	5380	3230	4860	2950	4440	2670	4010	2430	3660	2220	3330
9	3510	5280	3170	4760	2890	4350	2610	3920	2380	3580	2170	3260
10	3440	5160	3100	4660	2830	4250	2550	3830	2330	3500	2120	3180
11	3350	5040	3020	4540	2760	4140	2490	3740	2270	3400	2060	3100
12	3270	4910	2940	4420	2680	4030	2420	3630	2200	3310	2000	3010
13	3180	4770	2860	4300	2600	3910	2340	3520	2130	3210	1940	2920
14	3080	4630	2770	4160	2520	3790	2270	3410	2060	3100	1870	2820
15	2980	4480	2680	4020	2430	3660	2190	3290	1990	2990	1810	2720
16	2880	4320	2580	3880	2350	3530	2110	3170	1910	2880	1740	2610
17	2770	4170	2480	3730	2250	3390	2020	3040	1840	2760	1670	2500
18	2660	4000	2380	3580	2160	3250	1940	2910	1760	2640	1590	2390
19	2550	3840	2280	3430	2070	3110	1850	2780	1680	2520	1520	2280
20	2440	3670	2180	3280	1970	2970	1770	2650	1600	2400	1450	2170
22	2220	3340	1980	2970	1790	2680	1590	2390	1440	2160	1300	1950
24	2000	3010	1780	2670	1600	2400	1420	2140	1280	1930	1160	1740
26	1790	2680	1580	2370	1420	2130	1260	1890	1130	1700	1020	1530
28	1580	2370	1390	2090	1250	1870	1100	1650	988	1480	885	1330
30	1380	2080	1210	1820	1090	1630	959	1440	860	1290	771	1160
32	1210	1820	1070	1600	954	1430	843	1270	756	1140	678	1020
34	1080	1620	945	1420	845	1270	746	1120	670	1010	600	902
36	959	1440	843	1270	754	1130	666	1000	597	898	535	805
38	861	1290	757	1140	676	1020	598	898	536	806	481	722
40	777	1170	683	1030	610	917	539	811	484	727	434	652
Properties												
P_{wo}, kips	1370	2050	1170	1750	1020	1530	865	1300	746	1120	639	959
P_{wi}, kips/in.	77.1	116	70.6	106	66.3	99.5	60.7	91.0	55.9	83.9	51.1	76.7
P_{wb}, kips	11400	17200	8770	13200	7270	10900	5560	8350	4340	6530	3340	5020
P_{fb}, kips	2130	3200	1790	2690	1480	2230	1230	1850	1040	1570	878	1320
L_p, ft	10.7		10.6		10.5		10.3		10.3		10.2	
L_r, ft	116		105		96.8		88.0		80.7		73.9	
A_g, in.^{2}	98.9		89.5		81.9		74.1		67.7		61.8	
I_x, in.^{4}	4060		3550		3110		2720		2420		2140	
I_y, in.^{4}	1190		1050		937		828		742		664	
r_y, in.	3.47		3.42		3.38		3.34		3.31		3.28	
r_x/r_y	1.85		1.84		1.82		1.81		1.80		1.80	
$P_{ex}(KL)^2/10^4$, k-in.^{2}	116000		102000		89000		77900		69300		61300	
$P_{ey}(KL)^2/10^4$, k-in.^{2}	34100		30100		26800		23700		21200		19000	

ASD	LRFD
Ω_c = 1.67	ϕ_c = 0.90

[h] Flange thickness is greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

F_y = 65 ksi

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

W12

Shape		W12×											
lb/ft		190		170		152		136		120		106	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y	0	2180	3280	1950	2920	1740	2610	1550	2330	1370	2060	1210	1830
	6	2080	3130	1860	2790	1660	2490	1480	2220	1300	1960	1150	1730
	7	2050	3070	1820	2740	1630	2450	1450	2180	1280	1920	1130	1700
	8	2010	3020	1790	2690	1600	2400	1420	2140	1250	1880	1110	1670
	9	1960	2950	1750	2630	1560	2350	1390	2090	1220	1840	1080	1630
	10	1910	2880	1710	2560	1520	2290	1350	2040	1190	1790	1050	1580
	11	1860	2800	1660	2490	1480	2220	1320	1980	1160	1740	1020	1540
	12	1810	2720	1610	2420	1430	2150	1270	1920	1120	1680	990	1490
	13	1750	2630	1560	2340	1390	2080	1230	1850	1080	1630	956	1440
	14	1690	2540	1500	2260	1340	2010	1190	1780	1040	1570	920	1380
	15	1630	2450	1450	2170	1290	1930	1140	1710	1000	1500	883	1330
	16	1560	2350	1390	2090	1230	1850	1090	1640	958	1440	845	1270
	17	1500	2250	1330	2000	1180	1770	1050	1570	915	1380	807	1210
	18	1430	2150	1270	1910	1130	1690	996	1500	871	1310	768	1150
	19	1370	2050	1210	1820	1070	1610	947	1420	827	1240	729	1100
	20	1300	1950	1150	1730	1020	1530	898	1350	783	1180	689	1040
	22	1160	1750	1030	1540	907	1360	800	1200	697	1050	612	920
	24	1030	1550	910	1370	802	1210	705	1060	613	921	537	808
	26	908	1360	797	1200	701	1050	615	924	532	800	466	700
	28	788	1180	690	1040	606	910	530	797	459	690	402	604
	30	686	1030	601	904	528	793	462	695	400	601	350	526
	32	603	906	528	794	464	697	406	610	352	528	308	462
	34	534	803	468	704	411	617	360	541	311	468	272	410
	36	476	716	418	628	366	551	321	482	278	417	243	365
	38	428	643	375	563	329	494	288	433	249	375	218	328
	40	386	580	338	508	297	446	260	391	225	338	197	296
Properties													
P_{wo}, kips		535	803	449	674	377	566	317	475	262	392	210	315
P_{wi}, kips/in.		45.9	68.9	41.6	62.4	37.7	56.6	34.2	51.4	30.8	46.2	26.4	39.7
P_{wb}, kips		2420	3640	1800	2710	1330	2000	1000	1500	726	1090	462	694
P_{fb}, kips		737	1110	592	890	477	717	380	571	300	450	238	358
L_p, ft		10.1		9.98		9.88		9.79		9.70		9.63	
L_r, ft		67.4		60.7		54.8		49.1		44.2		39.9	
A_g, in.^{2}		56.0		50.0		44.7		39.9		35.2		31.2	
I_x, in.^{4}		1890		1650		1430		1240		1070		933	
I_y, in.^{4}		589		517		454		398		345		301	
r_y, in.		3.25		3.22		3.19		3.16		3.13		3.11	
r_x/r_y		1.79		1.78		1.77		1.77		1.76		1.76	
$P_{ex}(KL)^2/10^4$, k-in.^{2}		54100		47200		40900		35500		30600		26700	
$P_{ey}(KL)^2/10^4$, k-in.^{2}		16900		14800		13000		11400		9870		8620	

ASD	LRFD
Ω_c = 1.67	ϕ_c = 0.90

W12

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 65 ksi

Shape	W12×											
lb/ft	96		87		79		72		65		58	
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y: 0	1100	1650	996	1500	903	1360	821	1230	743	1120	662	994
6	1040	1570	946	1420	856	1290	779	1170	704	1060	612	920
7	1020	1540	928	1390	840	1260	764	1150	691	1040	595	894
8	1000	1510	908	1360	822	1240	747	1120	675	1020	576	865
9	977	1470	886	1330	802	1200	728	1090	658	989	555	834
10	951	1430	862	1300	779	1170	708	1060	640	962	532	800
11	923	1390	836	1260	756	1140	687	1030	620	932	509	765
12	893	1340	808	1210	731	1100	664	997	599	900	484	727
13	861	1290	780	1170	704	1060	639	961	577	867	458	689
14	829	1250	750	1130	677	1020	614	923	554	833	432	650
15	795	1190	719	1080	648	975	589	885	530	797	406	610
16	760	1140	687	1030	620	931	562	845	506	761	379	570
17	725	1090	655	984	590	887	535	805	482	724	353	531
18	690	1040	622	936	561	843	508	764	457	687	327	492
19	654	983	590	887	531	798	481	723	432	650	302	454
20	619	930	557	838	501	753	454	683	408	613	277	417
22	548	824	493	742	443	666	401	603	360	540	231	347
24	481	722	432	649	387	582	350	526	313	471	194	292
26	416	625	373	560	333	501	301	453	269	404	165	249
28	358	539	321	483	287	432	260	390	232	349	143	214
30	312	469	280	421	250	376	226	340	202	304	124	187
32	274	413	246	370	220	331	199	299	178	267	109	164
34	243	365	218	327	195	293	176	265	157	236	96.7	145
36	217	326	194	292	174	261	157	236	140	211	86.3	130
38	195	293	174	262	156	234	141	212	126	189	77.4	116
40	176	264	157	237	141	212	127	191	114	171	69.9	105
Properties												
P_{wo}, kips	179	268	157	236	135	203	118	177	101	152	96.7	145
P_{wi}, kips/in.	23.8	35.8	22.3	33.5	20.4	30.6	18.6	28.0	16.9	25.4	15.6	23.4
P_{wb}, kips	337	507	277	416	211	316	161	243	121	181	94.7	142
P_{fb}, kips	197	296	160	240	131	198	109	164	89.0	134	99.6	150
L_p, ft	9.57		9.51		9.45		9.42		9.36		7.78	
L_r, ft	37.0		34.4		32.1		30.4		28.8		24.4	
A_g, in.^{2}	28.2		25.6		23.2		21.1		19.1		17.0	
I_x, in.^{4}	833		740		662		597		533		475	
I_y, in.^{4}	270		241		216		195		174		107	
r_y, in.	3.09		3.07		3.05		3.04		3.02		2.51	
r_x/r_y	1.76		1.75		1.75		1.75		1.75		2.10	
$P_{ex}(KL)^2/10^4$, k-in.^{2}	23800		21200		18900		17100		15300		13600	
$P_{ey}(KL)^2/10^4$, k-in.^{2}	7730		6900		6180		5580		4980		3060	

ASD	LRFD
Ω_c = 1.67	ϕ_c = 0.90

F_y = 65 ksi

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

W12

Shape	W12×									
lb/ft	58		53		50		45		40[c]	
Design	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y: 0	662	994	607	913	568	854	510	766	450	676
6	612	920	560	842	500	751	448	673	400	600
7	595	894	544	818	477	717	427	642	381	573
8	576	865	527	791	452	680	405	609	361	542
9	555	834	507	762	426	640	381	573	339	510
10	532	800	486	731	398	598	356	535	317	476
11	509	765	464	697	369	555	330	496	293	441
12	484	727	441	662	340	511	304	456	270	405
13	458	689	417	627	311	468	278	417	246	370
14	432	650	393	590	283	425	252	378	223	336
15	406	610	368	553	255	383	227	341	201	302
16	379	570	343	516	228	343	203	305	179	270
17	353	531	319	480	203	304	180	270	159	239
18	327	492	295	444	181	272	160	241	142	213
19	302	454	272	409	162	244	144	216	127	191
20	277	417	249	375	146	220	130	195	115	173
22	231	347	207	311	121	182	107	161	95.0	143
24	194	292	174	261	102	153	90.3	136	79.8	120
26	165	249	148	223	86.6	130	76.9	116	68.0	102
28	143	214	128	192	74.7	112	66.3	100	58.6	88.1
30	124	187	111	167	65.0	97.8	57.8	86.8	51.1	76.8
32	109	164	97.8	147	57.2	85.9	50.8	76.3	44.9	67.5
34	96.7	145	86.6	130						
36	86.3	130	77.3	116						
38	77.4	116	69.4	104						
40	69.9	105	62.6	94.1						
Properties										
P_{wo}, kips	96.7	145	88.2	132	91.4	137	78.4	118	65.2	97.8
P_{wi}, kips/in.	15.6	23.4	15.0	22.4	16.0	24.1	14.5	21.8	12.8	19.2
P_{wb}, kips	94.7	142	83.6	126	101	151	74.8	112	51.1	76.8
P_{fb}, kips	99.6	150	80.4	121	99.6	150	80.4	121	64.5	97.0
L_p, ft	7.78		7.68		6.07		6.04		6.01	
L_r, ft	24.4		23.2		19.5		18.5		17.6	
A_g, in.^{2}	17.0		15.6		14.6		13.1		11.7	
I_x, in.^{4}	475		425		391		348		307	
I_y, in.^{4}	107		95.8		56.3		50.0		44.1	
r_y, in.	2.51		2.48		1.96		1.95		1.94	
r_x/r_y	2.10		2.11		2.64		2.64		2.64	
$P_{ex}(KL)^2/10^4$, k-in.^{2}	13600		12200		11200		9960		8790	
$P_{ey}(KL)^2/10^4$, k-in.^{2}	3060		2740		1610		1430		1260	

ASD	LRFD
Ω_c = 1.67	ϕ_c = 0.90

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W10

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 65 ksi

Shape	W10×											
lb/ft	112		100		88		77		68		60	
Design	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y												
0	1280	1920	1140	1710	1010	1520	884	1330	775	1160	689	1040
6	1200	1800	1060	1600	942	1420	821	1230	720	1080	639	961
7	1170	1750	1040	1560	918	1380	800	1200	701	1050	622	935
8	1130	1700	1010	1510	892	1340	776	1170	680	1020	603	907
9	1100	1650	974	1460	862	1300	750	1130	657	987	582	875
10	1060	1590	938	1410	830	1250	722	1080	632	949	560	842
11	1020	1530	901	1350	796	1200	692	1040	605	909	536	806
12	973	1460	861	1290	761	1140	660	992	577	868	511	768
13	928	1390	820	1230	724	1090	627	943	549	825	485	730
14	881	1320	778	1170	687	1030	594	893	519	780	459	690
15	834	1250	736	1110	648	974	560	842	489	736	432	650
16	786	1180	692	1040	610	916	526	791	459	690	405	609
17	738	1110	649	976	571	859	492	740	429	646	379	569
18	691	1040	606	912	533	801	458	689	400	601	352	529
19	644	967	564	848	495	745	425	639	371	557	326	490
20	597	898	523	786	459	689	393	591	342	515	301	452
22	509	765	444	667	388	583	331	497	288	433	252	379
24	428	644	373	560	326	490	278	418	242	364	212	318
26	365	548	318	478	278	417	237	356	206	310	181	271
28	315	473	274	412	239	360	204	307	178	267	156	234
30	274	412	239	359	209	313	178	267	155	233	136	204
32	241	362	210	315	183	276	156	235	136	205	119	179
34	213	321	186	279	162	244	139	208	121	181	106	159
36	190	286	166	249	145	218	124	186	108	162	94.2	142
38	171	257	149	224	130	195	111	167	96.5	145	84.5	127
40	154	232	134	202	117	176	100	150	87.1	131	76.3	115
Properties												
P_{wo}, kips	286	429	239	358	195	293	157	236	129	194	107	161
P_{wi}, kips/in.	32.7	49.1	29.5	44.2	26.2	39.3	23.0	34.5	20.4	30.6	18.2	27.3
P_{wb}, kips	1080	1630	786	1180	556	835	374	563	261	392	186	280
P_{fb}, kips	380	571	305	459	238	358	184	277	144	217	112	169
L_p, ft	8.30		8.21		8.15		8.05		8.02		7.96	
L_r, ft	49.6		44.8		39.9		35.5		32.1		29.2	
A_g, in.^{2}	32.9		29.3		26.0		22.7		19.9		17.7	
I_x, in.^{4}	716		623		534		455		394		341	
I_y, in.^{4}	236		207		179		154		134		116	
r_y, in.	2.68		2.65		2.63		2.60		2.59		2.57	
r_x/r_y	1.74		1.74		1.73		1.73		1.71		1.71	
$P_{ex}(KL)^2/10^4$, k-in.^{2}	20500		17800		15300		13000		11300		9760	
$P_{ey}(KL)^2/10^4$, k-in.^{2}	6750		5920		5120		4410		3840		3320	

ASD	LRFD
Ω_c = 1.67	ϕ_c = 0.90

F_y = 65 ksi

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

W10

Shape		W10×									
lb/ft		54		49		45		39		33	
Design		P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD	P_n/Ω_c ASD	$\phi_c P_n$ LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y	0	615	924	560	842	518	778	448	673	378	568
	6	570	857	519	780	458	689	395	593	332	498
	7	555	834	505	759	438	659	377	567	316	475
	8	538	809	489	735	417	626	358	538	299	450
	9	519	780	472	709	393	591	337	507	282	423
	10	499	750	453	681	369	554	316	474	263	395
	11	478	718	434	652	344	516	293	441	243	366
	12	455	684	413	621	318	478	271	407	224	336
	13	432	649	392	589	292	439	248	373	204	307
	14	408	614	370	556	266	401	226	339	185	278
	15	384	578	348	523	242	363	204	307	167	251
	16	360	542	326	489	217	327	183	275	149	224
	17	336	505	304	456	194	292	163	245	132	198
	18	313	470	282	424	173	260	145	218	118	177
	19	289	435	261	392	155	234	130	196	106	159
	20	267	401	240	361	140	211	118	177	95.4	143
	22	223	336	200	301	116	174	97.2	146	78.8	118
	24	188	282	168	253	97.4	146	81.7	123	66.2	100
	26	160	240	143	216	83.0	125	69.6	105	56.4	84.8
	28	138	207	124	186	71.5	108	60.0	90.2	48.7	73.1
	30	120	180	108	162	62.3	93.7	52.3	78.6	42.4	63.7
	32	106	159	94.7	142	54.8	82.3	46.0	69.1	37.3	56.0
	34	93.5	141	83.9	126						
	36	83.4	125	74.8	112						
	38	74.8	112	67.2	101						
	40	67.6	102	60.6	91.1						
Properties											
P_{wo}, kips		89.8	135	78.1	117	84.9	127	70.3	105	58.7	88.1
P_{wi}, kips/in.		16.0	24.1	14.7	22.1	15.2	22.8	13.7	20.5	12.6	18.9
P_{wb}, kips		127	192	98.7	148	107	161	78.3	118	61.2	92.0
P_{fb}, kips		92.0	138	76.3	115	93.5	141	68.3	103	46.0	69.2
L_p, ft		7.93		7.87		6.23		6.13		6.01	
L_r, ft		27.0		25.6		21.6		19.7		18.0	
A_g, in.^{2}		15.8		14.4		13.3		11.5		9.71	
I_x, in.^{4}		303		272		248		209		171	
I_y, in.^{4}		103		93.4		53.4		45.0		36.6	
r_y, in.		2.56		2.54		2.01		1.98		1.94	
r_x/r_y		1.71		1.71		2.15		2.16		2.16	
$P_{ex}(KL)^2/10^4$, k-in.^{2}		8670		7790		7100		5980		4890	
$P_{ey}(KL)^2/10^4$, k-in.^{2}		2950		2670		1530		1290		1050	

ASD	LRFD	
Ω_c = 1.67	ϕ_c = 0.90	Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W8

Table 4-1 (continued)
Available Strength in Axial Compression, kips
W-Shapes

F_y = 65 ksi

Shape		W8×											
lb/ft		67		58		48		40		35		31	
Design		P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$	P_n/Ω_c	$\phi_c P_n$
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y	0	767	1150	666	1000	549	825	455	684	401	603	355	534
	6	687	1030	595	895	490	736	405	608	356	535	315	473
	7	660	993	572	859	470	706	388	583	341	512	301	453
	8	631	948	546	820	448	674	369	555	324	487	287	431
	9	599	901	518	778	425	638	349	524	306	460	271	407
	10	565	850	488	733	400	601	328	493	288	432	254	382
	11	530	797	457	687	374	562	306	460	268	403	237	356
	12	495	743	426	640	348	523	284	426	248	373	219	329
	13	458	689	394	592	322	483	261	393	229	344	202	303
	14	422	634	362	544	295	444	239	359	209	314	184	277
	15	386	581	331	498	269	405	217	327	190	285	167	251
	16	352	528	301	452	244	367	196	295	171	257	151	226
	17	318	478	271	408	220	331	176	264	153	230	135	202
	18	285	429	243	365	197	295	157	236	137	206	120	180
	19	256	385	218	328	176	265	141	212	123	184	108	162
	20	231	347	197	296	159	239	127	191	111	166	97.2	146
	22	191	287	163	244	132	198	105	158	91.5	138	80.3	121
	24	160	241	137	205	111	166	88.2	133	76.9	116	67.5	101
	26	137	205	116	175	94.2	142	75.2	113	65.5	98.5	57.5	86.5
	28	118	177	100	151	81.2	122	64.8	97.4	56.5	84.9	49.6	74.5
	30	103	154	87.5	131	70.7	106	56.5	84.9	49.2	74.0	43.2	64.9
	32	90.3	136	76.9	116	62.2	93.5	49.6	74.6	43.3	65.0	38.0	57.1
	34	79.9	120	68.1	102	55.1	82.8						
Properties													
P_{wo}, kips		164	246	133	199	93.6	140	74.4	112	59.7	89.6	51.2	76.8
P_{wi}, kips/in.		24.7	37.1	22.1	33.2	17.3	26.0	15.6	23.4	13.4	20.2	12.4	18.5
P_{wb}, kips		578	868	414	622	199	298	145	218	92.5	139	71.9	108
P_{fb}, kips		213	320	160	240	114	172	76.3	115	59.6	89.6	46.0	69.2
L_p, ft		6.57		6.51		6.44		6.32		6.29		6.26	
L_r, ft		36.9		32.4		27.6		23.8		21.7		20.1	
A_g, in.^{2}		19.7		17.1		14.1		11.7		10.3		9.13	
I_x, in.^{4}		272		228		184		146		127		110	
I_y, in.^{4}		88.6		75.1		60.9		49.1		42.6		37.1	
r_y, in.		2.12		2.1		2.08		2.04		2.03		2.02	
r_x/r_y		1.75		1.74		1.74		1.73		1.73		1.72	
$P_{ex}(KL)^2/10^4$, k-in.^{2}		7790		6530		5270		4180		3630		3150	
$P_{ey}(KL)^2/10^4$, k-in.^{2}		2540		2150		1740		1410		1220		1060	

ASD	LRFD	
Ω_c = 1.67	ϕ_c = 0.90	Note: Heavy line indicates KL/r_y equal to or greater than 200.

DESIGN TABLE DISCUSSION

Table 6-1. W-Shapes in Combined Flexure and Axial Force, 65 ksi

Steel W-shapes with F_y = 65 ksi (ASTM A913 Grade 65) and subject to combined axial force (tension or compression) and flexure may be checked for compliance with the provisions of Section H1.1 and H1.2 of the AISC *Specification* using values listed in Table 6-1 and the appropriate interaction equations provided in the following sections.

Values p, b_x, b_y, t_y and t_r presented in Table 6-1 are defined as follows.

	LRFD	**ASD**
Axial Compression	$p = \dfrac{1}{\phi_c P_n}$, (kips)$^{-1}$	$p = \dfrac{\Omega_c}{P_n}$, (kips)$^{-1}$
Strong Axis Bending	$b_x = \dfrac{8}{9\phi_b M_{nx}}$, (kip-ft)$^{-1}$	$b_x = \dfrac{8\Omega_b}{9M_{nx}}$, (kip-ft)$^{-1}$
Weak Axis Bending	$b_y = \dfrac{8}{9\phi_b M_{ny}}$, (kip-ft)$^{-1}$	$b_y = \dfrac{8\Omega_b}{9M_{ny}}$, (kip-ft)$^{-1}$
Tension Yielding	$t_y = \dfrac{1}{\phi_t F_y A_g}$, (kips)$^{-1}$	$t_y = \dfrac{\Omega_t}{F_y A_g}$, (kips)$^{-1}$
Tension Rupture	$t_r = \dfrac{1}{\phi_t F_u \left(0.75 A_g\right)}$, (kips)$^{-1}$	$t_r = \dfrac{\Omega_t}{F_u \left(0.75 A_g\right)}$, (kips)$^{-1}$

Combined Flexure and Compression

Equations H1-1a and H1-1b of the AISC *Specification* may be written as follows using the coefficients listed in Table 6-1 and defined previously.

When $pP_r \geq 0.2$:

$$pP_r + b_x M_{rx} + b_y M_{ry} \leq 1.0 \tag{6-1}$$

When $pP_r < 0.2$:

$$\tfrac{1}{2} pP_r + \tfrac{9}{8}\left(b_x M_{rx} + b_y M_{ry}\right) \leq 1.0 \tag{6-2}$$

The designer may check acceptability of a given shape using the appropriate interaction equation from the preceding list. See Aminmansour (2000) for more information on this method, including an alternative approach for selection of a trial shape.

Combined Flexure and Tension

Equations H1-1a and H1-1b of the AISC *Specification* may be written as follows using the coefficients listed in Table 6-1 and defined previously.

When $pP_r \geq 0.2$:

$$\left(t_y \text{ or } t_r\right) P_r + b_x M_{rx} + b_y M_{ry} \leq 1.0 \tag{6-3}$$

When $pP_r < 0.2$:

$$\tfrac{1}{2}\left(t_y \text{ or } t_r\right) P_r + \tfrac{9}{8}\left(b_x M_{rx} + b_y M_{ry}\right) \leq 1.0 \tag{6-4}$$

The larger value of t_y and t_r should be used in the preceding equations.

The designer may check acceptability of a given shape using the appropriate interaction equation from above along with variables t_r, t_y, b_x and b_y. See Aminmansour (2006) for more information on this method.

It is noted that the values for t_r listed in Table 6-1 are based on the assumption that $A_e = 0.75A_g$. See Part 5 for more information on this assumption. When $A_e > 0.75A_g$, the tabulated values for t_r are conservative. When $A_e < 0.75A_g$, t_r must be calculated based upon the actual value of A_e.

General Considerations for Use of Values Listed in Table 6-1

The following remarks are offered for consideration in use of the values listed in Table 6-1.

1. Values of p, b_x and b_y already account for section compactness and can be used directly.
2. Tabulated values of b_x assumed $C_b = 1.0$. A procedure for determining b_x when $C_b > 1.0$ follows.
3. Given that the limit state of lateral-torsional buckling does not apply to W-shapes bent about their weak axis, values of b_y are independent of unbraced length and C_b.
4. Values of b_x equally apply to combined flexure and compression as well as combined flexure and tension.
5. Smaller values of variable p for a given KL and smaller values of b_x for a given L_b indicate higher strength for the type of load in question. For example, a section with a smaller p at a certain KL is more effective in carrying axial compression than another section with a larger value of p at the same KL. Similarly, a section with a smaller b_x is more effective for flexure at a given L_b than another section with a larger b_x for the same L_b. This information may be used to select more efficient shapes when relatively large amounts of axial load or bending are present.

Determination of b_x when C_b > 1.0

The tabulated values of b_x assume that $C_b = 1.0$. These values may be modified in accordance with AISC *Specification* Sections F1 and H1.2. The following procedure may be used to account for $C_b > 1.0$.

$$b_{x(C_b>1.0)} = \frac{b_{x(C_b=1.0)}}{C_b} \geq b_{xmin} \qquad (6\text{-}5)$$

Values of b_{xmin} are listed in Table 6-1 at $L_b = 0$ ft. See Aminmansour (2009) for more information on this method. Values for p, b_x, b_y, t_y and t_r presented in Table 6-1 have been multiplied by 10^3. Thus, when used in the appropriate interaction equation they must be multiplied by 10^{-3} (0.001).

Return to Table of Contents

F_y = 65 ksi

Table 6-1
Combined Flexure and Axial Force
W-Shapes

W44

Shape	W44×											
	335[c]				290[c]				262[c,v]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.276	0.184	0.169	0.113	0.333	0.221	0.194	0.129	0.378	0.251	0.216	0.144
11	0.309	0.205	0.170	0.113	0.370	0.246	0.195	0.130	0.420	0.279	0.217	0.144
12	0.315	0.210	0.173	0.115	0.378	0.251	0.199	0.132	0.428	0.285	0.221	0.147
13	0.323	0.215	0.176	0.117	0.387	0.257	0.203	0.135	0.438	0.291	0.226	0.150
14	0.331	0.221	0.179	0.119	0.396	0.264	0.207	0.138	0.449	0.299	0.231	0.154
15	0.341	0.227	0.183	0.122	0.407	0.271	0.211	0.140	0.461	0.307	0.236	0.157
16	0.352	0.234	0.187	0.124	0.419	0.279	0.215	0.143	0.474	0.316	0.241	0.160
17	0.363	0.242	0.190	0.127	0.433	0.288	0.220	0.146	0.489	0.325	0.246	0.164
18	0.376	0.250	0.194	0.129	0.447	0.298	0.225	0.150	0.506	0.336	0.252	0.168
19	0.391	0.260	0.198	0.132	0.464	0.309	0.230	0.153	0.524	0.349	0.258	0.172
20	0.409	0.272	0.203	0.135	0.482	0.321	0.235	0.156	0.544	0.362	0.264	0.176
22	0.449	0.299	0.212	0.141	0.525	0.349	0.246	0.164	0.591	0.393	0.278	0.185
24	0.498	0.332	0.222	0.147	0.577	0.384	0.258	0.172	0.649	0.432	0.292	0.195
26	0.558	0.371	0.233	0.155	0.643	0.428	0.272	0.181	0.721	0.480	0.309	0.206
28	0.629	0.419	0.245	0.163	0.726	0.483	0.287	0.191	0.812	0.540	0.327	0.218
30	0.719	0.478	0.258	0.172	0.829	0.552	0.304	0.202	0.928	0.617	0.348	0.232
32	0.818	0.544	0.273	0.182	0.943	0.628	0.328	0.218	1.06	0.702	0.384	0.256
34	0.923	0.614	0.296	0.197	1.06	0.708	0.361	0.240	1.19	0.793	0.424	0.282
36	1.03	0.689	0.323	0.215	1.19	0.794	0.395	0.263	1.34	0.889	0.465	0.310
38	1.15	0.767	0.350	0.233	1.33	0.885	0.429	0.286	1.49	0.990	0.507	0.337
40	1.28	0.850	0.377	0.251	1.47	0.980	0.464	0.309	1.65	1.10	0.549	0.365
42	1.41	0.937	0.404	0.269	1.62	1.08	0.499	0.332	1.82	1.21	0.592	0.394
44	1.55	1.03	0.431	0.287	1.78	1.19	0.534	0.355	2.00	1.33	0.635	0.423
46	1.69	1.12	0.459	0.305	1.95	1.30	0.570	0.379	2.18	1.45	0.679	0.452
48	1.84	1.22	0.486	0.323	2.12	1.41	0.605	0.403	2.37	1.58	0.722	0.481
50	2.00	1.33	0.514	0.342	2.30	1.53	0.641	0.426	2.58	1.71	0.766	0.510

Other Constants and Properties

	335 ASD	335 LRFD	290 ASD	290 LRFD	262 ASD	262 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.16	0.773	1.34	0.889	1.51	1.00
$t_y \times 10^3$ (kips)$^{-1}$	0.261	0.174	0.301	0.200	0.333	0.221
$t_r \times 10^3$ (kips)$^{-1}$	0.338	0.226	0.390	0.260	0.432	0.288
r_x/r_y	5.10		5.10		5.10	
r_y, in.	3.49		3.49		3.47	

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Return to Table of Contents

W44-W40

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W44×				W40×							
	230[c,v]				593[h]				503[h]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.443	0.295	0.249	0.166	0.148	0.0982	0.0993	0.0661	0.174	0.116	0.118	0.0786
11	0.492	0.327	0.251	0.167	0.166	0.110	0.0993	0.0661	0.196	0.130	0.118	0.0786
12	0.502	0.334	0.256	0.171	0.169	0.113	0.0995	0.0662	0.200	0.133	0.119	0.0791
13	0.513	0.342	0.262	0.174	0.173	0.115	0.101	0.0669	0.205	0.137	0.120	0.0801
14	0.526	0.350	0.268	0.178	0.178	0.118	0.102	0.0676	0.211	0.140	0.122	0.0811
15	0.540	0.359	0.273	0.182	0.183	0.122	0.103	0.0684	0.217	0.144	0.123	0.0821
16	0.555	0.369	0.280	0.186	0.188	0.125	0.104	0.0691	0.224	0.149	0.125	0.0832
17	0.573	0.381	0.286	0.190	0.194	0.129	0.105	0.0699	0.231	0.154	0.127	0.0843
18	0.592	0.394	0.293	0.195	0.201	0.134	0.106	0.0707	0.239	0.159	0.128	0.0854
19	0.613	0.408	0.300	0.200	0.208	0.138	0.107	0.0715	0.248	0.165	0.130	0.0866
20	0.636	0.423	0.308	0.205	0.216	0.144	0.109	0.0723	0.258	0.172	0.132	0.0878
22	0.690	0.459	0.324	0.215	0.234	0.155	0.111	0.0740	0.280	0.186	0.136	0.0903
24	0.758	0.504	0.342	0.227	0.255	0.170	0.114	0.0758	0.307	0.204	0.140	0.0929
26	0.841	0.560	0.362	0.241	0.280	0.186	0.117	0.0777	0.339	0.225	0.144	0.0957
28	0.946	0.630	0.384	0.256	0.310	0.207	0.120	0.0797	0.377	0.251	0.148	0.0986
30	1.08	0.719	0.417	0.277	0.347	0.231	0.123	0.0817	0.423	0.281	0.153	0.102
32	1.23	0.818	0.466	0.310	0.390	0.260	0.126	0.0839	0.479	0.319	0.158	0.105
34	1.39	0.924	0.516	0.343	0.441	0.293	0.130	0.0862	0.541	0.360	0.163	0.109
36	1.56	1.04	0.568	0.378	0.494	0.329	0.133	0.0887	0.606	0.403	0.169	0.113
38	1.73	1.15	0.621	0.413	0.551	0.366	0.137	0.0912	0.675	0.449	0.175	0.117
40	1.92	1.28	0.674	0.449	0.610	0.406	0.141	0.0940	0.748	0.498	0.182	0.121
42	2.12	1.41	0.729	0.485	0.673	0.448	0.146	0.0969	0.825	0.549	0.189	0.126
44	2.33	1.55	0.784	0.522	0.738	0.491	0.150	0.100	0.906	0.603	0.197	0.131
46	2.54	1.69	0.840	0.559	0.807	0.537	0.155	0.103	0.990	0.659	0.208	0.138
48	2.77	1.84	0.897	0.597	0.879	0.585	0.160	0.107	1.08	0.717	0.219	0.145
50	3.00	2.00	0.954	0.634	0.953	0.634	0.166	0.111	1.17	0.778	0.229	0.153

Other Constants and Properties

	W44×230 ASD	W44×230 LRFD	W40×593 ASD	W40×593 LRFD	W40×503 ASD	W40×503 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.75	1.16	0.570	0.379	0.696	0.463
$t_y \times 10^3$ (kips)$^{-1}$	0.379	0.252	0.148	0.0982	0.174	0.116
$t_r \times 10^3$ (kips)$^{-1}$	0.492	0.328	0.192	0.128	0.225	0.150
r_x/r_y	5.10		4.47		4.52	
r_y, in.	3.43		3.80		3.72	

[c] Shape is slender for compression with F_y = 65 ksi.

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W40

Shape		W40×											
		431[h]				397[h]				392[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.202	0.135	0.140	0.0930	0.220	0.146	0.152	0.101	0.221	0.147	0.160	0.107
	11	0.229	0.152	0.140	0.0930	0.249	0.166	0.152	0.101	0.281	0.187	0.169	0.112
	12	0.235	0.156	0.141	0.0939	0.255	0.170	0.154	0.102	0.294	0.196	0.172	0.115
	13	0.241	0.160	0.143	0.0953	0.261	0.174	0.156	0.104	0.309	0.205	0.176	0.117
	14	0.247	0.165	0.145	0.0967	0.269	0.179	0.159	0.105	0.325	0.217	0.179	0.119
	15	0.255	0.170	0.147	0.0981	0.277	0.184	0.161	0.107	0.345	0.229	0.183	0.122
	16	0.263	0.175	0.150	0.0995	0.286	0.190	0.164	0.109	0.366	0.244	0.187	0.124
	17	0.272	0.181	0.152	0.101	0.296	0.197	0.166	0.111	0.391	0.260	0.191	0.127
	18	0.282	0.188	0.154	0.103	0.307	0.204	0.169	0.112	0.418	0.278	0.195	0.130
	19	0.293	0.195	0.157	0.104	0.319	0.212	0.172	0.114	0.450	0.299	0.199	0.133
	20	0.305	0.203	0.159	0.106	0.332	0.221	0.175	0.116	0.486	0.323	0.204	0.136
	22	0.333	0.221	0.164	0.109	0.362	0.241	0.181	0.120	0.574	0.382	0.214	0.142
	24	0.366	0.243	0.170	0.113	0.398	0.265	0.187	0.125	0.683	0.454	0.224	0.149
	26	0.405	0.270	0.176	0.117	0.441	0.294	0.194	0.129	0.801	0.533	0.236	0.157
	28	0.453	0.301	0.182	0.121	0.494	0.328	0.202	0.134	0.929	0.618	0.250	0.166
	30	0.510	0.339	0.189	0.126	0.556	0.370	0.210	0.140	1.07	0.710	0.265	0.176
	32	0.580	0.386	0.196	0.131	0.633	0.421	0.219	0.146	1.21	0.807	0.285	0.189
	34	0.655	0.436	0.204	0.136	0.714	0.475	0.228	0.152	1.37	0.911	0.307	0.204
	36	0.734	0.488	0.213	0.142	0.801	0.533	0.239	0.159	1.54	1.02	0.329	0.219
	38	0.818	0.544	0.222	0.148	0.892	0.594	0.250	0.167	1.71	1.14	0.350	0.233
	40	0.906	0.603	0.234	0.155	0.989	0.658	0.268	0.178	1.90	1.26	0.372	0.248
	42	0.999	0.665	0.249	0.165	1.09	0.725	0.286	0.190	2.09	1.39	0.394	0.262
	44	1.10	0.729	0.264	0.175	1.20	0.796	0.303	0.202	2.29	1.53	0.415	0.276
	46	1.20	0.797	0.279	0.185	1.31	0.870	0.321	0.213				
	48	1.30	0.868	0.293	0.195	1.42	0.947	0.338	0.225				
	50	1.42	0.942	0.308	0.205	1.55	1.03	0.356	0.237				
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		0.836		0.556		0.914		0.608		1.32		0.877	
$t_y \times 10^3$ (kips)$^{-1}$		0.202		0.135		0.220		0.146		0.221		0.147	
$t_r \times 10^3$ (kips)$^{-1}$		0.262		0.175		0.285		0.190		0.287		0.192	
r_x/r_y		4.55				4.56				6.10			
r_y, in.		3.65				3.64				2.64			

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.
Note: Heavy line indicates KL/r_y equal to or greater than 200.

W40

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W40× 372[h]				W40× 362[h]				W40× 331[h]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.234	0.155	0.163	0.109	0.242	0.161	0.167	0.111	0.263	0.175	0.192	0.128
11	0.265	0.177	0.163	0.109	0.275	0.183	0.167	0.111	0.338	0.225	0.205	0.136
12	0.272	0.181	0.165	0.110	0.282	0.188	0.169	0.113	0.354	0.236	0.209	0.139
13	0.279	0.186	0.168	0.112	0.290	0.193	0.172	0.114	0.373	0.248	0.214	0.142
14	0.287	0.191	0.171	0.113	0.298	0.198	0.175	0.116	0.395	0.263	0.219	0.146
15	0.296	0.197	0.173	0.115	0.307	0.205	0.178	0.118	0.419	0.279	0.224	0.149
16	0.306	0.204	0.176	0.117	0.318	0.211	0.181	0.120	0.447	0.297	0.230	0.153
17	0.317	0.211	0.179	0.119	0.329	0.219	0.184	0.122	0.479	0.318	0.236	0.157
18	0.329	0.219	0.182	0.121	0.341	0.227	0.187	0.125	0.515	0.342	0.242	0.161
19	0.342	0.228	0.186	0.123	0.355	0.236	0.191	0.127	0.556	0.370	0.249	0.165
20	0.356	0.237	0.189	0.126	0.370	0.246	0.194	0.129	0.602	0.401	0.256	0.170
22	0.389	0.259	0.196	0.130	0.404	0.269	0.201	0.134	0.719	0.478	0.270	0.180
24	0.429	0.286	0.203	0.135	0.445	0.296	0.209	0.139	0.855	0.569	0.287	0.191
26	0.477	0.317	0.212	0.141	0.495	0.329	0.218	0.145	1.00	0.668	0.306	0.204
28	0.535	0.356	0.220	0.147	0.555	0.369	0.227	0.151	1.16	0.774	0.330	0.220
30	0.605	0.402	0.230	0.153	0.628	0.418	0.237	0.158	1.34	0.889	0.362	0.241
32	0.688	0.458	0.241	0.160	0.714	0.475	0.249	0.165	1.52	1.01	0.393	0.262
34	0.777	0.517	0.252	0.168	0.806	0.536	0.261	0.174	1.72	1.14	0.425	0.283
36	0.871	0.579	0.265	0.176	0.904	0.601	0.274	0.182	1.92	1.28	0.456	0.304
38	0.970	0.646	0.284	0.189	1.01	0.670	0.295	0.196	2.14	1.43	0.488	0.324
40	1.08	0.715	0.304	0.202	1.12	0.742	0.316	0.210	2.38	1.58	0.519	0.345
42	1.19	0.789	0.324	0.216	1.23	0.818	0.337	0.224	2.62	1.74	0.550	0.366
44	1.30	0.866	0.344	0.229	1.35	0.898	0.358	0.238				
46	1.42	0.946	0.365	0.243	1.48	0.982	0.380	0.253				
48	1.55	1.03	0.385	0.256	1.61	1.07	0.401	0.267				
50	1.68	1.12	0.405	0.270	1.74	1.16	0.422	0.281				
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	0.989	0.658			1.02	0.675			1.62	1.08		
$t_y \times 10^3$ (kips)$^{-1}$	0.234	0.155			0.242	0.161			0.263	0.175		
$t_r \times 10^3$ (kips)$^{-1}$	0.303	0.202			0.314	0.210			0.341	0.227		
r_x/r_y	4.58				4.58				6.19			
r_y, in.	3.60				3.60				2.57			

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W40

Shape	W40×											
	327[h]				324[c]				297[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending: 0	0.268	0.178	0.194	0.129	0.276	0.184	0.188	0.125	0.307	0.204	0.206	0.137
11	0.344	0.229	0.207	0.138	0.309	0.205	0.188	0.125	0.343	0.228	0.206	0.137
12	0.360	0.240	0.212	0.141	0.315	0.210	0.191	0.127	0.351	0.233	0.210	0.140
13	0.379	0.252	0.217	0.144	0.323	0.215	0.194	0.129	0.359	0.239	0.214	0.142
14	0.401	0.267	0.222	0.148	0.332	0.221	0.197	0.131	0.369	0.245	0.218	0.145
15	0.426	0.283	0.227	0.151	0.343	0.228	0.201	0.134	0.379	0.252	0.222	0.147
16	0.454	0.302	0.233	0.155	0.354	0.236	0.204	0.136	0.391	0.260	0.226	0.150
17	0.485	0.323	0.239	0.159	0.367	0.244	0.208	0.138	0.404	0.268	0.230	0.153
18	0.522	0.347	0.245	0.163	0.381	0.254	0.212	0.141	0.419	0.279	0.235	0.156
19	0.563	0.374	0.252	0.168	0.396	0.264	0.216	0.144	0.437	0.290	0.240	0.159
20	0.610	0.406	0.259	0.172	0.413	0.275	0.220	0.147	0.456	0.303	0.245	0.163
22	0.726	0.483	0.274	0.182	0.452	0.301	0.229	0.153	0.499	0.332	0.255	0.170
24	0.864	0.575	0.291	0.194	0.499	0.332	0.239	0.159	0.552	0.367	0.267	0.178
26	1.01	0.675	0.310	0.206	0.555	0.369	0.250	0.166	0.616	0.410	0.280	0.186
28	1.18	0.783	0.334	0.222	0.623	0.414	0.261	0.174	0.693	0.461	0.294	0.195
30	1.35	0.899	0.366	0.244	0.706	0.470	0.274	0.182	0.788	0.524	0.309	0.206
32	1.54	1.02	0.398	0.265	0.803	0.534	0.288	0.192	0.897	0.597	0.326	0.217
34	1.73	1.15	0.430	0.286	0.907	0.603	0.303	0.202	1.01	0.674	0.352	0.234
36	1.95	1.29	0.462	0.307	1.02	0.676	0.329	0.219	1.13	0.755	0.383	0.255
38	2.17	1.44	0.494	0.329	1.13	0.754	0.355	0.236	1.26	0.841	0.414	0.276
40	2.40	1.60	0.526	0.350	1.25	0.835	0.382	0.254	1.40	0.932	0.446	0.297
42	2.65	1.76	0.557	0.371	1.38	0.921	0.408	0.272	1.54	1.03	0.478	0.318
44					1.52	1.01	0.435	0.289	1.70	1.13	0.509	0.339
46					1.66	1.10	0.461	0.307	1.85	1.23	0.541	0.360
48					1.81	1.20	0.488	0.324	2.02	1.34	0.573	0.381
50					1.96	1.30	0.514	0.342	2.19	1.46	0.605	0.403

Other Constants and Properties

	327[h] ASD	327[h] LRFD	324[c] ASD	324[c] LRFD	297[c] ASD	297[c] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.63	1.09	1.15	0.763	1.27	0.848
$t_y \times 10^3$ (kips)$^{-1}$	0.268	0.178	0.270	0.179	0.294	0.196
$t_r \times 10^3$ (kips)$^{-1}$	0.348	0.232	0.350	0.233	0.382	0.255
r_x/r_y	6.20		4.58		4.60	
r_y, in.	2.58		3.58		3.54	

[c] Shape is slender for compression with F_y = 65 ksi.

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W40

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W40×											
	294[c]				278[c]				277[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.301	0.200	0.216	0.144	0.318	0.212	0.230	0.153	0.337	0.224	0.219	0.146
11	0.385	0.256	0.232	0.154	0.405	0.270	0.249	0.165	0.375	0.250	0.219	0.146
12	0.404	0.269	0.238	0.158	0.426	0.283	0.255	0.170	0.383	0.255	0.223	0.148
13	0.425	0.283	0.244	0.162	0.449	0.299	0.262	0.174	0.392	0.261	0.227	0.151
14	0.450	0.300	0.250	0.166	0.476	0.317	0.269	0.179	0.402	0.267	0.231	0.154
15	0.479	0.318	0.257	0.171	0.507	0.337	0.276	0.184	0.413	0.275	0.236	0.157
16	0.511	0.340	0.264	0.175	0.542	0.361	0.284	0.189	0.425	0.283	0.240	0.160
17	0.548	0.364	0.271	0.180	0.582	0.387	0.292	0.194	0.438	0.292	0.245	0.163
18	0.590	0.392	0.279	0.185	0.628	0.418	0.301	0.200	0.453	0.301	0.250	0.166
19	0.637	0.424	0.287	0.191	0.680	0.452	0.310	0.206	0.469	0.312	0.255	0.170
20	0.692	0.460	0.296	0.197	0.739	0.492	0.320	0.213	0.488	0.324	0.261	0.173
22	0.827	0.550	0.315	0.210	0.887	0.590	0.342	0.227	0.530	0.352	0.272	0.181
24	0.985	0.655	0.337	0.224	1.06	0.702	0.367	0.244	0.583	0.388	0.285	0.190
26	1.16	0.769	0.363	0.241	1.24	0.824	0.401	0.267	0.649	0.432	0.299	0.199
28	1.34	0.892	0.402	0.268	1.44	0.956	0.445	0.296	0.728	0.485	0.314	0.209
30	1.54	1.02	0.442	0.294	1.65	1.10	0.490	0.326	0.825	0.549	0.331	0.220
32	1.75	1.16	0.482	0.320	1.88	1.25	0.535	0.356	0.939	0.625	0.350	0.233
34	1.98	1.31	0.521	0.347	2.12	1.41	0.580	0.386	1.06	0.705	0.380	0.253
36	2.22	1.47	0.561	0.373	2.38	1.58	0.624	0.415	1.19	0.791	0.414	0.276
38	2.47	1.64	0.601	0.400	2.65	1.76	0.669	0.445	1.32	0.881	0.449	0.299
40	2.73	1.82	0.640	0.426	2.93	1.95	0.714	0.475	1.47	0.976	0.484	0.322
42	3.02	2.01	0.679	0.452	3.23	2.15	0.758	0.504	1.62	1.08	0.519	0.345
44									1.78	1.18	0.555	0.369
46									1.94	1.29	0.590	0.393
48									2.11	1.41	0.625	0.416
50									2.29	1.53	0.661	0.440
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.83		1.22		1.97		1.31		1.34		0.894	
$t_y \times 10^3$ (kips)$^{-1}$	0.298		0.198		0.312		0.208		0.315		0.210	
$t_r \times 10^3$ (kips)$^{-1}$	0.387		0.258		0.405		0.270		0.409		0.273	
r_x/r_y	6.24				6.27				4.58			
r_y, in.	2.55				2.52				3.58			

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W40

Shape		W40× 264[c]				W40× 249[c]				W40× 235[c]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.345	0.230	0.243	0.161	0.383	0.255	0.245	0.163	0.403	0.268	0.271	0.181
	11	0.431	0.287	0.262	0.174	0.426	0.283	0.245	0.163	0.496	0.330	0.294	0.196
	12	0.453	0.301	0.269	0.179	0.435	0.289	0.249	0.166	0.518	0.344	0.302	0.201
	13	0.478	0.318	0.276	0.184	0.445	0.296	0.254	0.169	0.542	0.361	0.310	0.207
	14	0.506	0.337	0.284	0.189	0.456	0.303	0.259	0.172	0.570	0.379	0.319	0.213
	15	0.539	0.359	0.292	0.194	0.468	0.311	0.264	0.176	0.603	0.401	0.329	0.219
	16	0.576	0.383	0.301	0.200	0.482	0.321	0.270	0.179	0.640	0.426	0.339	0.226
	17	0.619	0.412	0.310	0.206	0.497	0.331	0.275	0.183	0.686	0.457	0.350	0.233
	18	0.667	0.444	0.319	0.212	0.513	0.342	0.281	0.187	0.739	0.492	0.361	0.240
	19	0.723	0.481	0.330	0.219	0.532	0.354	0.287	0.191	0.800	0.532	0.373	0.248
	20	0.786	0.523	0.340	0.227	0.552	0.367	0.294	0.195	0.869	0.578	0.386	0.257
	22	0.943	0.628	0.365	0.243	0.599	0.399	0.307	0.204	1.04	0.692	0.415	0.276
	24	1.12	0.747	0.392	0.261	0.657	0.437	0.322	0.214	1.24	0.824	0.451	0.300
	26	1.32	0.877	0.434	0.289	0.729	0.485	0.339	0.225	1.45	0.967	0.510	0.339
	28	1.53	1.02	0.483	0.322	0.819	0.545	0.357	0.238	1.68	1.12	0.569	0.379
	30	1.75	1.17	0.533	0.354	0.931	0.619	0.377	0.251	1.93	1.29	0.629	0.419
	32	2.00	1.33	0.582	0.387	1.06	0.705	0.405	0.270	2.20	1.46	0.690	0.459
	34	2.25	1.50	0.632	0.420	1.20	0.795	0.446	0.297	2.48	1.65	0.750	0.499
	36	2.53	1.68	0.681	0.453	1.34	0.892	0.488	0.325	2.79	1.85	0.811	0.540
	38	2.81	1.87	0.730	0.486	1.49	0.994	0.530	0.353	3.10	2.06	0.872	0.580
	40	3.12	2.07	0.780	0.519	1.65	1.10	0.573	0.381	3.44	2.29	0.932	0.620
	42	3.44	2.29	0.829	0.552	1.82	1.21	0.616	0.410	3.79	2.52	0.993	0.661
	44					2.00	1.33	0.659	0.438				
	46					2.19	1.46	0.702	0.467				
	48					2.38	1.59	0.746	0.496				
	50					2.59	1.72	0.790	0.525				
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		2.08		1.38		1.51		1.00		2.32		1.55	
$t_y \times 10^3$ (kips)$^{-1}$		0.332		0.221		0.350		0.233		0.372		0.247	
$t_r \times 10^3$ (kips)$^{-1}$		0.431		0.287		0.454		0.302		0.482		0.322	
r_x/r_y		6.27				4.59				6.26			
r_y, in.		2.52				3.55				2.54			

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W40

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W40×											
		215[c,v]				211[c]				199[c,v]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.458	0.305	0.284	0.189	0.462	0.307	0.302	0.201	0.500	0.333	0.315	0.210
	11	0.508	0.338	0.284	0.189	0.568	0.378	0.330	0.219	0.556	0.370	0.317	0.211
	12	0.519	0.345	0.290	0.193	0.592	0.394	0.339	0.226	0.568	0.378	0.324	0.216
	13	0.530	0.353	0.296	0.197	0.619	0.412	0.350	0.233	0.581	0.387	0.331	0.220
	14	0.543	0.361	0.302	0.201	0.651	0.433	0.360	0.240	0.596	0.396	0.338	0.225
	15	0.557	0.371	0.308	0.205	0.688	0.458	0.372	0.247	0.612	0.407	0.346	0.230
	16	0.573	0.381	0.315	0.209	0.731	0.486	0.384	0.255	0.630	0.419	0.353	0.235
	17	0.590	0.393	0.322	0.214	0.781	0.519	0.397	0.264	0.649	0.432	0.362	0.241
	18	0.609	0.405	0.329	0.219	0.838	0.558	0.411	0.273	0.671	0.447	0.370	0.246
	19	0.630	0.419	0.336	0.224	0.906	0.603	0.426	0.283	0.696	0.463	0.379	0.252
	20	0.654	0.435	0.344	0.229	0.987	0.656	0.442	0.294	0.723	0.481	0.389	0.259
	22	0.708	0.471	0.361	0.240	1.19	0.789	0.478	0.318	0.785	0.523	0.409	0.272
	24	0.774	0.515	0.380	0.253	1.41	0.938	0.535	0.356	0.863	0.574	0.432	0.287
	26	0.855	0.569	0.401	0.267	1.66	1.10	0.606	0.403	0.959	0.638	0.458	0.304
	28	0.956	0.636	0.424	0.282	1.92	1.28	0.679	0.452	1.08	0.718	0.486	0.323
	30	1.08	0.721	0.450	0.300	2.20	1.47	0.753	0.501	1.23	0.820	0.526	0.350
	32	1.23	0.820	0.497	0.331	2.51	1.67	0.827	0.550	1.40	0.933	0.588	0.391
	34	1.39	0.926	0.549	0.365	2.83	1.88	0.902	0.600	1.58	1.05	0.651	0.433
	36	1.56	1.04	0.603	0.401	3.17	2.11	0.978	0.650	1.77	1.18	0.716	0.476
	38	1.74	1.16	0.657	0.437	3.54	2.35	1.05	0.701	1.98	1.32	0.782	0.520
	40	1.93	1.28	0.712	0.474	3.92	2.61	1.13	0.751	2.19	1.46	0.849	0.565
	42	2.12	1.41	0.768	0.511					2.41	1.61	0.918	0.610
	44	2.33	1.55	0.825	0.549					2.65	1.76	0.987	0.657
	46	2.55	1.69	0.882	0.587					2.90	1.93	1.06	0.703
	48	2.77	1.85	0.939	0.625					3.15	2.10	1.13	0.750
	50	3.01	2.00	0.997	0.663					3.42	2.28	1.20	0.797

Other Constants and Properties

	215 ASD	215 LRFD	211 ASD	211 LRFD	199 ASD	199 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.76	1.17	2.61	1.74	2.00	1.33
$t_y \times 10^3$ (kips)$^{-1}$	0.405	0.269	0.414	0.275	0.437	0.291
$t_r \times 10^3$ (kips)$^{-1}$	0.525	0.350	0.537	0.358	0.567	0.378
r_x/r_y	4.58		6.29		4.64	
r_y, in.	3.54		2.51		3.45	

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W40

Shape		W40×											
		183[c,v]				167[c,v]				149[c,v]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.560	0.373	0.354	0.236	0.614	0.408	0.395	0.263	0.709	0.472	0.458	0.305
	11	0.685	0.456	0.389	0.259	0.758	0.504	0.440	0.293	0.886	0.590	0.520	0.346
	12	0.713	0.475	0.401	0.267	0.791	0.526	0.454	0.302	0.927	0.617	0.538	0.358
	13	0.746	0.496	0.413	0.275	0.829	0.552	0.470	0.312	0.976	0.649	0.558	0.371
	14	0.783	0.521	0.427	0.284	0.874	0.581	0.486	0.323	1.03	0.687	0.579	0.385
	15	0.826	0.550	0.441	0.293	0.925	0.616	0.504	0.335	1.10	0.731	0.602	0.401
	16	0.876	0.583	0.457	0.304	0.985	0.656	0.523	0.348	1.18	0.782	0.627	0.417
	17	0.933	0.621	0.473	0.315	1.06	0.702	0.543	0.361	1.27	0.844	0.654	0.435
	18	1.00	0.665	0.491	0.327	1.14	0.757	0.565	0.376	1.38	0.916	0.684	0.455
	19	1.08	0.717	0.510	0.339	1.23	0.821	0.589	0.392	1.51	1.00	0.716	0.477
	20	1.17	0.778	0.531	0.353	1.35	0.898	0.615	0.409	1.67	1.11	0.752	0.500
	22	1.40	0.934	0.577	0.384	1.63	1.09	0.693	0.461	2.02	1.34	0.882	0.587
	24	1.67	1.11	0.669	0.445	1.94	1.29	0.804	0.535	2.40	1.60	1.03	0.683
	26	1.96	1.30	0.763	0.507	2.28	1.52	0.919	0.611	2.82	1.88	1.18	0.783
	28	2.27	1.51	0.859	0.571	2.65	1.76	1.04	0.690	3.27	2.18	1.33	0.887
	30	2.61	1.74	0.957	0.636	3.04	2.02	1.16	0.771	3.75	2.50	1.49	0.993
	32	2.97	1.98	1.06	0.702	3.45	2.30	1.28	0.853	4.27	2.84	1.66	1.10
	34	3.35	2.23	1.16	0.769	3.90	2.59	1.41	0.937	4.82	3.21	1.82	1.21
	36	3.76	2.50	1.26	0.837	4.37	2.91	1.53	1.02	5.41	3.60	1.99	1.33
	38	4.19	2.79	1.36	0.905	4.87	3.24	1.66	1.11	6.02	4.01	2.16	1.44
	40	4.64	3.09	1.46	0.973	5.40	3.59	1.79	1.19				

Other Constants and Properties

	183 ASD	183 LRFD	167 ASD	167 LRFD	149 ASD	149 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	3.10	2.06	3.61	2.40	4.41	2.94
$t_y \times 10^3$ (kips)$^{-1}$	0.482	0.321	0.521	0.347	0.587	0.390
$t_r \times 10^3$ (kips)$^{-1}$	0.625	0.417	0.676	0.451	0.761	0.507
r_x/r_y	6.31		6.38		6.55	
r_y, in.	2.49		2.40		2.29	

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W36

F_y = 65 ksi

Shape	W36× 652[h]				W36× 529[h]				W36× 487[h]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.134	0.0890	0.0942	0.0627	0.165	0.110	0.118	0.0783	0.180	0.120	0.129	0.0856
11	0.148	0.0982	0.0942	0.0627	0.183	0.122	0.118	0.0783	0.200	0.133	0.129	0.0856
12	0.150	0.100	0.0942	0.0627	0.186	0.124	0.118	0.0783	0.204	0.136	0.129	0.0856
13	0.154	0.102	0.0944	0.0628	0.190	0.127	0.118	0.0787	0.208	0.139	0.130	0.0863
14	0.157	0.104	0.0952	0.0634	0.195	0.130	0.120	0.0796	0.213	0.142	0.131	0.0873
15	0.161	0.107	0.0961	0.0639	0.200	0.133	0.121	0.0804	0.219	0.145	0.133	0.0883
16	0.165	0.110	0.0969	0.0645	0.205	0.136	0.122	0.0813	0.225	0.149	0.134	0.0893
17	0.169	0.113	0.0977	0.0650	0.211	0.140	0.124	0.0822	0.231	0.154	0.136	0.0904
18	0.174	0.116	0.0986	0.0656	0.217	0.145	0.125	0.0831	0.238	0.159	0.137	0.0915
19	0.180	0.119	0.0995	0.0662	0.224	0.149	0.126	0.0841	0.246	0.164	0.139	0.0926
20	0.185	0.123	0.100	0.0668	0.232	0.154	0.128	0.0850	0.255	0.169	0.141	0.0937
22	0.198	0.132	0.102	0.0680	0.249	0.166	0.131	0.0870	0.274	0.182	0.144	0.0961
24	0.214	0.142	0.104	0.0693	0.270	0.179	0.134	0.0890	0.297	0.198	0.148	0.0986
26	0.232	0.154	0.106	0.0706	0.294	0.195	0.137	0.0912	0.324	0.216	0.152	0.101
28	0.253	0.169	0.108	0.0720	0.322	0.214	0.141	0.0935	0.356	0.237	0.156	0.104
30	0.278	0.185	0.110	0.0734	0.356	0.237	0.144	0.0959	0.394	0.262	0.161	0.107
32	0.308	0.205	0.113	0.0749	0.395	0.263	0.148	0.0984	0.439	0.292	0.165	0.110
34	0.343	0.228	0.115	0.0765	0.444	0.295	0.152	0.101	0.494	0.329	0.170	0.113
36	0.385	0.256	0.117	0.0781	0.497	0.331	0.156	0.104	0.554	0.368	0.175	0.117
38	0.429	0.285	0.120	0.0798	0.554	0.369	0.161	0.107	0.617	0.410	0.181	0.120
40	0.475	0.316	0.123	0.0816	0.614	0.409	0.165	0.110	0.684	0.455	0.187	0.124
42	0.524	0.348	0.125	0.0834	0.677	0.450	0.170	0.113	0.754	0.501	0.193	0.128
44	0.575	0.382	0.128	0.0853	0.743	0.494	0.176	0.117	0.827	0.550	0.200	0.133
46	0.628	0.418	0.131	0.0873	0.812	0.540	0.181	0.121	0.904	0.601	0.207	0.138
48	0.684	0.455	0.134	0.0895	0.884	0.588	0.187	0.125	0.984	0.655	0.215	0.143
50	0.742	0.494	0.138	0.0917	0.960	0.638	0.194	0.129	1.07	0.711	0.226	0.150
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	0.472		0.314		0.604		0.402		0.665		0.443	
$t_y \times 10^3$ (kips)$^{-1}$	0.134		0.0890		0.165		0.110		0.180		0.120	
$t_r \times 10^3$ (kips)$^{-1}$	0.174		0.116		0.214		0.142		0.233		0.155	
r_x/r_y	3.95				4.00				3.99			
r_y, in.	4.10				4.00				3.96			

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W36

Shape		W36×											
		441[h]				395[h]				361[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.198	0.131	0.143	0.0955	0.221	0.147	0.160	0.107	0.242	0.161	0.177	0.118
	11	0.220	0.146	0.143	0.0955	0.247	0.164	0.160	0.107	0.271	0.180	0.177	0.118
	12	0.225	0.149	0.143	0.0955	0.252	0.168	0.160	0.107	0.277	0.184	0.177	0.118
	13	0.230	0.153	0.143	0.0955	0.258	0.172	0.162	0.108	0.283	0.189	0.180	0.120
	14	0.235	0.157	0.143	0.0955	0.265	0.176	0.165	0.110	0.290	0.193	0.182	0.121
	15	0.241	0.161	0.143	0.0955	0.272	0.181	0.167	0.111	0.298	0.199	0.185	0.123
	16	0.248	0.165	0.143	0.0955	0.280	0.186	0.169	0.113	0.307	0.204	0.188	0.125
	17	0.256	0.170	0.143	0.0955	0.288	0.192	0.172	0.114	0.317	0.211	0.191	0.127
	18	0.264	0.175	0.143	0.0955	0.297	0.198	0.174	0.116	0.327	0.218	0.194	0.129
	19	0.273	0.181	0.143	0.0955	0.308	0.205	0.177	0.118	0.338	0.225	0.197	0.131
	20	0.282	0.188	0.143	0.0955	0.319	0.212	0.179	0.119	0.351	0.233	0.200	0.133
	22	0.304	0.202	0.143	0.0955	0.344	0.229	0.185	0.123	0.379	0.252	0.206	0.137
	24	0.330	0.220	0.143	0.0955	0.374	0.249	0.191	0.127	0.413	0.274	0.213	0.142
	26	0.361	0.240	0.143	0.0955	0.410	0.272	0.197	0.131	0.452	0.301	0.221	0.147
	28	0.397	0.264	0.143	0.0955	0.452	0.301	0.204	0.135	0.500	0.333	0.229	0.152
	30	0.441	0.293	0.143	0.0955	0.502	0.334	0.211	0.140	0.556	0.370	0.238	0.158
	32	0.492	0.327	0.154	0.103	0.562	0.374	0.218	0.145	0.624	0.415	0.247	0.165
	34	0.554	0.369	0.168	0.111	0.634	0.422	0.227	0.151	0.705	0.469	0.258	0.171
	36	0.622	0.414	0.181	0.120	0.711	0.473	0.236	0.157	0.790	0.526	0.269	0.179
	38	0.693	0.461	0.194	0.129	0.792	0.527	0.245	0.163	0.880	0.586	0.281	0.187
	40	0.767	0.511	0.207	0.138	0.878	0.584	0.256	0.170	0.976	0.649	0.296	0.197
	42	0.846	0.563	0.221	0.147	0.968	0.644	0.269	0.179	1.08	0.716	0.316	0.210
	44	0.928	0.618	0.234	0.155	1.06	0.707	0.285	0.190	1.18	0.785	0.336	0.224
	46	1.01	0.675	0.247	0.164	1.16	0.772	0.302	0.201	1.29	0.858	0.356	0.237
	48	1.10	0.735	0.260	0.173	1.26	0.841	0.318	0.212	1.40	0.935	0.376	0.250
	50	1.20	0.798	0.273	0.182	1.37	0.913	0.334	0.222	1.52	1.01	0.395	0.263

Other Constants and Properties

Property	441[h] ASD	441[h] LRFD	395[h] ASD	395[h] LRFD	361[h] ASD	361[h] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	0.745	0.495	0.843	0.561	0.935	0.622
$t_y \times 10^3$ (kips)$^{-1}$	0.198	0.131	0.221	0.147	0.242	0.161
$t_r \times 10^3$ (kips)$^{-1}$	0.256	0.171	0.287	0.192	0.314	0.210
r_x/r_y	4.01		4.05		4.05	
r_y, in.	3.92		3.88		3.85	

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

W36

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W36×											
	330				302[c]				282[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.265	0.176	0.194	0.129	0.294	0.196	0.214	0.142	0.321	0.214	0.230	0.153
11	0.297	0.197	0.194	0.129	0.325	0.216	0.214	0.142	0.354	0.236	0.230	0.153
12	0.303	0.202	0.195	0.130	0.331	0.221	0.215	0.143	0.361	0.240	0.231	0.154
13	0.310	0.207	0.198	0.131	0.338	0.225	0.218	0.145	0.369	0.245	0.235	0.156
14	0.318	0.212	0.201	0.134	0.347	0.231	0.222	0.148	0.377	0.251	0.239	0.159
15	0.327	0.218	0.204	0.136	0.357	0.237	0.225	0.150	0.387	0.257	0.243	0.162
16	0.337	0.224	0.207	0.138	0.367	0.244	0.229	0.152	0.397	0.264	0.247	0.164
17	0.347	0.231	0.210	0.140	0.379	0.252	0.233	0.155	0.408	0.272	0.252	0.167
18	0.359	0.239	0.214	0.142	0.391	0.260	0.237	0.158	0.421	0.280	0.256	0.170
19	0.371	0.247	0.218	0.145	0.405	0.269	0.241	0.160	0.436	0.290	0.261	0.174
20	0.385	0.256	0.221	0.147	0.420	0.280	0.245	0.163	0.453	0.301	0.266	0.177
22	0.417	0.277	0.229	0.152	0.455	0.302	0.255	0.169	0.490	0.326	0.276	0.184
24	0.454	0.302	0.238	0.158	0.496	0.330	0.264	0.176	0.535	0.356	0.287	0.191
26	0.498	0.331	0.247	0.164	0.544	0.362	0.275	0.183	0.588	0.391	0.299	0.199
28	0.551	0.367	0.256	0.171	0.602	0.401	0.286	0.191	0.652	0.434	0.313	0.208
30	0.614	0.409	0.267	0.178	0.671	0.447	0.299	0.199	0.727	0.484	0.327	0.218
32	0.690	0.459	0.278	0.185	0.755	0.503	0.313	0.208	0.820	0.545	0.343	0.228
34	0.779	0.518	0.291	0.194	0.853	0.567	0.327	0.218	0.925	0.616	0.361	0.240
36	0.874	0.581	0.305	0.203	0.956	0.636	0.344	0.229	1.04	0.690	0.385	0.256
38	0.973	0.648	0.322	0.214	1.07	0.709	0.371	0.247	1.16	0.769	0.417	0.277
40	1.08	0.717	0.346	0.230	1.18	0.785	0.399	0.266	1.28	0.852	0.449	0.299
42	1.19	0.791	0.369	0.246	1.30	0.866	0.428	0.284	1.41	0.939	0.482	0.320
44	1.30	0.868	0.393	0.262	1.43	0.950	0.456	0.303	1.55	1.03	0.514	0.342
46	1.43	0.949	0.417	0.277	1.56	1.04	0.484	0.322	1.69	1.13	0.547	0.364
48	1.55	1.03	0.441	0.293	1.70	1.13	0.513	0.341	1.84	1.23	0.580	0.386
50	1.69	1.12	0.465	0.309	1.84	1.23	0.541	0.360	2.00	1.33	0.612	0.407
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.03		0.688		1.14		0.757		1.23		0.818	
$t_y \times 10^3$ (kips)$^{-1}$	0.265		0.176		0.289		0.192		0.310		0.206	
$t_r \times 10^3$ (kips)$^{-1}$	0.344		0.229		0.375		0.250		0.402		0.268	
r_x/r_y	4.05				4.03				4.05			
r_y, in.	3.83				3.82				3.80			

[c] Shape is slender for compression with F_y = 65 ksi.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W36

Shape	W36× 262[c]				W36× 256[c]				W36× 247[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.350	0.233	0.249	0.166	0.349	0.232	0.264	0.175	0.377	0.251	0.266	0.177
11	0.386	0.257	0.249	0.166	0.432	0.287	0.281	0.187	0.416	0.277	0.266	0.177
12	0.393	0.262	0.251	0.167	0.452	0.301	0.288	0.191	0.424	0.282	0.268	0.178
13	0.402	0.267	0.255	0.170	0.474	0.316	0.295	0.196	0.433	0.288	0.273	0.181
14	0.411	0.273	0.260	0.173	0.500	0.333	0.302	0.201	0.443	0.295	0.278	0.185
15	0.421	0.280	0.264	0.176	0.529	0.352	0.310	0.206	0.454	0.302	0.283	0.188
16	0.433	0.288	0.269	0.179	0.562	0.374	0.319	0.212	0.466	0.310	0.288	0.192
17	0.445	0.296	0.274	0.182	0.599	0.399	0.327	0.218	0.479	0.319	0.294	0.195
18	0.459	0.305	0.279	0.186	0.642	0.427	0.337	0.224	0.494	0.329	0.300	0.199
19	0.474	0.315	0.285	0.190	0.690	0.459	0.347	0.231	0.511	0.340	0.306	0.203
20	0.491	0.327	0.291	0.193	0.744	0.495	0.357	0.238	0.528	0.352	0.312	0.208
22	0.532	0.354	0.302	0.201	0.877	0.583	0.380	0.253	0.570	0.379	0.325	0.216
24	0.581	0.387	0.315	0.210	1.04	0.694	0.406	0.270	0.623	0.414	0.340	0.226
26	0.640	0.426	0.330	0.219	1.22	0.815	0.436	0.290	0.687	0.457	0.356	0.237
28	0.711	0.473	0.345	0.230	1.42	0.945	0.484	0.322	0.763	0.508	0.373	0.248
30	0.795	0.529	0.362	0.241	1.63	1.08	0.533	0.355	0.855	0.569	0.392	0.261
32	0.899	0.598	0.381	0.253	1.86	1.23	0.582	0.387	0.967	0.644	0.414	0.275
34	1.01	0.675	0.402	0.267	2.09	1.39	0.632	0.420	1.09	0.727	0.444	0.295
36	1.14	0.757	0.438	0.291	2.35	1.56	0.681	0.453	1.22	0.815	0.485	0.323
38	1.27	0.843	0.475	0.316	2.62	1.74	0.730	0.486	1.36	0.908	0.528	0.351
40	1.40	0.934	0.513	0.341	2.90	1.93	0.779	0.519	1.51	1.01	0.570	0.379
42	1.55	1.03	0.551	0.367	3.20	2.13	0.828	0.551	1.67	1.11	0.613	0.408
44	1.70	1.13	0.589	0.392	3.51	2.33	0.877	0.584	1.83	1.22	0.657	0.437
46	1.86	1.24	0.628	0.418					2.00	1.33	0.700	0.466
48	2.02	1.35	0.666	0.443					2.18	1.45	0.744	0.495
50	2.19	1.46	0.705	0.469					2.36	1.57	0.788	0.524

Other Constants and Properties

	262 ASD	262 LRFD	256 ASD	256 LRFD	247 ASD	247 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.34	0.894	2.00	1.33	1.44	0.960
$t_y \times 10^3$ (kips)$^{-1}$	0.333	0.221	0.341	0.227	0.354	0.236
$t_r \times 10^3$ (kips)$^{-1}$	0.432	0.288	0.443	0.295	0.460	0.307
r_x/r_y	4.07		5.62		4.06	
r_y, in.	3.76		2.65		3.74	

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W36

Table 6-1 (continued)

Combined Flexure and Axial Force

W-Shapes

F_y = 65 ksi

Shape	W36×											
	232[c]				231[c]				210[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.398	0.265	0.293	0.195	0.406	0.270	0.285	0.189	0.445	0.296	0.329	0.219
11	0.487	0.324	0.314	0.209	0.448	0.298	0.285	0.189	0.545	0.363	0.355	0.236
12	0.507	0.337	0.322	0.214	0.457	0.304	0.287	0.191	0.567	0.378	0.365	0.243
13	0.530	0.353	0.330	0.220	0.466	0.310	0.292	0.195	0.594	0.395	0.376	0.250
14	0.559	0.372	0.339	0.226	0.477	0.317	0.298	0.198	0.624	0.415	0.386	0.257
15	0.592	0.394	0.349	0.232	0.489	0.325	0.304	0.202	0.659	0.439	0.398	0.265
16	0.629	0.419	0.359	0.239	0.502	0.334	0.309	0.206	0.703	0.467	0.410	0.273
17	0.672	0.447	0.369	0.246	0.517	0.344	0.316	0.210	0.752	0.500	0.423	0.282
18	0.721	0.480	0.380	0.253	0.533	0.354	0.322	0.214	0.808	0.538	0.437	0.291
19	0.776	0.516	0.392	0.261	0.550	0.366	0.329	0.219	0.872	0.580	0.452	0.301
20	0.839	0.558	0.405	0.269	0.570	0.379	0.336	0.223	0.945	0.629	0.468	0.311
22	0.993	0.661	0.432	0.288	0.614	0.409	0.350	0.233	1.13	0.749	0.503	0.335
24	1.18	0.787	0.464	0.309	0.669	0.445	0.367	0.244	1.34	0.891	0.545	0.362
26	1.39	0.923	0.512	0.341	0.738	0.491	0.384	0.256	1.57	1.05	0.616	0.410
28	1.61	1.07	0.571	0.380	0.822	0.547	0.404	0.269	1.82	1.21	0.690	0.459
30	1.85	1.23	0.631	0.420	0.922	0.613	0.425	0.283	2.09	1.39	0.765	0.509
32	2.10	1.40	0.691	0.460	1.05	0.695	0.449	0.299	2.38	1.58	0.841	0.559
34	2.37	1.58	0.751	0.500	1.18	0.785	0.489	0.326	2.69	1.79	0.917	0.610
36	2.66	1.77	0.812	0.540	1.32	0.880	0.536	0.357	3.01	2.00	0.993	0.661
38	2.96	1.97	0.872	0.580	1.47	0.981	0.583	0.388	3.36	2.23	1.07	0.712
40	3.28	2.18	0.932	0.620	1.63	1.09	0.631	0.420	3.72	2.48	1.15	0.763
42	3.62	2.41	0.992	0.660	1.80	1.20	0.680	0.452	4.10	2.73	1.22	0.814
44					1.98	1.31	0.729	0.485				
46					2.16	1.44	0.778	0.518				
48					2.35	1.56	0.828	0.551				
50					2.55	1.70	0.878	0.584				
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	2.25		1.49		1.56		1.04		2.56		1.70	
$t_y \times 10^3$ (kips)$^{-1}$	0.378		0.251		0.377		0.251		0.415		0.276	
$t_r \times 10^3$ (kips)$^{-1}$	0.490		0.327		0.489		0.326		0.539		0.359	
r_x/r_y	5.65				4.07				5.66			
r_y, in.	2.62				3.71				2.58			

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W36

Shape		W36×											
		194[c]				182[c]				170[c,v]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.495	0.329	0.357	0.238	0.536	0.356	0.382	0.254	0.586	0.390	0.410	0.273
	11	0.605	0.403	0.388	0.258	0.653	0.435	0.415	0.276	0.713	0.474	0.448	0.298
	12	0.630	0.419	0.399	0.265	0.680	0.452	0.427	0.284	0.742	0.493	0.461	0.307
	13	0.658	0.438	0.410	0.273	0.710	0.472	0.440	0.293	0.775	0.516	0.476	0.317
	14	0.691	0.460	0.423	0.281	0.745	0.496	0.454	0.302	0.813	0.541	0.491	0.327
	15	0.730	0.485	0.436	0.290	0.786	0.523	0.468	0.311	0.857	0.570	0.507	0.337
	16	0.774	0.515	0.450	0.299	0.833	0.554	0.484	0.322	0.908	0.604	0.524	0.349
	17	0.824	0.548	0.465	0.309	0.887	0.590	0.500	0.333	0.967	0.643	0.543	0.361
	18	0.887	0.590	0.481	0.320	0.950	0.632	0.518	0.344	1.04	0.689	0.563	0.374
	19	0.958	0.637	0.498	0.331	1.02	0.682	0.537	0.357	1.11	0.742	0.584	0.389
	20	1.04	0.691	0.516	0.343	1.11	0.740	0.557	0.371	1.21	0.804	0.607	0.404
	22	1.24	0.826	0.557	0.370	1.33	0.885	0.603	0.401	1.45	0.964	0.659	0.439
	24	1.48	0.983	0.616	0.410	1.58	1.05	0.677	0.451	1.72	1.15	0.751	0.500
	26	1.73	1.15	0.700	0.466	1.86	1.24	0.771	0.513	2.02	1.35	0.857	0.570
	28	2.01	1.34	0.786	0.523	2.16	1.43	0.868	0.577	2.35	1.56	0.966	0.643
	30	2.31	1.54	0.873	0.581	2.47	1.65	0.966	0.642	2.69	1.79	1.08	0.717
	32	2.63	1.75	0.961	0.639	2.81	1.87	1.07	0.709	3.07	2.04	1.19	0.792
	34	2.96	1.97	1.05	0.699	3.18	2.11	1.17	0.775	3.46	2.30	1.31	0.869
	36	3.32	2.21	1.14	0.758	3.56	2.37	1.27	0.843	3.88	2.58	1.42	0.946
	38	3.70	2.46	1.23	0.818	3.97	2.64	1.37	0.911	4.32	2.88	1.54	1.02
	40	4.10	2.73	1.32	0.878	4.40	2.93	1.47	0.979	4.79	3.19	1.66	1.10
	42	4.52	3.01	1.41	0.938	4.85	3.23	1.57	1.05	5.28	3.51	1.77	1.18

Other Constants and Properties

	194 ASD	194 LRFD	182 ASD	182 LRFD	170 ASD	170 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	2.81	1.87	3.02	2.01	3.27	2.18
$t_y \times 10^3$ (kips)$^{-1}$	0.451	0.300	0.479	0.319	0.514	0.342
$t_r \times 10^3$ (kips)$^{-1}$	0.585	0.390	0.622	0.415	0.667	0.444
r_x/r_y	5.70		5.69		5.73	
r_y, in.	2.56		2.55		2.53	

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W36

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W36×											
	160[c,v]				150[c,v]				135[c,v]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending: 0	0.633	0.421	0.439	0.292	0.682	0.454	0.472	0.314	0.777	0.517	0.538	0.358
11	0.771	0.513	0.482	0.321	0.832	0.553	0.520	0.346	0.954	0.635	0.602	0.401
12	0.803	0.534	0.497	0.331	0.866	0.576	0.537	0.357	0.995	0.662	0.623	0.414
13	0.839	0.558	0.513	0.341	0.905	0.602	0.555	0.369	1.04	0.694	0.645	0.429
14	0.880	0.586	0.530	0.352	0.950	0.632	0.573	0.382	1.10	0.730	0.668	0.445
15	0.928	0.618	0.548	0.364	1.00	0.667	0.594	0.395	1.16	0.773	0.693	0.461
16	0.984	0.655	0.567	0.377	1.06	0.707	0.615	0.409	1.24	0.823	0.721	0.479
17	1.05	0.698	0.588	0.391	1.13	0.754	0.639	0.425	1.32	0.881	0.750	0.499
18	1.12	0.748	0.610	0.406	1.22	0.809	0.664	0.442	1.43	0.950	0.782	0.520
19	1.21	0.806	0.634	0.422	1.31	0.873	0.691	0.460	1.55	1.03	0.817	0.543
20	1.31	0.874	0.660	0.439	1.43	0.949	0.720	0.479	1.70	1.13	0.854	0.569
22	1.58	1.05	0.719	0.478	1.72	1.14	0.796	0.530	2.05	1.37	0.977	0.650
24	1.88	1.25	0.830	0.553	2.04	1.36	0.926	0.616	2.44	1.62	1.14	0.758
26	2.20	1.47	0.950	0.632	2.40	1.59	1.06	0.706	2.87	1.91	1.31	0.871
28	2.56	1.70	1.07	0.714	2.78	1.85	1.20	0.799	3.32	2.21	1.49	0.989
30	2.94	1.95	1.20	0.797	3.19	2.12	1.34	0.894	3.82	2.54	1.67	1.11
32	3.34	2.22	1.33	0.883	3.63	2.42	1.49	0.991	4.34	2.89	1.85	1.23
34	3.77	2.51	1.46	0.969	4.10	2.73	1.64	1.09	4.90	3.26	2.05	1.36
36	4.23	2.81	1.59	1.06	4.59	3.06	1.79	1.19	5.49	3.66	2.24	1.49
38	4.71	3.13	1.72	1.15	5.12	3.41	1.94	1.29	6.12	4.07	2.44	1.62
40	5.22	3.47	1.86	1.23	5.67	3.77	2.10	1.40				
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	3.55		2.36		3.87		2.57		4.59		3.05	
$t_y \times 10^3$ (kips)$^{-1}$	0.547		0.364		0.580		0.386		0.644		0.428	
$t_r \times 10^3$ (kips)$^{-1}$	0.709		0.473		0.752		0.502		0.835		0.557	
r_x/r_y	5.76				5.79				5.88			
r_y, in.	2.50				2.47				2.38			

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W33

Shape	W33× 387[h]				W33× 354[h]				W33× 318			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.225	0.150	0.176	0.117	0.247	0.164	0.193	0.128	0.274	0.182	0.216	0.144
11	0.253	0.168	0.176	0.117	0.278	0.185	0.193	0.128	0.309	0.206	0.216	0.144
12	0.259	0.172	0.176	0.117	0.284	0.189	0.194	0.129	0.316	0.211	0.217	0.145
13	0.265	0.176	0.179	0.119	0.291	0.194	0.197	0.131	0.324	0.216	0.221	0.147
14	0.272	0.181	0.181	0.120	0.299	0.199	0.199	0.133	0.333	0.222	0.224	0.149
15	0.280	0.186	0.183	0.122	0.308	0.205	0.202	0.135	0.343	0.228	0.227	0.151
16	0.288	0.192	0.186	0.124	0.317	0.211	0.205	0.137	0.354	0.235	0.231	0.154
17	0.298	0.198	0.188	0.125	0.328	0.218	0.208	0.139	0.365	0.243	0.235	0.156
18	0.308	0.205	0.191	0.127	0.339	0.226	0.211	0.141	0.378	0.252	0.238	0.159
19	0.319	0.212	0.194	0.129	0.352	0.234	0.214	0.143	0.393	0.261	0.242	0.161
20	0.331	0.220	0.196	0.131	0.365	0.243	0.218	0.145	0.408	0.272	0.246	0.164
22	0.359	0.239	0.202	0.134	0.397	0.264	0.225	0.149	0.444	0.295	0.255	0.170
24	0.392	0.261	0.208	0.138	0.434	0.289	0.232	0.154	0.486	0.323	0.264	0.176
26	0.432	0.288	0.215	0.143	0.479	0.318	0.240	0.160	0.537	0.357	0.274	0.182
28	0.480	0.319	0.221	0.147	0.532	0.354	0.248	0.165	0.598	0.398	0.285	0.189
30	0.536	0.357	0.229	0.152	0.596	0.397	0.257	0.171	0.671	0.446	0.296	0.197
32	0.605	0.403	0.237	0.157	0.674	0.449	0.267	0.177	0.761	0.506	0.308	0.205
34	0.684	0.455	0.245	0.163	0.761	0.507	0.277	0.184	0.859	0.571	0.322	0.214
36	0.766	0.510	0.254	0.169	0.854	0.568	0.288	0.192	0.963	0.641	0.337	0.224
38	0.854	0.568	0.264	0.175	0.951	0.633	0.300	0.200	1.07	0.714	0.353	0.235
40	0.946	0.629	0.274	0.182	1.05	0.701	0.314	0.209	1.19	0.791	0.378	0.251
42	1.04	0.694	0.285	0.190	1.16	0.773	0.333	0.221	1.31	0.872	0.403	0.268
44	1.14	0.762	0.300	0.200	1.28	0.848	0.353	0.235	1.44	0.957	0.428	0.285
46	1.25	0.832	0.317	0.211	1.39	0.927	0.372	0.248	1.57	1.05	0.453	0.301
48	1.36	0.906	0.333	0.222	1.52	1.01	0.392	0.261	1.71	1.14	0.477	0.318
50	1.48	0.984	0.350	0.233	1.65	1.10	0.412	0.274	1.86	1.24	0.502	0.334

Other Constants and Properties

	387[h] ASD	387[h] LRFD	354[h] ASD	354[h] LRFD	318 ASD	318 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	0.878	0.584	0.972	0.647	1.10	0.729
$t_y \times 10^3$ (kips)$^{-1}$	0.225	0.150	0.247	0.164	0.274	0.182
$t_r \times 10^3$ (kips)$^{-1}$	0.292	0.195	0.321	0.214	0.356	0.237
r_x/r_y	3.87		3.88		3.91	
r_y, in.	3.77		3.74		3.71	

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

W33

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W33×											
	291				263[c]				241[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.300	0.200	0.236	0.157	0.339	0.226	0.264	0.175	0.374	0.249	0.292	0.194
11	0.339	0.226	0.236	0.157	0.378	0.252	0.264	0.175	0.417	0.277	0.292	0.194
12	0.347	0.231	0.238	0.159	0.386	0.257	0.267	0.177	0.426	0.283	0.296	0.197
13	0.356	0.237	0.242	0.161	0.395	0.262	0.271	0.180	0.436	0.290	0.301	0.200
14	0.366	0.243	0.246	0.164	0.406	0.270	0.276	0.183	0.447	0.297	0.306	0.204
15	0.377	0.251	0.250	0.167	0.418	0.278	0.281	0.187	0.459	0.305	0.312	0.208
16	0.389	0.259	0.254	0.169	0.431	0.287	0.285	0.190	0.472	0.314	0.318	0.211
17	0.402	0.267	0.259	0.172	0.446	0.297	0.291	0.193	0.489	0.325	0.324	0.216
18	0.416	0.277	0.263	0.175	0.462	0.308	0.296	0.197	0.507	0.337	0.330	0.220
19	0.432	0.288	0.268	0.178	0.480	0.319	0.302	0.201	0.527	0.351	0.337	0.224
20	0.450	0.299	0.273	0.181	0.500	0.332	0.307	0.204	0.549	0.365	0.344	0.229
22	0.490	0.326	0.283	0.188	0.544	0.362	0.319	0.213	0.599	0.399	0.358	0.238
24	0.537	0.357	0.294	0.196	0.598	0.398	0.333	0.221	0.660	0.439	0.374	0.249
26	0.594	0.395	0.306	0.203	0.662	0.441	0.347	0.231	0.732	0.487	0.391	0.260
28	0.663	0.441	0.319	0.212	0.740	0.492	0.363	0.241	0.820	0.545	0.410	0.273
30	0.745	0.496	0.333	0.221	0.833	0.554	0.380	0.253	0.925	0.616	0.431	0.287
32	0.846	0.563	0.348	0.231	0.946	0.630	0.399	0.265	1.05	0.701	0.455	0.302
34	0.955	0.636	0.364	0.242	1.07	0.711	0.419	0.279	1.19	0.791	0.487	0.324
36	1.07	0.713	0.383	0.255	1.20	0.797	0.452	0.301	1.33	0.887	0.530	0.353
38	1.19	0.794	0.413	0.275	1.33	0.888	0.489	0.325	1.48	0.988	0.574	0.382
40	1.32	0.880	0.443	0.295	1.48	0.984	0.525	0.349	1.65	1.09	0.619	0.412
42	1.46	0.970	0.473	0.315	1.63	1.08	0.562	0.374	1.81	1.21	0.663	0.441
44	1.60	1.06	0.503	0.335	1.79	1.19	0.598	0.398	1.99	1.32	0.708	0.471
46	1.75	1.16	0.533	0.354	1.96	1.30	0.635	0.422	2.18	1.45	0.753	0.501
48	1.90	1.27	0.563	0.374	2.13	1.42	0.672	0.447	2.37	1.58	0.797	0.530
50	2.07	1.37	0.592	0.394	2.31	1.54	0.708	0.471	2.57	1.71	0.842	0.560
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.21		0.807		1.36		0.903		1.51		1.00	
$t_y \times 10^3$ (kips)$^{-1}$	0.300		0.200		0.332		0.221		0.361		0.240	
$t_r \times 10^3$ (kips)$^{-1}$	0.389		0.260		0.431		0.287		0.469		0.313	
r_x/r_y	3.91				3.91				3.90			
r_y, in.	3.68				3.66				3.62			

[c] Shape is slender for compression with F_y = 65 ksi.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W33

Shape	W33×											
	221[c]				201[c]				169[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.414	0.276	0.320	0.213	0.468	0.311	0.355	0.236	0.575	0.382	0.436	0.290
11	0.462	0.307	0.320	0.213	0.521	0.346	0.355	0.236	0.710	0.472	0.476	0.317
12	0.472	0.314	0.325	0.216	0.531	0.354	0.361	0.240	0.740	0.493	0.490	0.326
13	0.482	0.321	0.331	0.220	0.544	0.362	0.368	0.245	0.776	0.516	0.505	0.336
14	0.495	0.329	0.337	0.224	0.557	0.371	0.376	0.250	0.817	0.543	0.521	0.347
15	0.508	0.338	0.344	0.229	0.573	0.381	0.383	0.255	0.864	0.575	0.538	0.358
16	0.523	0.348	0.351	0.233	0.589	0.392	0.391	0.260	0.918	0.611	0.556	0.370
17	0.540	0.359	0.358	0.238	0.608	0.405	0.399	0.266	0.982	0.653	0.575	0.383
18	0.558	0.372	0.365	0.243	0.629	0.418	0.408	0.272	1.06	0.702	0.596	0.396
19	0.579	0.385	0.373	0.248	0.652	0.433	0.417	0.278	1.14	0.761	0.618	0.411
20	0.602	0.400	0.381	0.253	0.677	0.450	0.427	0.284	1.25	0.829	0.642	0.427
22	0.658	0.438	0.398	0.265	0.736	0.489	0.447	0.297	1.50	0.997	0.695	0.463
24	0.725	0.483	0.417	0.277	0.810	0.539	0.469	0.312	1.78	1.19	0.786	0.523
26	0.807	0.537	0.437	0.291	0.902	0.600	0.493	0.328	2.09	1.39	0.891	0.593
28	0.905	0.602	0.460	0.306	1.01	0.675	0.520	0.346	2.43	1.62	0.999	0.664
30	1.02	0.682	0.485	0.323	1.15	0.766	0.551	0.366	2.79	1.85	1.11	0.737
32	1.17	0.776	0.513	0.341	1.31	0.871	0.596	0.397	3.17	2.11	1.22	0.810
34	1.32	0.876	0.562	0.374	1.48	0.984	0.657	0.437	3.58	2.38	1.33	0.883
36	1.48	0.982	0.614	0.408	1.66	1.10	0.719	0.479	4.01	2.67	1.44	0.957
38	1.64	1.09	0.666	0.443	1.85	1.23	0.782	0.520	4.47	2.98	1.55	1.03
40	1.82	1.21	0.719	0.478	2.05	1.36	0.846	0.563	4.95	3.30	1.66	1.10
42	2.01	1.34	0.772	0.514	2.26	1.50	0.910	0.606				
44	2.20	1.47	0.825	0.549	2.48	1.65	0.975	0.649				
46	2.41	1.60	0.879	0.585	2.71	1.80	1.04	0.692				
48	2.62	1.75	0.932	0.620	2.95	1.96	1.11	0.736				
50	2.85	1.89	0.986	0.656	3.20	2.13	1.17	0.780				
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.67		1.11		1.86		1.24		3.25		2.16	
$t_y \times 10^3$ (kips)$^{-1}$	0.393		0.262		0.435		0.289		0.519		0.345	
$t_r \times 10^3$ (kips)$^{-1}$	0.510		0.340		0.564		0.376		0.673		0.449	
r_x / r_y	3.93				3.93				5.48			
r_y, in.	3.59				3.56				2.50			

[c] Shape is slender for compression with F_y = 50 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W33

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W33× 152[c]				W33× 141[c,v]				W33× 130[c,v]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.647	0.430	0.490	0.326	0.713	0.474	0.533	0.355	0.787	0.523	0.587	0.390
11	0.798	0.531	0.540	0.359	0.881	0.586	0.591	0.393	0.974	0.648	0.656	0.436
12	0.832	0.554	0.557	0.370	0.920	0.612	0.610	0.406	1.02	0.677	0.678	0.451
13	0.872	0.580	0.574	0.382	0.964	0.642	0.630	0.419	1.07	0.710	0.702	0.467
14	0.918	0.611	0.593	0.395	1.02	0.676	0.652	0.434	1.13	0.749	0.727	0.484
15	0.971	0.646	0.614	0.408	1.08	0.716	0.675	0.449	1.19	0.793	0.754	0.502
16	1.03	0.687	0.636	0.423	1.15	0.762	0.700	0.466	1.27	0.846	0.784	0.521
17	1.10	0.735	0.659	0.438	1.23	0.816	0.727	0.483	1.36	0.907	0.816	0.543
18	1.19	0.791	0.684	0.455	1.32	0.879	0.756	0.503	1.47	0.979	0.850	0.566
19	1.29	0.856	0.711	0.473	1.43	0.953	0.787	0.524	1.60	1.06	0.888	0.591
20	1.40	0.934	0.741	0.493	1.56	1.04	0.821	0.546	1.75	1.17	0.929	0.618
22	1.69	1.13	0.810	0.539	1.89	1.26	0.918	0.611	2.12	1.41	1.06	0.707
24	2.01	1.34	0.936	0.623	2.25	1.50	1.06	0.708	2.52	1.68	1.23	0.821
26	2.36	1.57	1.07	0.709	2.64	1.76	1.21	0.808	2.96	1.97	1.41	0.939
28	2.74	1.82	1.20	0.798	3.07	2.04	1.37	0.911	3.43	2.28	1.60	1.06
30	3.15	2.09	1.33	0.888	3.52	2.34	1.53	1.02	3.94	2.62	1.78	1.19
32	3.58	2.38	1.47	0.979	4.00	2.66	1.69	1.12	4.48	2.98	1.98	1.32
34	4.04	2.69	1.61	1.07	4.52	3.01	1.85	1.23	5.06	3.37	2.17	1.45
36	4.53	3.02	1.75	1.16	5.07	3.37	2.02	1.34	5.68	3.78	2.37	1.58
38	5.05	3.36	1.89	1.26	5.65	3.76	2.18	1.45	6.32	4.21	2.57	1.71
40	5.60	3.72	2.03	1.35	6.26	4.16	2.35	1.56				

Other Constants and Properties

	152 ASD	152 LRFD	141 ASD	141 LRFD	130 ASD	130 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	3.71	2.47	4.10	2.73	4.61	3.06
$t_y \times 10^3$ (kips)$^{-1}$	0.572	0.381	0.619	0.412	0.671	0.446
$t_r \times 10^3$ (kips)$^{-1}$	0.742	0.495	0.803	0.535	0.870	0.580
r_x/r_y	5.47		5.51		5.52	
r_y, in.	2.47		2.43		2.39	

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W33-W30

Shape		W33×				W30×							
		118[c,v]				391[h]				357[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.890	0.592	0.660	0.439	0.223	0.149	0.189	0.126	0.245	0.163	0.208	0.138
	11	1.11	0.738	0.747	0.497	0.253	0.168	0.189	0.126	0.277	0.184	0.208	0.138
	12	1.16	0.771	0.773	0.514	0.259	0.172	0.190	0.127	0.284	0.189	0.209	0.139
	13	1.22	0.811	0.802	0.533	0.265	0.176	0.192	0.128	0.291	0.194	0.212	0.141
	14	1.29	0.856	0.832	0.554	0.273	0.181	0.195	0.130	0.300	0.199	0.215	0.143
	15	1.37	0.910	0.865	0.576	0.281	0.187	0.197	0.131	0.309	0.205	0.218	0.145
	16	1.46	0.972	0.901	0.600	0.290	0.193	0.199	0.133	0.319	0.212	0.220	0.147
	17	1.57	1.05	0.940	0.625	0.300	0.199	0.202	0.134	0.330	0.219	0.223	0.149
	18	1.70	1.13	0.982	0.654	0.311	0.207	0.204	0.136	0.342	0.228	0.226	0.151
	19	1.86	1.24	1.03	0.684	0.322	0.215	0.207	0.138	0.355	0.236	0.229	0.153
	20	2.05	1.37	1.08	0.718	0.335	0.223	0.209	0.139	0.370	0.246	0.233	0.155
	22	2.48	1.65	1.27	0.845	0.365	0.243	0.215	0.143	0.403	0.268	0.239	0.159
	24	2.95	1.97	1.48	0.984	0.401	0.267	0.220	0.147	0.444	0.295	0.246	0.164
	26	3.47	2.31	1.70	1.13	0.444	0.295	0.226	0.151	0.492	0.327	0.254	0.169
	28	4.02	2.68	1.92	1.28	0.496	0.330	0.233	0.155	0.550	0.366	0.261	0.174
	30	4.62	3.07	2.16	1.44	0.558	0.371	0.239	0.159	0.620	0.413	0.270	0.179
	32	5.25	3.49	2.40	1.59	0.633	0.421	0.246	0.164	0.705	0.469	0.279	0.185
	34	5.93	3.95	2.64	1.76	0.715	0.476	0.254	0.169	0.796	0.530	0.288	0.192
	36	6.65	4.42	2.89	1.92	0.802	0.533	0.262	0.174	0.892	0.594	0.298	0.199
	38	7.41	4.93	3.14	2.09	0.893	0.594	0.270	0.180	0.994	0.662	0.309	0.206
	40					0.990	0.658	0.279	0.186	1.10	0.733	0.321	0.214
	42					1.09	0.726	0.289	0.192	1.21	0.808	0.334	0.222
	44					1.20	0.797	0.299	0.199	1.33	0.887	0.349	0.233
	46					1.31	0.871	0.310	0.206	1.46	0.969	0.368	0.245
	48					1.43	0.948	0.325	0.216	1.59	1.06	0.387	0.257
	50					1.55	1.03	0.340	0.226	1.72	1.15	0.406	0.270

Other Constants and Properties

Property	W33×118 ASD	W33×118 LRFD	W30×391 ASD	W30×391 LRFD	W30×357 ASD	W30×357 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	5.34	3.55	0.884	0.588	0.982	0.654
$t_y \times 10^3$ (kips)$^{-1}$	0.740	0.493	0.223	0.149	0.245	0.163
$t_r \times 10^3$ (kips)$^{-1}$	0.961	0.640	0.290	0.193	0.317	0.212
r_x/r_y	5.60		3.65		3.65	
r_y, in.	2.32		3.67		3.64	

[c] Shape is slender for compression with F_y = 65 ksi.

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

W30

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W30× 326[h]				W30× 292				W30× 261			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.268	0.178	0.230	0.153	0.299	0.199	0.259	0.172	0.334	0.222	0.291	0.193
	11	0.304	0.203	0.230	0.153	0.340	0.226	0.259	0.172	0.381	0.254	0.291	0.194
	12	0.312	0.208	0.233	0.155	0.348	0.232	0.262	0.174	0.391	0.260	0.296	0.197
	13	0.320	0.213	0.236	0.157	0.358	0.238	0.266	0.177	0.402	0.267	0.300	0.200
	14	0.330	0.219	0.239	0.159	0.368	0.245	0.270	0.179	0.414	0.275	0.305	0.203
	15	0.340	0.226	0.242	0.161	0.380	0.253	0.274	0.182	0.427	0.284	0.311	0.207
	16	0.351	0.234	0.246	0.164	0.393	0.261	0.278	0.185	0.442	0.294	0.316	0.210
	17	0.364	0.242	0.249	0.166	0.407	0.271	0.282	0.188	0.458	0.305	0.321	0.214
	18	0.377	0.251	0.253	0.168	0.422	0.281	0.287	0.191	0.476	0.317	0.327	0.218
	19	0.392	0.261	0.257	0.171	0.439	0.292	0.292	0.194	0.496	0.330	0.333	0.221
	20	0.409	0.272	0.261	0.173	0.458	0.305	0.296	0.197	0.518	0.344	0.339	0.225
	22	0.447	0.297	0.269	0.179	0.501	0.333	0.306	0.204	0.568	0.378	0.352	0.234
	24	0.492	0.328	0.277	0.184	0.553	0.368	0.317	0.211	0.628	0.418	0.365	0.243
	26	0.547	0.364	0.286	0.190	0.615	0.409	0.329	0.219	0.701	0.466	0.380	0.253
	28	0.613	0.408	0.296	0.197	0.690	0.459	0.341	0.227	0.789	0.525	0.397	0.264
	30	0.694	0.462	0.306	0.204	0.782	0.520	0.355	0.236	0.899	0.598	0.414	0.276
	32	0.789	0.525	0.318	0.211	0.890	0.592	0.369	0.246	1.02	0.680	0.434	0.289
	34	0.891	0.593	0.330	0.219	1.00	0.669	0.385	0.256	1.15	0.768	0.455	0.303
	36	0.999	0.665	0.342	0.228	1.13	0.749	0.402	0.268	1.29	0.861	0.481	0.320
	38	1.11	0.741	0.356	0.237	1.26	0.835	0.421	0.280	1.44	0.959	0.517	0.344
	40	1.23	0.821	0.372	0.247	1.39	0.925	0.450	0.299	1.60	1.06	0.554	0.368
	42	1.36	0.905	0.392	0.261	1.53	1.02	0.479	0.318	1.76	1.17	0.590	0.392
	44	1.49	0.993	0.415	0.276	1.68	1.12	0.507	0.337	1.93	1.29	0.626	0.416
	46	1.63	1.09	0.438	0.291	1.84	1.22	0.535	0.356	2.11	1.41	0.662	0.440
	48	1.78	1.18	0.460	0.306	2.00	1.33	0.564	0.375	2.30	1.53	0.698	0.464
	50	1.93	1.28	0.483	0.321	2.17	1.45	0.592	0.394	2.50	1.66	0.734	0.488

Other Constants and Properties

	326[h] ASD	326[h] LRFD	292 ASD	292 LRFD	261 ASD	261 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.09	0.724	1.23	0.818	1.40	0.930
$t_y \times 10^3$ (kips)$^{-1}$	0.268	0.178	0.299	0.199	0.334	0.222
$t_r \times 10^3$ (kips)$^{-1}$	0.348	0.232	0.388	0.258	0.433	0.289
r_x/r_y	3.67		3.69		3.71	
r_y, in.	3.60		3.58		3.53	

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W30

Shape	W30× 235[c]				W30× 211[c]				W30× 191[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.373	0.248	0.324	0.215	0.422	0.281	0.365	0.243	0.479	0.319	0.406	0.270
11	0.424	0.282	0.324	0.216	0.475	0.316	0.366	0.244	0.539	0.359	0.408	0.272
12	0.435	0.289	0.330	0.219	0.487	0.324	0.373	0.248	0.551	0.367	0.416	0.277
13	0.447	0.298	0.335	0.223	0.499	0.332	0.380	0.253	0.565	0.376	0.424	0.282
14	0.461	0.307	0.341	0.227	0.514	0.342	0.387	0.257	0.581	0.387	0.433	0.288
15	0.476	0.317	0.347	0.231	0.531	0.353	0.394	0.262	0.599	0.398	0.441	0.294
16	0.493	0.328	0.353	0.235	0.550	0.366	0.402	0.267	0.618	0.411	0.450	0.300
17	0.511	0.340	0.360	0.240	0.571	0.380	0.410	0.273	0.640	0.426	0.460	0.306
18	0.531	0.354	0.367	0.244	0.594	0.395	0.418	0.278	0.663	0.441	0.470	0.313
19	0.554	0.368	0.374	0.249	0.619	0.412	0.427	0.284	0.692	0.460	0.480	0.319
20	0.578	0.385	0.381	0.254	0.646	0.430	0.436	0.290	0.724	0.481	0.491	0.327
22	0.635	0.422	0.397	0.264	0.710	0.473	0.455	0.303	0.796	0.530	0.514	0.342
24	0.703	0.468	0.413	0.275	0.788	0.524	0.476	0.317	0.885	0.589	0.540	0.359
26	0.786	0.523	0.432	0.287	0.882	0.587	0.499	0.332	0.992	0.660	0.568	0.378
28	0.886	0.589	0.452	0.301	0.995	0.662	0.524	0.349	1.12	0.747	0.599	0.398
30	1.01	0.672	0.474	0.315	1.14	0.756	0.552	0.368	1.28	0.854	0.634	0.422
32	1.15	0.764	0.498	0.331	1.29	0.860	0.584	0.388	1.46	0.972	0.686	0.457
34	1.30	0.863	0.527	0.350	1.46	0.971	0.635	0.423	1.65	1.10	0.753	0.501
36	1.45	0.968	0.571	0.380	1.64	1.09	0.690	0.459	1.85	1.23	0.821	0.546
38	1.62	1.08	0.615	0.409	1.82	1.21	0.746	0.496	2.06	1.37	0.889	0.591
40	1.80	1.19	0.659	0.439	2.02	1.34	0.802	0.533	2.28	1.52	0.957	0.637
42	1.98	1.32	0.704	0.468	2.23	1.48	0.858	0.571	2.52	1.67	1.03	0.683
44	2.17	1.45	0.748	0.498	2.44	1.63	0.914	0.608	2.76	1.84	1.10	0.729
46	2.37	1.58	0.792	0.527	2.67	1.78	0.970	0.645	3.02	2.01	1.16	0.775
48	2.59	1.72	0.837	0.557	2.91	1.94	1.03	0.683	3.29	2.19	1.23	0.821
50	2.81	1.87	0.881	0.586	3.16	2.10	1.08	0.720	3.57	2.37	1.30	0.867

Other Constants and Properties

	235[c] ASD	235[c] LRFD	211[c] ASD	211[c] LRFD	191[c] ASD	191[c] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.57	1.04	1.77	1.18	1.99	1.32
$t_y \times 10^3$ (kips)$^{-1}$	0.371	0.247	0.412	0.274	0.458	0.305
$t_r \times 10^3$ (kips)$^{-1}$	0.481	0.321	0.535	0.357	0.594	0.396
r_x/r_y	3.70		3.70		3.70	
r_y, in.	3.51		3.49		3.46	

[c] Shape is slender for compression with F_y = 65 ksi.

Return to Table of Contents

W30

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W30×											
		173[c]				148[c]				132[c]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.538	0.358	0.451	0.300	0.639	0.425	0.548	0.365	0.734	0.488	0.627	0.417
	11	0.606	0.403	0.455	0.303	0.831	0.553	0.616	0.410	0.953	0.634	0.712	0.474
	12	0.620	0.412	0.464	0.309	0.876	0.583	0.636	0.423	1.00	0.668	0.737	0.490
	13	0.636	0.423	0.474	0.315	0.929	0.618	0.657	0.437	1.07	0.709	0.763	0.508
	14	0.653	0.435	0.483	0.322	0.991	0.659	0.680	0.452	1.14	0.757	0.792	0.527
	15	0.673	0.448	0.494	0.328	1.07	0.709	0.704	0.469	1.22	0.813	0.822	0.547
	16	0.695	0.462	0.504	0.336	1.16	0.769	0.731	0.486	1.32	0.880	0.856	0.569
	17	0.719	0.478	0.515	0.343	1.26	0.839	0.759	0.505	1.45	0.962	0.892	0.593
	18	0.746	0.496	0.527	0.351	1.38	0.920	0.789	0.525	1.59	1.06	0.931	0.619
	19	0.776	0.516	0.539	0.359	1.53	1.02	0.823	0.547	1.76	1.17	0.974	0.648
	20	0.810	0.539	0.552	0.367	1.69	1.12	0.859	0.571	1.95	1.30	1.02	0.679
	22	0.889	0.592	0.580	0.386	2.05	1.36	0.967	0.643	2.36	1.57	1.18	0.787
	24	0.990	0.659	0.610	0.406	2.43	1.62	1.10	0.735	2.81	1.87	1.36	0.904
	26	1.11	0.741	0.644	0.428	2.86	1.90	1.25	0.828	3.30	2.19	1.54	1.02
	28	1.26	0.841	0.681	0.453	3.31	2.20	1.39	0.923	3.82	2.54	1.72	1.15
	30	1.45	0.964	0.724	0.481	3.80	2.53	1.53	1.02	4.39	2.92	1.91	1.27
	32	1.65	1.10	0.802	0.533	4.33	2.88	1.67	1.11	4.99	3.32	2.09	1.39
	34	1.86	1.24	0.882	0.587	4.89	3.25	1.82	1.21	5.64	3.75	2.28	1.52
	36	2.09	1.39	0.964	0.641	5.48	3.64	1.96	1.30	6.32	4.21	2.47	1.64
	38	2.32	1.55	1.05	0.696								
	40	2.57	1.71	1.13	0.751								
	42	2.84	1.89	1.21	0.807								
	44	3.12	2.07	1.30	0.863								
	46	3.41	2.27	1.38	0.919								
	48	3.71	2.47	1.47	0.976								
	50	4.02	2.68	1.55	1.03								
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		2.23		1.48		4.03		2.68		4.69		3.12	
$t_y \times 10^3$ (kips)$^{-1}$		0.505		0.336		0.589		0.392		0.662		0.441	
$t_r \times 10^3$ (kips)$^{-1}$		0.655		0.437		0.765		0.510		0.859		0.573	
r_x/r_y		3.71				5.44				5.42			
r_y, in.		3.42				2.28				2.25			

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W30

Shape	W30× 124[c]				W30× 116[c,v]				W30× 108[c,v]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.793	0.527	0.672	0.447	0.858	0.571	0.725	0.482	0.942	0.627	0.792	0.527
11	1.03	0.685	0.767	0.510	1.12	0.745	0.834	0.555	1.23	0.820	0.921	0.613
12	1.09	0.722	0.794	0.528	1.18	0.786	0.865	0.575	1.30	0.867	0.957	0.637
13	1.15	0.766	0.824	0.548	1.26	0.835	0.898	0.597	1.39	0.922	0.996	0.663
14	1.23	0.817	0.856	0.569	1.34	0.893	0.934	0.621	1.48	0.988	1.04	0.691
15	1.32	0.878	0.890	0.592	1.45	0.962	0.973	0.647	1.60	1.07	1.08	0.721
16	1.43	0.951	0.927	0.617	1.57	1.04	1.01	0.675	1.74	1.16	1.13	0.754
17	1.56	1.04	0.968	0.644	1.72	1.14	1.06	0.706	1.91	1.27	1.19	0.791
18	1.72	1.14	1.01	0.673	1.89	1.26	1.11	0.739	2.12	1.41	1.25	0.831
19	1.91	1.27	1.06	0.706	2.11	1.40	1.17	0.776	2.36	1.57	1.32	0.877
20	2.11	1.40	1.12	0.745	2.34	1.55	1.26	0.837	2.62	1.74	1.44	0.959
22	2.55	1.70	1.31	0.873	2.83	1.88	1.48	0.983	3.16	2.11	1.70	1.13
24	3.04	2.02	1.51	1.01	3.36	2.24	1.70	1.13	3.77	2.51	1.96	1.31
26	3.57	2.37	1.72	1.14	3.95	2.63	1.94	1.29	4.42	2.94	2.24	1.49
28	4.14	2.75	1.92	1.28	4.58	3.05	2.18	1.45	5.13	3.41	2.52	1.68
30	4.75	3.16	2.13	1.42	5.26	3.50	2.42	1.61	5.88	3.91	2.81	1.87
32	5.40	3.60	2.35	1.56	5.98	3.98	2.67	1.78	6.69	4.45	3.10	2.06
34	6.10	4.06	2.56	1.70	6.75	4.49	2.92	1.94	7.56	5.03	3.40	2.26
36	6.84	4.55	2.78	1.85	7.57	5.04	3.17	2.11				

Other Constants and Properties

	124 ASD	124 LRFD	116 ASD	116 LRFD	108 ASD	108 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	5.08	3.38	5.57	3.71	6.24	4.15
$t_y \times 10^3$ (kips)$^{-1}$	0.704	0.468	0.751	0.500	0.810	0.539
$t_r \times 10^3$ (kips)$^{-1}$	0.913	0.609	0.975	0.650	1.05	0.701
r_x/r_y	5.43		5.48		5.53	
r_y, in.	2.23		2.19		2.15	

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

W30-W27

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W30×								W27×			
		99[c,v]				90[c,f,v]				539[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	1.05	0.700	0.878	0.584	1.20	0.797	0.983	0.654	0.162	0.108	0.145	0.0965
	11	1.38	0.921	1.03	0.687	1.57	1.04	1.14	0.761	0.183	0.122	0.145	0.0965
	12	1.46	0.975	1.08	0.715	1.65	1.10	1.19	0.793	0.187	0.125	0.146	0.0970
	13	1.56	1.04	1.12	0.746	1.76	1.17	1.24	0.827	0.192	0.128	0.147	0.0977
	14	1.68	1.11	1.17	0.779	1.89	1.25	1.30	0.865	0.198	0.131	0.148	0.0984
	15	1.81	1.21	1.23	0.815	2.03	1.35	1.36	0.906	0.204	0.135	0.149	0.0992
	16	1.98	1.32	1.29	0.855	2.21	1.47	1.43	0.951	0.210	0.140	0.150	0.0999
	17	2.18	1.45	1.35	0.899	2.43	1.62	1.51	1.00	0.217	0.145	0.151	0.101
	18	2.43	1.61	1.42	0.947	2.70	1.80	1.59	1.06	0.225	0.150	0.152	0.101
	19	2.70	1.80	1.55	1.03	3.01	2.00	1.75	1.17	0.234	0.156	0.154	0.102
	20	3.00	1.99	1.69	1.13	3.34	2.22	1.92	1.28	0.244	0.162	0.155	0.103
	22	3.63	2.41	2.00	1.33	4.04	2.69	2.28	1.52	0.266	0.177	0.157	0.105
	24	4.31	2.87	2.32	1.54	4.80	3.20	2.65	1.76	0.292	0.194	0.160	0.106
	26	5.06	3.37	2.65	1.76	5.64	3.75	3.04	2.02	0.324	0.215	0.162	0.108
	28	5.87	3.91	2.99	1.99	6.54	4.35	3.44	2.29	0.362	0.241	0.165	0.110
	30	6.74	4.49	3.34	2.22	7.51	4.99	3.85	2.56	0.407	0.271	0.168	0.112
	32	7.67	5.10	3.69	2.46	8.54	5.68	4.27	2.84	0.463	0.308	0.171	0.114
	34	8.66	5.76	4.06	2.70	9.64	6.41	4.70	3.13	0.523	0.348	0.174	0.116
	36									0.586	0.390	0.177	0.118
	38									0.653	0.435	0.180	0.120
	40									0.724	0.481	0.183	0.122
	42									0.798	0.531	0.187	0.124
	44									0.876	0.583	0.190	0.127
	46									0.957	0.637	0.194	0.129
	48									1.04	0.693	0.198	0.132
	50									1.13	0.752	0.202	0.134
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		7.10		4.72		8.07		5.37		0.627		0.417	
$t_y \times 10^3$ (kips)$^{-1}$		0.886		0.589		0.977		0.650		0.162		0.108	
$t_r \times 10^3$ (kips)$^{-1}$		1.15		0.766		1.27		0.845		0.210		0.140	
r_x/r_y		5.57				5.60				3.48			
r_y, in.		2.10				2.09				3.65			

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W27

Shape		W27×											
		368[h]				336[h]				307[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.236	0.157	0.221	0.147	0.259	0.172	0.243	0.161	0.285	0.190	0.266	0.177
	11	0.270	0.180	0.221	0.147	0.298	0.198	0.243	0.162	0.328	0.219	0.267	0.178
	12	0.277	0.185	0.224	0.149	0.306	0.203	0.246	0.164	0.337	0.225	0.271	0.180
	13	0.285	0.190	0.226	0.151	0.315	0.209	0.249	0.166	0.348	0.231	0.274	0.183
	14	0.294	0.196	0.229	0.152	0.324	0.216	0.252	0.168	0.359	0.239	0.278	0.185
	15	0.304	0.202	0.231	0.154	0.335	0.223	0.255	0.170	0.371	0.247	0.282	0.188
	16	0.315	0.209	0.234	0.156	0.348	0.231	0.258	0.172	0.385	0.256	0.286	0.190
	17	0.327	0.217	0.237	0.157	0.361	0.240	0.262	0.174	0.400	0.266	0.290	0.193
	18	0.340	0.226	0.239	0.159	0.376	0.250	0.265	0.176	0.417	0.278	0.294	0.196
	19	0.354	0.236	0.242	0.161	0.392	0.261	0.268	0.179	0.436	0.290	0.298	0.198
	20	0.370	0.246	0.245	0.163	0.410	0.273	0.272	0.181	0.456	0.303	0.302	0.201
	22	0.407	0.271	0.251	0.167	0.452	0.301	0.279	0.186	0.504	0.335	0.311	0.207
	24	0.452	0.301	0.257	0.171	0.502	0.334	0.287	0.191	0.561	0.373	0.321	0.214
	26	0.506	0.337	0.264	0.175	0.564	0.375	0.295	0.196	0.631	0.420	0.331	0.220
	28	0.572	0.380	0.270	0.180	0.638	0.425	0.303	0.202	0.717	0.477	0.342	0.228
	30	0.653	0.435	0.278	0.185	0.730	0.486	0.312	0.208	0.822	0.547	0.354	0.235
	32	0.743	0.494	0.285	0.190	0.831	0.553	0.322	0.214	0.935	0.622	0.366	0.244
	34	0.839	0.558	0.293	0.195	0.938	0.624	0.332	0.221	1.06	0.703	0.379	0.252
	36	0.941	0.626	0.302	0.201	1.05	0.700	0.343	0.228	1.18	0.788	0.394	0.262
	38	1.05	0.697	0.311	0.207	1.17	0.780	0.355	0.236	1.32	0.878	0.409	0.272
	40	1.16	0.773	0.320	0.213	1.30	0.864	0.367	0.244	1.46	0.972	0.426	0.283
	42	1.28	0.852	0.331	0.220	1.43	0.952	0.381	0.253	1.61	1.07	0.445	0.296
	44	1.41	0.935	0.341	0.227	1.57	1.05	0.395	0.263	1.77	1.18	0.469	0.312
	46	1.54	1.02	0.353	0.235	1.72	1.14	0.413	0.275	1.93	1.29	0.494	0.329
	48	1.67	1.11	0.365	0.243	1.87	1.24	0.434	0.289	2.10	1.40	0.519	0.345
	50	1.81	1.21	0.381	0.254	2.03	1.35	0.454	0.302	2.28	1.52	0.543	0.361

Other Constants and Properties

	368[h] ASD	368[h] LRFD	336[h] ASD	336[h] LRFD	307[h] ASD	307[h] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	0.982	0.654	1.09	0.724	1.21	0.803
$t_y \times 10^3$ (kips)$^{-1}$	0.236	0.157	0.259	0.172	0.285	0.190
$t_r \times 10^3$ (kips)$^{-1}$	0.306	0.204	0.336	0.224	0.370	0.246
r_x/r_y	3.51		3.51		3.52	
r_y, in.	3.48		3.45		3.41	

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

W27

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W27×											
		281				258				235			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.309	0.206	0.293	0.195	0.338	0.225	0.322	0.214	0.370	0.246	0.355	0.236
	11	0.357	0.238	0.295	0.196	0.391	0.260	0.324	0.216	0.430	0.286	0.359	0.239
	12	0.367	0.244	0.299	0.199	0.402	0.267	0.329	0.219	0.442	0.294	0.365	0.243
	13	0.378	0.252	0.303	0.202	0.414	0.276	0.334	0.222	0.456	0.303	0.371	0.247
	14	0.390	0.260	0.307	0.205	0.428	0.285	0.339	0.226	0.472	0.314	0.377	0.251
	15	0.404	0.269	0.312	0.207	0.443	0.295	0.345	0.229	0.489	0.325	0.383	0.255
	16	0.419	0.279	0.316	0.211	0.460	0.306	0.350	0.233	0.508	0.338	0.390	0.259
	17	0.436	0.290	0.321	0.214	0.479	0.319	0.356	0.237	0.529	0.352	0.397	0.264
	18	0.455	0.303	0.326	0.217	0.500	0.333	0.362	0.241	0.552	0.367	0.404	0.269
	19	0.475	0.316	0.331	0.220	0.523	0.348	0.368	0.245	0.578	0.385	0.411	0.274
	20	0.498	0.331	0.336	0.224	0.548	0.365	0.374	0.249	0.607	0.404	0.419	0.279
	22	0.550	0.366	0.347	0.231	0.607	0.404	0.387	0.258	0.673	0.448	0.435	0.289
	24	0.614	0.408	0.359	0.239	0.679	0.452	0.401	0.267	0.754	0.501	0.452	0.301
	26	0.692	0.460	0.371	0.247	0.766	0.510	0.416	0.277	0.853	0.567	0.471	0.313
	28	0.787	0.523	0.384	0.256	0.874	0.582	0.433	0.288	0.976	0.649	0.492	0.327
	30	0.903	0.601	0.398	0.265	1.00	0.668	0.450	0.300	1.12	0.745	0.514	0.342
	32	1.03	0.683	0.414	0.275	1.14	0.760	0.470	0.312	1.27	0.848	0.538	0.358
	34	1.16	0.772	0.430	0.286	1.29	0.858	0.490	0.326	1.44	0.957	0.565	0.376
	36	1.30	0.865	0.448	0.298	1.45	0.962	0.513	0.342	1.61	1.07	0.603	0.401
	38	1.45	0.964	0.467	0.311	1.61	1.07	0.544	0.362	1.80	1.20	0.646	0.430
	40	1.61	1.07	0.492	0.327	1.78	1.19	0.580	0.386	1.99	1.33	0.690	0.459
	42	1.77	1.18	0.522	0.347	1.97	1.31	0.615	0.409	2.20	1.46	0.733	0.487
	44	1.94	1.29	0.551	0.367	2.16	1.44	0.650	0.433	2.41	1.60	0.776	0.516
	46	2.12	1.41	0.580	0.386	2.36	1.57	0.685	0.456	2.63	1.75	0.818	0.544
	48	2.31	1.54	0.610	0.406	2.57	1.71	0.721	0.479	2.87	1.91	0.861	0.573
	50	2.51	1.67	0.639	0.425	2.79	1.85	0.756	0.503	3.11	2.07	0.904	0.601

Other Constants and Properties

	281 ASD	281 LRFD	258 ASD	258 LRFD	235 ASD	235 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.33	0.885	1.47	0.975	1.63	1.09
$t_y \times 10^3$ (kips)$^{-1}$	0.309	0.206	0.338	0.225	0.370	0.246
$t_r \times 10^3$ (kips)$^{-1}$	0.401	0.267	0.438	0.292	0.480	0.320
r_x/r_y	3.54		3.54		3.54	
r_y, in.	3.39		3.36		3.33	

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W27

Shape	W27×											
	217				194[c]				178[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.402	0.268	0.385	0.256	0.452	0.301	0.434	0.289	0.495	0.330	0.481	0.320
11	0.467	0.311	0.390	0.259	0.524	0.349	0.441	0.293	0.572	0.381	0.489	0.325
12	0.481	0.320	0.397	0.264	0.540	0.359	0.449	0.299	0.590	0.392	0.499	0.332
13	0.496	0.330	0.403	0.268	0.557	0.371	0.457	0.304	0.609	0.405	0.509	0.338
14	0.513	0.341	0.411	0.273	0.577	0.384	0.466	0.310	0.631	0.420	0.519	0.345
15	0.532	0.354	0.418	0.278	0.598	0.398	0.475	0.316	0.655	0.436	0.530	0.352
16	0.553	0.368	0.426	0.283	0.622	0.414	0.484	0.322	0.682	0.454	0.541	0.360
17	0.576	0.383	0.434	0.288	0.648	0.431	0.494	0.329	0.712	0.474	0.552	0.368
18	0.601	0.400	0.442	0.294	0.678	0.451	0.504	0.335	0.745	0.495	0.565	0.376
19	0.629	0.419	0.450	0.300	0.710	0.473	0.515	0.342	0.781	0.520	0.577	0.384
20	0.661	0.440	0.459	0.306	0.746	0.496	0.526	0.350	0.822	0.547	0.591	0.393
22	0.733	0.488	0.478	0.318	0.830	0.552	0.549	0.366	0.916	0.610	0.619	0.412
24	0.822	0.547	0.499	0.332	0.932	0.620	0.575	0.383	1.03	0.687	0.651	0.433
26	0.931	0.619	0.521	0.347	1.06	0.704	0.604	0.402	1.18	0.782	0.686	0.456
28	1.07	0.710	0.545	0.363	1.22	0.809	0.635	0.422	1.35	0.901	0.725	0.482
30	1.22	0.814	0.572	0.381	1.40	0.928	0.670	0.446	1.55	1.03	0.768	0.511
32	1.39	0.927	0.602	0.400	1.59	1.06	0.714	0.475	1.77	1.18	0.840	0.559
34	1.57	1.05	0.640	0.426	1.79	1.19	0.778	0.517	2.00	1.33	0.917	0.610
36	1.76	1.17	0.690	0.459	2.01	1.34	0.841	0.560	2.24	1.49	0.995	0.662
38	1.96	1.31	0.741	0.493	2.24	1.49	0.905	0.602	2.49	1.66	1.07	0.713
40	2.18	1.45	0.791	0.527	2.48	1.65	0.968	0.644	2.76	1.84	1.15	0.765
42	2.40	1.60	0.842	0.560	2.73	1.82	1.03	0.687	3.05	2.03	1.23	0.817
44	2.63	1.75	0.892	0.593	3.00	2.00	1.10	0.729	3.34	2.23	1.31	0.869
46	2.88	1.91	0.942	0.627	3.28	2.18	1.16	0.771	3.66	2.43	1.38	0.920
48	3.13	2.09	0.992	0.660	3.57	2.38	1.22	0.813	3.98	2.65	1.46	0.972
50	3.40	2.26	1.04	0.693	3.88	2.58	1.29	0.855	4.32	2.87	1.54	1.02
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.78		1.18		2.02		1.34		2.25		1.49	
$t_y \times 10^3$ (kips)$^{-1}$	0.402		0.268		0.450		0.299		0.489		0.326	
$t_r \times 10^3$ (kips)$^{-1}$	0.522		0.348		0.584		0.389		0.635		0.423	
r_x/r_y	3.55				3.56				3.57			
r_y, in.	3.32				3.29				3.25			

[c] Shape is slender for compression with F_y = 65 ksi.

Return to Table of Contents

W27

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W27×											
	161[c]				146[c]				129[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.558	0.371	0.532	0.354	0.628	0.418	0.591	0.393	0.726	0.483	0.694	0.462
11	0.641	0.426	0.543	0.361	0.720	0.479	0.604	0.402	0.966	0.643	0.788	0.524
12	0.658	0.438	0.554	0.369	0.740	0.492	0.617	0.411	1.02	0.681	0.814	0.542
13	0.678	0.451	0.566	0.376	0.762	0.507	0.631	0.420	1.09	0.726	0.842	0.560
14	0.700	0.466	0.578	0.384	0.787	0.523	0.645	0.429	1.18	0.783	0.873	0.581
15	0.725	0.482	0.590	0.393	0.814	0.542	0.66	0.439	1.28	0.850	0.905	0.602
16	0.755	0.502	0.604	0.402	0.845	0.562	0.675	0.449	1.39	0.927	0.941	0.626
17	0.789	0.525	0.618	0.411	0.880	0.586	0.691	0.460	1.53	1.02	0.979	0.651
18	0.826	0.549	0.632	0.421	0.919	0.611	0.708	0.471	1.69	1.12	1.02	0.679
19	0.867	0.577	0.647	0.431	0.964	0.641	0.726	0.483	1.87	1.25	1.06	0.708
20	0.912	0.607	0.663	0.441	1.02	0.675	0.745	0.496	2.08	1.38	1.11	0.741
22	1.02	0.678	0.698	0.464	1.14	0.756	0.786	0.523	2.51	1.67	1.28	0.850
24	1.15	0.765	0.736	0.489	1.28	0.855	0.831	0.553	2.99	1.99	1.46	0.969
26	1.31	0.872	0.778	0.518	1.47	0.977	0.882	0.587	3.51	2.33	1.64	1.09
28	1.51	1.01	0.826	0.550	1.70	1.13	0.939	0.625	4.07	2.71	1.82	1.21
30	1.74	1.16	0.895	0.596	1.95	1.30	1.04	0.695	4.67	3.11	2.00	1.33
32	1.98	1.31	0.987	0.657	2.22	1.48	1.16	0.769	5.31	3.54	2.18	1.45
34	2.23	1.48	1.08	0.719	2.50	1.67	1.27	0.843	6.00	3.99	2.36	1.57
36	2.50	1.66	1.17	0.781	2.81	1.87	1.38	0.919	6.73	4.47	2.54	1.69
38	2.79	1.85	1.27	0.844	3.13	2.08	1.50	0.995				
40	3.09	2.05	1.36	0.907	3.47	2.31	1.61	1.07				
42	3.40	2.26	1.46	0.970	3.82	2.54	1.73	1.15				
44	3.73	2.48	1.55	1.03	4.19	2.79	1.84	1.23				
46	4.08	2.72	1.65	1.10	4.58	3.05	1.96	1.30				
48	4.44	2.96	1.74	1.16	4.99	3.32	2.07	1.38				
50	4.82	3.21	1.84	1.22	5.41	3.60	2.19	1.46				

Other Constants and Properties

	161[c] ASD	161[c] LRFD	146[c] ASD	146[c] LRFD	129[c] ASD	129[c] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	2.51	1.67	2.81	1.87	4.76	3.17
$t_y \times 10^3$ (kips)$^{-1}$	0.540	0.359	0.595	0.396	0.680	0.452
$t_r \times 10^3$ (kips)$^{-1}$	0.700	0.467	0.772	0.514	0.882	0.588
r_x / r_y	3.56		3.59		5.07	
r_y, in.	3.23		3.20		2.21	

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL / r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W27

Shape		W27×											
		114[c]				102[c]				94[c,v]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.836	0.556	0.799	0.532	0.965	0.642	0.899	0.598	1.07	0.711	0.986	0.656
	11	1.11	0.740	0.917	0.610	1.28	0.852	1.04	0.692	1.42	0.945	1.15	0.766
	12	1.18	0.784	0.950	0.632	1.36	0.903	1.08	0.718	1.51	1.00	1.20	0.797
	13	1.26	0.836	0.986	0.656	1.45	0.963	1.12	0.746	1.61	1.07	1.25	0.829
	14	1.35	0.898	1.02	0.681	1.55	1.03	1.17	0.777	1.73	1.15	1.30	0.865
	15	1.46	0.973	1.07	0.709	1.68	1.12	1.22	0.810	1.87	1.24	1.36	0.904
	16	1.60	1.06	1.11	0.739	1.83	1.22	1.27	0.846	2.04	1.36	1.42	0.946
	17	1.76	1.17	1.16	0.771	2.02	1.34	1.33	0.885	2.25	1.49	1.49	0.993
	18	1.94	1.29	1.21	0.807	2.24	1.49	1.39	0.928	2.50	1.66	1.57	1.04
	19	2.17	1.44	1.27	0.846	2.49	1.66	1.47	0.976	2.79	1.86	1.69	1.12
	20	2.40	1.60	1.35	0.900	2.76	1.84	1.60	1.07	3.09	2.06	1.84	1.23
	22	2.90	1.93	1.58	1.05	3.34	2.22	1.87	1.25	3.74	2.49	2.16	1.44
	24	3.46	2.30	1.80	1.20	3.98	2.65	2.15	1.43	4.45	2.96	2.50	1.66
	26	4.06	2.70	2.04	1.36	4.67	3.11	2.44	1.63	5.22	3.47	2.84	1.89
	28	4.70	3.13	2.27	1.51	5.42	3.60	2.74	1.82	6.06	4.03	3.19	2.12
	30	5.40	3.59	2.51	1.67	6.22	4.14	3.03	2.02	6.95	4.62	3.54	2.36
	32	6.14	4.09	2.75	1.83	7.07	4.71	3.33	2.22	7.91	5.26	3.90	2.59
	34	6.94	4.61	2.99	1.99	7.99	5.31	3.63	2.42	8.93	5.94	4.26	2.83
	36	7.78	5.17	3.23	2.15								

Other Constants and Properties

	114[c] ASD	114[c] LRFD	102[c] ASD	102[c] LRFD	94[c,v] ASD	94[c,v] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	5.56	3.70	6.31	4.20	7.06	4.70
$t_y \times 10^3$ (kips)$^{-1}$	0.765	0.509	0.856	0.570	0.931	0.619
$t_r \times 10^3$ (kips)$^{-1}$	0.992	0.661	1.11	0.741	1.21	0.805
r_x/r_y	5.05		5.12		5.14	
r_y, in.	2.18		2.15		2.12	

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W27-W24

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W27× 84[c,v]				W24× 370[h]				W24× 335[h]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	1.23	0.816	1.12	0.747	0.236	0.157	0.243	0.161	0.261	0.174	0.269	0.179
11	1.64	1.09	1.33	0.885	0.275	0.183	0.245	0.163	0.306	0.204	0.271	0.181
12	1.74	1.16	1.39	0.922	0.283	0.189	0.247	0.164	0.316	0.210	0.274	0.183
13	1.86	1.23	1.45	0.962	0.293	0.195	0.249	0.166	0.326	0.217	0.277	0.185
14	2.00	1.33	1.51	1.01	0.303	0.202	0.252	0.167	0.338	0.225	0.280	0.187
15	2.17	1.44	1.58	1.05	0.314	0.209	0.254	0.169	0.351	0.234	0.283	0.189
16	2.37	1.58	1.66	1.11	0.327	0.218	0.257	0.171	0.366	0.243	0.287	0.191
17	2.62	1.75	1.75	1.16	0.341	0.227	0.259	0.172	0.382	0.254	0.290	0.193
18	2.93	1.95	1.86	1.24	0.357	0.237	0.262	0.174	0.400	0.266	0.293	0.195
19	3.27	2.17	2.05	1.36	0.374	0.249	0.264	0.176	0.420	0.279	0.296	0.197
20	3.62	2.41	2.24	1.49	0.393	0.262	0.267	0.178	0.442	0.294	0.300	0.199
22	4.38	2.91	2.64	1.76	0.438	0.291	0.273	0.181	0.493	0.328	0.307	0.204
24	5.21	3.47	3.06	2.04	0.493	0.328	0.279	0.185	0.556	0.370	0.314	0.209
26	6.12	4.07	3.49	2.32	0.560	0.373	0.285	0.189	0.634	0.422	0.322	0.214
28	7.10	4.72	3.93	2.62	0.644	0.429	0.291	0.194	0.732	0.487	0.330	0.220
30	8.15	5.42	4.38	2.92	0.740	0.492	0.298	0.198	0.841	0.559	0.339	0.226
32	9.27	6.17	4.84	3.22	0.842	0.560	0.305	0.203	0.957	0.636	0.348	0.232
34	10.5	6.96	5.31	3.53	0.950	0.632	0.312	0.208	1.08	0.718	0.358	0.238
36					1.07	0.709	0.320	0.213	1.21	0.806	0.368	0.245
38					1.19	0.790	0.328	0.218	1.35	0.897	0.379	0.252
40					1.32	0.875	0.336	0.224	1.49	0.994	0.390	0.260
42					1.45	0.965	0.345	0.230	1.65	1.10	0.402	0.268
44					1.59	1.06	0.354	0.236	1.81	1.20	0.415	0.276
46					1.74	1.16	0.364	0.242	1.98	1.32	0.429	0.285
48					1.89	1.26	0.375	0.249	2.15	1.43	0.444	0.295
50					2.05	1.37	0.386	0.257	2.34	1.55	0.461	0.307
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	8.25		5.49		1.03		0.683		1.15		0.766	
$t_y \times 10^3$ (kips)$^{-1}$	1.04		0.692		0.236		0.157		0.261		0.174	
$t_r \times 10^3$ (kips)$^{-1}$	1.35		0.900		0.306		0.204		0.339		0.226	
r_x/r_y	5.17				3.39				3.41			
r_y, in.	2.07				3.27				3.23			

[c] Shape is slender for compression with F_y = 65 ksi.

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W24

Shape	W24×											
	306[h]				279[h]				250			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.286	0.191	0.297	0.198	0.314	0.209	0.328	0.218	0.350	0.233	0.368	0.245
11	0.337	0.224	0.301	0.200	0.370	0.246	0.333	0.222	0.413	0.275	0.375	0.249
12	0.347	0.231	0.304	0.203	0.382	0.254	0.337	0.224	0.427	0.284	0.380	0.253
13	0.359	0.239	0.308	0.205	0.395	0.263	0.342	0.227	0.442	0.294	0.385	0.256
14	0.372	0.248	0.312	0.207	0.410	0.273	0.346	0.230	0.459	0.305	0.391	0.260
15	0.387	0.257	0.315	0.210	0.426	0.284	0.351	0.233	0.478	0.318	0.397	0.264
16	0.403	0.268	0.319	0.212	0.445	0.296	0.355	0.236	0.499	0.332	0.403	0.268
17	0.421	0.280	0.323	0.215	0.465	0.309	0.360	0.240	0.522	0.347	0.409	0.272
18	0.442	0.294	0.327	0.218	0.488	0.325	0.365	0.243	0.548	0.365	0.415	0.276
19	0.464	0.309	0.331	0.220	0.513	0.341	0.370	0.246	0.577	0.384	0.422	0.280
20	0.489	0.325	0.335	0.223	0.541	0.360	0.375	0.250	0.609	0.405	0.428	0.285
22	0.547	0.364	0.344	0.229	0.606	0.404	0.386	0.257	0.684	0.455	0.442	0.294
24	0.619	0.412	0.354	0.235	0.687	0.457	0.398	0.265	0.778	0.517	0.457	0.304
26	0.707	0.470	0.363	0.242	0.788	0.524	0.410	0.273	0.893	0.594	0.473	0.315
28	0.818	0.544	0.374	0.249	0.913	0.607	0.423	0.281	1.04	0.690	0.490	0.326
30	0.939	0.625	0.385	0.256	1.05	0.697	0.437	0.291	1.19	0.792	0.509	0.338
32	1.07	0.711	0.396	0.264	1.19	0.793	0.452	0.301	1.35	0.901	0.528	0.352
34	1.21	0.802	0.408	0.272	1.35	0.895	0.468	0.311	1.53	1.02	0.550	0.366
36	1.35	0.899	0.421	0.280	1.51	1.00	0.485	0.322	1.71	1.14	0.573	0.381
38	1.51	1.00	0.435	0.290	1.68	1.12	0.503	0.335	1.91	1.27	0.599	0.398
40	1.67	1.11	0.450	0.300	1.86	1.24	0.523	0.348	2.12	1.41	0.633	0.421
42	1.84	1.22	0.466	0.310	2.05	1.37	0.544	0.362	2.33	1.55	0.669	0.445
44	2.02	1.34	0.483	0.322	2.25	1.50	0.574	0.382	2.56	1.70	0.706	0.469
46	2.21	1.47	0.504	0.335	2.46	1.64	0.603	0.401	2.80	1.86	0.742	0.494
48	2.40	1.60	0.528	0.351	2.68	1.78	0.631	0.420	3.05	2.03	0.778	0.518
50	2.61	1.73	0.552	0.367	2.91	1.94	0.660	0.439	3.31	2.20	0.814	0.541

Other Constants and Properties

	306[h] ASD	306[h] LRFD	279[h] ASD	279[h] LRFD	250 ASD	250 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.28	0.852	1.42	0.945	1.60	1.07
$t_y \times 10^3$ (kips)$^{-1}$	0.286	0.191	0.314	0.209	0.350	0.233
$t_r \times 10^3$ (kips)$^{-1}$	0.372	0.248	0.407	0.271	0.454	0.302
r_x/r_y	3.41		3.41		3.41	
r_y, in.	3.20		3.17		3.14	

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

W24

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W24×											
	229				207				192			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.382	0.254	0.406	0.270	0.423	0.282	0.452	0.301	0.455	0.303	0.490	0.326
11	0.454	0.302	0.414	0.276	0.504	0.335	0.463	0.308	0.542	0.361	0.503	0.335
12	0.469	0.312	0.421	0.280	0.521	0.347	0.471	0.313	0.561	0.373	0.512	0.341
13	0.486	0.323	0.427	0.284	0.540	0.359	0.479	0.319	0.581	0.387	0.521	0.347
14	0.505	0.336	0.434	0.289	0.562	0.374	0.487	0.324	0.604	0.402	0.531	0.353
15	0.526	0.350	0.441	0.293	0.586	0.390	0.496	0.330	0.630	0.419	0.541	0.360
16	0.549	0.365	0.448	0.298	0.612	0.407	0.505	0.336	0.660	0.439	0.551	0.367
17	0.576	0.383	0.455	0.303	0.642	0.427	0.514	0.342	0.692	0.460	0.562	0.374
18	0.605	0.402	0.463	0.308	0.676	0.449	0.523	0.348	0.728	0.484	0.573	0.381
19	0.637	0.424	0.471	0.313	0.713	0.474	0.533	0.355	0.768	0.511	0.584	0.389
20	0.673	0.448	0.479	0.319	0.754	0.502	0.544	0.362	0.813	0.541	0.596	0.397
22	0.758	0.505	0.497	0.330	0.851	0.566	0.565	0.376	0.918	0.611	0.622	0.414
24	0.864	0.575	0.515	0.343	0.972	0.647	0.589	0.392	1.05	0.698	0.650	0.433
26	0.996	0.663	0.535	0.356	1.12	0.748	0.615	0.409	1.22	0.809	0.681	0.453
28	1.16	0.769	0.557	0.370	1.30	0.868	0.643	0.428	1.41	0.938	0.715	0.475
30	1.33	0.883	0.580	0.386	1.50	0.996	0.674	0.448	1.62	1.08	0.752	0.500
32	1.51	1.00	0.606	0.403	1.70	1.13	0.708	0.471	1.84	1.23	0.793	0.528
34	1.70	1.13	0.634	0.422	1.92	1.28	0.748	0.497	2.08	1.38	0.856	0.569
36	1.91	1.27	0.664	0.442	2.16	1.43	0.803	0.534	2.33	1.55	0.920	0.612
38	2.13	1.42	0.708	0.471	2.40	1.60	0.857	0.571	2.60	1.73	0.984	0.655
40	2.36	1.57	0.752	0.501	2.66	1.77	0.912	0.607	2.88	1.92	1.05	0.697
42	2.60	1.73	0.796	0.530	2.93	1.95	0.967	0.643	3.17	2.11	1.11	0.740
44	2.85	1.90	0.840	0.559	3.22	2.14	1.02	0.679	3.48	2.32	1.17	0.782
46	3.12	2.08	0.884	0.588	3.52	2.34	1.07	0.715	3.81	2.53	1.24	0.824
48	3.40	2.26	0.928	0.617	3.83	2.55	1.13	0.751	4.15	2.76	1.30	0.866
50	3.68	2.45	0.971	0.646	4.16	2.77	1.18	0.787	4.50	2.99	1.36	0.908

Other Constants and Properties

	229 ASD	229 LRFD	207 ASD	207 LRFD	192 ASD	192 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.78	1.18	2.00	1.33	2.18	1.45
$t_y \times 10^3$ (kips)$^{-1}$	0.382	0.254	0.423	0.282	0.455	0.303
$t_r \times 10^3$ (kips)$^{-1}$	0.496	0.331	0.549	0.366	0.590	0.393
r_x/r_y	3.44		3.44		3.42	
r_y, in.	3.11		3.08		3.07	

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W24

Shape	W24×											
	176				162				146[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.497	0.331	0.536	0.357	0.537	0.358	0.586	0.390	0.606	0.403	0.656	0.436
11	0.594	0.396	0.552	0.367	0.642	0.427	0.603	0.401	0.717	0.477	0.679	0.452
12	0.615	0.409	0.563	0.374	0.664	0.442	0.615	0.409	0.743	0.494	0.693	0.461
13	0.638	0.425	0.573	0.382	0.689	0.459	0.628	0.418	0.771	0.513	0.708	0.471
14	0.664	0.442	0.585	0.389	0.717	0.477	0.641	0.426	0.803	0.535	0.724	0.482
15	0.693	0.461	0.597	0.397	0.748	0.498	0.655	0.436	0.839	0.558	0.740	0.493
16	0.726	0.483	0.609	0.405	0.783	0.521	0.669	0.445	0.880	0.585	0.758	0.504
17	0.762	0.507	0.622	0.414	0.822	0.547	0.684	0.455	0.925	0.615	0.776	0.516
18	0.803	0.534	0.635	0.422	0.866	0.576	0.699	0.465	0.975	0.649	0.795	0.529
19	0.848	0.564	0.649	0.432	0.914	0.608	0.715	0.476	1.03	0.686	0.815	0.542
20	0.899	0.598	0.663	0.441	0.968	0.644	0.732	0.487	1.09	0.727	0.836	0.556
22	1.02	0.677	0.695	0.462	1.10	0.729	0.769	0.512	1.24	0.826	0.881	0.586
24	1.17	0.776	0.729	0.485	1.25	0.835	0.809	0.538	1.43	0.949	0.931	0.620
26	1.36	0.902	0.766	0.510	1.46	0.969	0.854	0.568	1.66	1.11	0.988	0.657
28	1.57	1.05	0.808	0.538	1.69	1.12	0.904	0.602	1.93	1.28	1.05	0.700
30	1.80	1.20	0.855	0.569	1.94	1.29	0.969	0.645	2.21	1.47	1.17	0.775
32	2.05	1.37	0.923	0.614	2.21	1.47	1.06	0.705	2.52	1.68	1.28	0.850
34	2.32	1.54	1.00	0.665	2.49	1.66	1.15	0.765	2.84	1.89	1.39	0.926
36	2.60	1.73	1.08	0.716	2.79	1.86	1.24	0.826	3.19	2.12	1.50	1.00
38	2.90	1.93	1.15	0.767	3.11	2.07	1.33	0.886	3.55	2.36	1.62	1.08
40	3.21	2.13	1.23	0.818	3.45	2.29	1.42	0.947	3.93	2.62	1.73	1.15
42	3.54	2.35	1.31	0.869	3.80	2.53	1.51	1.01	4.34	2.89	1.85	1.23
44	3.88	2.58	1.38	0.920	4.17	2.78	1.60	1.07	4.76	3.17	1.96	1.30
46	4.24	2.82	1.46	0.970	4.56	3.03	1.69	1.13	5.20	3.46	2.07	1.38
48	4.62	3.07	1.53	1.02	4.96	3.30	1.78	1.19	5.67	3.77	2.19	1.45
50	5.01	3.34	1.61	1.07	5.39	3.58	1.87	1.25	6.15	4.09	2.30	1.53
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	2.38		1.59		2.61		1.74		2.94		1.96	
$t_y \times 10^3$ (kips)$^{-1}$	0.497		0.331		0.537		0.358		0.597		0.398	
$t_r \times 10^3$ (kips)$^{-1}$	0.645		0.430		0.697		0.465		0.775		0.517	
r_x/r_y	3.45				3.41				3.42			
r_y, in.	3.04				3.05				3.01			

[c] Shape is slender for compression with F_y = 65 ksi.

Return to Table of Contents

W24

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W24×											
		131[c]				117[c]				104[c,f]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.687	0.457	0.741	0.493	0.789	0.525	0.838	0.558	0.905	0.602	0.961	0.640
	11	0.809	0.538	0.770	0.513	0.928	0.618	0.875	0.582	1.06	0.708	0.995	0.662
	12	0.835	0.556	0.788	0.524	0.958	0.638	0.897	0.597	1.10	0.731	1.02	0.679
	13	0.865	0.576	0.806	0.537	0.992	0.660	0.919	0.611	1.14	0.757	1.05	0.696
	14	0.902	0.600	0.826	0.549	1.03	0.686	0.942	0.627	1.18	0.786	1.07	0.715
	15	0.944	0.628	0.846	0.563	1.07	0.715	0.967	0.643	1.23	0.819	1.10	0.735
	16	0.990	0.659	0.867	0.577	1.12	0.748	0.993	0.660	1.29	0.857	1.14	0.755
	17	1.04	0.693	0.890	0.592	1.18	0.785	1.02	0.679	1.35	0.899	1.17	0.777
	18	1.10	0.732	0.913	0.607	1.25	0.830	1.05	0.698	1.42	0.947	1.20	0.801
	19	1.17	0.775	0.938	0.624	1.32	0.880	1.08	0.718	1.50	1.00	1.24	0.826
	20	1.24	0.824	0.964	0.641	1.41	0.936	1.11	0.740	1.60	1.06	1.28	0.852
	22	1.41	0.938	1.02	0.679	1.61	1.07	1.18	0.787	1.83	1.22	1.37	0.910
	24	1.63	1.08	1.09	0.722	1.86	1.24	1.26	0.840	2.12	1.41	1.47	0.976
	26	1.90	1.27	1.16	0.770	2.18	1.45	1.36	0.908	2.49	1.66	1.64	1.09
	28	2.21	1.47	1.28	0.849	2.53	1.68	1.54	1.02	2.89	1.92	1.85	1.23
	30	2.53	1.68	1.42	0.942	2.90	1.93	1.71	1.14	3.32	2.21	2.06	1.37
	32	2.88	1.92	1.56	1.04	3.30	2.20	1.89	1.26	3.77	2.51	2.29	1.52
	34	3.25	2.16	1.70	1.13	3.72	2.48	2.07	1.38	4.26	2.83	2.51	1.67
	36	3.65	2.43	1.84	1.23	4.18	2.78	2.25	1.50	4.78	3.18	2.74	1.82
	38	4.06	2.70	1.99	1.32	4.65	3.10	2.43	1.62	5.32	3.54	2.97	1.98
	40	4.50	3.00	2.13	1.42	5.16	3.43	2.62	1.74	5.90	3.92	3.20	2.13
	42	4.96	3.30	2.28	1.52	5.68	3.78	2.80	1.86	6.50	4.33	3.44	2.29
	44	5.45	3.62	2.42	1.61	6.24	4.15	2.98	1.99	7.13	4.75	3.67	2.44
	46	5.95	3.96	2.57	1.71	6.82	4.54	3.17	2.11	7.80	5.19	3.91	2.60
	48	6.48	4.31	2.71	1.80	7.42	4.94	3.35	2.23	8.49	5.65	4.14	2.76

Other Constants and Properties

	131 ASD	131 LRFD	117 ASD	117 LRFD	104 ASD	104 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	3.36	2.24	3.84	2.55	4.48	2.98
$t_y \times 10^3$ (kips)$^{-1}$	0.666	0.443	0.747	0.497	0.837	0.557
$t_r \times 10^3$ (kips)$^{-1}$	0.864	0.576	0.969	0.646	1.09	0.724
r_x/r_y	3.43		3.44		3.47	
r_y, in.	2.97		2.94		2.91	

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W24

Shape		W24×											
		103[c]				94[c]				84[c]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.903	0.601	0.979	0.651	1.01	0.670	1.08	0.718	1.16	0.773	1.22	0.814
	11	1.29	0.860	1.16	0.769	1.43	0.954	1.28	0.854	1.65	1.10	1.48	0.982
	12	1.39	0.928	1.20	0.799	1.54	1.03	1.34	0.888	1.78	1.18	1.54	1.02
	13	1.52	1.01	1.25	0.831	1.67	1.11	1.39	0.926	1.93	1.28	1.61	1.07
	14	1.67	1.11	1.30	0.866	1.84	1.22	1.45	0.967	2.11	1.41	1.69	1.12
	15	1.85	1.23	1.36	0.904	2.03	1.35	1.52	1.01	2.34	1.56	1.77	1.18
	16	2.05	1.37	1.42	0.946	2.27	1.51	1.59	1.06	2.61	1.74	1.86	1.24
	17	2.31	1.54	1.49	0.991	2.55	1.70	1.67	1.11	2.95	1.96	1.96	1.31
	18	2.59	1.72	1.57	1.04	2.86	1.90	1.76	1.17	3.30	2.20	2.14	1.42
	19	2.88	1.92	1.68	1.12	3.18	2.12	1.93	1.29	3.68	2.45	2.33	1.55
	20	3.19	2.12	1.82	1.21	3.53	2.35	2.10	1.39	4.08	2.71	2.54	1.69
	22	3.86	2.57	2.09	1.39	4.27	2.84	2.43	1.61	4.94	3.28	2.95	1.96
	24	4.60	3.06	2.37	1.58	5.08	3.38	2.76	1.84	5.88	3.91	3.37	2.24
	26	5.40	3.59	2.65	1.77	5.96	3.97	3.10	2.06	6.90	4.59	3.80	2.53
	28	6.26	4.16	2.94	1.95	6.92	4.60	3.44	2.29	8.00	5.32	4.24	2.82
	30	7.19	4.78	3.22	2.14	7.94	5.28	3.79	2.52	9.18	6.11	4.67	3.11
	32	8.18	5.44	3.50	2.33	9.03	6.01	4.13	2.75	10.4	6.95	5.11	3.40

Other Constants and Properties

	103 ASD	103 LRFD	94 ASD	94 LRFD	84 ASD	84 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	6.60	4.39	7.31	4.86	8.41	5.59
$t_y \times 10^3$ (kips)$^{-1}$	0.848	0.564	0.928	0.617	1.04	0.692
$t_r \times 10^3$ (kips)$^{-1}$	1.10	0.733	1.20	0.802	1.35	0.900
r_x/r_y	5.03		4.98		5.02	
r_y, in.	1.99		1.98		1.95	

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W24

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W24× 76[c,v]				W24× 68[c,v]				W24× 62[c,v]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	1.31	0.872	1.37	0.912	1.50	0.996	1.55	1.03	1.67	1.11	1.79	1.19
	6	1.45	0.963	1.37	0.913	1.66	1.10	1.56	1.04	2.01	1.34	1.96	1.30
	7	1.50	1.00	1.42	0.947	1.72	1.15	1.62	1.08	2.16	1.43	2.07	1.38
	8	1.57	1.04	1.48	0.984	1.80	1.20	1.69	1.12	2.35	1.56	2.19	1.46
	9	1.65	1.10	1.54	1.02	1.90	1.26	1.76	1.17	2.60	1.73	2.33	1.55
	10	1.75	1.16	1.60	1.07	2.01	1.34	1.84	1.22	2.93	1.95	2.50	1.66
	11	1.87	1.24	1.67	1.11	2.15	1.43	1.92	1.28	3.37	2.25	2.68	1.78
	12	2.01	1.34	1.75	1.16	2.32	1.54	2.02	1.34	3.98	2.65	2.89	1.93
	13	2.18	1.45	1.83	1.22	2.53	1.68	2.12	1.41	4.67	3.11	3.25	2.16
	14	2.39	1.59	1.92	1.28	2.78	1.85	2.24	1.49	5.42	3.60	3.69	2.46
	15	2.65	1.76	2.02	1.35	3.09	2.06	2.37	1.57	6.22	4.14	4.15	2.76
	16	2.97	1.98	2.14	1.42	3.49	2.32	2.51	1.67	7.08	4.71	4.62	3.08
	17	3.35	2.23	2.30	1.53	3.94	2.62	2.76	1.84	7.99	5.31	5.11	3.40
	18	3.76	2.50	2.53	1.68	4.42	2.94	3.05	2.03	8.96	5.96	5.60	3.72
	19	4.19	2.79	2.77	1.85	4.92	3.27	3.35	2.23	9.98	6.64	6.10	4.06
	20	4.64	3.09	3.02	2.01	5.45	3.63	3.66	2.43	11.1	7.36	6.61	4.40
	22	5.62	3.74	3.53	2.35	6.60	4.39	4.29	2.85	13.4	8.90	7.64	5.08
	24	6.68	4.45	4.05	2.69	7.85	5.22	4.94	3.29				
	26	7.84	5.22	4.58	3.05	9.21	6.13	5.61	3.74				
	28	9.10	6.05	5.12	3.41	10.7	7.11	6.30	4.19				
	30	10.4	6.95	5.66	3.77	12.3	8.16	6.99	4.65				
	32	11.9	7.90	6.21	4.13								
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		9.58		6.38		11.20		7.44		17.50		11.60	
$t_y \times 10^3$ (kips)$^{-1}$		1.15		0.763		1.28		0.850		1.41		0.939	
$t_r \times 10^3$ (kips)$^{-1}$		1.49		0.992		1.66		1.11		1.83		1.22	
r_x/r_y		5.05				5.11				6.69			
r_y, in.		1.92				1.87				1.38			

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W24-W21

Shape	W24×				W21×							
	55[c,v]				201				182			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	1.95	1.30	2.05	1.36	0.433	0.288	0.517	0.344	0.479	0.319	0.576	0.383
6	2.35	1.56	2.26	1.50	0.457	0.304	0.517	0.344	0.506	0.337	0.576	0.383
7	2.52	1.68	2.40	1.59	0.466	0.310	0.517	0.344	0.516	0.344	0.576	0.383
8	2.75	1.83	2.55	1.70	0.477	0.317	0.517	0.344	0.528	0.352	0.576	0.383
9	3.06	2.03	2.73	1.81	0.489	0.326	0.517	0.344	0.542	0.361	0.576	0.383
10	3.46	2.30	2.93	1.95	0.503	0.335	0.522	0.347	0.558	0.371	0.582	0.387
11	4.00	2.66	3.16	2.10	0.520	0.346	0.530	0.352	0.576	0.383	0.591	0.394
12	4.74	3.16	3.43	2.28	0.538	0.358	0.537	0.358	0.597	0.397	0.601	0.400
13	5.57	3.70	3.96	2.63	0.558	0.371	0.545	0.363	0.620	0.412	0.611	0.406
14	6.46	4.29	4.51	3.00	0.581	0.387	0.554	0.368	0.646	0.430	0.621	0.413
15	7.41	4.93	5.08	3.38	0.607	0.404	0.562	0.374	0.675	0.449	0.632	0.420
16	8.43	5.61	5.68	3.78	0.636	0.423	0.571	0.380	0.707	0.471	0.643	0.428
17	9.52	6.33	6.29	4.18	0.669	0.445	0.581	0.386	0.744	0.495	0.654	0.435
18	10.7	7.10	6.91	4.60	0.705	0.469	0.590	0.393	0.785	0.522	0.666	0.443
19	11.9	7.91	7.55	5.02	0.745	0.496	0.600	0.399	0.830	0.552	0.678	0.451
20	13.2	8.77	8.20	5.46	0.790	0.525	0.610	0.406	0.881	0.586	0.691	0.459
22	15.9	10.6	9.52	6.34	0.896	0.596	0.631	0.420	1.00	0.666	0.717	0.477
24					1.03	0.684	0.654	0.435	1.15	0.766	0.746	0.496
26					1.20	0.797	0.679	0.452	1.34	0.893	0.777	0.517
28					1.39	0.924	0.705	0.469	1.56	1.04	0.811	0.540
30					1.59	1.06	0.734	0.488	1.79	1.19	0.849	0.565
32					1.81	1.21	0.765	0.509	2.03	1.35	0.889	0.592
34					2.05	1.36	0.799	0.531	2.30	1.53	0.934	0.622
36					2.30	1.53	0.836	0.556	2.57	1.71	1.00	0.666
38					2.56	1.70	0.885	0.589	2.87	1.91	1.07	0.710
40					2.83	1.89	0.939	0.625	3.18	2.11	1.13	0.754
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	20.6		13.7		2.06		1.37		2.30		1.53	
$t_y \times 10^3$ (kips)$^{-1}$	1.59		1.06		0.433		0.288		0.479		0.319	
$t_r \times 10^3$ (kips)$^{-1}$	2.06		1.37		0.562		0.375		0.622		0.415	
r_x/r_y	6.80				3.14				3.13			
r_y, in.	1.34				3.02				3.00			

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W21

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W21×											
		166				147				132			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.526	0.350	0.634	0.422	0.595	0.396	0.735	0.489	0.662	0.441	0.823	0.548
	6	0.556	0.370	0.634	0.422	0.629	0.419	0.735	0.489	0.701	0.467	0.823	0.548
	7	0.567	0.378	0.634	0.422	0.642	0.427	0.735	0.489	0.716	0.476	0.823	0.548
	8	0.581	0.386	0.634	0.422	0.658	0.438	0.735	0.489	0.733	0.488	0.823	0.548
	9	0.596	0.397	0.634	0.422	0.676	0.449	0.735	0.489	0.753	0.501	0.823	0.548
	10	0.614	0.408	0.642	0.427	0.696	0.463	0.747	0.497	0.777	0.517	0.838	0.558
	11	0.634	0.422	0.653	0.435	0.719	0.479	0.761	0.506	0.803	0.534	0.856	0.569
	12	0.656	0.437	0.665	0.442	0.746	0.496	0.776	0.516	0.833	0.554	0.874	0.581
	13	0.682	0.454	0.677	0.450	0.776	0.516	0.792	0.527	0.867	0.577	0.892	0.594
	14	0.711	0.473	0.689	0.458	0.809	0.539	0.808	0.537	0.905	0.602	0.912	0.607
	15	0.743	0.494	0.702	0.467	0.847	0.564	0.825	0.549	0.948	0.631	0.933	0.621
	16	0.779	0.518	0.715	0.476	0.890	0.592	0.842	0.560	0.996	0.663	0.954	0.635
	17	0.819	0.545	0.729	0.485	0.937	0.623	0.861	0.573	1.05	0.698	0.977	0.650
	18	0.865	0.575	0.743	0.494	0.990	0.659	0.880	0.585	1.11	0.739	1.00	0.665
	19	0.915	0.609	0.758	0.504	1.05	0.698	0.900	0.599	1.18	0.783	1.02	0.682
	20	0.971	0.646	0.774	0.515	1.12	0.742	0.921	0.613	1.25	0.834	1.05	0.699
	22	1.10	0.735	0.806	0.537	1.27	0.847	0.966	0.643	1.43	0.953	1.11	0.737
	24	1.27	0.846	0.842	0.560	1.47	0.979	1.02	0.676	1.66	1.10	1.17	0.778
	26	1.48	0.988	0.882	0.587	1.72	1.15	1.07	0.712	1.94	1.29	1.24	0.825
	28	1.72	1.15	0.925	0.615	2.00	1.33	1.13	0.753	2.25	1.50	1.32	0.877
	30	1.98	1.31	0.972	0.647	2.29	1.53	1.21	0.803	2.59	1.72	1.45	0.962
	32	2.25	1.50	1.02	0.682	2.61	1.74	1.31	0.875	2.95	1.96	1.58	1.05
	34	2.54	1.69	1.10	0.735	2.95	1.96	1.42	0.947	3.32	2.21	1.71	1.14
	36	2.85	1.89	1.18	0.788	3.30	2.20	1.53	1.02	3.73	2.48	1.85	1.23
	38	3.17	2.11	1.26	0.842	3.68	2.45	1.64	1.09	4.15	2.76	1.98	1.32
	40	3.51	2.34	1.34	0.895	4.08	2.71	1.75	1.16	4.60	3.06	2.12	1.41

Other Constants and Properties

	166 ASD	166 LRFD	147 ASD	147 LRFD	132 ASD	132 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	2.54	1.69	2.96	1.97	3.33	2.22
$t_y \times 10^3$ (kips)$^{-1}$	0.526	0.350	0.595	0.396	0.662	0.441
$t_r \times 10^3$ (kips)$^{-1}$	0.683	0.455	0.772	0.514	0.859	0.573
r_x/r_y	3.13		3.11		3.11	
r_y, in.	2.99		2.95		2.93	

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W21

Shape	W21×											
	122				111[c]				101[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.716	0.476	0.893	0.594	0.804	0.535	0.982	0.654	0.898	0.597	1.08	0.721
6	0.758	0.504	0.893	0.594	0.846	0.563	0.982	0.654	0.944	0.628	1.08	0.721
7	0.774	0.515	0.893	0.594	0.862	0.574	0.982	0.654	0.962	0.640	1.08	0.721
8	0.793	0.528	0.893	0.594	0.881	0.586	0.982	0.654	0.983	0.654	1.08	0.721
9	0.815	0.542	0.893	0.594	0.903	0.601	0.983	0.654	1.01	0.670	1.08	0.722
10	0.840	0.559	0.911	0.606	0.927	0.617	1.00	0.668	1.03	0.689	1.11	0.738
11	0.869	0.578	0.930	0.619	0.960	0.638	1.03	0.684	1.07	0.710	1.14	0.756
12	0.902	0.600	0.951	0.633	0.996	0.663	1.05	0.700	1.10	0.734	1.16	0.774
13	0.939	0.625	0.973	0.647	1.04	0.690	1.08	0.716	1.15	0.762	1.19	0.793
14	0.980	0.652	0.995	0.662	1.08	0.721	1.10	0.734	1.19	0.794	1.22	0.814
15	1.03	0.683	1.02	0.678	1.14	0.756	1.13	0.752	1.25	0.829	1.25	0.835
16	1.08	0.718	1.04	0.694	1.20	0.795	1.16	0.772	1.31	0.873	1.29	0.857
17	1.14	0.757	1.07	0.712	1.26	0.839	1.19	0.792	1.38	0.921	1.32	0.881
18	1.20	0.801	1.10	0.730	1.34	0.888	1.22	0.813	1.47	0.976	1.36	0.906
19	1.28	0.850	1.13	0.749	1.42	0.944	1.26	0.836	1.56	1.04	1.40	0.933
20	1.36	0.905	1.16	0.769	1.51	1.01	1.29	0.860	1.66	1.10	1.44	0.961
22	1.56	1.04	1.22	0.813	1.73	1.15	1.37	0.912	1.91	1.27	1.54	1.02
24	1.80	1.20	1.30	0.862	2.01	1.34	1.46	0.972	2.22	1.48	1.64	1.09
26	2.12	1.41	1.38	0.917	2.36	1.57	1.56	1.04	2.60	1.73	1.80	1.19
28	2.45	1.63	1.50	0.996	2.74	1.82	1.74	1.16	3.02	2.01	2.01	1.34
30	2.82	1.87	1.65	1.10	3.14	2.09	1.93	1.28	3.46	2.30	2.24	1.49
32	3.20	2.13	1.81	1.20	3.58	2.38	2.12	1.41	3.94	2.62	2.46	1.64
34	3.62	2.41	1.97	1.31	4.04	2.69	2.31	1.53	4.45	2.96	2.69	1.79
36	4.06	2.70	2.12	1.41	4.53	3.01	2.50	1.66	4.99	3.32	2.92	1.94
38	4.52	3.01	2.28	1.52	5.05	3.36	2.69	1.79	5.56	3.70	3.14	2.09
40	5.01	3.33	2.44	1.62	5.59	3.72	2.88	1.91	6.16	4.10	3.37	2.24

Other Constants and Properties

	122 ASD	122 LRFD	111 ASD	111 LRFD	101 ASD	101 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	3.63	2.41	4.02	2.67	4.44	2.96
$t_y \times 10^3$ (kips)$^{-1}$	0.716	0.476	0.788	0.524	0.862	0.574
$t_r \times 10^3$ (kips)$^{-1}$	0.929	0.619	1.02	0.682	1.12	0.746
r_x/r_y	3.11		3.12		3.12	
r_y, in.	2.92		2.90		2.89	

[c] Shape is slender for compression with F_y = 65 ksi.

Return to Table of Contents

W21

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W21×											
	93[c]				83[c]				73[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.950	0.632	1.24	0.825	1.10	0.731	1.40	0.930	1.29	0.859	1.59	1.06
6	1.09	0.724	1.25	0.833	1.24	0.827	1.41	0.941	1.46	0.970	1.62	1.08
7	1.15	0.763	1.29	0.861	1.30	0.866	1.47	0.975	1.52	1.01	1.68	1.12
8	1.22	0.811	1.34	0.891	1.37	0.914	1.52	1.01	1.61	1.07	1.74	1.16
9	1.31	0.869	1.39	0.924	1.47	0.976	1.58	1.05	1.71	1.14	1.82	1.21
10	1.41	0.938	1.44	0.959	1.58	1.05	1.64	1.09	1.84	1.22	1.89	1.26
11	1.53	1.02	1.50	0.996	1.73	1.15	1.71	1.14	1.99	1.32	1.98	1.32
12	1.68	1.12	1.56	1.04	1.90	1.26	1.79	1.19	2.18	1.45	2.07	1.38
13	1.86	1.24	1.62	1.08	2.10	1.40	1.87	1.24	2.42	1.61	2.17	1.45
14	2.08	1.38	1.70	1.13	2.35	1.56	1.96	1.30	2.71	1.80	2.28	1.52
15	2.34	1.56	1.77	1.18	2.64	1.76	2.06	1.37	3.06	2.04	2.41	1.60
16	2.65	1.77	1.86	1.24	3.00	2.00	2.17	1.44	3.48	2.32	2.55	1.70
17	3.00	1.99	1.96	1.30	3.39	2.25	2.29	1.52	3.93	2.62	2.78	1.85
18	3.36	2.23	2.08	1.38	3.80	2.53	2.51	1.67	4.41	2.93	3.04	2.02
19	3.74	2.49	2.25	1.50	4.23	2.82	2.72	1.81	4.91	3.27	3.31	2.20
20	4.15	2.76	2.42	1.61	4.69	3.12	2.94	1.96	5.44	3.62	3.58	2.38
22	5.02	3.34	2.77	1.84	5.67	3.78	3.37	2.24	6.58	4.38	4.13	2.75
24	5.97	3.97	3.12	2.07	6.75	4.49	3.81	2.53	7.83	5.21	4.68	3.12
26	7.01	4.66	3.46	2.30	7.93	5.27	4.25	2.83	9.19	6.12	5.24	3.49
28	8.13	5.41	3.81	2.54	9.19	6.12	4.69	3.12	10.7	7.09	5.81	3.86
30	9.33	6.21	4.16	2.77	10.6	7.02	5.13	3.41	12.2	8.14	6.37	4.24
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	7.90		5.25		8.99		5.98		10.3		6.85	
$t_y \times 10^3$ (kips)$^{-1}$	0.941		0.626		1.05		0.701		1.19		0.795	
$t_r \times 10^3$ (kips)$^{-1}$	1.22		0.814		1.37		0.911		1.55		1.03	
r_x/r_y	4.73				4.74				4.77			
r_y, in.	1.84				1.83				1.81			

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W21

Shape		W21×											
		68[c]				62[c]				57[c]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	1.41	0.939	1.71	1.14	1.58	1.05	1.90	1.27	1.74	1.16	2.12	1.41
	6	1.59	1.06	1.74	1.16	1.78	1.18	1.94	1.29	2.14	1.43	2.34	1.55
	7	1.66	1.11	1.81	1.20	1.86	1.24	2.02	1.34	2.32	1.54	2.47	1.64
	8	1.75	1.17	1.88	1.25	1.96	1.31	2.11	1.40	2.55	1.70	2.62	1.74
	9	1.87	1.24	1.97	1.31	2.09	1.39	2.20	1.46	2.86	1.90	2.79	1.86
	10	2.00	1.33	2.05	1.37	2.24	1.49	2.30	1.53	3.27	2.17	2.99	1.99
	11	2.17	1.44	2.15	1.43	2.43	1.62	2.41	1.60	3.82	2.54	3.21	2.14
	12	2.37	1.58	2.26	1.50	2.66	1.77	2.53	1.69	4.53	3.02	3.47	2.31
	13	2.62	1.75	2.37	1.58	2.95	1.96	2.67	1.78	5.32	3.54	3.92	2.61
	14	2.94	1.96	2.50	1.67	3.31	2.20	2.82	1.88	6.17	4.10	4.42	2.94
	15	3.33	2.21	2.65	1.76	3.76	2.50	2.99	1.99	7.08	4.71	4.94	3.29
	16	3.78	2.52	2.82	1.88	4.28	2.85	3.26	2.17	8.06	5.36	5.47	3.64
	17	4.27	2.84	3.11	2.07	4.83	3.21	3.61	2.40	9.10	6.05	6.01	4.00
	18	4.79	3.19	3.42	2.27	5.41	3.60	3.97	2.64	10.2	6.79	6.55	4.36
	19	5.34	3.55	3.72	2.48	6.03	4.01	4.33	2.88	11.4	7.56	7.10	4.72
	20	5.91	3.93	4.03	2.68	6.68	4.45	4.70	3.13	12.6	8.38	7.65	5.09
	22	7.16	4.76	4.66	3.10	8.09	5.38	5.46	3.63	15.2	10.1	8.76	5.83
	24	8.52	5.67	5.31	3.53	9.63	6.40	6.24	4.15				
	26	9.99	6.65	5.95	3.96	11.3	7.52	7.02	4.67				
	28	11.6	7.71	6.60	4.39	13.1	8.72	7.81	5.20				
	30	13.3	8.85	7.26	4.83								
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		11.2		7.47		12.6		8.40		18.5		12.3	
$t_y \times 10^3$ (kips)$^{-1}$		1.28		0.855		1.40		0.934		1.54		1.02	
$t_r \times 10^3$ (kips)$^{-1}$		1.67		1.11		1.82		1.21		2.00		1.33	
r_x/r_y		4.78				4.82				6.19			
r_y, in.		1.80				1.77				1.35			

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W21

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W21×											
	55[c,v]				50[c,v]				48[c,f,v]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	1.83	1.22	2.18	1.45	2.04	1.36	2.49	1.66	2.17	1.44	2.68	1.78
6	2.07	1.37	2.23	1.49	2.53	1.68	2.78	1.85	2.46	1.64	2.68	1.78
7	2.16	1.44	2.33	1.55	2.75	1.83	2.95	1.96	2.58	1.72	2.78	1.85
8	2.28	1.52	2.44	1.62	3.04	2.02	3.14	2.09	2.73	1.82	2.92	1.94
9	2.43	1.62	2.55	1.70	3.42	2.28	3.36	2.23	2.92	1.94	3.07	2.04
10	2.61	1.74	2.68	1.78	3.94	2.62	3.61	2.40	3.15	2.10	3.23	2.15
11	2.84	1.89	2.82	1.88	4.67	3.10	3.91	2.60	3.44	2.29	3.42	2.27
12	3.11	2.07	2.98	1.98	5.55	3.69	4.36	2.90	3.80	2.52	3.62	2.41
13	3.46	2.30	3.16	2.10	6.52	4.34	5.00	3.33	4.25	2.83	3.86	2.57
14	3.89	2.59	3.36	2.23	7.56	5.03	5.67	3.77	4.83	3.22	4.12	2.74
15	4.45	2.96	3.59	2.39	8.68	5.77	6.36	4.23	5.55	3.69	4.60	3.06
16	5.06	3.37	4.02	2.67	9.87	6.57	7.06	4.70	6.31	4.20	5.16	3.44
17	5.71	3.80	4.46	2.97	11.1	7.42	7.78	5.17	7.13	4.74	5.75	3.82
18	6.40	4.26	4.92	3.27	12.5	8.31	8.51	5.66	7.99	5.32	6.35	4.22
19	7.13	4.75	5.38	3.58	13.9	9.26	9.24	6.15	8.90	5.92	6.97	4.63
20	7.90	5.26	5.86	3.90	15.4	10.3	9.99	6.65	9.86	6.56	7.60	5.06
21	8.71	5.80	6.34	4.22	17.0	11.3	10.7	7.15	10.9	7.23	8.25	5.49
22	9.56	6.36	6.84	4.55					11.9	7.94	8.91	5.93
23	10.5	6.95	7.34	4.88					13.0	8.68	9.58	6.37
24	11.4	7.57	7.84	5.22					14.2	9.45	10.3	6.82
25	12.3	8.22	8.35	5.56					15.4	10.3	10.9	7.28
26	13.4	8.89	8.87	5.90					16.7	11.1	11.6	7.75
27	14.4	9.58	9.38	6.24					18.0	12.0	12.3	8.22
28	15.5	10.3	9.90	6.59								

Row label: Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending

Other Constants and Properties

	55 ASD	55 LRFD	50 ASD	50 LRFD	48 ASD	48 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	14.9	9.91	22.5	14.9	19.6	13.0
$t_y \times 10^3$ (kips)$^{-1}$	1.59	1.06	1.75	1.16	1.82	1.21
$t_r \times 10^3$ (kips)$^{-1}$	2.06	1.37	2.27	1.51	2.36	1.58
r_x/r_y	4.86		6.29		4.96	
r_y, in.	1.73		1.30		1.66	

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W21-W18

Shape		W21×				W18×							
		44[c,v]				311[h]				283[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	2.39	1.59	2.87	1.91	0.280	0.187	0.363	0.242	0.308	0.205	0.405	0.270
	6	2.97	1.98	3.25	2.16	0.297	0.197	0.363	0.242	0.327	0.218	0.405	0.270
	7	3.24	2.15	3.46	2.30	0.303	0.202	0.363	0.242	0.334	0.222	0.405	0.270
	8	3.59	2.39	3.71	2.47	0.310	0.206	0.363	0.242	0.342	0.228	0.405	0.270
	9	4.06	2.70	3.99	2.66	0.319	0.212	0.363	0.242	0.352	0.234	0.405	0.270
	10	4.71	3.13	4.32	2.88	0.328	0.218	0.366	0.243	0.363	0.241	0.409	0.272
	11	5.62	3.74	4.71	3.13	0.339	0.226	0.369	0.245	0.375	0.250	0.412	0.274
	12	6.68	4.45	5.43	3.61	0.352	0.234	0.372	0.247	0.389	0.259	0.416	0.277
	13	7.84	5.22	6.25	4.16	0.366	0.243	0.375	0.249	0.405	0.270	0.420	0.279
	14	9.10	6.05	7.11	4.73	0.382	0.254	0.378	0.251	0.423	0.282	0.424	0.282
	15	10.4	6.95	7.99	5.32	0.400	0.266	0.381	0.254	0.444	0.295	0.428	0.284
	16	11.9	7.91	8.90	5.92	0.420	0.279	0.384	0.256	0.467	0.310	0.431	0.287
	17	13.4	8.93	9.83	6.54	0.442	0.294	0.387	0.258	0.492	0.327	0.435	0.290
	18	15.0	10.0	10.8	7.18	0.467	0.311	0.391	0.260	0.521	0.346	0.440	0.292
	19	16.8	11.1	11.8	7.82	0.495	0.329	0.394	0.262	0.553	0.368	0.444	0.295
	20	18.6	12.4	12.7	8.47	0.526	0.350	0.397	0.264	0.589	0.392	0.448	0.298
	22					0.601	0.400	0.404	0.269	0.674	0.449	0.457	0.304
	24					0.694	0.462	0.412	0.274	0.783	0.521	0.466	0.310
	26					0.812	0.541	0.419	0.279	0.918	0.611	0.475	0.316
	28					0.942	0.627	0.427	0.284	1.06	0.708	0.485	0.323
	30					1.08	0.720	0.435	0.289	1.22	0.813	0.495	0.330
	32					1.23	0.819	0.443	0.295	1.39	0.925	0.506	0.337
	34					1.39	0.924	0.452	0.301	1.57	1.04	0.517	0.344
	36					1.56	1.04	0.461	0.307	1.76	1.17	0.529	0.352
	38					1.74	1.15	0.470	0.313	1.96	1.30	0.541	0.360
	40					1.92	1.28	0.480	0.319	2.17	1.45	0.554	0.368

Other Constants and Properties

	W21×44 ASD	W21×44 LRFD	W18×311 ASD	W18×311 LRFD	W18×283 ASD	W18×283 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	26.9	17.9	1.32	0.881	1.48	0.986
$t_y \times 10^3$ (kips)$^{-1}$	1.98	1.31	0.280	0.187	0.308	0.205
$t_r \times 10^3$ (kips)$^{-1}$	2.56	1.71	0.364	0.243	0.400	0.267
r_x/r_y	6.40		2.96		2.96	
r_y, in.	1.26		2.95		2.91	

[c] Shape is slender for compression with F_y = 65 ksi.

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W18

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W18×											
	258[h]				234[h]				211			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.338	0.225	0.449	0.298	0.375	0.249	0.499	0.332	0.412	0.274	0.559	0.372
6	0.359	0.239	0.449	0.298	0.398	0.265	0.499	0.332	0.439	0.292	0.559	0.372
7	0.367	0.244	0.449	0.298	0.407	0.271	0.499	0.332	0.449	0.299	0.559	0.372
8	0.376	0.250	0.449	0.298	0.417	0.278	0.499	0.332	0.460	0.306	0.559	0.372
9	0.386	0.257	0.449	0.299	0.429	0.286	0.500	0.333	0.474	0.315	0.561	0.373
10	0.399	0.265	0.453	0.302	0.443	0.295	0.505	0.336	0.490	0.326	0.568	0.378
11	0.413	0.275	0.458	0.304	0.459	0.306	0.511	0.340	0.508	0.338	0.574	0.382
12	0.429	0.285	0.462	0.307	0.477	0.318	0.516	0.343	0.528	0.352	0.581	0.387
13	0.447	0.297	0.467	0.310	0.498	0.331	0.522	0.347	0.552	0.367	0.588	0.391
14	0.467	0.311	0.471	0.314	0.521	0.347	0.528	0.351	0.578	0.384	0.596	0.396
15	0.490	0.326	0.476	0.317	0.547	0.364	0.534	0.355	0.607	0.404	0.603	0.401
16	0.516	0.343	0.481	0.320	0.577	0.384	0.540	0.359	0.641	0.426	0.611	0.406
17	0.545	0.362	0.486	0.323	0.610	0.406	0.546	0.363	0.678	0.451	0.618	0.411
18	0.577	0.384	0.491	0.327	0.647	0.430	0.552	0.367	0.720	0.479	0.626	0.417
19	0.613	0.408	0.496	0.330	0.688	0.458	0.558	0.372	0.768	0.511	0.635	0.422
20	0.654	0.435	0.501	0.334	0.735	0.489	0.565	0.376	0.821	0.546	0.643	0.428
22	0.751	0.500	0.512	0.341	0.847	0.563	0.579	0.385	0.949	0.631	0.661	0.440
24	0.875	0.582	0.524	0.348	0.990	0.659	0.593	0.395	1.11	0.741	0.679	0.452
26	1.03	0.684	0.536	0.356	1.16	0.773	0.608	0.405	1.31	0.870	0.699	0.465
28	1.19	0.793	0.548	0.365	1.35	0.897	0.624	0.415	1.52	1.01	0.720	0.479
30	1.37	0.910	0.561	0.373	1.55	1.03	0.641	0.426	1.74	1.16	0.742	0.494
32	1.56	1.04	0.575	0.382	1.76	1.17	0.658	0.438	1.98	1.32	0.766	0.509
34	1.76	1.17	0.589	0.392	1.99	1.32	0.677	0.451	2.24	1.49	0.791	0.526
36	1.97	1.31	0.604	0.402	2.23	1.48	0.697	0.464	2.51	1.67	0.817	0.544
38	2.19	1.46	0.620	0.413	2.48	1.65	0.718	0.478	2.79	1.86	0.846	0.563
40	2.43	1.62	0.637	0.424	2.75	1.83	0.740	0.492	3.09	2.06	0.877	0.583

Other Constants and Properties

	258[h] ASD	258[h] LRFD	234[h] ASD	234[h] LRFD	211 ASD	211 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.65	1.10	1.84	1.22	2.08	1.38
$t_y \times 10^3$ (kips)$^{-1}$	0.338	0.225	0.375	0.249	0.412	0.274
$t_r \times 10^3$ (kips)$^{-1}$	0.439	0.292	0.486	0.324	0.535	0.357
r_x/r_y	2.96		2.96		2.96	
r_y, in.	2.88		2.85		2.82	

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W18

Shape		W18× 192				W18× 175				W18× 158			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.457	0.304	0.620	0.413	0.500	0.333	0.689	0.458	0.555	0.369	0.770	0.512
	6	0.487	0.324	0.620	0.413	0.533	0.355	0.689	0.458	0.593	0.394	0.770	0.512
	7	0.498	0.332	0.620	0.413	0.546	0.363	0.689	0.458	0.607	0.404	0.770	0.512
	8	0.512	0.340	0.620	0.413	0.561	0.373	0.689	0.458	0.624	0.415	0.770	0.512
	9	0.527	0.351	0.623	0.414	0.578	0.385	0.693	0.461	0.643	0.428	0.776	0.516
	10	0.545	0.363	0.631	0.420	0.598	0.398	0.703	0.468	0.666	0.443	0.788	0.524
	11	0.566	0.376	0.639	0.425	0.621	0.413	0.713	0.474	0.692	0.460	0.801	0.533
	12	0.589	0.392	0.648	0.431	0.647	0.431	0.723	0.481	0.722	0.480	0.814	0.541
	13	0.615	0.409	0.656	0.437	0.677	0.451	0.734	0.488	0.755	0.502	0.827	0.550
	14	0.645	0.429	0.665	0.443	0.711	0.473	0.745	0.496	0.793	0.528	0.841	0.560
	15	0.679	0.452	0.675	0.449	0.749	0.498	0.757	0.503	0.836	0.556	0.855	0.569
	16	0.717	0.477	0.684	0.455	0.792	0.527	0.768	0.511	0.885	0.589	0.870	0.579
	17	0.760	0.506	0.694	0.462	0.840	0.559	0.780	0.519	0.940	0.625	0.886	0.589
	18	0.808	0.538	0.704	0.468	0.895	0.595	0.793	0.528	1.00	0.667	0.902	0.600
	19	0.862	0.574	0.714	0.475	0.956	0.636	0.806	0.536	1.07	0.713	0.918	0.611
	20	0.924	0.615	0.725	0.482	1.03	0.682	0.819	0.545	1.15	0.766	0.935	0.622
	22	1.07	0.712	0.747	0.497	1.19	0.794	0.847	0.564	1.34	0.892	0.971	0.646
	24	1.26	0.839	0.771	0.513	1.41	0.938	0.877	0.584	1.59	1.06	1.01	0.672
	26	1.48	0.985	0.796	0.530	1.65	1.10	0.910	0.605	1.86	1.24	1.05	0.701
	28	1.72	1.14	0.823	0.548	1.92	1.28	0.945	0.628	2.16	1.44	1.10	0.731
	30	1.97	1.31	0.852	0.567	2.20	1.47	0.982	0.653	2.48	1.65	1.15	0.765
	32	2.24	1.49	0.883	0.587	2.51	1.67	1.02	0.680	2.82	1.88	1.21	0.802
	34	2.53	1.68	0.916	0.609	2.83	1.88	1.07	0.710	3.19	2.12	1.27	0.843
	36	2.84	1.89	0.952	0.633	3.17	2.11	1.11	0.742	3.57	2.38	1.35	0.900
	38	3.16	2.10	0.990	0.659	3.53	2.35	1.18	0.784	3.98	2.65	1.44	0.957
	40	3.50	2.33	1.03	0.688	3.91	2.60	1.25	0.830	4.41	2.93	1.52	1.01

Other Constants and Properties

	W18×192 ASD	W18×192 LRFD	W18×175 ASD	W18×175 LRFD	W18×158 ASD	W18×158 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	2.30	1.53	2.59	1.72	2.89	1.92
$t_y \times 10^3$ (kips)$^{-1}$	0.457	0.304	0.500	0.333	0.555	0.369
$t_r \times 10^3$ (kips)$^{-1}$	0.593	0.395	0.649	0.432	0.720	0.480
r_x/r_y	2.97		2.97		2.96	
r_y, in.	2.79		2.76		2.74	

Return to Table of Contents

W18

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W18×											
	143				130				119			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending: 0	0.612	0.407	0.851	0.566	0.671	0.446	0.945	0.629	0.732	0.487	1.05	0.696
6	0.654	0.435	0.851	0.566	0.718	0.478	0.945	0.629	0.784	0.521	1.05	0.696
7	0.670	0.446	0.851	0.566	0.735	0.489	0.945	0.629	0.803	0.534	1.05	0.696
8	0.689	0.458	0.851	0.566	0.756	0.503	0.945	0.629	0.826	0.550	1.05	0.696
9	0.711	0.473	0.859	0.572	0.781	0.520	0.956	0.636	0.853	0.568	1.06	0.705
10	0.736	0.490	0.874	0.582	0.809	0.539	0.974	0.648	0.884	0.588	1.08	0.719
11	0.765	0.509	0.889	0.592	0.842	0.560	0.992	0.660	0.920	0.612	1.10	0.734
12	0.798	0.531	0.905	0.602	0.879	0.585	1.01	0.673	0.961	0.639	1.13	0.750
13	0.836	0.556	0.921	0.613	0.921	0.613	1.03	0.686	1.01	0.670	1.15	0.766
14	0.879	0.585	0.938	0.624	0.969	0.645	1.05	0.700	1.06	0.706	1.18	0.783
15	0.928	0.617	0.956	0.636	1.02	0.681	1.07	0.714	1.12	0.745	1.20	0.800
16	0.982	0.654	0.974	0.648	1.08	0.722	1.10	0.729	1.19	0.790	1.23	0.819
17	1.04	0.695	0.993	0.661	1.15	0.768	1.12	0.745	1.26	0.841	1.26	0.838
18	1.11	0.741	1.01	0.674	1.23	0.820	1.14	0.761	1.35	0.899	1.29	0.858
19	1.19	0.794	1.03	0.688	1.32	0.879	1.17	0.778	1.45	0.964	1.32	0.880
20	1.28	0.853	1.05	0.702	1.42	0.946	1.20	0.796	1.56	1.04	1.36	0.902
22	1.50	0.996	1.10	0.732	1.66	1.11	1.25	0.834	1.83	1.22	1.43	0.950
24	1.78	1.18	1.15	0.765	1.98	1.31	1.32	0.877	2.17	1.45	1.51	1.00
26	2.08	1.39	1.20	0.801	2.32	1.54	1.39	0.923	2.55	1.70	1.60	1.06
28	2.42	1.61	1.26	0.841	2.69	1.79	1.47	0.975	2.96	1.97	1.71	1.14
30	2.77	1.85	1.33	0.885	3.09	2.05	1.56	1.04	3.39	2.26	1.87	1.24
32	3.16	2.10	1.41	0.938	3.51	2.34	1.69	1.13	3.86	2.57	2.03	1.35
34	3.56	2.37	1.51	1.01	3.97	2.64	1.82	1.21	4.36	2.90	2.18	1.45
36	4.00	2.66	1.62	1.08	4.45	2.96	1.95	1.30	4.89	3.25	2.34	1.56
38	4.45	2.96	1.72	1.15	4.95	3.30	2.08	1.38	5.45	3.62	2.50	1.66
40	4.93	3.28	1.82	1.21	5.49	3.65	2.20	1.47	6.04	4.02	2.65	1.77

Other Constants and Properties

	143 ASD	143 LRFD	130 ASD	130 LRFD	119 ASD	119 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	3.21	2.14	3.57	2.38	3.97	2.64
$t_y \times 10^3$ (kips)$^{-1}$	0.612	0.407	0.671	0.446	0.732	0.487
$t_r \times 10^3$ (kips)$^{-1}$	0.794	0.529	0.870	0.580	0.950	0.633
r_x/r_y	2.97		2.97		2.94	
r_y, in.	2.72		2.70		2.69	

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W18

Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending

Shape	W18× 106				W18× 97				W18× 86[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.826	0.550	1.19	0.793	0.901	0.600	1.30	0.864	1.03	0.685	1.47	0.980
6	0.886	0.589	1.19	0.793	0.967	0.643	1.30	0.864	1.10	0.730	1.47	0.980
7	0.908	0.604	1.19	0.793	0.992	0.660	1.30	0.864	1.12	0.747	1.47	0.980
8	0.935	0.622	1.19	0.793	1.02	0.679	1.30	0.864	1.15	0.767	1.47	0.980
9	0.966	0.643	1.21	0.806	1.06	0.702	1.32	0.880	1.19	0.793	1.50	1.00
10	1.00	0.667	1.24	0.824	1.10	0.729	1.35	0.900	1.24	0.824	1.54	1.03
11	1.04	0.695	1.27	0.842	1.14	0.759	1.38	0.921	1.29	0.858	1.58	1.05
12	1.09	0.726	1.29	0.862	1.19	0.794	1.42	0.944	1.35	0.898	1.62	1.08
13	1.15	0.762	1.33	0.882	1.25	0.834	1.45	0.967	1.42	0.944	1.67	1.11
14	1.21	0.803	1.36	0.903	1.32	0.879	1.49	0.992	1.50	0.996	1.71	1.14
15	1.28	0.849	1.39	0.926	1.40	0.930	1.53	1.02	1.59	1.05	1.76	1.17
16	1.36	0.902	1.43	0.949	1.48	0.988	1.57	1.05	1.69	1.12	1.81	1.21
17	1.44	0.961	1.46	0.974	1.58	1.05	1.62	1.07	1.80	1.20	1.87	1.24
18	1.55	1.03	1.50	1.00	1.70	1.13	1.66	1.11	1.93	1.28	1.93	1.28
19	1.66	1.11	1.54	1.03	1.82	1.21	1.71	1.14	2.07	1.38	1.99	1.32
20	1.79	1.19	1.59	1.06	1.97	1.31	1.76	1.17	2.24	1.49	2.05	1.37
22	2.11	1.40	1.68	1.12	2.32	1.54	1.87	1.25	2.65	1.76	2.20	1.46
24	2.51	1.67	1.79	1.19	2.76	1.83	2.00	1.33	3.15	2.10	2.38	1.58
26	2.94	1.96	1.91	1.27	3.24	2.15	2.19	1.46	3.70	2.46	2.68	1.78
28	3.41	2.27	2.11	1.40	3.75	2.50	2.43	1.61	4.29	2.86	2.98	1.98
30	3.92	2.61	2.31	1.54	4.31	2.87	2.67	1.77	4.93	3.28	3.29	2.19
32	4.46	2.97	2.51	1.67	4.90	3.26	2.91	1.93	5.61	3.73	3.59	2.39
34	5.03	3.35	2.72	1.81	5.53	3.68	3.15	2.09	6.33	4.21	3.90	2.60
36	5.64	3.75	2.92	1.94	6.20	4.13	3.38	2.25	7.10	4.72	4.21	2.80
38	6.29	4.18	3.12	2.08	6.91	4.60	3.62	2.41	7.91	5.26	4.51	3.00
40	6.97	4.63	3.32	2.21	7.66	5.10	3.86	2.57	8.76	5.83	4.82	3.21

Other Constants and Properties

	W18×106 ASD	W18×106 LRFD	W18×97 ASD	W18×97 LRFD	W18×86 ASD	W18×86 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	4.53	3.01	4.96	3.30	5.66	3.77
$t_y \times 10^3$ (kips)$^{-1}$	0.826	0.550	0.901	0.600	1.02	0.676
$t_r \times 10^3$ (kips)$^{-1}$	1.07	0.715	1.17	0.780	1.32	0.878
r_x/r_y	2.95		2.95		2.95	
r_y, in.	2.66		2.65		2.63	

[c] Shape is slender for compression with F_y = 65 ksi.

Return to Table of Contents

W18

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W18× 76[c,f]				W18× 71[c]				W18× 65[c]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	1.20	0.799	1.69	1.12	1.24	0.826	1.88	1.25	1.39	0.927	2.06	1.37
	6	1.28	0.850	1.69	1.12	1.46	0.970	1.93	1.28	1.62	1.07	2.12	1.41
	7	1.31	0.869	1.69	1.12	1.55	1.03	2.00	1.33	1.71	1.14	2.20	1.47
	8	1.34	0.893	1.69	1.12	1.66	1.11	2.08	1.38	1.83	1.22	2.29	1.53
	9	1.38	0.920	1.72	1.15	1.80	1.20	2.16	1.44	1.98	1.32	2.39	1.59
	10	1.43	0.952	1.77	1.18	1.97	1.31	2.26	1.50	2.17	1.45	2.50	1.66
	11	1.49	0.989	1.82	1.21	2.18	1.45	2.36	1.57	2.40	1.60	2.61	1.74
	12	1.55	1.03	1.87	1.24	2.43	1.62	2.47	1.64	2.68	1.78	2.74	1.82
	13	1.62	1.08	1.92	1.28	2.74	1.82	2.59	1.72	3.02	2.01	2.88	1.92
	14	1.71	1.14	1.98	1.32	3.11	2.07	2.72	1.81	3.44	2.29	3.04	2.02
	15	1.81	1.20	2.04	1.36	3.57	2.37	2.87	1.91	3.95	2.63	3.21	2.14
	16	1.93	1.28	2.10	1.40	4.06	2.70	3.03	2.02	4.50	2.99	3.43	2.28
	17	2.06	1.37	2.17	1.45	4.58	3.05	3.27	2.18	5.08	3.38	3.76	2.50
	18	2.21	1.47	2.25	1.49	5.14	3.42	3.55	2.36	5.69	3.79	4.09	2.72
	19	2.38	1.58	2.32	1.55	5.73	3.81	3.84	2.55	6.34	4.22	4.43	2.95
	20	2.57	1.71	2.41	1.60	6.34	4.22	4.12	2.74	7.02	4.67	4.76	3.17
	22	3.05	2.03	2.60	1.73	7.68	5.11	4.69	3.12	8.50	5.66	5.44	3.62
	24	3.63	2.42	2.90	1.93	9.14	6.08	5.25	3.50	10.1	6.73	6.11	4.07
	26	4.26	2.84	3.28	2.18	10.7	7.13	5.82	3.87	11.9	7.90	6.79	4.51
	28	4.94	3.29	3.67	2.44	12.4	8.27	6.38	4.25	13.8	9.16	7.46	4.96
	30	5.68	3.78	4.06	2.70								
	32	6.46	4.30	4.45	2.96								
	34	7.29	4.85	4.85	3.22								
	36	8.17	5.44	5.24	3.49								
	38	9.11	6.06	5.64	3.75								
	40	10.1	6.71	6.04	4.02								
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		6.52		4.34		11.1		7.38		12.2		8.10	
$t_y \times 10^3$ (kips)$^{-1}$		1.15		0.767		1.23		0.818		1.35		0.895	
$t_r \times 10^3$ (kips)$^{-1}$		1.49		0.997		1.59		1.06		1.75		1.16	
r_x/r_y		2.96				4.41				4.43			
r_y, in.		2.61				1.70				1.69			

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W18

Shape	W18×											
	60[c]				55[c]				50[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	1.55	1.03	2.23	1.48	1.71	1.14	2.45	1.63	1.93	1.29	2.71	1.81
6	1.79	1.19	2.30	1.53	1.97	1.31	2.53	1.68	2.23	1.48	2.82	1.87
7	1.89	1.26	2.39	1.59	2.08	1.39	2.64	1.76	2.35	1.57	2.94	1.96
8	2.01	1.34	2.49	1.66	2.22	1.48	2.76	1.83	2.51	1.67	3.08	2.05
9	2.17	1.44	2.60	1.73	2.39	1.59	2.88	1.92	2.70	1.80	3.23	2.15
10	2.37	1.58	2.72	1.81	2.60	1.73	3.03	2.01	2.94	1.95	3.40	2.26
11	2.63	1.75	2.85	1.90	2.87	1.91	3.18	2.12	3.23	2.15	3.59	2.39
12	2.93	1.95	3.00	1.99	3.22	2.14	3.36	2.23	3.61	2.40	3.79	2.52
13	3.31	2.20	3.16	2.10	3.64	2.42	3.55	2.36	4.09	2.72	4.03	2.68
14	3.78	2.52	3.34	2.22	4.16	2.77	3.76	2.50	4.69	3.12	4.29	2.85
15	4.34	2.89	3.54	2.35	4.77	3.17	4.04	2.69	5.39	3.58	4.70	3.13
16	4.94	3.28	3.87	2.58	5.43	3.61	4.48	2.98	6.13	4.08	5.22	3.48
17	5.57	3.71	4.25	2.83	6.13	4.08	4.93	3.28	6.92	4.60	5.76	3.83
18	6.25	4.16	4.63	3.08	6.87	4.57	5.39	3.59	7.76	5.16	6.31	4.20
19	6.96	4.63	5.02	3.34	7.65	5.09	5.85	3.89	8.64	5.75	6.86	4.57
20	7.71	5.13	5.41	3.60	8.48	5.64	6.32	4.20	9.58	6.37	7.43	4.94
22	9.33	6.21	6.19	4.12	10.3	6.83	7.26	4.83	11.6	7.71	8.56	5.70
24	11.1	7.39	6.98	4.64	12.2	8.13	8.20	5.46	13.8	9.17	9.72	6.47
26	13.0	8.67	7.76	5.16	14.3	9.54	9.16	6.09	16.2	10.8	10.9	7.24
28	15.1	10.1	8.55	5.69								
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	13.3		8.85		14.8		9.86		16.5		11.0	
$t_y \times 10^3$ (kips)$^{-1}$	1.46		0.971		1.59		1.06		1.75		1.16	
$t_r \times 10^3$ (kips)$^{-1}$	1.89		1.26		2.06		1.37		2.27		1.51	
r_x/r_y	4.45				4.44				4.47			
r_y, in.	1.68				1.67				1.65			

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W18

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W18×											
	46[c]				40[c,v]				35[c,v]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	2.12	1.41	3.02	2.01	2.52	1.67	3.50	2.33	2.97	1.98	4.12	2.74
6	2.68	1.78	3.37	2.24	3.17	2.11	3.93	2.62	3.78	2.51	4.73	3.14
7	2.93	1.95	3.57	2.38	3.47	2.31	4.18	2.78	4.15	2.76	5.06	3.37
8	3.27	2.17	3.81	2.53	3.87	2.57	4.47	2.97	4.65	3.10	5.45	3.62
9	3.71	2.47	4.07	2.71	4.39	2.92	4.80	3.19	5.33	3.55	5.90	3.92
10	4.33	2.88	4.37	2.91	5.10	3.40	5.18	3.45	6.27	4.17	6.43	4.28
11	5.16	3.43	4.73	3.15	6.09	4.05	5.63	3.74	7.56	5.03	7.23	4.81
12	6.14	4.09	5.23	3.48	7.25	4.82	6.43	4.28	9.00	5.99	8.43	5.61
13	7.21	4.80	5.95	3.96	8.51	5.66	7.35	4.89	10.6	7.03	9.67	6.43
14	8.36	5.56	6.69	4.45	9.87	6.56	8.30	5.52	12.2	8.15	11.0	7.29
15	9.60	6.38	7.45	4.95	11.3	7.54	9.27	6.17	14.1	9.36	12.3	8.17
16	10.9	7.26	8.21	5.46	12.9	8.57	10.3	6.83	16.0	10.6	13.6	9.07
17	12.3	8.20	8.98	5.97	14.5	9.68	11.3	7.50	18.1	12.0	15.0	10.0
18	13.8	9.19	9.75	6.49	16.3	10.9	12.3	8.17	20.2	13.5	16.4	10.9
19	15.4	10.2	10.5	7.01	18.2	12.1	13.3	8.85	22.6	15.0	17.9	11.9
20	17.1	11.3	11.3	7.53	20.1	13.4	14.3	9.54	25.0	16.6	19.3	12.8
21	18.8	12.5	12.1	8.05	22.2	14.8	15.4	10.2				
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	23.4		15.6		27.4		18.2		34.0		22.6	
$t_y \times 10^3$ (kips)$^{-1}$	1.90		1.27		2.18		1.45		2.49		1.66	
$t_r \times 10^3$ (kips)$^{-1}$	2.47		1.65		2.82		1.88		3.24		2.16	
r_x/r_y	5.62				5.68				5.77			
r_y, in.	1.29				1.27				1.22			

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W16

Shape	W16× 100				W16× 89				W16× 77			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending: 0	0.874	0.581	1.38	0.921	0.981	0.652	1.57	1.04	1.14	0.756	1.83	1.22
6	0.945	0.629	1.38	0.921	1.06	0.706	1.57	1.04	1.23	0.820	1.83	1.22
7	0.972	0.647	1.38	0.921	1.09	0.727	1.57	1.04	1.27	0.844	1.83	1.22
8	1.00	0.668	1.39	0.925	1.13	0.751	1.58	1.05	1.31	0.873	1.84	1.23
9	1.04	0.693	1.42	0.944	1.17	0.780	1.61	1.07	1.36	0.907	1.89	1.26
10	1.09	0.723	1.45	0.965	1.22	0.814	1.65	1.10	1.42	0.947	1.94	1.29
11	1.14	0.756	1.48	0.986	1.28	0.852	1.69	1.12	1.49	0.992	1.99	1.32
12	1.19	0.795	1.51	1.01	1.35	0.897	1.73	1.15	1.57	1.04	2.04	1.36
13	1.26	0.839	1.55	1.03	1.42	0.947	1.78	1.18	1.66	1.11	2.10	1.40
14	1.34	0.890	1.58	1.05	1.51	1.01	1.82	1.21	1.76	1.17	2.16	1.44
15	1.42	0.948	1.62	1.08	1.61	1.07	1.87	1.24	1.88	1.25	2.22	1.48
16	1.52	1.01	1.66	1.11	1.73	1.15	1.92	1.28	2.02	1.34	2.29	1.52
17	1.64	1.09	1.70	1.13	1.86	1.23	1.98	1.32	2.17	1.45	2.36	1.57
18	1.77	1.18	1.75	1.16	2.01	1.33	2.03	1.35	2.35	1.56	2.44	1.62
19	1.91	1.27	1.79	1.19	2.18	1.45	2.10	1.39	2.56	1.70	2.52	1.68
20	2.08	1.39	1.84	1.23	2.37	1.58	2.16	1.44	2.79	1.86	2.61	1.74
22	2.50	1.67	1.95	1.30	2.85	1.90	2.30	1.53	3.36	2.24	2.80	1.86
24	2.98	1.98	2.07	1.38	3.40	2.26	2.46	1.64	4.00	2.66	3.09	2.06
26	3.50	2.33	2.20	1.46	3.99	2.65	2.71	1.80	4.70	3.13	3.46	2.30
28	4.06	2.70	2.40	1.60	4.62	3.08	2.99	1.99	5.45	3.62	3.83	2.55
30	4.66	3.10	2.62	1.74	5.31	3.53	3.26	2.17	6.25	4.16	4.20	2.80
32	5.30	3.52	2.83	1.88	6.04	4.02	3.54	2.36	7.12	4.73	4.57	3.04
34	5.98	3.98	3.04	2.03	6.82	4.54	3.82	2.54	8.03	5.34	4.94	3.29
36	6.70	4.46	3.26	2.17	7.64	5.09	4.09	2.72	9.01	5.99	5.31	3.53
38	7.47	4.97	3.47	2.31	8.52	5.67	4.36	2.90	10.0	6.68	5.68	3.78
40	8.28	5.51	3.68	2.45	9.44	6.28	4.63	3.08	11.1	7.40	6.04	4.02

Other Constants and Properties

	100 ASD	100 LRFD	89 ASD	89 LRFD	77 ASD	77 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	4.99	3.32	5.70	3.79	6.67	4.44
$t_y \times 10^3$ (kips)$^{-1}$	0.874	0.581	0.981	0.652	1.14	0.756
$t_r \times 10^3$ (kips)$^{-1}$	1.13	0.756	1.27	0.848	1.47	0.983
r_x/r_y	2.83		2.83		2.83	
r_y, in.	2.51		2.49		2.47	

Return to Table of Contents

W16

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W16×											
		67[c]				57[c]				50[c]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	1.35	0.897	2.11	1.40	1.55	1.03	2.61	1.74	1.83	1.22	2.98	1.98
	6	1.45	0.963	2.11	1.40	1.85	1.23	2.72	1.81	2.16	1.44	3.11	2.07
	7	1.49	0.989	2.11	1.40	1.99	1.32	2.82	1.88	2.30	1.53	3.25	2.16
	8	1.53	1.02	2.13	1.42	2.15	1.43	2.94	1.96	2.47	1.64	3.40	2.26
	9	1.59	1.06	2.19	1.45	2.36	1.57	3.07	2.04	2.71	1.80	3.57	2.37
	10	1.65	1.10	2.25	1.50	2.61	1.74	3.21	2.14	3.00	2.00	3.75	2.49
	11	1.72	1.15	2.31	1.54	2.92	1.94	3.37	2.24	3.37	2.24	3.95	2.63
	12	1.82	1.21	2.38	1.58	3.30	2.20	3.54	2.35	3.81	2.54	4.17	2.78
	13	1.92	1.28	2.45	1.63	3.78	2.51	3.73	2.48	4.36	2.90	4.43	2.94
	14	2.04	1.36	2.53	1.68	4.37	2.90	3.94	2.62	5.05	3.36	4.71	3.13
	15	2.18	1.45	2.61	1.74	5.01	3.33	4.17	2.77	5.80	3.86	5.13	3.41
	16	2.34	1.56	2.70	1.80	5.70	3.79	4.55	3.03	6.60	4.39	5.66	3.77
	17	2.52	1.68	2.79	1.86	6.44	4.28	4.97	3.31	7.45	4.96	6.20	4.12
	18	2.73	1.81	2.89	1.92	7.22	4.80	5.39	3.58	8.35	5.56	6.74	4.48
	19	2.97	1.97	3.00	1.99	8.04	5.35	5.81	3.86	9.31	6.19	7.28	4.85
	20	3.24	2.16	3.11	2.07	8.91	5.93	6.23	4.14	10.3	6.86	7.83	5.21
	22	3.91	2.60	3.39	2.25	10.8	7.17	7.07	4.70	12.5	8.30	8.93	5.94
	24	4.65	3.10	3.86	2.57	12.8	8.54	7.90	5.26	14.8	9.88	10.0	6.67
	26	5.46	3.63	4.34	2.89	15.1	10.0	8.74	5.82	17.4	11.6	11.1	7.40
	28	6.33	4.21	4.82	3.21								
	30	7.27	4.84	5.31	3.53								
	32	8.27	5.50	5.80	3.86								
	34	9.34	6.21	6.29	4.18								
	36	10.5	6.96	6.77	4.51								
	38	11.7	7.76	7.26	4.83								
	40	12.9	8.60	7.75	5.15								
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		7.72		5.14		14.5		9.65		16.8		11.2	
$t_y \times 10^3$ (kips)$^{-1}$		1.31		0.872		1.53		1.02		1.75		1.16	
$t_r \times 10^3$ (kips)$^{-1}$		1.70		1.13		1.98		1.32		2.27		1.51	
r_x / r_y		2.83				4.20				4.20			
r_y, in.		2.46				1.60				1.59			

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W16

Shape	W16× 45[c]				40[c]				36[c,f,v]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	2.07	1.38	3.33	2.22	2.41	1.60	3.75	2.50	2.73	1.82	4.29	2.86
6	2.45	1.63	3.50	2.33	2.83	1.88	3.95	2.63	3.22	2.14	4.55	3.03
7	2.60	1.73	3.66	2.44	3.00	2.00	4.14	2.76	3.43	2.28	4.78	3.18
8	2.80	1.86	3.84	2.55	3.22	2.15	4.35	2.90	3.70	2.46	5.04	3.35
9	3.05	2.03	4.04	2.68	3.50	2.33	4.59	3.05	4.03	2.68	5.32	3.54
10	3.37	2.24	4.25	2.83	3.85	2.56	4.85	3.23	4.45	2.96	5.64	3.75
11	3.78	2.52	4.50	2.99	4.29	2.85	5.14	3.42	4.99	3.32	6.00	3.99
12	4.30	2.86	4.77	3.17	4.84	3.22	5.47	3.64	5.69	3.78	6.41	4.26
13	4.94	3.29	5.07	3.38	5.57	3.70	5.84	3.89	6.61	4.40	6.88	4.58
14	5.73	3.81	5.43	3.61	6.46	4.30	6.40	4.26	7.67	5.10	7.82	5.20
15	6.58	4.37	6.06	4.03	7.41	4.93	7.17	4.77	8.80	5.86	8.79	5.85
16	7.48	4.98	6.71	4.46	8.43	5.61	7.96	5.30	10.0	6.66	9.79	6.51
17	8.45	5.62	7.36	4.90	9.52	6.33	8.76	5.83	11.3	7.52	10.8	7.19
18	9.47	6.30	8.03	5.34	10.7	7.10	9.58	6.38	12.7	8.43	11.9	7.89
19	10.5	7.02	8.70	5.79	11.9	7.91	10.4	6.93	14.1	9.40	12.9	8.59
20	11.7	7.78	9.37	6.23	13.2	8.77	11.2	7.48	15.6	10.4	14.0	9.31
21	12.9	8.57	10.0	6.68	14.5	9.66	12.1	8.04	17.3	11.5	15.1	10.0
22	14.1	9.41	10.7	7.14	15.9	10.6	12.9	8.61	18.9	12.6	16.2	10.8
23	15.5	10.3	11.4	7.59	17.4	11.6	13.8	9.17	20.7	13.8	17.3	11.5
24	16.8	11.2	12.1	8.04	19.0	12.6	14.6	9.74	22.5	15.0	18.4	12.2
25	18.3	12.2	12.8	8.50	20.6	13.7	15.5	10.3	24.4	16.3	19.5	13.0
26	19.8	13.1	13.5	8.95	22.3	14.8	16.4	10.9				

Row labels: Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending

Other Constants and Properties

	45 ASD	45 LRFD	40 ASD	40 LRFD	36 ASD	36 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	18.9	12.6	21.6	14.4	25.5	16.9
$t_y \times 10^3$ (kips)$^{-1}$	1.93	1.29	2.18	1.45	2.42	1.61
$t_r \times 10^3$ (kips)$^{-1}$	2.51	1.67	2.82	1.88	3.14	2.10
r_x/r_y	4.24		4.22		4.28	
r_y, in.	1.57		1.57		1.52	

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W16

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W16×							
		31[c,v]				26[c,v]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	3.27	2.17	5.08	3.38	4.05	2.69	6.20	4.13
	6	4.30	2.86	5.92	3.94	5.38	3.58	7.39	4.91
	7	4.79	3.18	6.36	4.23	6.02	4.01	7.99	5.32
	8	5.45	3.62	6.88	4.58	6.91	4.60	8.70	5.79
	9	6.36	4.23	7.49	4.98	8.15	5.42	9.56	6.36
	10	7.67	5.10	8.21	5.47	9.94	6.62	10.9	7.26
	11	9.28	6.17	9.55	6.35	12.0	8.01	12.9	8.60
	12	11.0	7.34	11.1	7.35	14.3	9.53	15.0	10.0
	13	13.0	8.62	12.6	8.39	16.8	11.2	17.2	11.5
	14	15.0	10.0	14.2	9.45	19.5	13.0	19.5	13.0
	15	17.2	11.5	15.8	10.5	22.4	14.9	21.9	14.6
	16	19.6	13.1	17.5	11.6	25.5	16.9	24.3	16.2
	17	22.2	14.7	19.2	12.8	28.7	19.1	26.7	17.8
	18	24.8	16.5	20.9	13.9	32.2	21.4	29.2	19.4
	19	27.7	18.4	22.6	15.0				

Other Constants and Properties				
$b_y \times 10^3$ (kip-ft)$^{-1}$	39.0	25.9	50.0	33.3
$t_y \times 10^3$ (kips)$^{-1}$	2.81	1.87	3.35	2.23
$t_r \times 10^3$ (kips)$^{-1}$	3.65	2.43	4.34	2.89
r_x/r_y	5.48		5.59	
r_y, in.	1.17		1.12	

[c] Shape is slender for compression with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W14

Shape	W14×											
	730[h]				665[h]				605[h]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.119	0.0795	0.165	0.110	0.131	0.0872	0.185	0.123	0.144	0.0960	0.208	0.138
11	0.129	0.0857	0.165	0.110	0.142	0.0943	0.185	0.123	0.156	0.104	0.208	0.138
12	0.131	0.0870	0.165	0.110	0.144	0.0957	0.185	0.123	0.159	0.106	0.208	0.138
13	0.133	0.0883	0.165	0.110	0.146	0.0972	0.185	0.123	0.161	0.107	0.208	0.138
14	0.135	0.0898	0.165	0.110	0.149	0.0989	0.185	0.123	0.164	0.109	0.208	0.138
15	0.137	0.0915	0.165	0.110	0.151	0.101	0.185	0.123	0.167	0.111	0.208	0.138
16	0.140	0.0932	0.166	0.110	0.154	0.103	0.186	0.124	0.171	0.114	0.209	0.139
17	0.143	0.0952	0.166	0.110	0.158	0.105	0.186	0.124	0.175	0.116	0.209	0.139
18	0.146	0.0973	0.166	0.111	0.161	0.107	0.187	0.124	0.179	0.119	0.210	0.140
19	0.150	0.0995	0.167	0.111	0.165	0.110	0.187	0.125	0.183	0.122	0.210	0.140
20	0.153	0.102	0.167	0.111	0.169	0.113	0.188	0.125	0.188	0.125	0.211	0.140
22	0.161	0.107	0.168	0.112	0.179	0.119	0.189	0.126	0.199	0.132	0.212	0.141
24	0.171	0.114	0.169	0.112	0.190	0.126	0.190	0.126	0.211	0.141	0.213	0.142
26	0.182	0.121	0.170	0.113	0.202	0.135	0.191	0.127	0.226	0.150	0.215	0.143
28	0.195	0.130	0.170	0.113	0.217	0.144	0.192	0.128	0.242	0.161	0.216	0.144
30	0.209	0.139	0.171	0.114	0.233	0.155	0.193	0.128	0.262	0.174	0.217	0.144
32	0.226	0.150	0.172	0.115	0.253	0.168	0.194	0.129	0.284	0.189	0.218	0.145
34	0.245	0.163	0.173	0.115	0.275	0.183	0.195	0.130	0.310	0.206	0.220	0.146
36	0.268	0.178	0.174	0.116	0.301	0.200	0.196	0.130	0.340	0.226	0.221	0.147
38	0.293	0.195	0.175	0.116	0.331	0.220	0.197	0.131	0.375	0.250	0.222	0.148
40	0.324	0.216	0.176	0.117	0.366	0.244	0.198	0.132	0.416	0.277	0.223	0.149
42	0.357	0.238	0.176	0.117	0.404	0.269	0.199	0.132	0.459	0.305	0.225	0.150
44	0.392	0.261	0.177	0.118	0.443	0.295	0.200	0.133	0.503	0.335	0.226	0.150
46	0.429	0.285	0.178	0.119	0.485	0.322	0.201	0.134	0.550	0.366	0.227	0.151
48	0.467	0.311	0.179	0.119	0.528	0.351	0.202	0.135	0.599	0.399	0.229	0.152
50	0.506	0.337	0.180	0.120	0.573	0.381	0.204	0.135	0.650	0.432	0.230	0.153

Other Constants and Properties

	730[h] ASD	730[h] LRFD	665[h] ASD	665[h] LRFD	605[h] ASD	605[h] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	0.336	0.223	0.375	0.250	0.420	0.280
$t_y \times 10^3$ (kips)$^{-1}$	0.119	0.0795	0.131	0.0872	0.144	0.0960
$t_r \times 10^3$ (kips)$^{-1}$	0.155	0.103	0.170	0.113	0.187	0.125
r_x/r_y	1.74		1.73		1.71	
r_y, in.	4.69		4.62		4.55	

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

W14

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W14×											
		550[h]				500[h]				455[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.159	0.106	0.232	0.155	0.175	0.116	0.261	0.174	0.192	0.128	0.293	0.195
	11	0.172	0.115	0.232	0.155	0.190	0.127	0.261	0.174	0.209	0.139	0.293	0.195
	12	0.175	0.116	0.232	0.155	0.193	0.129	0.261	0.174	0.212	0.141	0.293	0.195
	13	0.178	0.118	0.232	0.155	0.197	0.131	0.261	0.174	0.216	0.144	0.293	0.195
	14	0.181	0.121	0.232	0.155	0.200	0.133	0.261	0.174	0.221	0.147	0.293	0.195
	15	0.185	0.123	0.233	0.155	0.204	0.136	0.262	0.174	0.225	0.150	0.294	0.196
	16	0.189	0.126	0.234	0.155	0.209	0.139	0.263	0.175	0.230	0.153	0.295	0.196
	17	0.193	0.128	0.234	0.156	0.214	0.142	0.264	0.176	0.236	0.157	0.296	0.197
	18	0.198	0.131	0.235	0.156	0.219	0.146	0.265	0.176	0.242	0.161	0.297	0.198
	19	0.203	0.135	0.236	0.157	0.225	0.150	0.266	0.177	0.248	0.165	0.298	0.199
	20	0.208	0.138	0.237	0.157	0.231	0.154	0.266	0.177	0.255	0.170	0.299	0.199
	22	0.220	0.147	0.238	0.158	0.245	0.163	0.268	0.178	0.271	0.180	0.302	0.201
	24	0.234	0.156	0.239	0.159	0.261	0.174	0.270	0.180	0.289	0.192	0.304	0.202
	26	0.251	0.167	0.241	0.160	0.280	0.186	0.272	0.181	0.311	0.207	0.306	0.204
	28	0.270	0.180	0.242	0.161	0.302	0.201	0.274	0.182	0.335	0.223	0.308	0.205
	30	0.292	0.194	0.244	0.162	0.327	0.218	0.275	0.183	0.364	0.242	0.311	0.207
	32	0.318	0.211	0.246	0.163	0.357	0.238	0.277	0.185	0.398	0.265	0.313	0.208
	34	0.348	0.231	0.247	0.164	0.391	0.260	0.279	0.186	0.437	0.291	0.315	0.210
	36	0.382	0.254	0.249	0.165	0.432	0.287	0.281	0.187	0.483	0.322	0.318	0.211
	38	0.424	0.282	0.250	0.166	0.480	0.319	0.283	0.188	0.538	0.358	0.320	0.213
	40	0.469	0.312	0.252	0.168	0.531	0.354	0.285	0.190	0.596	0.397	0.323	0.215
	42	0.517	0.344	0.253	0.169	0.586	0.390	0.287	0.191	0.657	0.437	0.325	0.216
	44	0.568	0.378	0.255	0.170	0.643	0.428	0.289	0.192	0.721	0.480	0.328	0.218
	46	0.621	0.413	0.257	0.171	0.703	0.468	0.291	0.194	0.789	0.525	0.330	0.220
	48	0.676	0.450	0.259	0.172	0.765	0.509	0.293	0.195	0.859	0.571	0.333	0.221
	50	0.733	0.488	0.260	0.173	0.830	0.552	0.295	0.197	0.932	0.620	0.335	0.223
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		0.470		0.313		0.525		0.349		0.586		0.390	
$t_y \times 10^3$ (kips)$^{-1}$		0.159		0.106		0.175		0.116		0.192		0.128	
$t_r \times 10^3$ (kips)$^{-1}$		0.206		0.137		0.227		0.151		0.249		0.166	
r_x/r_y		1.70				1.69				1.67			
r_y, in.		4.49				4.43				4.38			

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W14

Shape		W14×											
		426[h]				398[h]				370[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.206	0.137	0.315	0.210	0.220	0.146	0.342	0.228	0.236	0.157	0.372	0.248
	11	0.224	0.149	0.315	0.210	0.240	0.160	0.342	0.228	0.258	0.172	0.372	0.248
	12	0.228	0.152	0.315	0.210	0.244	0.162	0.342	0.228	0.263	0.175	0.372	0.248
	13	0.232	0.155	0.315	0.210	0.249	0.165	0.342	0.228	0.268	0.178	0.372	0.248
	14	0.237	0.158	0.316	0.210	0.254	0.169	0.343	0.228	0.273	0.182	0.374	0.249
	15	0.242	0.161	0.317	0.211	0.259	0.172	0.344	0.229	0.279	0.186	0.375	0.250
	16	0.248	0.165	0.318	0.212	0.265	0.176	0.346	0.230	0.286	0.190	0.377	0.251
	17	0.254	0.169	0.320	0.213	0.272	0.181	0.347	0.231	0.293	0.195	0.378	0.252
	18	0.260	0.173	0.321	0.213	0.279	0.185	0.348	0.232	0.301	0.200	0.380	0.253
	19	0.267	0.178	0.322	0.214	0.287	0.191	0.350	0.233	0.309	0.206	0.382	0.254
	20	0.275	0.183	0.323	0.215	0.295	0.196	0.351	0.234	0.318	0.212	0.383	0.255
	22	0.292	0.194	0.326	0.217	0.314	0.209	0.354	0.236	0.339	0.226	0.387	0.257
	24	0.312	0.208	0.328	0.218	0.336	0.223	0.357	0.238	0.363	0.242	0.390	0.259
	26	0.336	0.223	0.331	0.220	0.361	0.240	0.360	0.239	0.392	0.261	0.393	0.262
	28	0.363	0.242	0.333	0.222	0.391	0.260	0.363	0.241	0.425	0.283	0.397	0.264
	30	0.395	0.263	0.336	0.224	0.426	0.284	0.366	0.244	0.463	0.308	0.401	0.267
	32	0.433	0.288	0.339	0.225	0.467	0.311	0.369	0.246	0.508	0.338	0.404	0.269
	34	0.476	0.317	0.341	0.227	0.515	0.342	0.372	0.248	0.561	0.374	0.408	0.271
	36	0.527	0.351	0.344	0.229	0.571	0.380	0.375	0.250	0.625	0.416	0.412	0.274
	38	0.588	0.391	0.347	0.231	0.637	0.423	0.379	0.252	0.696	0.463	0.416	0.276
	40	0.651	0.433	0.350	0.233	0.705	0.469	0.382	0.254	0.771	0.513	0.419	0.279
	42	0.718	0.478	0.353	0.235	0.778	0.517	0.385	0.256	0.850	0.566	0.423	0.282
	44	0.788	0.524	0.356	0.237	0.853	0.568	0.389	0.259	0.933	0.621	0.428	0.284
	46	0.861	0.573	0.359	0.239	0.933	0.621	0.392	0.261	1.02	0.679	0.432	0.287
	48	0.938	0.624	0.362	0.241	1.02	0.676	0.396	0.263	1.11	0.739	0.436	0.290
	50	1.02	0.677	0.365	0.243	1.10	0.733	0.400	0.266	1.21	0.802	0.440	0.293

Other Constants and Properties

	426 ASD	426 LRFD	398 ASD	398 LRFD	370 ASD	370 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	0.631	0.420	0.682	0.454	0.741	0.493
$t_y \times 10^3$ (kips)$^{-1}$	0.206	0.137	0.220	0.146	0.236	0.157
$t_r \times 10^3$ (kips)$^{-1}$	0.267	0.178	0.285	0.190	0.306	0.204
r_x/r_y	1.67		1.66		1.66	
r_y, in.	4.34		4.31		4.27	

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

W14

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W14×											
		342[h]				311[h]				283[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.254	0.169	0.408	0.271	0.281	0.187	0.454	0.302	0.308	0.205	0.506	0.336
	11	0.279	0.186	0.408	0.271	0.309	0.205	0.454	0.302	0.339	0.226	0.506	0.336
	12	0.284	0.189	0.408	0.271	0.314	0.209	0.454	0.302	0.345	0.230	0.506	0.336
	13	0.289	0.192	0.408	0.271	0.320	0.213	0.454	0.302	0.352	0.234	0.506	0.337
	14	0.295	0.196	0.409	0.272	0.327	0.218	0.457	0.304	0.360	0.239	0.509	0.338
	15	0.302	0.201	0.411	0.274	0.335	0.223	0.459	0.305	0.368	0.245	0.511	0.340
	16	0.309	0.206	0.413	0.275	0.343	0.228	0.461	0.307	0.377	0.251	0.514	0.342
	17	0.317	0.211	0.415	0.276	0.352	0.234	0.464	0.308	0.387	0.258	0.517	0.344
	18	0.326	0.217	0.417	0.277	0.361	0.240	0.466	0.310	0.398	0.265	0.520	0.346
	19	0.335	0.223	0.419	0.279	0.372	0.247	0.468	0.312	0.410	0.273	0.523	0.348
	20	0.345	0.230	0.421	0.280	0.383	0.255	0.471	0.313	0.423	0.281	0.526	0.350
	22	0.368	0.245	0.425	0.283	0.409	0.272	0.476	0.316	0.451	0.300	0.532	0.354
	24	0.394	0.262	0.429	0.285	0.440	0.292	0.481	0.320	0.485	0.323	0.538	0.358
	26	0.426	0.283	0.433	0.288	0.475	0.316	0.486	0.323	0.525	0.349	0.544	0.362
	28	0.462	0.307	0.437	0.291	0.516	0.344	0.491	0.327	0.572	0.380	0.551	0.366
	30	0.505	0.336	0.441	0.294	0.565	0.376	0.496	0.330	0.626	0.417	0.557	0.371
	32	0.555	0.369	0.446	0.297	0.622	0.414	0.501	0.334	0.691	0.459	0.564	0.375
	34	0.613	0.408	0.450	0.300	0.689	0.459	0.507	0.337	0.766	0.510	0.571	0.380
	36	0.684	0.455	0.455	0.303	0.770	0.512	0.513	0.341	0.857	0.570	0.578	0.384
	38	0.762	0.507	0.459	0.306	0.858	0.571	0.518	0.345	0.955	0.635	0.585	0.389
	40	0.844	0.562	0.464	0.309	0.951	0.633	0.524	0.349	1.06	0.704	0.592	0.394
	42	0.931	0.619	0.469	0.312	1.05	0.697	0.530	0.353	1.17	0.776	0.600	0.399
	44	1.02	0.680	0.474	0.315	1.15	0.765	0.536	0.357	1.28	0.852	0.608	0.404
	46	1.12	0.743	0.479	0.319	1.26	0.837	0.543	0.361	1.40	0.931	0.616	0.410
	48	1.22	0.809	0.484	0.322	1.37	0.911	0.549	0.365	1.52	1.01	0.624	0.415
	50	1.32	0.878	0.489	0.325	1.49	0.988	0.556	0.370	1.65	1.10	0.632	0.421

Other Constants and Properties

	342[h] ASD	342[h] LRFD	311[h] ASD	311[h] LRFD	283[h] ASD	283[h] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	0.811	0.539	0.901	0.600	1.00	0.665
$t_y \times 10^3$ (kips)$^{-1}$	0.254	0.169	0.281	0.187	0.308	0.205
$t_r \times 10^3$ (kips)$^{-1}$	0.330	0.220	0.365	0.243	0.400	0.267
r_x/r_y	1.65		1.64		1.63	
r_y, in.	4.24		4.20		4.17	

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W14

Shape	W14×											
	257				233				211			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.340	0.226	0.563	0.374	0.375	0.250	0.629	0.418	0.414	0.276	0.703	0.468
11	0.374	0.249	0.563	0.374	0.414	0.275	0.629	0.418	0.458	0.305	0.703	0.468
12	0.381	0.254	0.563	0.374	0.422	0.281	0.629	0.418	0.467	0.311	0.703	0.468
13	0.389	0.259	0.563	0.375	0.430	0.286	0.630	0.419	0.476	0.317	0.705	0.469
14	0.398	0.265	0.567	0.377	0.440	0.293	0.634	0.422	0.487	0.324	0.710	0.472
15	0.407	0.271	0.570	0.379	0.450	0.300	0.638	0.425	0.499	0.332	0.715	0.476
16	0.417	0.278	0.574	0.382	0.462	0.307	0.642	0.427	0.512	0.341	0.720	0.479
17	0.429	0.285	0.577	0.384	0.475	0.316	0.647	0.430	0.526	0.350	0.726	0.483
18	0.441	0.293	0.581	0.386	0.488	0.325	0.651	0.433	0.542	0.360	0.731	0.486
19	0.454	0.302	0.584	0.389	0.503	0.335	0.656	0.436	0.558	0.372	0.737	0.490
20	0.468	0.312	0.588	0.391	0.519	0.346	0.660	0.439	0.577	0.384	0.742	0.494
22	0.501	0.333	0.595	0.396	0.556	0.370	0.669	0.445	0.618	0.411	0.754	0.501
24	0.540	0.359	0.603	0.401	0.600	0.399	0.679	0.452	0.667	0.444	0.766	0.509
26	0.585	0.389	0.611	0.406	0.650	0.433	0.688	0.458	0.724	0.482	0.778	0.517
28	0.638	0.424	0.619	0.412	0.710	0.472	0.699	0.465	0.792	0.527	0.790	0.526
30	0.700	0.466	0.627	0.417	0.781	0.519	0.709	0.472	0.872	0.580	0.803	0.535
32	0.773	0.514	0.635	0.423	0.863	0.574	0.719	0.479	0.966	0.643	0.817	0.543
34	0.859	0.572	0.644	0.428	0.962	0.640	0.730	0.486	1.08	0.717	0.831	0.553
36	0.963	0.641	0.653	0.434	1.08	0.717	0.742	0.493	1.21	0.804	0.845	0.562
38	1.07	0.714	0.662	0.440	1.20	0.799	0.753	0.501	1.35	0.896	0.860	0.572
40	1.19	0.791	0.671	0.447	1.33	0.886	0.765	0.509	1.49	0.993	0.876	0.583
42	1.31	0.872	0.681	0.453	1.47	0.976	0.778	0.517	1.65	1.09	0.892	0.593
44	1.44	0.957	0.691	0.460	1.61	1.07	0.790	0.526	1.81	1.20	0.908	0.604
46	1.57	1.05	0.701	0.466	1.76	1.17	0.804	0.535	1.97	1.31	0.925	0.616
48	1.71	1.14	0.712	0.473	1.92	1.28	0.817	0.544	2.15	1.43	0.943	0.628
50	1.86	1.24	0.722	0.481	2.08	1.38	0.832	0.553	2.33	1.55	0.962	0.640
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.11		0.741		1.24		0.825		1.38		0.921	
$t_y \times 10^3$ (kips)$^{-1}$	0.340		0.226		0.375		0.250		0.414		0.276	
$t_r \times 10^3$ (kips)$^{-1}$	0.441		0.294		0.487		0.324		0.538		0.358	
r_x/r_y	1.62				1.62				1.61			
r_y, in.	4.13				4.10				4.07			

W14

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W14×											
	193				176				159			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.452	0.301	0.772	0.514	0.496	0.330	0.856	0.570	0.550	0.366	0.955	0.635
11	0.500	0.333	0.772	0.514	0.550	0.366	0.856	0.570	0.610	0.406	0.955	0.635
12	0.510	0.339	0.772	0.514	0.560	0.373	0.856	0.570	0.622	0.414	0.955	0.635
13	0.521	0.347	0.775	0.516	0.572	0.381	0.860	0.572	0.636	0.423	0.960	0.639
14	0.533	0.354	0.781	0.520	0.586	0.390	0.868	0.577	0.651	0.433	0.970	0.645
15	0.546	0.363	0.787	0.524	0.600	0.399	0.875	0.582	0.667	0.444	0.979	0.651
16	0.560	0.373	0.794	0.528	0.616	0.410	0.883	0.588	0.685	0.456	0.989	0.658
17	0.576	0.383	0.800	0.533	0.634	0.422	0.891	0.593	0.704	0.469	0.999	0.664
18	0.593	0.394	0.807	0.537	0.653	0.434	0.899	0.598	0.726	0.483	1.01	0.671
19	0.611	0.407	0.814	0.541	0.673	0.448	0.907	0.604	0.749	0.498	1.02	0.678
20	0.632	0.420	0.820	0.546	0.696	0.463	0.916	0.609	0.775	0.515	1.03	0.685
22	0.677	0.451	0.834	0.555	0.747	0.497	0.933	0.621	0.832	0.554	1.05	0.699
24	0.731	0.487	0.849	0.565	0.808	0.538	0.951	0.633	0.901	0.599	1.07	0.714
26	0.795	0.529	0.864	0.575	0.879	0.585	0.969	0.645	0.981	0.653	1.10	0.730
28	0.870	0.579	0.879	0.585	0.964	0.641	0.989	0.658	1.08	0.716	1.12	0.746
30	0.959	0.638	0.896	0.596	1.06	0.707	1.01	0.671	1.19	0.790	1.15	0.763
32	1.06	0.707	0.912	0.607	1.18	0.786	1.03	0.685	1.32	0.879	1.17	0.781
34	1.19	0.791	0.930	0.618	1.32	0.880	1.05	0.700	1.48	0.986	1.20	0.800
36	1.33	0.887	0.948	0.630	1.48	0.987	1.07	0.715	1.66	1.11	1.23	0.819
38	1.48	0.988	0.966	0.643	1.65	1.10	1.10	0.731	1.85	1.23	1.26	0.840
40	1.65	1.09	0.986	0.656	1.83	1.22	1.12	0.747	2.05	1.36	1.29	0.861
42	1.81	1.21	1.01	0.669	2.02	1.34	1.15	0.765	2.26	1.50	1.33	0.884
44	1.99	1.32	1.03	0.683	2.22	1.47	1.18	0.783	2.48	1.65	1.37	0.908
46	2.18	1.45	1.05	0.698	2.42	1.61	1.21	0.802	2.71	1.81	1.40	0.934
48	2.37	1.58	1.07	0.713	2.64	1.75	1.24	0.822	2.95	1.97	1.44	0.961
50	2.57	1.71	1.10	0.729	2.86	1.90	1.27	0.843	3.21	2.13	1.49	0.989

Other Constants and Properties

	193 ASD	193 LRFD	176 ASD	176 LRFD	159 ASD	159 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.52	1.01	1.68	1.12	1.88	1.25
$t_y \times 10^3$ (kips)$^{-1}$	0.452	0.301	0.496	0.330	0.550	0.366
$t_r \times 10^3$ (kips)$^{-1}$	0.587	0.391	0.644	0.429	0.714	0.476
r_x/r_y	1.60		1.60		1.60	
r_y, in.	4.05		4.02		4.00	

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W14

Shape	W14×											
	145				132				120			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	0.602	0.400	1.05	0.701	0.662	0.441	1.17	0.779	0.728	0.484	1.29	0.860
11	0.668	0.444	1.05	0.701	0.744	0.495	1.17	0.779	0.819	0.545	1.29	0.860
12	0.681	0.453	1.05	0.701	0.761	0.506	1.18	0.783	0.838	0.558	1.30	0.864
13	0.696	0.463	1.06	0.706	0.780	0.519	1.19	0.792	0.859	0.571	1.32	0.875
14	0.713	0.474	1.07	0.714	0.801	0.533	1.20	0.801	0.882	0.587	1.33	0.887
15	0.731	0.486	1.08	0.721	0.823	0.548	1.22	0.811	0.907	0.604	1.35	0.898
16	0.751	0.499	1.10	0.729	0.848	0.564	1.23	0.821	0.935	0.622	1.37	0.910
17	0.772	0.514	1.11	0.737	0.876	0.583	1.25	0.831	0.966	0.643	1.39	0.922
18	0.796	0.530	1.12	0.745	0.906	0.603	1.26	0.841	0.999	0.665	1.40	0.935
19	0.822	0.547	1.13	0.753	0.939	0.625	1.28	0.851	1.04	0.689	1.42	0.947
20	0.850	0.566	1.14	0.762	0.975	0.649	1.30	0.862	1.08	0.716	1.44	0.961
22	0.914	0.608	1.17	0.779	1.06	0.704	1.33	0.885	1.17	0.778	1.49	0.988
24	0.990	0.659	1.20	0.798	1.16	0.769	1.37	0.908	1.28	0.851	1.53	1.02
26	1.08	0.718	1.23	0.817	1.27	0.848	1.40	0.933	1.41	0.938	1.58	1.05
28	1.18	0.788	1.26	0.837	1.41	0.941	1.44	0.960	1.57	1.04	1.62	1.08
30	1.31	0.871	1.29	0.858	1.58	1.05	1.48	0.987	1.76	1.17	1.68	1.12
32	1.46	0.970	1.32	0.880	1.79	1.19	1.53	1.02	1.99	1.32	1.73	1.15
34	1.64	1.09	1.36	0.903	2.02	1.34	1.58	1.05	2.24	1.49	1.79	1.19
36	1.84	1.22	1.39	0.928	2.26	1.51	1.63	1.08	2.51	1.67	1.86	1.24
38	2.05	1.36	1.43	0.954	2.52	1.68	1.68	1.12	2.80	1.86	1.93	1.28
40	2.27	1.51	1.48	0.982	2.79	1.86	1.74	1.16	3.10	2.07	2.00	1.33
42	2.50	1.66	1.52	1.01	3.08	2.05	1.80	1.20	3.42	2.28	2.09	1.39
44	2.74	1.82	1.57	1.04	3.38	2.25	1.86	1.24	3.76	2.50	2.21	1.47
46	3.00	1.99	1.62	1.07	3.70	2.46	1.96	1.30	4.11	2.73	2.33	1.55
48	3.26	2.17	1.67	1.11	4.02	2.68	2.06	1.37	4.47	2.97	2.45	1.63
50	3.54	2.36	1.74	1.16	4.37	2.91	2.15	1.43	4.85	3.23	2.57	1.71

Other Constants and Properties

	145 ASD	145 LRFD	132 ASD	132 LRFD	120 ASD	120 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	2.06	1.37	2.43	1.61	2.69	1.79
$t_y \times 10^3$ (kips)$^{-1}$	0.602	0.400	0.662	0.441	0.728	0.484
$t_r \times 10^3$ (kips)$^{-1}$	0.781	0.520	0.859	0.573	0.944	0.630
r_x/r_y	1.59		1.67		1.67	
r_y, in.	3.98		3.76		3.74	

Return to Table of Contents

W14

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W14×											
	109[f]				99[f]				90[f]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending: 0	0.803	0.534	1.45	0.962	0.883	0.587	1.64	1.09	0.970	0.645	1.86	1.24
11	0.904	0.602	1.45	0.962	0.996	0.663	1.64	1.09	1.09	0.728	1.86	1.24
12	0.925	0.615	1.45	0.962	1.02	0.678	1.64	1.09	1.12	0.745	1.86	1.24
13	0.948	0.631	1.45	0.968	1.04	0.695	1.64	1.09	1.15	0.764	1.86	1.24
14	0.974	0.648	1.47	0.981	1.07	0.714	1.64	1.09	1.18	0.785	1.86	1.24
15	1.00	0.667	1.50	0.995	1.10	0.735	1.67	1.11	1.21	0.808	1.86	1.24
16	1.03	0.687	1.52	1.01	1.14	0.758	1.69	1.13	1.25	0.833	1.88	1.25
17	1.07	0.710	1.54	1.02	1.18	0.783	1.72	1.14	1.29	0.861	1.91	1.27
18	1.10	0.735	1.56	1.04	1.22	0.811	1.75	1.16	1.34	0.892	1.94	1.29
19	1.15	0.762	1.58	1.05	1.26	0.841	1.78	1.18	1.39	0.925	1.97	1.31
20	1.19	0.792	1.61	1.07	1.31	0.874	1.81	1.20	1.45	0.962	2.01	1.34
22	1.29	0.860	1.66	1.10	1.43	0.951	1.87	1.24	1.57	1.05	2.08	1.39
24	1.41	0.941	1.71	1.14	1.57	1.04	1.93	1.29	1.72	1.15	2.16	1.44
26	1.56	1.04	1.77	1.18	1.73	1.15	2.00	1.33	1.91	1.27	2.25	1.50
28	1.74	1.16	1.83	1.22	1.93	1.28	2.08	1.38	2.12	1.41	2.34	1.56
30	1.95	1.29	1.90	1.26	2.16	1.44	2.16	1.44	2.38	1.59	2.45	1.63
32	2.20	1.47	1.97	1.31	2.45	1.63	2.25	1.50	2.70	1.80	2.56	1.70
34	2.49	1.66	2.04	1.36	2.77	1.84	2.34	1.56	3.05	2.03	2.68	1.78
36	2.79	1.86	2.12	1.41	3.10	2.06	2.45	1.63	3.42	2.28	2.86	1.90
38	3.11	2.07	2.21	1.47	3.45	2.30	2.60	1.73	3.81	2.54	3.08	2.05
40	3.44	2.29	2.33	1.55	3.83	2.55	2.78	1.85	4.23	2.81	3.29	2.19
42	3.80	2.53	2.48	1.65	4.22	2.81	2.96	1.97	4.66	3.10	3.51	2.33
44	4.17	2.77	2.62	1.74	4.63	3.08	3.13	2.08	5.11	3.40	3.72	2.48
46	4.55	3.03	2.76	1.84	5.06	3.37	3.31	2.20	5.59	3.72	3.94	2.62
48	4.96	3.30	2.91	1.93	5.51	3.67	3.48	2.32	6.08	4.05	4.15	2.76
50	5.38	3.58	3.05	2.03	5.98	3.98	3.66	2.43	6.60	4.39	4.36	2.90

Other Constants and Properties

	109[f] ASD	109[f] LRFD	99[f] ASD	99[f] LRFD	90[f] ASD	90[f] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	3.01	2.01	3.47	2.31	3.98	2.65
$t_y \times 10^3$ (kips)$^{-1}$	0.803	0.534	0.883	0.587	0.970	0.645
$t_r \times 10^3$ (kips)$^{-1}$	1.04	0.694	1.15	0.764	1.26	0.839
r_x/r_y	1.67		1.66		1.66	
r_y, in.	3.73		3.71		3.70	

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W14

Shape		W14×											
		82				74				68			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	1.07	0.712	1.97	1.31	1.18	0.784	2.18	1.45	1.28	0.855	2.38	1.59
	6	1.16	0.772	1.97	1.31	1.28	0.850	2.18	1.45	1.39	0.927	2.38	1.59
	7	1.19	0.794	1.97	1.31	1.31	0.874	2.18	1.45	1.44	0.955	2.38	1.59
	8	1.23	0.821	1.98	1.32	1.36	0.904	2.19	1.46	1.48	0.988	2.40	1.60
	9	1.28	0.853	2.03	1.35	1.41	0.939	2.24	1.49	1.54	1.03	2.46	1.64
	10	1.34	0.890	2.07	1.38	1.47	0.980	2.29	1.52	1.61	1.07	2.52	1.68
	11	1.40	0.932	2.11	1.41	1.54	1.03	2.34	1.56	1.69	1.12	2.58	1.72
	12	1.47	0.981	2.16	1.44	1.62	1.08	2.40	1.60	1.78	1.18	2.65	1.76
	13	1.56	1.04	2.21	1.47	1.72	1.14	2.46	1.63	1.88	1.25	2.72	1.81
	14	1.66	1.10	2.26	1.50	1.82	1.21	2.52	1.68	2.00	1.33	2.79	1.86
	15	1.77	1.18	2.31	1.54	1.94	1.29	2.58	1.72	2.14	1.42	2.86	1.91
	16	1.89	1.26	2.37	1.57	2.08	1.39	2.65	1.76	2.29	1.53	2.95	1.96
	17	2.04	1.36	2.42	1.61	2.24	1.49	2.72	1.81	2.47	1.64	3.03	2.02
	18	2.20	1.46	2.48	1.65	2.42	1.61	2.80	1.86	2.67	1.78	3.12	2.08
	19	2.39	1.59	2.55	1.70	2.63	1.75	2.88	1.91	2.91	1.93	3.22	2.14
	20	2.61	1.73	2.62	1.74	2.87	1.91	2.96	1.97	3.17	2.11	3.32	2.21
	22	3.14	2.09	2.76	1.84	3.46	2.30	3.15	2.09	3.83	2.55	3.54	2.36
	24	3.74	2.49	2.93	1.95	4.12	2.74	3.36	2.23	4.56	3.03	3.80	2.53
	26	4.39	2.92	3.11	2.07	4.83	3.21	3.64	2.42	5.35	3.56	4.24	2.82
	28	5.09	3.39	3.38	2.25	5.60	3.73	4.00	2.66	6.21	4.13	4.67	3.11
	30	5.84	3.89	3.67	2.44	6.43	4.28	4.36	2.90	7.12	4.74	5.10	3.39
	32	6.65	4.42	3.97	2.64	7.32	4.87	4.72	3.14	8.11	5.39	5.53	3.68
	34	7.50	4.99	4.26	2.84	8.26	5.50	5.07	3.38	9.15	6.09	5.96	3.96
	36	8.41	5.60	4.56	3.03	9.26	6.16	5.43	3.61	10.3	6.83	6.38	4.25
	38	9.37	6.24	4.85	3.22	10.3	6.86	5.78	3.85	11.4	7.60	6.81	4.53
	40	10.4	6.91	5.14	3.42	11.4	7.61	6.14	4.08	12.7	8.43	7.23	4.81

Other Constants and Properties

	82 ASD	82 LRFD	74 ASD	74 LRFD	68 ASD	68 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	6.12	4.07	6.77	4.50	7.43	4.94
$t_y \times 10^3$ (kips)$^{-1}$	1.07	0.712	1.18	0.784	1.28	0.855
$t_r \times 10^3$ (kips)$^{-1}$	1.39	0.926	1.53	1.02	1.67	1.11
r_x/r_y	2.44		2.44		2.44	
r_y, in.	2.48		2.48		2.46	

Return to Table of Contents

W14

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W14×											
	61				53				48[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	1.44	0.955	2.69	1.79	1.65	1.10	3.15	2.09	1.85	1.23	3.50	2.33
6	1.56	1.04	2.69	1.79	1.88	1.25	3.15	2.10	2.09	1.39	3.50	2.33
7	1.61	1.07	2.69	1.79	1.98	1.31	3.25	2.16	2.19	1.46	3.62	2.41
8	1.66	1.11	2.71	1.81	2.09	1.39	3.35	2.23	2.32	1.54	3.74	2.49
9	1.73	1.15	2.78	1.85	2.22	1.48	3.46	2.31	2.47	1.64	3.88	2.58
10	1.80	1.20	2.85	1.90	2.39	1.59	3.58	2.38	2.65	1.76	4.02	2.67
11	1.89	1.26	2.93	1.95	2.58	1.72	3.71	2.47	2.87	1.91	4.17	2.78
12	1.99	1.33	3.01	2.00	2.81	1.87	3.85	2.56	3.13	2.08	4.34	2.89
13	2.11	1.40	3.09	2.06	3.08	2.05	3.99	2.66	3.44	2.29	4.52	3.01
14	2.24	1.49	3.18	2.12	3.41	2.27	4.15	2.76	3.80	2.53	4.71	3.14
15	2.40	1.60	3.28	2.18	3.80	2.53	4.32	2.88	4.24	2.82	4.92	3.28
16	2.57	1.71	3.38	2.25	4.26	2.84	4.51	3.00	4.77	3.17	5.16	3.43
17	2.77	1.85	3.49	2.32	4.81	3.20	4.71	3.14	5.38	3.58	5.41	3.60
18	3.00	2.00	3.60	2.39	5.40	3.59	4.94	3.28	6.03	4.01	5.79	3.85
19	3.27	2.18	3.72	2.48	6.01	4.00	5.28	3.51	6.72	4.47	6.26	4.17
20	3.57	2.38	3.85	2.56	6.66	4.43	5.67	3.77	7.45	4.96	6.74	4.48
22	4.32	2.87	4.14	2.75	8.06	5.36	6.44	4.29	9.01	6.00	7.69	5.12
24	5.14	3.42	4.59	3.05	9.60	6.38	7.22	4.80	10.7	7.14	8.64	5.75
26	6.03	4.01	5.13	3.41	11.3	7.49	7.99	5.32	12.6	8.38	9.59	6.38
28	6.99	4.65	5.66	3.77	13.1	8.69	8.76	5.83	14.6	9.72	10.5	7.01
30	8.02	5.34	6.20	4.13	15.0	9.98	9.53	6.34	16.8	11.2	11.5	7.65
32	9.13	6.07	6.74	4.48	17.1	11.3	10.3	6.85				
34	10.3	6.86	7.27	4.84								
36	11.6	7.69	7.81	5.20								
38	12.9	8.57	8.34	5.55								
40	14.3	9.49	8.87	5.90								

Other Constants and Properties

	61		53		48[c]	
$b_y \times 10^3$ (kip-ft)$^{-1}$	8.36	5.56	12.5	8.29	14.0	9.30
$t_y \times 10^3$ (kips)$^{-1}$	1.44	0.955	1.65	1.10	1.82	1.21
$t_r \times 10^3$ (kips)$^{-1}$	1.86	1.24	2.14	1.42	2.36	1.58
r_x / r_y	2.44		3.07		3.06	
r_y, in.	2.45		1.92		1.91	

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W14

Shape	W14×											
	43[c]				38[c]				34[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	2.11	1.40	3.94	2.62	2.43	1.61	4.46	2.96	2.78	1.85	5.02	3.34
6	2.38	1.59	3.96	2.63	2.89	1.92	4.70	3.12	3.31	2.20	5.32	3.54
7	2.49	1.66	4.10	2.72	3.08	2.05	4.92	3.27	3.53	2.35	5.58	3.71
8	2.63	1.75	4.25	2.82	3.33	2.22	5.16	3.43	3.81	2.54	5.87	3.91
9	2.79	1.86	4.41	2.93	3.64	2.42	5.43	3.61	4.17	2.78	6.20	4.12
10	2.99	1.99	4.58	3.05	4.06	2.70	5.72	3.81	4.62	3.08	6.56	4.36
11	3.24	2.16	4.77	3.17	4.57	3.04	6.05	4.03	5.21	3.47	6.96	4.63
12	3.54	2.36	4.97	3.31	5.21	3.47	6.42	4.27	5.96	3.97	7.42	4.94
13	3.90	2.59	5.19	3.45	6.02	4.00	6.84	4.55	6.92	4.60	7.94	5.28
14	4.32	2.88	5.44	3.62	6.98	4.64	7.42	4.94	8.02	5.34	8.85	5.89
15	4.83	3.21	5.70	3.79	8.01	5.33	8.26	5.50	9.21	6.13	9.89	6.58
16	5.45	3.63	6.00	3.99	9.11	6.06	9.12	6.07	10.5	6.97	11.0	7.29
17	6.15	4.09	6.37	4.24	10.3	6.85	9.99	6.65	11.8	7.87	12.0	8.01
18	6.90	4.59	6.95	4.62	11.5	7.68	10.9	7.23	13.3	8.82	13.1	8.73
19	7.68	5.11	7.53	5.01	12.9	8.55	11.8	7.82	14.8	9.83	14.2	9.47
20	8.51	5.66	8.12	5.40	14.2	9.48	12.6	8.41	16.4	10.9	15.3	10.2
21	9.39	6.25	8.71	5.80	15.7	10.4	13.5	9.00	18.0	12.0	16.5	11.0
22	10.3	6.85	9.31	6.19	17.2	11.5	14.4	9.60	19.8	13.2	17.6	11.7
23	11.3	7.49	9.90	6.59	18.8	12.5	15.3	10.2	21.6	14.4	18.7	12.4
24	12.3	8.16	10.5	6.99	20.5	13.6	16.2	10.8	23.6	15.7	19.8	13.2
25	13.3	8.85	11.1	7.39	22.3	14.8	17.1	11.4	25.6	17.0	21.0	13.9
26	14.4	9.57	11.7	7.78								
27	15.5	10.3	12.3	8.18								
28	16.7	11.1	12.9	8.58								
29	17.9	11.9	13.5	8.98								
30	19.2	12.7	14.1	9.37								

Other Constants and Properties

	43[c] ASD	43[c] LRFD	38[c] ASD	38[c] LRFD	34[c] ASD	34[c] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	15.8	10.5	22.6	15.1	25.9	17.2
$t_y \times 10^3$ (kips)$^{-1}$	2.04	1.36	2.29	1.53	2.57	1.71
$t_r \times 10^3$ (kips)$^{-1}$	2.65	1.76	2.98	1.98	3.33	2.22
r_x/r_y	3.08		3.79		3.81	
r_y, in.	1.89		1.55		1.53	

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W14

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W14×											
		30[c,f]				26[c,v]				22[c,v]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	3.20	2.13	5.92	3.94	3.78	2.51	6.82	4.54	4.65	3.09	8.25	5.49
	6	3.83	2.55	6.19	4.12	5.29	3.52	8.20	5.45	6.58	4.38	10.2	6.76
	7	4.09	2.72	6.52	4.34	6.05	4.02	8.87	5.90	7.57	5.03	11.1	7.38
	8	4.43	2.95	6.88	4.58	7.11	4.73	9.67	6.44	8.97	5.97	12.2	8.11
	9	4.86	3.24	7.29	4.85	8.65	5.76	10.6	7.07	11.1	7.36	13.6	9.05
	10	5.41	3.60	7.75	5.15	10.7	7.11	12.3	8.16	13.6	9.08	16.4	10.9
	11	6.12	4.07	8.26	5.50	12.9	8.60	14.4	9.55	16.5	11.0	19.2	12.8
	12	7.05	4.69	8.85	5.89	15.4	10.2	16.5	11.0	19.7	13.1	22.3	14.8
	13	8.24	5.48	9.70	6.45	18.1	12.0	18.7	12.4	23.1	15.3	25.4	16.9
	14	9.56	6.36	11.0	7.31	20.9	13.9	20.9	13.9	26.8	17.8	28.5	19.0
	15	11.0	7.30	12.3	8.20	24.0	16.0	23.2	15.4	30.7	20.4	31.8	21.2
	16	12.5	8.31	13.7	9.12	27.3	18.2	25.5	17.0	34.9	23.2	35.1	23.3
	17	14.1	9.38	15.1	10.0	30.9	20.5	27.8	18.5	39.4	26.2	38.4	25.6
	18	15.8	10.5	16.5	11.0	34.6	23.0	30.1	20.0				
	19	17.6	11.7	18.0	12.0								
	20	19.5	13.0	19.4	12.9								
	22	23.6	15.7	22.4	14.9								
	24	28.1	18.7	25.4	16.9								
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		31.4		20.9		49.5		32.9		62.4		41.5	
$t_y \times 10^3$ (kips)$^{-1}$		2.90		1.93		3.34		2.22		3.96		2.63	
$t_r \times 10^3$ (kips)$^{-1}$		3.77		2.51		4.33		2.89		5.14		3.42	
r_x/r_y		3.85				5.23				5.33			
r_y, in.		1.49				1.08				1.04			

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes


W12

Shape		W12×											
		336[h]				305[h]				279[h]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.260	0.173	0.454	0.302	0.287	0.191	0.510	0.340	0.314	0.209	0.570	0.379
	6	0.271	0.180	0.454	0.302	0.299	0.199	0.510	0.340	0.328	0.218	0.570	0.379
	7	0.275	0.183	0.454	0.302	0.304	0.202	0.510	0.340	0.333	0.221	0.570	0.379
	8	0.279	0.186	0.454	0.302	0.309	0.206	0.510	0.340	0.339	0.225	0.570	0.379
	9	0.285	0.190	0.454	0.302	0.316	0.210	0.510	0.340	0.346	0.230	0.570	0.379
	10	0.291	0.194	0.454	0.302	0.323	0.215	0.510	0.340	0.354	0.235	0.570	0.379
	11	0.298	0.198	0.455	0.303	0.331	0.220	0.511	0.340	0.363	0.241	0.571	0.380
	12	0.306	0.204	0.457	0.304	0.340	0.226	0.514	0.342	0.373	0.248	0.574	0.382
	13	0.315	0.209	0.459	0.305	0.350	0.233	0.516	0.343	0.384	0.256	0.577	0.384
	14	0.325	0.216	0.461	0.307	0.361	0.240	0.518	0.345	0.397	0.264	0.580	0.386
	15	0.335	0.223	0.463	0.308	0.374	0.249	0.521	0.347	0.411	0.273	0.583	0.388
	16	0.348	0.231	0.465	0.309	0.387	0.258	0.523	0.348	0.426	0.284	0.586	0.390
	17	0.361	0.240	0.467	0.311	0.403	0.268	0.526	0.350	0.444	0.295	0.589	0.392
	18	0.375	0.250	0.469	0.312	0.419	0.279	0.528	0.351	0.462	0.308	0.592	0.394
	19	0.392	0.261	0.471	0.313	0.438	0.291	0.531	0.353	0.483	0.322	0.595	0.396
	20	0.409	0.272	0.473	0.315	0.458	0.305	0.533	0.355	0.507	0.337	0.598	0.398
	22	0.450	0.300	0.477	0.317	0.506	0.337	0.538	0.358	0.560	0.373	0.604	0.402
	24	0.500	0.333	0.481	0.320	0.563	0.375	0.544	0.362	0.626	0.416	0.611	0.406
	26	0.560	0.373	0.485	0.323	0.633	0.421	0.549	0.365	0.705	0.469	0.617	0.411
	28	0.633	0.421	0.490	0.326	0.719	0.478	0.555	0.369	0.803	0.534	0.624	0.415
	30	0.724	0.482	0.494	0.329	0.824	0.548	0.560	0.373	0.922	0.613	0.631	0.420
	32	0.824	0.548	0.499	0.332	0.937	0.624	0.566	0.376	1.05	0.698	0.638	0.424
	34	0.930	0.619	0.503	0.335	1.06	0.704	0.572	0.380	1.18	0.788	0.645	0.429
	36	1.04	0.694	0.508	0.338	1.19	0.789	0.578	0.384	1.33	0.883	0.652	0.434
	38	1.16	0.773	0.513	0.341	1.32	0.879	0.584	0.388	1.48	0.984	0.660	0.439
	40	1.29	0.856	0.518	0.344	1.46	0.974	0.590	0.392	1.64	1.09	0.667	0.444
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		1.00		0.665		1.12		0.747		1.25		0.829	
$t_y \times 10^3$ (kips)$^{-1}$		0.260		0.173		0.287		0.191		0.314		0.209	
$t_r \times 10^3$ (kips)$^{-1}$		0.337		0.225		0.372		0.248		0.407		0.271	
r_x/r_y		1.85				1.84				1.82			
r_y, in.		3.47				3.42				3.38			

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

W12

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W12×											
	252[h]				230[h]				210			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.347	0.231	0.640	0.426	0.380	0.252	0.710	0.472	0.416	0.277	0.788	0.524
6	0.362	0.241	0.640	0.426	0.397	0.264	0.710	0.472	0.435	0.290	0.788	0.524
7	0.368	0.245	0.640	0.426	0.403	0.268	0.710	0.472	0.442	0.294	0.788	0.524
8	0.375	0.250	0.640	0.426	0.411	0.274	0.710	0.472	0.451	0.300	0.788	0.524
9	0.383	0.255	0.640	0.426	0.420	0.279	0.710	0.472	0.461	0.307	0.788	0.524
10	0.392	0.261	0.640	0.426	0.430	0.286	0.710	0.472	0.472	0.314	0.788	0.524
11	0.402	0.268	0.643	0.428	0.441	0.294	0.713	0.474	0.485	0.323	0.792	0.527
12	0.414	0.275	0.646	0.430	0.454	0.302	0.717	0.477	0.499	0.332	0.797	0.530
13	0.427	0.284	0.650	0.432	0.469	0.312	0.722	0.480	0.515	0.343	0.802	0.534
14	0.441	0.293	0.653	0.435	0.485	0.323	0.726	0.483	0.533	0.355	0.807	0.537
15	0.457	0.304	0.657	0.437	0.503	0.334	0.730	0.486	0.554	0.368	0.813	0.541
16	0.475	0.316	0.661	0.440	0.523	0.348	0.735	0.489	0.576	0.383	0.818	0.544
17	0.494	0.329	0.665	0.442	0.545	0.362	0.739	0.492	0.600	0.400	0.824	0.548
18	0.516	0.343	0.668	0.445	0.569	0.378	0.744	0.495	0.628	0.418	0.829	0.552
19	0.540	0.359	0.672	0.447	0.596	0.396	0.749	0.498	0.658	0.438	0.835	0.556
20	0.566	0.377	0.676	0.450	0.626	0.416	0.753	0.501	0.692	0.460	0.841	0.559
22	0.628	0.418	0.684	0.455	0.695	0.462	0.763	0.508	0.770	0.512	0.853	0.567
24	0.703	0.468	0.692	0.460	0.779	0.519	0.773	0.514	0.865	0.576	0.865	0.575
26	0.795	0.529	0.700	0.466	0.883	0.588	0.783	0.521	0.982	0.654	0.877	0.584
28	0.909	0.605	0.709	0.471	1.01	0.674	0.793	0.528	1.13	0.752	0.890	0.592
30	1.04	0.694	0.717	0.477	1.16	0.773	0.804	0.535	1.30	0.863	0.903	0.601
32	1.19	0.790	0.726	0.483	1.32	0.880	0.815	0.542	1.48	0.982	0.917	0.610
34	1.34	0.891	0.735	0.489	1.49	0.993	0.826	0.550	1.67	1.11	0.931	0.619
36	1.50	0.999	0.744	0.495	1.67	1.11	0.838	0.557	1.87	1.24	0.946	0.629
38	1.67	1.11	0.754	0.502	1.87	1.24	0.850	0.565	2.08	1.38	0.960	0.639
40	1.85	1.23	0.764	0.508	2.07	1.37	0.862	0.573	2.31	1.53	0.976	0.649
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.40		0.930		1.55		1.03		1.72		1.15	
$t_y \times 10^3$ (kips)$^{-1}$	0.347		0.231		0.380		0.252		0.416		0.277	
$t_r \times 10^3$ (kips)$^{-1}$	0.450		0.300		0.492		0.328		0.539		0.360	
r_x/r_y	1.81				1.80				1.80			
r_y, in.	3.34				3.31				3.28			

[h] Flange thickness greater than 2 in. Special requirements may apply per AISC *Specification* Section A3.1c.

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W12

Shape	W12×											
	190				170				152			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	0.459	0.305	0.881	0.586	0.514	0.342	0.997	0.663	0.575	0.382	1.13	0.750
6	0.481	0.320	0.881	0.586	0.539	0.359	0.997	0.663	0.603	0.401	1.13	0.750
7	0.489	0.325	0.881	0.586	0.548	0.365	0.997	0.663	0.614	0.408	1.13	0.750
8	0.498	0.332	0.881	0.586	0.559	0.372	0.997	0.663	0.626	0.417	1.13	0.750
9	0.510	0.339	0.881	0.586	0.572	0.380	0.997	0.663	0.641	0.426	1.13	0.750
10	0.522	0.347	0.881	0.586	0.586	0.390	0.997	0.663	0.658	0.437	1.13	0.751
11	0.537	0.357	0.887	0.590	0.603	0.401	1.00	0.668	0.676	0.450	1.14	0.758
12	0.553	0.368	0.893	0.594	0.621	0.413	1.01	0.674	0.698	0.464	1.15	0.765
13	0.571	0.380	0.900	0.599	0.642	0.427	1.02	0.679	0.721	0.480	1.16	0.772
14	0.591	0.394	0.906	0.603	0.666	0.443	1.03	0.685	0.748	0.498	1.17	0.779
15	0.614	0.409	0.913	0.607	0.692	0.460	1.04	0.691	0.778	0.518	1.18	0.786
16	0.639	0.425	0.920	0.612	0.720	0.479	1.05	0.696	0.811	0.540	1.19	0.793
17	0.667	0.444	0.927	0.617	0.753	0.501	1.06	0.702	0.848	0.564	1.20	0.801
18	0.698	0.465	0.934	0.621	0.788	0.524	1.06	0.708	0.889	0.591	1.22	0.809
19	0.732	0.487	0.941	0.626	0.828	0.551	1.07	0.714	0.934	0.621	1.23	0.816
20	0.770	0.513	0.948	0.631	0.871	0.580	1.08	0.720	0.984	0.655	1.24	0.824
22	0.859	0.572	0.963	0.641	0.973	0.648	1.10	0.733	1.10	0.733	1.26	0.841
24	0.968	0.644	0.978	0.651	1.10	0.731	1.12	0.746	1.25	0.830	1.29	0.858
26	1.10	0.733	0.994	0.661	1.25	0.835	1.14	0.759	1.43	0.949	1.32	0.875
28	1.27	0.845	1.01	0.672	1.45	0.964	1.16	0.773	1.65	1.10	1.34	0.894
30	1.46	0.970	1.03	0.683	1.66	1.11	1.18	0.788	1.90	1.26	1.37	0.913
32	1.66	1.10	1.04	0.695	1.89	1.26	1.21	0.803	2.16	1.43	1.40	0.933
34	1.87	1.25	1.06	0.707	2.14	1.42	1.23	0.819	2.43	1.62	1.43	0.954
36	2.10	1.40	1.08	0.719	2.39	1.59	1.26	0.835	2.73	1.82	1.47	0.976
38	2.34	1.56	1.10	0.732	2.67	1.78	1.28	0.852	3.04	2.02	1.50	0.999
40	2.59	1.72	1.12	0.745	2.96	1.97	1.31	0.870	3.37	2.24	1.54	1.02

Other Constants and Properties

	190 ASD	190 LRFD	170 ASD	170 LRFD	152 ASD	152 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	1.92	1.28	2.18	1.45	2.47	1.64
$t_y \times 10^3$ (kips)$^{-1}$	0.459	0.305	0.514	0.342	0.575	0.382
$t_r \times 10^3$ (kips)$^{-1}$	0.595	0.397	0.667	0.444	0.746	0.497
r_x/r_y	1.79		1.78		1.77	
r_y, in.	3.25		3.22		3.19	

Return to Table of Contents

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W12

F_y = 65 ksi

Shape		W12× 136				W12× 120				W12× 106			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.644	0.428	1.28	0.852	0.730	0.486	1.47	0.980	0.823	0.548	1.67	1.11
	6	0.676	0.450	1.28	0.852	0.768	0.511	1.47	0.980	0.867	0.577	1.67	1.11
	7	0.689	0.458	1.28	0.852	0.782	0.520	1.47	0.980	0.883	0.587	1.67	1.11
	8	0.703	0.468	1.28	0.852	0.798	0.531	1.47	0.980	0.902	0.600	1.67	1.11
	9	0.720	0.479	1.28	0.852	0.817	0.544	1.47	0.980	0.923	0.614	1.67	1.11
	10	0.739	0.491	1.28	0.854	0.839	0.558	1.48	0.984	0.949	0.631	1.68	1.12
	11	0.760	0.506	1.30	0.862	0.864	0.575	1.50	0.995	0.977	0.650	1.70	1.13
	12	0.784	0.522	1.31	0.871	0.893	0.594	1.51	1.01	1.01	0.672	1.72	1.15
	13	0.812	0.540	1.32	0.880	0.924	0.615	1.53	1.02	1.05	0.696	1.75	1.16
	14	0.842	0.560	1.34	0.889	0.960	0.639	1.55	1.03	1.09	0.723	1.77	1.18
	15	0.877	0.583	1.35	0.899	1.00	0.665	1.57	1.04	1.13	0.753	1.79	1.19
	16	0.915	0.609	1.37	0.908	1.04	0.694	1.59	1.05	1.18	0.787	1.82	1.21
	17	0.957	0.637	1.38	0.918	1.09	0.727	1.60	1.07	1.24	0.825	1.84	1.23
	18	1.00	0.668	1.39	0.928	1.15	0.764	1.62	1.08	1.30	0.867	1.87	1.24
	19	1.06	0.703	1.41	0.938	1.21	0.804	1.64	1.09	1.37	0.913	1.89	1.26
	20	1.11	0.741	1.43	0.949	1.28	0.849	1.67	1.11	1.45	0.965	1.92	1.28
	22	1.25	0.832	1.46	0.970	1.44	0.955	1.71	1.14	1.63	1.09	1.98	1.32
	24	1.42	0.944	1.49	0.993	1.63	1.09	1.75	1.17	1.86	1.24	2.04	1.36
	26	1.63	1.08	1.53	1.02	1.88	1.25	1.80	1.20	2.15	1.43	2.10	1.40
	28	1.89	1.25	1.56	1.04	2.18	1.45	1.85	1.23	2.49	1.66	2.17	1.45
	30	2.16	1.44	1.60	1.07	2.50	1.66	1.91	1.27	2.86	1.90	2.25	1.50
	32	2.46	1.64	1.64	1.09	2.84	1.89	1.96	1.31	3.25	2.16	2.33	1.55
	34	2.78	1.85	1.69	1.12	3.21	2.14	2.02	1.35	3.67	2.44	2.41	1.60
	36	3.12	2.07	1.73	1.15	3.60	2.40	2.09	1.39	4.11	2.74	2.50	1.66
	38	3.47	2.31	1.78	1.19	4.01	2.67	2.16	1.44	4.58	3.05	2.60	1.73
	40	3.85	2.56	1.83	1.22	4.44	2.96	2.23	1.48	5.08	3.38	2.71	1.80
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		2.80		1.86		3.21		2.14		3.65		2.43	
$t_y \times 10^3$ (kips)$^{-1}$		0.644		0.428		0.730		0.486		0.823		0.548	
$t_r \times 10^3$ (kips)$^{-1}$		0.835		0.557		0.947		0.631		1.07		0.712	
r_x/r_y		1.77				1.76				1.76			
r_y, in.		3.16				3.13				3.11			

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W12

Shape		W12×											
		96				87				79[f]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.911	0.606	1.86	1.24	1.00	0.668	2.08	1.38	1.11	0.737	2.32	1.54
	6	0.959	0.638	1.86	1.24	1.06	0.704	2.08	1.38	1.17	0.777	2.32	1.54
	7	0.977	0.650	1.86	1.24	1.08	0.717	2.08	1.38	1.19	0.792	2.32	1.54
	8	0.999	0.664	1.86	1.24	1.10	0.733	2.08	1.38	1.22	0.810	2.32	1.54
	9	1.02	0.681	1.86	1.24	1.13	0.751	2.08	1.38	1.25	0.830	2.32	1.54
	10	1.05	0.700	1.88	1.25	1.16	0.772	2.09	1.39	1.28	0.854	2.32	1.55
	11	1.08	0.721	1.90	1.27	1.20	0.796	2.12	1.41	1.32	0.880	2.36	1.57
	12	1.12	0.745	1.93	1.28	1.24	0.823	2.16	1.44	1.37	0.911	2.40	1.60
	13	1.16	0.772	1.96	1.30	1.28	0.853	2.19	1.46	1.42	0.945	2.44	1.63
	14	1.21	0.803	1.98	1.32	1.33	0.888	2.23	1.48	1.48	0.983	2.49	1.66
	15	1.26	0.837	2.01	1.34	1.39	0.926	2.26	1.51	1.54	1.03	2.53	1.69
	16	1.32	0.875	2.04	1.36	1.46	0.968	2.30	1.53	1.61	1.07	2.58	1.72
	17	1.38	0.917	2.08	1.38	1.53	1.02	2.34	1.56	1.69	1.13	2.63	1.75
	18	1.45	0.965	2.11	1.40	1.61	1.07	2.38	1.58	1.78	1.19	2.68	1.78
	19	1.53	1.02	2.14	1.42	1.70	1.13	2.42	1.61	1.88	1.25	2.73	1.82
	20	1.62	1.08	2.18	1.45	1.79	1.19	2.46	1.64	1.99	1.33	2.78	1.85
	22	1.82	1.21	2.25	1.50	2.03	1.35	2.56	1.70	2.26	1.50	2.90	1.93
	24	2.08	1.38	2.32	1.55	2.32	1.54	2.65	1.77	2.58	1.72	3.02	2.01
	26	2.41	1.60	2.41	1.60	2.68	1.79	2.76	1.84	3.00	2.00	3.16	2.10
	28	2.79	1.86	2.49	1.66	3.11	2.07	2.88	1.91	3.48	2.32	3.31	2.20
	30	3.20	2.13	2.59	1.72	3.57	2.38	3.00	2.00	4.00	2.66	3.47	2.31
	32	3.64	2.42	2.69	1.79	4.07	2.71	3.14	2.09	4.55	3.02	3.65	2.43
	34	4.11	2.74	2.80	1.87	4.59	3.05	3.29	2.19	5.13	3.41	3.93	2.61
	36	4.61	3.07	2.92	1.95	5.15	3.42	3.51	2.33	5.75	3.83	4.21	2.80
	38	5.14	3.42	3.09	2.05	5.73	3.81	3.74	2.49	6.41	4.26	4.50	2.99
	40	5.69	3.79	3.27	2.18	6.35	4.23	3.97	2.64	7.10	4.73	4.78	3.18

Other Constants and Properties

	96 ASD	96 LRFD	87 ASD	87 LRFD	79[f] ASD	79[f] LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	4.06	2.70	4.54	3.02	5.09	3.38
$t_y \times 10^3$ (kips)$^{-1}$	0.911	0.606	1.00	0.668	1.11	0.737
$t_r \times 10^3$ (kips)$^{-1}$	1.18	0.788	1.30	0.868	1.44	0.958
r_x/r_y	1.76		1.75		1.75	
r_y, in.	3.09		3.07		3.05	

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Return to Table of Contents

W12

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W12×											
		72[f]				65[f]				58			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	1.22	0.810	2.61	1.74	1.35	0.895	2.99	1.99	1.51	1.01	3.17	2.11
	6	1.28	0.855	2.61	1.74	1.42	0.945	2.99	1.99	1.63	1.09	3.17	2.11
	7	1.31	0.871	2.61	1.74	1.45	0.963	2.99	1.99	1.68	1.12	3.17	2.11
	8	1.34	0.891	2.61	1.74	1.48	0.985	2.99	1.99	1.74	1.16	3.19	2.12
	9	1.37	0.913	2.61	1.74	1.52	1.01	2.99	1.99	1.80	1.20	3.26	2.17
	10	1.41	0.939	2.61	1.74	1.56	1.04	2.99	1.99	1.88	1.25	3.34	2.22
	11	1.46	0.969	2.61	1.74	1.61	1.07	2.99	1.99	1.97	1.31	3.42	2.27
	12	1.51	1.00	2.66	1.77	1.67	1.11	2.99	1.99	2.07	1.37	3.50	2.33
	13	1.56	1.04	2.71	1.80	1.73	1.15	3.04	2.02	2.18	1.45	3.59	2.39
	14	1.63	1.08	2.76	1.84	1.81	1.20	3.10	2.06	2.31	1.54	3.68	2.45
	15	1.70	1.13	2.81	1.87	1.89	1.25	3.17	2.11	2.46	1.64	3.78	2.51
	16	1.78	1.18	2.87	1.91	1.98	1.31	3.23	2.15	2.64	1.75	3.88	2.58
	17	1.87	1.24	2.93	1.95	2.08	1.38	3.30	2.20	2.83	1.88	3.99	2.65
	18	1.97	1.31	2.99	1.99	2.19	1.46	3.38	2.25	3.06	2.03	4.10	2.73
	19	2.08	1.38	3.05	2.03	2.31	1.54	3.46	2.30	3.31	2.20	4.22	2.81
	20	2.20	1.47	3.12	2.07	2.45	1.63	3.54	2.35	3.60	2.40	4.35	2.89
	22	2.49	1.66	3.26	2.17	2.78	1.85	3.71	2.47	4.33	2.88	4.63	3.08
	24	2.86	1.90	3.41	2.27	3.19	2.12	3.90	2.60	5.15	3.43	4.95	3.29
	26	3.32	2.21	3.58	2.38	3.72	2.47	4.11	2.74	6.05	4.02	5.47	3.64
	28	3.85	2.56	3.77	2.51	4.31	2.87	4.35	2.90	7.01	4.67	6.02	4.01
	30	4.42	2.94	3.98	2.64	4.95	3.29	4.71	3.13	8.05	5.36	6.57	4.37
	32	5.03	3.35	4.29	2.86	5.63	3.75	5.13	3.41	9.16	6.09	7.12	4.74
	34	5.68	3.78	4.63	3.08	6.36	4.23	5.55	3.69	10.3	6.88	7.66	5.10
	36	6.37	4.24	4.98	3.31	7.13	4.74	5.97	3.98	11.6	7.71	8.21	5.46
	38	7.09	4.72	5.32	3.54	7.94	5.28	6.39	4.25	12.9	8.59	8.75	5.82
	40	7.86	5.23	5.66	3.76	8.80	5.85	6.81	4.53	14.3	9.52	9.29	6.18

Other Constants and Properties

Property	72 ASD	72 LRFD	65 ASD	65 LRFD	58 ASD	58 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	5.80	3.86	6.74	4.48	8.43	5.61
$t_y \times 10^3$ (kips)$^{-1}$	1.22	0.810	1.35	0.895	1.51	1.01
$t_r \times 10^3$ (kips)$^{-1}$	1.58	1.05	1.75	1.16	1.96	1.31
r_x/r_y	1.75		1.75		2.10	
r_y, in.	3.04		3.02		2.51	

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W12

Shape	W12× 53[f]				W12× 50				W12× 45			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	1.65	1.10	3.58	2.38	1.76	1.17	3.81	2.54	1.96	1.30	4.27	2.84
6	1.78	1.19	3.58	2.38	2.00	1.33	3.81	2.54	2.23	1.49	4.27	2.84
7	1.84	1.22	3.58	2.38	2.10	1.39	3.91	2.60	2.34	1.56	4.39	2.92
8	1.90	1.26	3.58	2.38	2.21	1.47	4.03	2.68	2.47	1.64	4.53	3.02
9	1.97	1.31	3.63	2.42	2.35	1.56	4.15	2.76	2.63	1.75	4.68	3.11
10	2.06	1.37	3.72	2.48	2.51	1.67	4.28	2.85	2.81	1.87	4.84	3.22
11	2.16	1.43	3.82	2.54	2.71	1.80	4.42	2.94	3.03	2.02	5.01	3.33
12	2.27	1.51	3.92	2.61	2.94	1.96	4.57	3.04	3.29	2.19	5.19	3.45
13	2.40	1.60	4.02	2.68	3.21	2.14	4.73	3.14	3.60	2.40	5.38	3.58
14	2.55	1.69	4.13	2.75	3.54	2.35	4.90	3.26	3.97	2.64	5.59	3.72
15	2.72	1.81	4.25	2.83	3.92	2.61	5.08	3.38	4.41	2.93	5.82	3.87
16	2.91	1.94	4.38	2.91	4.38	2.91	5.27	3.51	4.93	3.28	6.07	4.04
17	3.13	2.08	4.51	3.00	4.94	3.28	5.49	3.65	5.56	3.70	6.34	4.22
18	3.39	2.25	4.65	3.09	5.53	3.68	5.72	3.80	6.23	4.15	6.63	4.41
19	3.68	2.45	4.80	3.19	6.17	4.10	5.96	3.97	6.94	4.62	7.06	4.70
20	4.01	2.67	4.96	3.30	6.83	4.55	6.33	4.21	7.69	5.12	7.58	5.04
22	4.83	3.22	5.31	3.53	8.27	5.50	7.17	4.77	9.31	6.19	8.62	5.73
24	5.75	3.83	5.81	3.86	9.84	6.55	8.01	5.33	11.1	7.37	9.66	6.43
26	6.75	4.49	6.48	4.31	11.5	7.68	8.84	5.88	13.0	8.65	10.7	7.11
28	7.83	5.21	7.15	4.75	13.4	8.91	9.67	6.44	15.1	10.0	11.7	7.80
30	8.99	5.98	7.81	5.20	15.4	10.2	10.5	6.99	17.3	11.5	12.8	8.48
32	10.2	6.80	8.48	5.64	17.5	11.6	11.3	7.53	19.7	13.1	13.8	9.16
34	11.5	7.68	9.15	6.09								
36	12.9	8.61	9.81	6.53								
38	14.4	9.59	10.5	6.97								
40	16.0	10.6	11.1	7.41								
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	9.68	6.44			12.9	8.56			14.4	9.60		
$t_y \times 10^3$ (kips)$^{-1}$	1.65	1.10			1.76	1.17			1.96	1.30		
$t_r \times 10^3$ (kips)$^{-1}$	2.14	1.42			2.28	1.52			2.54	1.70		
r_x / r_y	2.11				2.64				2.64			
r_y, in.	2.48				1.96				1.95			

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W12

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W12×											
		40[c]				35[c]				30[c]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	2.22	1.48	4.81	3.20	2.58	1.71	5.35	3.56	3.12	2.08	6.36	4.23
	6	2.50	1.67	4.81	3.20	3.09	2.06	5.64	3.75	3.74	2.49	6.74	4.48
	7	2.62	1.75	4.96	3.30	3.31	2.20	5.89	3.92	4.00	2.66	7.07	4.70
	8	2.77	1.84	5.13	3.41	3.61	2.40	6.18	4.11	4.33	2.88	7.43	4.95
	9	2.95	1.96	5.31	3.53	3.98	2.65	6.48	4.31	4.75	3.16	7.84	5.21
	10	3.16	2.10	5.50	3.66	4.44	2.96	6.82	4.54	5.29	3.52	8.28	5.51
	11	3.41	2.27	5.71	3.80	5.01	3.34	7.20	4.79	5.99	3.98	8.79	5.85
	12	3.71	2.47	5.94	3.95	5.73	3.81	7.63	5.07	6.86	4.56	9.36	6.22
	13	4.06	2.70	6.18	4.11	6.63	4.41	8.10	5.39	7.97	5.30	10.0	6.65
	14	4.48	2.98	6.44	4.28	7.69	5.11	8.65	5.76	9.25	6.15	11.1	7.41
	15	4.98	3.31	6.73	4.47	8.82	5.87	9.60	6.39	10.6	7.06	12.4	8.26
	16	5.57	3.71	7.04	4.68	10.0	6.68	10.6	7.02	12.1	8.04	13.7	9.13
	17	6.29	4.18	7.38	4.91	11.3	7.54	11.5	7.66	13.6	9.07	15.0	10.0
	18	7.05	4.69	7.87	5.24	12.7	8.45	12.5	8.30	15.3	10.2	16.4	10.9
	19	7.85	5.23	8.52	5.67	14.2	9.42	13.4	8.94	17.0	11.3	17.7	11.8
	20	8.70	5.79	9.16	6.10	15.7	10.4	14.4	9.59	18.9	12.6	19.0	12.7
	22	10.5	7.01	10.5	6.96	19.0	12.6	16.3	10.9	22.8	15.2	21.7	14.5
	24	12.5	8.34	11.8	7.83	22.6	15.0	18.3	12.2	27.2	18.1	24.4	16.3
	26	14.7	9.79	13.1	8.69								
	28	17.1	11.3	14.4	9.56								
	30	19.6	13.0	15.7	10.4								
	32	22.3	14.8	16.9	11.3								
Other Constants and Properties													
$b_y \times 10^3$ (kip-ft)$^{-1}$		16.3		10.9		23.8		15.9		28.7		19.1	
$t_y \times 10^3$ (kips)$^{-1}$		2.20		1.46		2.49		1.66		2.92		1.94	
$t_r \times 10^3$ (kips)$^{-1}$		2.85		1.90		3.24		2.16		3.79		2.53	
r_x/r_y		2.64				3.41				3.43			
r_y, in.		1.94				1.54				1.52			

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W12

Shape	W12×											
	26[c,f]				22[c]				19[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	3.69	2.45	7.48	4.97	4.34	2.89	9.35	6.22	5.23	3.48	11.1	7.38
1	3.70	2.46	7.48	4.97	4.40	2.93	9.35	6.22	5.31	3.53	11.1	7.38
2	3.76	2.50	7.48	4.97	4.60	3.06	9.35	6.22	5.55	3.70	11.1	7.38
3	3.85	2.56	7.48	4.97	4.96	3.30	9.63	6.41	6.00	3.99	11.5	7.67
4	3.98	2.65	7.48	4.97	5.54	3.69	10.5	6.96	6.71	4.47	12.6	8.38
5	4.17	2.77	7.48	4.98	6.44	4.28	11.4	7.61	7.83	5.21	13.9	9.25
6	4.40	2.93	7.85	5.22	7.87	5.23	12.6	8.40	9.59	6.38	15.5	10.3
7	4.71	3.13	8.25	5.49	10.1	6.70	14.1	9.38	12.5	8.30	17.5	11.7
8	5.09	3.39	8.71	5.79	13.2	8.75	16.3	10.8	16.3	10.8	21.2	14.1
9	5.57	3.71	9.21	6.13	16.7	11.1	19.6	13.0	20.6	13.7	25.7	17.1
10	6.19	4.12	9.78	6.50	20.6	13.7	23.0	15.3	25.5	16.9	30.4	20.2
11	6.97	4.63	10.4	6.93	24.9	16.6	26.5	17.6	30.8	20.5	35.2	23.4
12	7.97	5.30	11.1	7.42	29.6	19.7	30.0	20.0	36.7	24.4	40.1	26.7
13	9.28	6.18	12.2	8.12	34.7	23.1	33.5	22.3	43.0	28.6	45.1	30.0
14	10.8	7.16	13.8	9.19	40.3	26.8	37.1	24.7				
15	12.4	8.22	15.4	10.3								
16	14.1	9.36	17.1	11.4								
17	15.9	10.6	18.8	12.5								
18	17.8	11.8	20.6	13.7								
19	19.8	13.2	22.3	14.9								
20	22.0	14.6	24.1	16.0								
21	24.2	16.1	25.9	17.2								
22	26.6	17.7	27.7	18.4								
23	29.1	19.3	29.5	19.6								
24	31.6	21.0	31.3	20.8								
25	34.3	22.8	33.1	22.0								
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	34.3		22.8		74.9		49.8		92.0		61.2	
$t_y \times 10^3$ (kips)$^{-1}$	3.36		2.23		3.96		2.64		4.61		3.07	
$t_r \times 10^3$ (kips)$^{-1}$	4.36		2.90		5.14		3.43		5.98		3.99	
r_x/r_y	3.42				5.79				5.86			
r_y, in.	1.51				0.848				0.822			

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W12

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape		W12×							
		16[c,v]				14[c,f,v]			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	6.43	4.28	13.6	9.07	7.57	5.03	16.1	10.7
	1	6.53	4.34	13.6	9.07	7.68	5.11	16.1	10.7
	2	6.85	4.56	13.6	9.07	8.06	5.36	16.1	10.7
	3	7.43	4.95	14.4	9.59	8.75	5.82	16.8	11.2
	4	8.39	5.59	15.9	10.6	9.90	6.58	18.6	12.4
	5	9.94	6.61	17.8	11.8	11.7	7.82	20.9	13.9
	6	12.5	8.30	20.1	13.4	14.8	9.86	23.8	15.8
	7	16.7	11.1	23.4	15.5	19.9	13.2	28.7	19.1
	8	21.8	14.5	29.5	19.6	26.0	17.3	36.4	24.2
	9	27.6	18.3	36.1	24.0	32.9	21.9	44.6	29.7
	10	34.0	22.6	42.9	28.5	40.6	27.0	53.3	35.5
	11	41.2	27.4	50.0	33.3	49.1	32.7	62.4	41.5
	12	49.0	32.6	57.2	38.1	58.5	38.9	71.8	47.8
Other Constants and Properties									
$b_y \times 10^3$ (kip-ft)$^{-1}$		121		80.8		149		99.3	
$t_y \times 10^3$ (kips)$^{-1}$		5.45		3.63		6.18		4.11	
$t_r \times 10^3$ (kips)$^{-1}$		7.08		4.72		8.01		5.34	
r_x/r_y		6.04				6.14			
r_y, in.		0.773				0.753			

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

[v] Shape does not meet the h/t_w limit for shear in AISC *Specification* Section G2.1(a) with F_y = 65 ksi; therefore, ϕ_v = 0.90 and Ω_v = 1.67.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W10

Shape		W10×											
		112				100				88			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	0.781	0.520	1.86	1.24	0.877	0.583	2.11	1.40	0.988	0.657	2.43	1.61
	6	0.836	0.556	1.86	1.24	0.941	0.626	2.11	1.40	1.06	0.706	2.43	1.61
	7	0.857	0.570	1.86	1.24	0.965	0.642	2.11	1.40	1.09	0.724	2.43	1.61
	8	0.882	0.587	1.86	1.24	0.993	0.661	2.11	1.40	1.12	0.746	2.43	1.61
	9	0.911	0.606	1.88	1.25	1.03	0.683	2.13	1.41	1.16	0.772	2.45	1.63
	10	0.945	0.629	1.90	1.26	1.07	0.709	2.15	1.43	1.20	0.801	2.48	1.65
	11	0.983	0.654	1.91	1.27	1.11	0.739	2.17	1.45	1.26	0.835	2.51	1.67
	12	1.03	0.684	1.93	1.29	1.16	0.772	2.20	1.46	1.31	0.874	2.55	1.69
	13	1.08	0.717	1.95	1.30	1.22	0.811	2.22	1.48	1.38	0.919	2.58	1.72
	14	1.13	0.755	1.97	1.31	1.28	0.855	2.25	1.50	1.46	0.969	2.61	1.74
	15	1.20	0.798	1.99	1.33	1.36	0.905	2.28	1.51	1.54	1.03	2.65	1.76
	16	1.27	0.846	2.01	1.34	1.44	0.961	2.30	1.53	1.64	1.09	2.68	1.79
	17	1.35	0.901	2.04	1.35	1.54	1.02	2.33	1.55	1.75	1.16	2.72	1.81
	18	1.45	0.963	2.06	1.37	1.65	1.10	2.36	1.57	1.88	1.25	2.76	1.84
	19	1.55	1.03	2.08	1.38	1.77	1.18	2.39	1.59	2.02	1.34	2.80	1.86
	20	1.67	1.11	2.10	1.40	1.91	1.27	2.42	1.61	2.18	1.45	2.84	1.89
	22	1.96	1.31	2.15	1.43	2.25	1.50	2.48	1.65	2.58	1.72	2.92	1.94
	24	2.34	1.55	2.20	1.46	2.68	1.78	2.54	1.69	3.07	2.04	3.01	2.00
	26	2.74	1.82	2.25	1.50	3.15	2.09	2.61	1.74	3.60	2.40	3.11	2.07
	28	3.18	2.11	2.30	1.53	3.65	2.43	2.68	1.79	4.18	2.78	3.21	2.13
	30	3.65	2.43	2.36	1.57	4.19	2.79	2.76	1.84	4.79	3.19	3.31	2.21
	32	4.15	2.76	2.42	1.61	4.77	3.17	2.84	1.89	5.46	3.63	3.43	2.28
	34	4.69	3.12	2.48	1.65	5.38	3.58	2.93	1.95	6.16	4.10	3.55	2.36
	36	5.25	3.50	2.55	1.70	6.03	4.01	3.02	2.01	6.90	4.59	3.69	2.45
	38	5.85	3.90	2.62	1.74	6.72	4.47	3.11	2.07	7.69	5.12	3.83	2.55
	40	6.49	4.32	2.69	1.79	7.45	4.96	3.22	2.14	8.52	5.67	3.99	2.65

Other Constants and Properties

	112 ASD	112 LRFD	100 ASD	100 LRFD	88 ASD	88 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	3.96	2.63	4.49	2.99	5.16	3.43
$t_y \times 10^3$ (kips)$^{-1}$	0.781	0.520	0.877	0.583	0.988	0.657
$t_r \times 10^3$ (kips)$^{-1}$	1.01	0.675	1.14	0.758	1.28	0.855
r_x/r_y	1.75		1.74		1.71	
r_y, in.	2.68		2.65		2.63	

W10

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W10×											
	77				68				60			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending: 0	1.13	0.753	2.81	1.87	1.29	0.859	3.21	2.14	1.45	0.966	3.67	2.44
6	1.22	0.810	2.81	1.87	1.39	0.924	3.21	2.14	1.56	1.04	3.67	2.44
7	1.25	0.832	2.81	1.87	1.43	0.949	3.21	2.14	1.61	1.07	3.67	2.44
8	1.29	0.857	2.81	1.87	1.47	0.979	3.21	2.14	1.66	1.10	3.68	2.45
9	1.33	0.887	2.85	1.89	1.52	1.01	3.26	2.17	1.72	1.14	3.74	2.49
10	1.39	0.922	2.89	1.92	1.58	1.05	3.32	2.21	1.79	1.19	3.81	2.54
11	1.45	0.962	2.93	1.95	1.65	1.10	3.37	2.24	1.87	1.24	3.88	2.58
12	1.51	1.01	2.97	1.98	1.73	1.15	3.43	2.28	1.96	1.30	3.96	2.63
13	1.59	1.06	3.02	2.01	1.82	1.21	3.49	2.32	2.06	1.37	4.03	2.68
14	1.68	1.12	3.06	2.04	1.93	1.28	3.55	2.36	2.18	1.45	4.11	2.74
15	1.78	1.19	3.11	2.07	2.04	1.36	3.61	2.40	2.31	1.54	4.19	2.79
16	1.90	1.26	3.16	2.10	2.18	1.45	3.67	2.44	2.47	1.64	4.28	2.85
17	2.03	1.35	3.21	2.14	2.33	1.55	3.74	2.49	2.64	1.76	4.37	2.91
18	2.18	1.45	3.26	2.17	2.50	1.66	3.81	2.54	2.84	1.89	4.46	2.97
19	2.35	1.56	3.32	2.21	2.70	1.79	3.88	2.58	3.07	2.04	4.56	3.03
20	2.54	1.69	3.37	2.24	2.92	1.94	3.96	2.63	3.33	2.21	4.66	3.10
22	3.02	2.01	3.49	2.32	3.47	2.31	4.12	2.74	3.97	2.64	4.88	3.25
24	3.60	2.39	3.61	2.40	4.13	2.75	4.29	2.86	4.72	3.14	5.12	3.41
26	4.22	2.81	3.75	2.49	4.85	3.23	4.48	2.98	5.54	3.69	5.38	3.58
28	4.89	3.26	3.89	2.59	5.63	3.74	4.69	3.12	6.42	4.27	5.68	3.78
30	5.62	3.74	4.05	2.70	6.46	4.30	4.91	3.27	7.38	4.91	6.07	4.04
32	6.39	4.25	4.22	2.81	7.35	4.89	5.16	3.43	8.39	5.58	6.55	4.36
34	7.22	4.80	4.41	2.93	8.30	5.52	5.53	3.68	9.47	6.30	7.02	4.67
36	8.09	5.38	4.64	3.09	9.30	6.19	5.89	3.92	10.6	7.07	7.49	4.98
38	9.02	6.00	4.92	3.27	10.4	6.90	6.25	4.16	11.8	7.87	7.96	5.30
40	9.99	6.65	5.20	3.46	11.5	7.64	6.61	4.40	13.1	8.72	8.43	5.61

Other Constants and Properties

	77 ASD	77 LRFD	68 ASD	68 LRFD	60 ASD	60 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	5.97	3.97	6.83	4.55	7.83	5.21
$t_y \times 10^3$ (kips)$^{-1}$	1.13	0.753	1.29	0.859	1.45	0.966
$t_r \times 10^3$ (kips)$^{-1}$	1.47	0.979	1.68	1.12	1.88	1.26
r_x/r_y	1.73		1.70		1.71	
r_y, in.	2.60		2.59		2.57	

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W10

Shape		W10×											
		54[f]				49[f]				45			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	1.63	1.08	4.13	2.75	1.78	1.19	4.66	3.10	1.93	1.29	4.99	3.32
	6	1.75	1.17	4.13	2.75	1.93	1.28	4.66	3.10	2.18	1.45	4.99	3.32
	7	1.80	1.20	4.13	2.75	1.98	1.32	4.66	3.10	2.28	1.52	5.09	3.38
	8	1.86	1.24	4.13	2.75	2.04	1.36	4.66	3.10	2.40	1.60	5.22	3.47
	9	1.93	1.28	4.20	2.80	2.12	1.41	4.66	3.10	2.54	1.69	5.35	3.56
	10	2.00	1.33	4.29	2.85	2.21	1.47	4.75	3.16	2.71	1.80	5.50	3.66
	11	2.09	1.39	4.38	2.91	2.31	1.53	4.85	3.23	2.91	1.94	5.65	3.76
	12	2.20	1.46	4.47	2.97	2.42	1.61	4.96	3.30	3.15	2.09	5.81	3.86
	13	2.31	1.54	4.56	3.04	2.55	1.70	5.08	3.38	3.42	2.28	5.98	3.98
	14	2.45	1.63	4.66	3.10	2.70	1.80	5.20	3.46	3.75	2.50	6.16	4.10
	15	2.60	1.73	4.77	3.17	2.88	1.91	5.32	3.54	4.14	2.75	6.35	4.22
	16	2.78	1.85	4.88	3.25	3.07	2.04	5.46	3.63	4.60	3.06	6.55	4.36
	17	2.97	1.98	4.99	3.32	3.29	2.19	5.60	3.72	5.15	3.43	6.76	4.50
	18	3.20	2.13	5.11	3.40	3.55	2.36	5.74	3.82	5.78	3.84	6.99	4.65
	19	3.46	2.30	5.24	3.49	3.84	2.55	5.90	3.92	6.44	4.28	7.24	4.82
	20	3.75	2.49	5.37	3.57	4.17	2.77	6.06	4.03	7.13	4.75	7.51	4.99
	22	4.48	2.98	5.66	3.76	4.99	3.32	6.41	4.27	8.63	5.74	8.15	5.42
	24	5.33	3.55	5.97	3.98	5.94	3.95	6.81	4.53	10.3	6.83	9.06	6.03
	26	6.25	4.16	6.33	4.21	6.97	4.64	7.31	4.87	12.1	8.02	9.96	6.63
	28	7.25	4.83	6.82	4.54	8.08	5.38	8.03	5.35	14.0	9.30	10.9	7.22
	30	8.33	5.54	7.42	4.94	9.28	6.17	8.75	5.82	16.0	10.7	11.7	7.82
	32	9.47	6.30	8.01	5.33	10.6	7.03	9.47	6.30	18.3	12.1	12.6	8.41
	34	10.7	7.12	8.60	5.72	11.9	7.93	10.2	6.77				
	36	12.0	7.98	9.19	6.11	13.4	8.89	10.9	7.24				
	38	13.4	8.89	9.77	6.50	14.9	9.91	11.6	7.71				
	40	14.8	9.85	10.4	6.89	16.5	11.0	12.3	8.18				

Other Constants and Properties

	54 ASD	54 LRFD	49 ASD	49 LRFD	45 ASD	45 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	8.80	5.86	10.1	6.69	13.5	8.98
$t_y \times 10^3$ (kips)$^{-1}$	1.63	1.08	1.78	1.19	1.93	1.29
$t_r \times 10^3$ (kips)$^{-1}$	2.11	1.41	2.31	1.54	2.51	1.67
r_x/r_y	1.72		1.73		2.14	
r_y, in.	2.56		2.54		2.01	

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W10

F_y = 65 ksi

Shape	W10×											
	39				33[f]				30			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	2.23	1.49	5.86	3.90	2.65	1.76	7.29	4.85	2.91	1.93	7.49	4.98
6	2.53	1.69	5.86	3.90	3.02	2.01	7.29	4.85	3.78	2.51	8.09	5.38
7	2.65	1.76	6.00	3.99	3.16	2.10	7.29	4.85	4.15	2.76	8.47	5.63
8	2.79	1.86	6.17	4.11	3.34	2.22	7.52	5.01	4.63	3.08	8.89	5.92
9	2.96	1.97	6.35	4.23	3.55	2.36	7.78	5.18	5.25	3.49	9.36	6.23
10	3.17	2.11	6.55	4.36	3.81	2.53	8.05	5.36	6.03	4.01	9.88	6.57
11	3.41	2.27	6.75	4.49	4.11	2.73	8.34	5.55	7.02	4.67	10.5	6.96
12	3.69	2.46	6.97	4.64	4.47	2.97	8.66	5.76	8.31	5.53	11.1	7.39
13	4.03	2.68	7.21	4.79	4.89	3.26	9.00	5.99	9.76	6.49	11.8	7.88
14	4.43	2.95	7.46	4.96	5.40	3.59	9.36	6.23	11.3	7.53	13.0	8.65
15	4.90	3.26	7.73	5.14	6.00	3.99	9.76	6.49	13.0	8.64	14.3	9.50
16	5.46	3.63	8.01	5.33	6.71	4.47	10.2	6.78	14.8	9.83	15.6	10.4
17	6.14	4.09	8.33	5.54	7.58	5.04	10.7	7.10	16.7	11.1	16.8	11.2
18	6.88	4.58	8.66	5.76	8.49	5.65	11.2	7.44	18.7	12.4	18.1	12.1
19	7.67	5.10	9.03	6.01	9.46	6.30	12.1	8.04	20.8	13.9	19.4	12.9
20	8.50	5.66	9.47	6.30	10.5	6.98	13.0	8.63	23.1	15.4	20.7	13.8
22	10.3	6.84	10.7	7.13	12.7	8.44	14.8	9.82	27.9	18.6	23.2	15.4
24	12.2	8.14	11.9	7.95	15.1	10.0	16.5	11.0				
26	14.4	9.56	13.2	8.77	17.7	11.8	18.3	12.2				
28	16.7	11.1	14.4	9.58	20.6	13.7	20.1	13.4				
30	19.1	12.7	15.6	10.4	23.6	15.7	21.9	14.5				
32	21.8	14.5	16.8	11.2	26.8	17.9	23.6	15.7				

Other Constants and Properties

	39 ASD	39 LRFD	33[f] ASD	33[f] LRFD	30 ASD	30 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	15.9	10.6	20.5	13.7	31.0	20.6
$t_y \times 10^3$ (kips)$^{-1}$	2.23	1.49	2.65	1.76	2.91	1.93
$t_r \times 10^3$ (kips)$^{-1}$	2.90	1.93	3.43	2.29	3.77	2.51
r_x/r_y	2.17		2.16		3.20	
r_y, in.	1.98		1.94		1.37	

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W10

Shape	W10×											
	26[c]				22[c]				19[c]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, *KL* (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	3.44	2.29	8.76	5.83	4.11	2.74	10.5	7.01	4.73	3.15	12.7	8.44
1	3.46	2.30	8.76	5.83	4.14	2.75	10.5	7.01	4.80	3.20	12.7	8.44
2	3.53	2.35	8.76	5.83	4.22	2.81	10.5	7.01	5.03	3.34	12.7	8.44
3	3.65	2.43	8.76	5.83	4.37	2.90	10.5	7.01	5.43	3.61	13.0	8.62
4	3.82	2.54	8.76	5.83	4.58	3.04	10.5	7.01	6.09	4.05	14.0	9.30
5	4.06	2.70	9.08	6.04	4.87	3.24	11.0	7.34	7.16	4.76	15.2	10.1
6	4.41	2.93	9.53	6.34	5.25	3.50	11.6	7.74	8.71	5.80	16.6	11.0
7	4.85	3.23	10.0	6.67	5.78	3.85	12.3	8.20	11.0	7.32	18.3	12.2
8	5.42	3.61	10.6	7.03	6.50	4.32	13.1	8.71	14.3	9.50	20.4	13.6
9	6.15	4.09	11.2	7.44	7.41	4.93	14.0	9.29	18.1	12.0	24.2	16.1
10	7.08	4.71	11.9	7.90	8.58	5.71	15.0	9.95	22.3	14.8	28.2	18.8
11	8.27	5.50	12.7	8.42	10.1	6.72	16.1	10.7	27.0	18.0	32.3	21.5
12	9.80	6.52	13.5	9.01	12.0	8.00	17.7	11.8	32.1	21.4	36.4	24.2
13	11.5	7.65	15.0	9.96	14.1	9.38	20.1	13.4	37.7	25.1	40.5	26.9
14	13.3	8.88	16.7	11.1	16.4	10.9	22.5	15.0	43.7	29.1	44.6	29.7
15	15.3	10.2	18.4	12.2	18.8	12.5	25.0	16.6				
16	17.4	11.6	20.1	13.4	21.4	14.2	27.4	18.2				
17	19.7	13.1	21.8	14.5	24.1	16.0	29.9	19.9				
18	22.1	14.7	23.6	15.7	27.0	18.0	32.4	21.6				
19	24.6	16.3	25.3	16.8	30.1	20.0	34.9	23.2				
20	27.2	18.1	27.0	18.0	33.4	22.2	37.4	24.9				
21	30.0	20.0	28.7	19.1	36.8	24.5	39.9	26.5				
22	32.9	21.9	30.5	20.3	40.4	26.9	42.4	28.2				
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	36.5		24.3		44.9		29.9		81.8		54.4	
$t_y \times 10^3$ (kips)$^{-1}$	3.38		2.25		3.96		2.63		4.57		3.04	
$t_r \times 10^3$ (kips)$^{-1}$	4.38		2.92		5.14		3.42		5.93		3.95	
r_x/r_y	3.20				3.21				4.74			
r_y, in.	1.36				1.33				0.874			

[c] Shape is slender for compression with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

W10

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W10×											
	17[c]				15[c]				12[c,f]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	5.41	3.60	14.7	9.75	6.24	4.15	17.1	11.4	8.30	5.52	22.7	15.1
1	5.50	3.66	14.7	9.75	6.34	4.22	17.1	11.4	8.43	5.61	22.7	15.1
2	5.76	3.83	14.7	9.75	6.66	4.43	17.1	11.4	8.85	5.89	22.7	15.1
3	6.24	4.15	15.1	10.0	7.25	4.82	17.8	11.9	9.62	6.40	22.9	15.2
4	7.02	4.67	16.4	10.9	8.21	5.46	19.5	13.0	10.9	7.24	25.2	16.8
5	8.31	5.53	17.9	11.9	9.81	6.53	21.5	14.3	12.9	8.57	28.1	18.7
6	10.3	6.83	19.8	13.2	12.3	8.21	24.0	16.0	16.1	10.7	31.7	21.1
7	13.2	8.76	22.1	14.7	16.2	10.8	27.1	18.0	21.5	14.3	36.6	24.4
8	17.2	11.4	25.4	16.9	21.2	14.1	32.8	21.8	28.1	18.7	46.2	30.8
9	21.8	14.5	30.6	20.4	26.8	17.8	39.6	26.4	35.6	23.7	56.5	37.6
10	26.9	17.9	36.0	23.9	33.1	22.0	46.8	31.1	43.9	29.2	67.2	44.7
11	32.5	21.6	41.4	27.5	40.1	26.7	54.0	35.9	53.1	35.4	78.3	52.1
12	38.7	25.8	46.8	31.2	47.7	31.7	61.4	40.9	63.2	42.1	89.6	59.6
13	45.4	30.2	52.3	34.8	56.0	37.2	68.8	45.8	74.2	49.4	101	67.3
14	52.7	35.1	57.8	38.5								
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	97.9		65.1		119		79.3		168		111	
$t_y \times 10^3$ (kips)$^{-1}$	5.15		3.43		5.83		3.88		7.26		4.83	
$t_r \times 10^3$ (kips)$^{-1}$	6.68		4.45		7.56		5.04		9.42		6.28	
r_x / r_y	4.79				4.88				4.97			
r_y, in.	0.845				0.810				0.785			

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

Return to Table of Contents

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W8

Shape		W8× 67				W8× 58				W8× 48			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	1.30	0.868	3.91	2.60	1.50	1.00	4.58	3.05	1.82	1.21	5.59	3.72
	6	1.46	0.968	3.91	2.60	1.68	1.12	4.58	3.05	2.04	1.36	5.59	3.72
	7	1.51	1.01	3.93	2.62	1.75	1.16	4.62	3.07	2.13	1.42	5.65	3.76
	8	1.58	1.05	3.98	2.65	1.83	1.22	4.69	3.12	2.23	1.48	5.76	3.83
	9	1.67	1.11	4.04	2.69	1.93	1.29	4.76	3.17	2.35	1.57	5.86	3.90
	10	1.77	1.18	4.09	2.72	2.05	1.36	4.84	3.22	2.50	1.66	5.98	3.98
	11	1.89	1.25	4.15	2.76	2.19	1.46	4.92	3.27	2.67	1.78	6.10	4.06
	12	2.02	1.35	4.21	2.80	2.35	1.56	5.00	3.32	2.87	1.91	6.22	4.14
	13	2.18	1.45	4.27	2.84	2.54	1.69	5.08	3.38	3.11	2.07	6.35	4.22
	14	2.37	1.58	4.33	2.88	2.76	1.84	5.17	3.44	3.39	2.25	6.48	4.31
	15	2.59	1.72	4.39	2.92	3.02	2.01	5.26	3.50	3.71	2.47	6.62	4.40
	16	2.84	1.89	4.46	2.97	3.33	2.21	5.35	3.56	4.10	2.72	6.76	4.50
	17	3.14	2.09	4.53	3.01	3.68	2.45	5.45	3.62	4.55	3.02	6.91	4.60
	18	3.51	2.33	4.60	3.06	4.12	2.74	5.55	3.69	5.09	3.39	7.07	4.71
	19	3.91	2.60	4.67	3.11	4.59	3.05	5.65	3.76	5.67	3.77	7.24	4.82
	20	4.33	2.88	4.74	3.16	5.08	3.38	5.76	3.83	6.28	4.18	7.41	4.93
	22	5.24	3.48	4.90	3.26	6.15	4.09	5.98	3.98	7.60	5.06	7.78	5.18
	24	6.23	4.15	5.06	3.37	7.32	4.87	6.23	4.14	9.05	6.02	8.20	5.45
	26	7.31	4.87	5.24	3.49	8.59	5.71	6.50	4.32	10.6	7.06	8.66	5.76
	28	8.48	5.64	5.43	3.61	9.96	6.63	6.79	4.52	12.3	8.19	9.20	6.12
	30	9.74	6.48	5.64	3.75	11.4	7.61	7.11	4.73	14.1	9.40	9.92	6.60
	32	11.1	7.37	5.86	3.90	13.0	8.66	7.45	4.96	16.1	10.7	10.6	7.08
	34	12.5	8.32	6.10	4.06	14.7	9.77	7.93	5.28	18.2	12.1	11.3	7.55

Other Constants and Properties

	W8×67 ASD	W8×67 LRFD	W8×58 ASD	W8×58 LRFD	W8×48 ASD	W8×48 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	8.38	5.58	9.82	6.54	12.0	7.96
$t_y \times 10^3$ (kips)$^{-1}$	1.30	0.868	1.50	1.00	1.82	1.21
$t_r \times 10^3$ (kips)$^{-1}$	1.69	1.13	1.95	1.30	2.36	1.58
r_x/r_y	1.75		1.74		1.74	
r_y, in.	2.12		2.10		2.08	

Note: Heavy line indicates KL/r_y greater than or equal to 200.

Return to Table of Contents

W8

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W8×							
	40				35[f]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending								
0	2.20	1.46	6.89	4.58	2.49	1.66	7.91	5.27
6	2.47	1.64	6.89	4.58	2.81	1.87	7.91	5.27
7	2.58	1.72	6.99	4.65	2.94	1.95	8.03	5.35
8	2.71	1.80	7.14	4.75	3.09	2.05	8.24	5.48
9	2.87	1.91	7.31	4.86	3.26	2.17	8.45	5.62
10	3.05	2.03	7.48	4.97	3.48	2.31	8.67	5.77
11	3.27	2.18	7.66	5.09	3.73	2.48	8.91	5.93
12	3.53	2.35	7.84	5.22	4.02	2.68	9.15	6.09
13	3.83	2.55	8.04	5.35	4.37	2.91	9.42	6.27
14	4.18	2.78	8.25	5.49	4.78	3.18	9.70	6.45
15	4.60	3.06	8.46	5.63	5.27	3.50	9.99	6.65
16	5.10	3.39	8.69	5.78	5.84	3.88	10.3	6.86
17	5.69	3.78	8.94	5.95	6.52	4.34	10.6	7.08
18	6.38	4.24	9.19	6.12	7.31	4.87	11.0	7.31
19	7.10	4.73	9.46	6.30	8.15	5.42	11.4	7.57
20	7.87	5.24	9.75	6.49	9.03	6.01	11.8	7.84
22	9.52	6.34	10.4	6.91	10.9	7.27	12.8	8.50
24	11.3	7.54	11.1	7.42	13.0	8.65	14.2	9.45
26	13.3	8.85	12.2	8.14	15.3	10.2	15.6	10.4
28	15.4	10.3	13.3	8.85	17.7	11.8	17.0	11.3
30	17.7	11.8	14.4	9.57	20.3	13.5	18.4	12.3
32	20.1	13.4	15.4	10.3	23.1	15.4	19.8	13.2
34	22.7	15.1	16.5	11.0				
Other Constants and Properties								
$b_y \times 10^3$ (kip-ft)$^{-1}$	14.8		9.86		17.1		11.4	
$t_y \times 10^3$ (kips)$^{-1}$	2.20		1.46		2.49		1.66	
$t_r \times 10^3$ (kips)$^{-1}$	2.85		1.90		3.24		2.16	
r_x/r_y	1.73				1.73			
r_y, in.	2.04				2.03			

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

F_y = 65 ksi

Table 6-1 (continued)

Combined Flexure and Axial Force

W-Shapes

W8

Shape		W8×							
		31[f]				28			
Design		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
		ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	0	2.81	1.87	9.32	6.20	3.11	2.07	10.1	6.70
	6	3.18	2.11	9.32	6.20	3.76	2.50	10.4	6.92
	7	3.32	2.21	9.32	6.20	4.02	2.68	10.7	7.15
	8	3.49	2.32	9.45	6.29	4.35	2.89	11.1	7.39
	9	3.69	2.46	9.72	6.47	4.75	3.16	11.5	7.66
	10	3.94	2.62	10.0	6.66	5.25	3.49	11.9	7.94
	11	4.22	2.81	10.3	6.86	5.85	3.89	12.4	8.25
	12	4.56	3.03	10.6	7.07	6.60	4.39	12.9	8.58
	13	4.96	3.30	11.0	7.30	7.52	5.00	13.4	8.93
	14	5.43	3.61	11.3	7.55	8.67	5.77	14.0	9.32
	15	5.99	3.98	11.7	7.81	9.96	6.62	14.6	9.74
	16	6.64	4.42	12.2	8.08	11.3	7.54	15.3	10.2
	17	7.43	4.94	12.6	8.38	12.8	8.51	16.1	10.7
	18	8.33	5.54	13.1	8.71	14.3	9.54	17.4	11.6
	19	9.28	6.18	13.6	9.06	16.0	10.6	18.6	12.4
	20	10.3	6.84	14.2	9.43	17.7	11.8	19.8	13.2
	22	12.4	8.28	16.0	10.6	21.4	14.2	22.1	14.7
	24	14.8	9.86	17.8	11.8	25.5	17.0	24.5	16.3
	26	17.4	11.6	19.6	13.1	29.9	19.9	26.9	17.9
	28	20.2	13.4	21.4	14.3				
	30	23.1	15.4	23.3	15.5				
	32	26.3	17.5	25.1	16.7				

Other Constants and Properties				
$b_y \times 10^3$ (kip-ft)$^{-1}$	20.4	13.6	27.1	18.1
$t_y \times 10^3$ (kips)$^{-1}$	2.81	1.87	3.11	2.07
$t_r \times 10^3$ (kips)$^{-1}$	3.65	2.43	4.04	2.69
r_x/r_y	1.72		2.13	
r_y, in.	2.02		1.62	

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y greater than or equal to 200.

Return to Table of Contents

W8

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

F_y = 65 ksi

Shape	W8× 24[f]				W8× 21				W8× 18			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
0	3.63	2.41	11.9	7.91	4.17	2.78	13.4	8.94	4.88	3.25	16.1	10.7
1	3.65	2.43	11.9	7.91	4.21	2.80	13.4	8.94	4.93	3.28	16.1	10.7
2	3.71	2.47	11.9	7.91	4.32	2.87	13.4	8.94	5.06	3.37	16.1	10.7
3	3.81	2.53	11.9	7.91	4.51	3.00	13.4	8.94	5.30	3.53	16.1	10.7
4	3.95	2.63	11.9	7.91	4.79	3.19	13.5	8.98	5.65	3.76	16.3	10.8
5	4.14	2.76	11.9	7.91	5.17	3.44	14.1	9.41	6.12	4.07	17.1	11.4
6	4.39	2.92	12.3	8.18	5.69	3.78	14.8	9.88	6.76	4.50	18.1	12.0
7	4.70	3.13	12.8	8.49	6.36	4.23	15.6	10.4	7.61	5.06	19.2	12.8
8	5.09	3.39	13.3	8.82	7.24	4.82	16.5	11.0	8.72	5.80	20.4	13.6
9	5.57	3.70	13.8	9.18	8.39	5.58	17.5	11.6	10.2	6.76	21.8	14.5
10	6.15	4.09	14.4	9.57	9.88	6.57	18.6	12.3	12.1	8.03	23.3	15.5
11	6.87	4.57	15.0	9.99	11.9	7.89	19.8	13.2	14.6	9.69	25.2	16.7
12	7.76	5.16	15.7	10.4	14.1	9.39	21.2	14.1	17.3	11.5	28.3	18.9
13	8.86	5.89	16.5	11.0	16.6	11.0	23.5	15.7	20.3	13.5	31.8	21.2
14	10.2	6.81	17.3	11.5	19.2	12.8	26.0	17.3	23.6	15.7	35.3	23.5
15	11.7	7.81	18.2	12.1	22.0	14.7	28.5	18.9	27.1	18.0	38.8	25.8
16	13.4	8.89	19.6	13.0	25.1	16.7	30.9	20.6	30.8	20.5	42.4	28.2
17	15.1	10.0	21.2	14.1	28.3	18.8	33.4	22.2	34.8	23.1	45.9	30.5
18	16.9	11.3	22.9	15.2	31.7	21.1	35.9	23.9	39.0	26.0	49.4	32.9
19	18.8	12.5	24.5	16.3	35.4	23.5	38.3	25.5	43.5	28.9	52.9	35.2
20	20.9	13.9	26.1	17.4	39.2	26.1	40.7	27.1	48.2	32.0	56.4	37.5
21	23.0	15.3	27.8	18.5	43.2	28.7	43.2	28.7				
22	25.3	16.8	29.4	19.6								
23	27.6	18.4	31.0	20.6								
24	30.1	20.0	32.6	21.7								
25	32.6	21.7	34.2	22.8								

Other Constants and Properties

	W8×24 ASD	W8×24 LRFD	W8×21 ASD	W8×21 LRFD	W8×18 ASD	W8×18 LRFD
$b_y \times 10^3$ (kip-ft)$^{-1}$	32.1	21.4	48.2	32.0	58.8	39.1
$t_y \times 10^3$ (kips)$^{-1}$	3.63	2.41	4.17	2.78	4.88	3.25
$t_r \times 10^3$ (kips)$^{-1}$	4.71	3.14	5.41	3.61	6.34	4.22
r_x / r_y	2.12		2.77		2.79	
r_y, in.	1.61		1.26		1.23	

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y equal to or greater than 200.

F_y = 65 ksi

Table 6-1 (continued)
Combined Flexure and Axial Force
W-Shapes

W8

Shape	W8× 15				13				10[c,f]			
Design	$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$		$p \times 10^3$ (kips)$^{-1}$		$b_x \times 10^3$ (kip-ft)$^{-1}$	
	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD	ASD	LRFD
Effective length, KL (ft), with respect to least radius of gyration, r_y, or Unbraced Length, L_b (ft), for X-X axis bending												
0	5.79	3.85	20.2	13.4	6.69	4.45	24.0	16.0	9.33	6.21	32.4	21.6
1	5.89	3.92	20.2	13.4	6.82	4.54	24.0	16.0	9.47	6.30	32.4	21.6
2	6.21	4.13	20.2	13.4	7.23	4.81	24.0	16.0	9.93	6.61	32.4	21.6
3	6.79	4.52	20.6	13.7	7.96	5.29	24.8	16.5	10.8	7.15	32.4	21.6
4	7.70	5.12	22.1	14.7	9.11	6.06	26.8	17.9	12.1	8.03	34.9	23.2
5	9.04	6.01	23.9	15.9	10.8	7.20	29.3	19.5	14.1	9.39	38.4	25.6
6	11.0	7.32	26.1	17.4	13.4	8.91	32.3	21.5	17.4	11.6	42.8	28.5
7	13.9	9.23	28.7	19.1	17.2	11.4	35.9	23.9	22.4	14.9	48.3	32.2
8	18.0	12.0	31.8	21.2	22.5	14.9	41.0	27.3	29.3	19.5	58.8	39.1
9	22.8	15.2	36.9	24.5	28.4	18.9	49.1	32.7	37.1	24.7	71.3	47.4
10	28.1	18.7	42.9	28.5	35.1	23.4	57.4	38.2	45.8	30.4	84.3	56.1
11	34.0	22.6	48.9	32.5	42.5	28.3	65.8	43.8	55.4	36.8	97.6	64.9
12	40.5	26.9	54.9	36.5	50.6	33.6	74.3	49.4	65.9	43.8	111	73.9
13	47.5	31.6	60.9	40.5	59.3	39.5	82.7	55.0	77.3	51.5	125	83.0
14	55.1	36.7	66.9	44.5	68.8	45.8	91.2	60.7	89.7	59.7	139	92.2
Other Constants and Properties												
$b_y \times 10^3$ (kip-ft)$^{-1}$	103		68.3		127		84.8		177		118	
$t_y \times 10^3$ (kips)$^{-1}$	5.79		3.85		6.69		4.45		8.68		5.78	
$t_r \times 10^3$ (kips)$^{-1}$	7.51		5.00		8.68		5.79		11.3		7.50	
r_x/r_y	3.76				3.81				3.83			
r_y, in.	0.876				0.843				0.841			

[c] Shape is slender for compression with F_y = 65 ksi.

[f] Shape does not meet compact limit for flexure with F_y = 65 ksi.

Note: Heavy line indicates KL/r_y greater than or equal to 200.