

Geometric Design

Geometric design for transportation facilities includes the design of geometric cross sections, horizontal alignment, vertical alignment, intersections, and various design details. These basic elements are common to all linear facilities, such as roadways, railways, and airport runways and taxiways. Although the details of design standards vary with the mode and the class of facility, most of the issues involved in geometric design are similar for all modes. In all cases, the goals of geometric design are to maximize the comfort, safety, and economy of facilities, while minimizing their environmental impacts. This chapter focuses on the fundamentals of geometric design, and presents standards and examples from different modes.

The order of presentation of material in this chapter is to consider geometric cross sections first, then vertical alignment, horizontal alignment, superelevation, intersections, and various design details. For purposes of exposition, the order of the topics is not very important. In a typical design project, on the other hand, there is a definite order of tasks, in which the establishment of a tentative horizontal centerline usually precedes establishment of vertical alignment. This is because the elevation of the existing ground along the centerline is an important consideration in establishing the vertical alignment. The process of designing the vertical alignment begins with plotting a profile of the existing terrain, and a tentative horizontal centerline must already be established in order to do this.

4.1 GEOMETRIC CROSS SECTION

The primary consideration in the design of geometric cross sections for highways, runways, and taxiways is drainage. Details vary depending on the type of facility and agency.^{1,2}

Highway cross sections consist of traveled way, shoulders (or parking lanes), and drainage channels. Shoulders are intended primarily as a safety feature. They provide for accommodation of stopped vehicles, emergency use, and lateral support of the pavement. Shoulders may be either paved or unpaved. Drainage channels may consist of ditches (usually grassed swales) or of paved shoulders with berms or curbs and gutters. Figures 4.1 to 4.4 show various types of geometric cross sections. For so-called normal crown sections (that is, no superelevation or banking of the roadway, as in a horizontal curve) the traveled way slopes down from the centerline or other profile grade line; roadway cross-slopes may either be constant or varying. Where cross-slopes vary, the cross section may consist of either a parabolic section or one in which

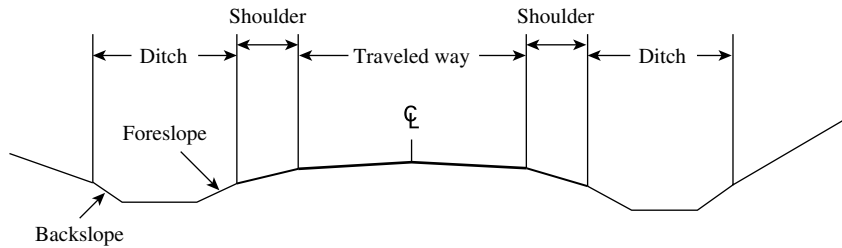


FIGURE 4.1
Two-lane highway cross section, with ditches.

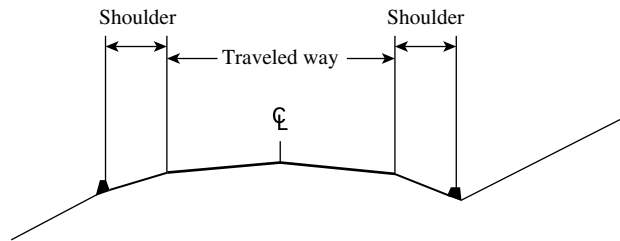


FIGURE 4.2
Two-lane highway cross section, curbed.

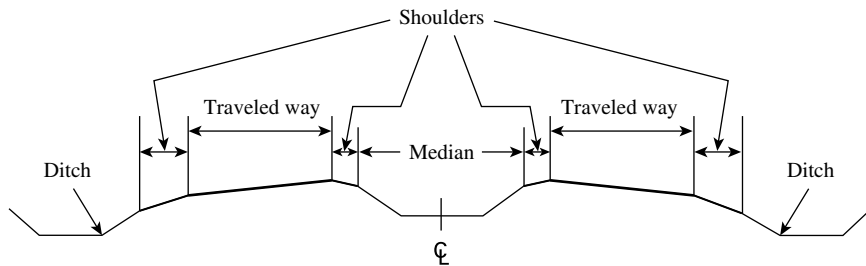


FIGURE 4.3
Divided highway cross section, depressed median, with ditches.

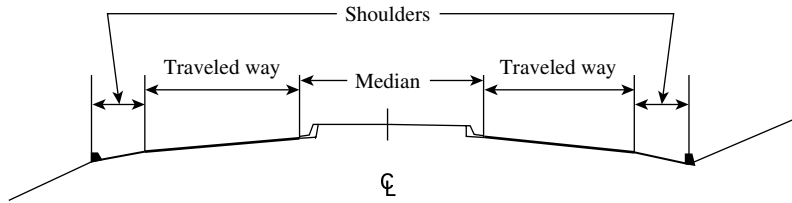


FIGURE 4.4

Divided highway cross section, raised median, curbed.

each lane has a constant cross-slope, but those of the outer lanes are greater than those of the inner lanes. For high-type roadways (such as freeways), cross-slopes are normally 1.5 to 2.0 percent. Shoulders or parking lanes slope away from the centerline at 2 to 6 percent. As a general rule, superelevated sections will be constructed in a single plane (including shoulders) if the rate of superelevation exceeds the normal cross-slope of the shoulder.

Where ditches are used, foreslopes should normally be 1:4 or flatter to prevent damage to vehicles or injury to occupants when traversed. Backslopes (cut or fill slopes) are discussed in more detail in Chapter 5. These are normally on the order of 1:2 or 1:1.5 except in rock cuts, where they may sometimes be even vertical. Use of 1:3 or flatter ditch backslopes facilitates the use of motorized equipment for maintenance, however; also, composite slopes (that is, two different slopes at different distances from the roadway) are sometimes used.

Standard lane widths are normally 3.6 m (12 ft.), although narrower lanes are common on older roadways, and may still be provided in cases where the standard lane width is not economical. Shoulders or parking lanes for heavily traveled roads are normally 2.4 to 3.6 m (8 to 12 ft.) in width; narrower shoulders are sometimes used on lightly traveled roads.

Runway and taxiway cross sections are similar to those for highways, except that cross-slopes are limited to 1 percent, with slopes of 1.5 percent on turf-covered areas immediately outside the runway or taxiway.³

Railway cross sections are as shown in Figure 4.5. The distance between the rails, as shown in the figure, is what is known as *standard gage* in North America. Other

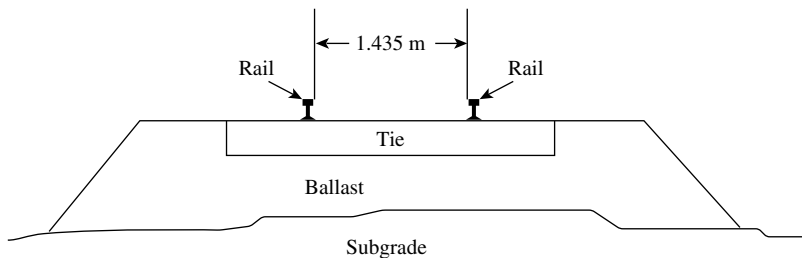


FIGURE 4.5

Railroad track geometric cross section.

gages are sometimes used. In the case of railroad track, the shape of the cross section is not intended to provide drainage, since the ballast, which is permeable material, serves this purpose.⁴

4.2 VERTICAL ALIGNMENT

The vertical alignment of a transportation facility consists of *tangent grades* (straight lines in the vertical plane) and *vertical curves*. Vertical alignment is documented by the *profile*. To repeat what was said in Chapter 3, the profile is a graph that has elevation as its vertical axis and distance, measured in stations along the centerline or other horizontal reference line of the facility, as its horizontal axis.

4.2.1 Tangent Grades

Tangent grades are designated according to their slopes or grades. Maximum grades vary, depending on the type of facility, and usually do not constitute an absolute standard. The effect of a steep grade is to slow down the heavier vehicles (which typically have the lowest power/weight ratios) and increase operating costs. Furthermore, the extent to which any vehicle (with a given power/weight ratio) is slowed depends on both the steepness and length of the grade. The effect of the slowing of the heavier vehicles depends on the situation, and is often more a matter of traffic analysis than simple geometric design. Chapter 10, for instance, discusses the effects of slow-moving vehicles on highway capacity and level of service. As a result, the maximum grade for a given facility is a matter of judgment, with the tradeoffs usually being cost of construction versus speed. In the case of railroads, on the other hand, the tradeoff is an economic one, involving travel time, construction cost, and minimum power/weight ratios for trains on various grades.

Table 4.1 gives the maximum grades recommended for various classes of roadway by AASHTO. It should be understood that considerably steeper grades can be

TABLE 4.1
Recommended standards for maximum grades, percent

Type of terrain	Freeways	Rural highways	Urban highways
Level	3–4	3–5	5–8
Rolling	4–5	5–6	6–9
Mountainous	5–6	5–8	8–11

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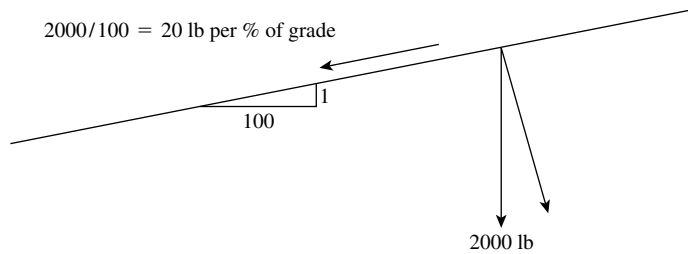


FIGURE 4.6
Force diagram for calculating grade resistance.

negotiated by passenger cars. In some urban areas, grades of minor streets may be as steep as 25 percent.

In the case of railways, older freight lines were built with grades of up to 2 percent. Modern practice is to limit grades against the load to 0.5 percent where possible, and grades against empty cars to 1.5 to 2.0 percent. The reason is that the rolling resistance of rail vehicles is very low compared with the grade resistance. Consequently, it is economical to build very flat grades and use low power/weight ratios for trains. Figure 4.6 illustrates the calculation of grade resistance. As can be seen, the resistance of the grade is approximately 20 lb/ton per 1 percent of grade. Rolling resistance, meanwhile, is around 3 lb/ton for level, tangent track in warm climates, where modern roller bearings are used. With this disproportion between rolling resistance and grade resistance, it makes sense to “underpower” trains and construct very level track, or else route lines so as to gain elevation gradually. The calculation of the distance the line needs to traverse in order to gain a certain amount of elevation, given a maximum grade, is known in railway practice as *development*.

Rail lines used for passenger traffic only, particularly urban rail transit systems, often use much steeper grades. Grades of up to 10 percent can be negotiated. Maximum grades for light rail systems are reported to be in the range of 6 to 9 percent; those for conventional “heavy” rail rapid transit systems are in the range of 3 to 4 percent. In the case of urban rail systems, lighter equipment, higher power/weight ratios to provide rapid acceleration, and numerous vertical controls (station elevations, streets to be passed over or under, etc.) account for the decision to allow steeper maximum grades.

In the case of runways used by aircraft with approach speeds of 121 knots or more, the maximum grade at any point is limited to 1.5 percent, and grades exceeding 0.8 percent are permitted only on the inner two quarters of the runway.

Minimum grades are sometimes specified for highways. These are normally intended to provide for drainage on curbed facilities. On such facilities, the grade of the edge of shoulder is normally the same as the profile grade, so that the centerline must have some minimum grade. For curbed roadways, AASHTO recommends a minimum grade of 0.50 percent, unless pavements are constructed with a high degree of quality control, in which case 0.30 percent is acceptable.

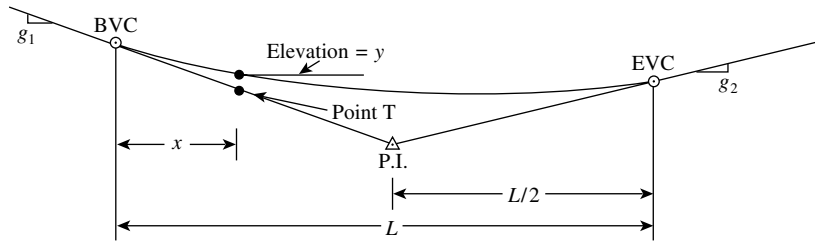


FIGURE 4.7
Symmetrical vertical curve.

4.2.2 Vertical Curves

Vertical tangents with different grades are joined by vertical curves such as the one shown in Figure 4.7.

Vertical curves are normally parabolas centered about the point of intersection (P.I.) of the vertical tangents they join. Vertical curves are thus of the form

$$y = y_0 + g_1x + \frac{rx^2}{2} \quad (4.1)$$

where y = elevation of a point on the curve

y_0 = elevation of the beginning of the vertical curve (BVC)

g_1 = grade just prior to the curve

x = horizontal distance from the BVC to the point on the curve

r = rate of change of grade

The rate of change of grade, in turn, is given by

$$r = \frac{g_2 - g_1}{L} \quad (4.2)$$

where g_2 is the grade just beyond the end of the vertical curve (EVC) and L is the length of the curve. Also, vertical curves are sometimes described by K , the reciprocal of r . K is the distance in meters required to achieve a 1 percent change in grade. Vertical curves are classified as *sags* where $g_2 > g_1$ and *crests* otherwise. Note that r (and hence the term $rx^2/2$) will be positive for sags and negative for crests.

Also note that vertical distances in the vertical curve formulas are the product of grade times a horizontal distance. In consistent units, if vertical distances are to be in meters, horizontal distances should also be in meters, and grades should be dimensionless ratios. In many cases, however, it is more convenient to represent grades in percent and horizontal distance in stations. This produces the correct result, because the grade is multiplied by 100 and the horizontal distance divided by 100, and the two factors of 100 cancel. It is very important not to mix the two methods, however. If grades are in percent, horizontal distances *must* be in stations; likewise, if grades are dimensionless ratios, horizontal distances *must* be in meters.

The parabola is selected as the vertical curve so that the rate of change of grade, which is the second derivative of the curve, will be constant with distance. Note that the first derivative is the grade itself, and since the rate of change of grade is constant, the grade of any point in the vertical curve is a linear function of the distance from the BVC to the point. That is,

$$g = \frac{dy}{dx} = g_1 + rx \quad (4.3)$$

The quantity $rx^2/2$ is the distance from the tangent to the curve and is known as the *offset*. If x is always measured from the BVC, the offset given by $rx^2/2$ will be measured from the g_1 tangent. To determine offsets from the g_2 tangent, x should be measured backward from the EVC. Since the curve is symmetrical about its center, the offsets from the g_1 and g_2 tangents, respectively, are also symmetrical about the center of the curve, which occurs at the station of its P.I.

Other properties of the vertical curve may be used to sketch it. For instance, at its center, the curve passes halfway between the P.I. and a chord joining the BVC and EVC. At the quarter points, it passes one quarter of the way between the tangents and the chord. Normal drafting practice is to show the P.I. by means of a triangular symbol, as in Figure 4.7, although the extended vertical tangents shown in the figure are often omitted. The BVC and EVC are shown by means of circular symbols. The P.I., BVC, and EVC are identified by notes. The stations of the BVC and EVC are given in notes, as are the station and elevation of the P.I., the two tangent grades, and the length of the vertical curve.

Elevations on vertical curves are easily calculated by means of a calculator, computer, or spreadsheet program. One traditional way of representing them is in the form of a table, such as Table 4.2. The table represents a 300 m sag vertical curve between

TABLE 4.2
Tabular form for profile calculations in vertical curve

Station	Grade	Tangent elevation	Offset	Profile elevation
99 + 75	+1%	149.75		149.75
100 + 00	BVC	150.00		150.00
100 + 25		150.25	+0.05	150.30
100 + 50		150.50	+0.21	150.71
100 + 75		150.75	+0.47	151.22
101 + 00		151.00	+0.83	151.83
101 + 25		151.25	+1.30	152.55
101 + 50	P.I.	151.50	+1.88	153.38
101 + 75		153.00	+1.30	154.30
102 + 00		154.50	+0.83	155.33
102 + 25		156.00	+0.47	156.47
102 + 50		157.50	+0.21	157.71
102 + 75		159.00	+0.05	159.00
103 + 00	EVC	160.50		160.50
103 + 25	+6%	162.00		162.00

a +1.0% grade and a +6.0% grade. The first column gives the station. The second column gives the intersecting grades and the locations of the BVC, P.I., and EVC. The third column gives the elevation of each point on the tangent grades, calculated as BVC elevation plus g_1x for the first tangent grade and P.I. elevation plus $g_2(x - L/2)$ for the second. The fourth column gives the offset, calculated as $rx^2/2$, with x measured from either the BVC or EVC as appropriate: since offsets are symmetrical about the P.I., however, they need be calculated only from the BVC to the P.I. The last column gives the curve elevation, which is the tangent elevation plus the offset. It should be noted that curve elevations can also be calculated by using only offsets from the g_1 tangent, and that in many cases it may be more convenient to use only one tangent.

EXAMPLE PROBLEM 4.1 A -2.5% grade is connected to a +1.0% grade by means of a 180 m vertical curve. The P.I. station is 100 + 00 and the P.I. elevation is 100.0 m above sea level. What are the station and elevation of the lowest point on the vertical curve?

Rate of change of grade:

$$r = \frac{g_2 - g_1}{L} = \frac{1.0\% - (-2.5\%)}{1.8 \text{ sta}} = 1.944\%/sta$$

Station of the low point:

At low point, $g = 0$

$$g = g_1 + rx = 0$$

or

$$x = \frac{-g_1}{r} = -\left(\frac{-2.5}{1.944}\right) = 1.29 = 1 + 29 \text{ sta}$$

$$\text{Station of BVC} = (100 + 00) - (0 + 90) = 99 + 10$$

$$\text{Station of low point} = (99 + 10) + (1 + 29) = 100 + 39$$

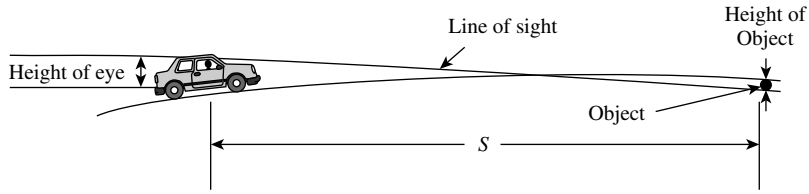
Elevation of BVC:

$$y_0 = 100.0 \text{ m} + (-0.9 \text{ sta})(-2.5\%) = 102.25 \text{ m}$$

Elevation of low point:

$$\begin{aligned} y &= y_0 + g_1x + \frac{rx^2}{2} \\ &= 102.25 \text{ m} + (-2.5\%)(1.29 \text{ sta}) + \frac{(1.944\%/sta)(1.29 \text{ sta})^2}{2} \\ &= 100.64 \text{ m} \end{aligned}$$

Design standards for vertical curves establish their minimum lengths for specific circumstances. For highways, minimum length of vertical curve may be based on sight distance, on comfort standards involving vertical acceleration, or appearance criteria. For passenger railways (especially urban rail transit systems), minimum vertical curve lengths will often be based on vertical acceleration standards; for freight railways, much more stringent standards may be maintained to avoid undue stress on couplings

**FIGURE 4.8**

Stopping sight distance diagram for crest vertical curve.

in sag vertical curves. For airport runways and taxiways, minimum vertical curve lengths are based on sight distance.

In most cases, sight distance or appearance standards will govern for highways. The equations used to calculate minimum lengths of vertical curves based on sight distance depend on whether the sight distance is greater than or less than the vertical curve length. For crest vertical curves, the minimum length depends on the sight distance, the height of the driver's eye, and the height of the object to be seen over the crest of the curve, as illustrated in Figure 4.8. The minimum length is given by the formula

$$L_{\min} = \begin{cases} \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} & \text{when } S \leq L \\ 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A} & \text{when } S \geq L \end{cases} \quad (4.4)$$

where S = sight distance (from Table 3.3)

L = vertical curve length

A = absolute value of the algebraic difference in grades, in percent, $|g_1 - g_2|$

h_1 = height of eye

h_2 = height of object

For stopping sight distance, the height of object is normally taken to be 0.150 m. For passing sight distance, the height of object used by AASHTO is 1.300 m. Height of eye is assumed to be 1.070 m.

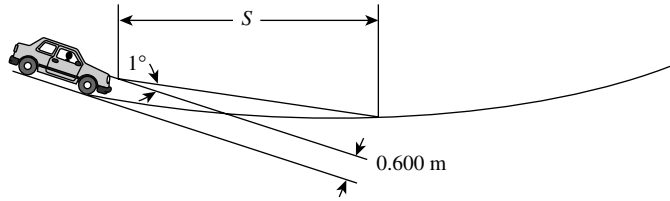
Inserting these standard values for h_1 and h_2 , Equation (4.4) may be reduced to

$$L_{\min} = \begin{cases} \frac{AS^2}{404} & \text{when } S \leq L \\ 2S - \frac{404}{A} & \text{when } S \geq L \end{cases} \quad (4.5)$$

for stopping sight distance and

$$L_{\min} = \begin{cases} \frac{AS^2}{946} & \text{when } S \leq L \\ 2S - \frac{946}{A} & \text{when } S \geq L \end{cases} \quad (4.6)$$

for passing sight distance.

**FIGURE 4.9**

Stopping sight distance diagram for sag vertical curve.

For sag vertical curves, stopping sight distance is based on the distance illuminated by the headlights at night. Design standards are based on an assumed headlight height of 0.600 m and an upward divergence of the headlight beam of 1° . This is illustrated by Figure 4.9. As in the case of crest vertical curves, the formulas for minimum length of vertical curve depend on whether the length of the curve is greater or less than the sight distance. For sag vertical curves, the formula is

$$L_{\min} = \begin{cases} \frac{AS^2}{200[0.6 + S(\tan 1^\circ)]} = \frac{AS^2}{120 + 3.5S} & \text{when } S \leq L \\ 2S - \frac{200[0.6 + S(\tan 1^\circ)]}{A} = 2S - \frac{120 + 3.5S}{A} & \text{when } S \geq L \end{cases} \quad (4.7)$$

Design charts or tables are used to determine minimum length of vertical curve to provide stopping sight distance for both crest and sag vertical curves, and passing sight distance on crests. These may be found in the *AASHTO Policy on Geometric Design of Highways and Streets*.

In some cases, sag vertical curves with a small total grade change can be sharp enough to cause discomfort without violating sight distance standards. In this case, it is necessary to establish a comfort criterion of the form

$$r \leq \frac{a}{v^2} \quad (4.8)$$

where r is the rate of change of grade, a is the maximum radial acceleration permitted, and v is speed. There is no general agreement as to the maximum value of radial acceleration that can be tolerated without producing discomfort. AASHTO suggests a value of 0.3 m/s^2 , and suggests the standard

$$L \geq \frac{AV^2}{395} \quad (4.9)$$

where L = length of vertical curve, m

$A = g_2 - g_1$, percent

V = design speed, km/h

Comfort standards for passenger railways are based on similar vertical accelerations.

For high-speed main tracks for freight railways, the specifications of the American Railway Engineering Association (AREA) are much more restrictive. The rate of change of grade is limited to 0.10 percent per station for crest vertical curves and 0.05 percent per station for sags. If the train is traveling down a grade steep enough to overcome rolling resistance, the couplings between the cars will be compressed. If the center of gravity of the train passes the low point of the curve while it is still compressed, the slack is suddenly pulled out of the couplers, and the resulting jerk may break the train apart. The design standard is intended to ensure that as the train approaches the low point, cars will be on grades too flat to overcome rolling resistance, and the slack will be pulled out of the couplers gradually.

For airports, rates of change of grade are limited to 0.33 percent per station on runways at airports used for high-speed aircraft. In addition, the Federal Aviation Administration (FAA) requires that for airports with no control tower, or at which the control tower is not always in operation, a 1.5 m object be visible from any point on the runway with a 1.5 m eye height.

Minimum vertical curve standards for highways may also be based on appearance. This problem arises because short vertical curves tend to look like kinks when viewed from a distance. Appearance standards vary from agency to agency. Current California standards, for instance, require a minimum vertical curve length of 60 m where grade breaks are less than 2 percent or design speeds are less than 60 km/h. Where the grade break is greater than 2 percent and the design speed is greater than 60 km/h, the minimum vertical curve is given by $L = 2V$, where L is the vertical curve length in meters and V is the design speed in km/h.

EXAMPLE PROBLEM 4.2 Determine the minimum length of a crest vertical curve between a +0.5% grade and a -1.0% grade for a road with a 100-km/h design speed. The vertical curve must provide 190-m stopping sight distance and meet the California appearance criteria. Round up to the next greatest 20 m interval.

Stopping sight distance criterion:

Assume $S \leq L$

$$L = \frac{AS^2}{200(\sqrt{h_1} + \sqrt{h_2})^2} = \frac{[0.5 - (-1.0)](190^2)}{200(\sqrt{1.070} + \sqrt{0.150})^2} = 134.0 \text{ m}$$

134.0 m < 190 m, so $S > L$

$$\begin{aligned} L &= 2S - \frac{200(\sqrt{h_1} + \sqrt{h_2})^2}{A} = 2(190) - \frac{200(\sqrt{1.070} + \sqrt{0.150})^2}{[0.5 - (-1.0)]} \\ &= 380.0 - 269.5 = 110.5 \text{ m} \end{aligned}$$

Appearance criterion:

Design speed = 100 km/h > 60 km/h but grade break = 1.5% < 2%. Use 60 m.

Conclusion:

Sight distance criterion governs. Use 120 m vertical curve.

EXAMPLE PROBLEM 4.3 Determine the minimum length of a sag vertical curve between a -0.7% grade and a $+0.5\%$ grade for a road with a 110 km/h design speed. The vertical curve must provide 220 m stopping sight distance and meet the California appearance criteria and the AASHTO comfort standard. Round up to the next greatest 20 m interval.

Stopping sight distance criterion:

Assume $S \leq L$

$$L = \frac{AS^2}{120 + 3.5S} = \frac{[0.5 - (-0.7)](220^2)}{120 + 3.5(220)} = 65.3 \text{ m}$$

$65.3 \text{ m} < 220 \text{ m}$, so $S > L$

$$\begin{aligned} L &= 2S - \frac{120 + 3.5S}{A} = 2(220) - \frac{120 + 3.5(220)}{[0.5 - (-0.7)]} \\ &= 440 - 741.7 = -301.7 \text{ m} \end{aligned}$$

Since $L < 0$, no vertical curve is needed to provide stopping sight distance.

Comfort criterion:

$$L = \frac{AV^2}{395} = \frac{[0.5 - (-0.7)](110^2)}{395} = 36.8 \text{ m}$$

Appearance criterion:

Design speed = 110 km/h $>$ 60 km/h but grade break = $1.2\% < 2\%$. Use 60 m.

Conclusion:

Appearance criterion governs. Use 60 m vertical curve.

Finally, vertical curve lengths may be limited by the need to provide clearances over or under objects such as overpasses or drainage structures. In the case of sag vertical curves passing over objects or crest vertical curves passing under them, the required clearances establish minimum lengths; in the case of crest vertical curves passing over objects or sags passing under them, the clearances establish maximum lengths. Where clearances limit vertical curve lengths, adequate sight distance should still be provided.

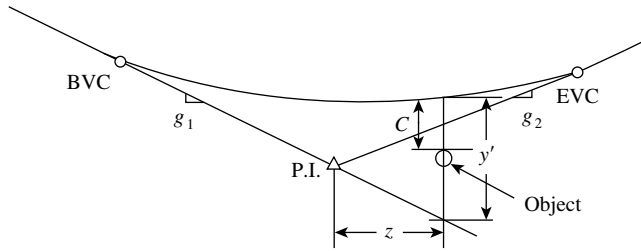
In either case, the maximum or minimum length of the vertical curve may be determined by assuming that the clearance is barely met and calculating the length of the vertical curve passing through the critical point thus established. It is easiest to do this as illustrated by Figure 4.10. In the figure, C represents the critical clearance, z the horizontal distance from the P.I. to the critical point, and y' the offset between the critical point and the tangent passing through the BVC.

The equation for the offset is

$$y' = \frac{rx^2}{2} \quad (4.10)$$

where r , as before, is

$$r = \frac{g_2 - g_1}{L} = \frac{A}{L} \quad (4.11)$$

**FIGURE 4.10**

Calculation of lengths of vertical curves constrained by clearances.

and

$$x = \frac{L}{2} + z \quad (4.12)$$

Substituting Equations (4.9) and (4.10) into Equation (4.8) results in

$$y' = \frac{A(L/2 + z)^2}{2L} \quad (4.13)$$

Expansion and rearrangement of Equation (4.11) leads to the quadratic equation

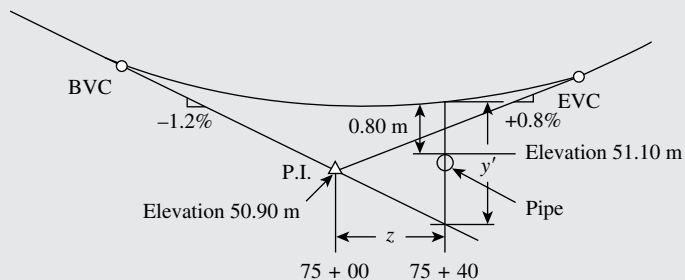
$$AL^2 + (4Az - 8y')L + 4Az^2 = 0 \quad (4.14)$$

Solving Equation (4.14) results in two roots. The smaller of these represents a vertical curve that is tangent between the P.I. and the critical point. Discarding this solution and letting $w = y'/A$ to simplify the notation, the solution for the in the larger root leads to

$$L = 4w - 2z + 4\sqrt{w^2 - wz} \quad (4.15)$$

as an expression for the maximum or minimum vertical curve length.

EXAMPLE PROBLEM 4.4 A vertical curve joins a -1.2% grade to a $+0.8\%$ grade. The P.I. of the vertical curve is at station $75 + 00$ and elevation 50.90 m above sea level. The centerline of the roadway must clear a pipe located at station $75 + 40$ by 0.80 m. The elevation of the top of the pipe is 51.10 m above sea level. What is the minimum length of the vertical curve that can be used?



Determine z :

$$z = (75 + 40) - (75 + 00) = 0.40 \text{ sta.}$$

Determine y' :

$$\text{Elevation of tangent} = 50.90 + (-1.2)(0.4) = 50.42 \text{ m}$$

$$\text{Elevation of roadway} = 51.10 + 0.80 = 51.90 \text{ m}$$

$$y' = 51.90 - 50.42 = 1.48 \text{ m}$$

Determine w :

$$A = g_2 - g_1 = (+0.8) - (-1.2) = 2.0$$

$$w = \frac{y'}{A} = \frac{1.48}{2} = 0.74$$

Determine L :

$$\begin{aligned} L &= 4w - 2z + 4\sqrt{w^2 - wz} \\ &= 4(0.74) - 2(0.4) + 4\sqrt{0.74^2 - (0.74)(0.4)} = 4.17 \text{ sta} = 417 \text{ m} \end{aligned}$$

Check y' :

$$x = \frac{4.17}{2} + 0.4 = 2.485 \text{ sta}$$

$$r = \frac{A}{L} = \frac{2}{4.17} = 0.48$$

$$y' = \frac{rx^2}{2} = \frac{(0.48)(2.485^2)}{2} = 1.48 \quad \text{Check}$$

4.3 HORIZONTAL ALIGNMENT

Horizontal alignment for linear transportation facilities such as highways and railways consists of horizontal tangents, circular curves, and possibly transition curves. In the case of highways, transition curves are not always used. Figure 4.11 illustrates horizontal alignments with and without transition curves.

4.3.1 Horizontal Tangents

Horizontal tangents are described in terms of their lengths (as expressed in the stationing of the job) and their directions. Directions may be either expressed as bearings or as azimuths and are always defined in the direction of increasing station. Azimuths are expressed as angles turned clockwise from due north; bearings are expressed as angles turned either clockwise or counterclockwise from either north or south. For instance, the azimuth 280° is equivalent to the bearing north 80° west (or $N80^\circ W$). Figure 4.12 illustrates azimuths and bearings.

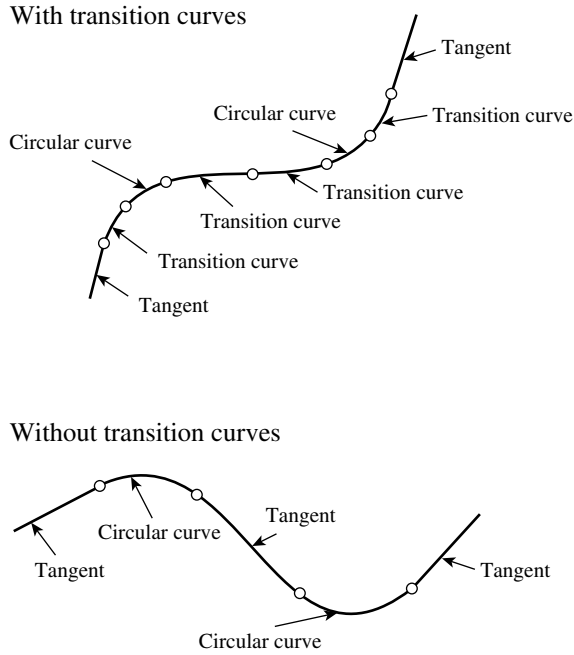


FIGURE 4.11
Horizontal alignments with and without transition curves.

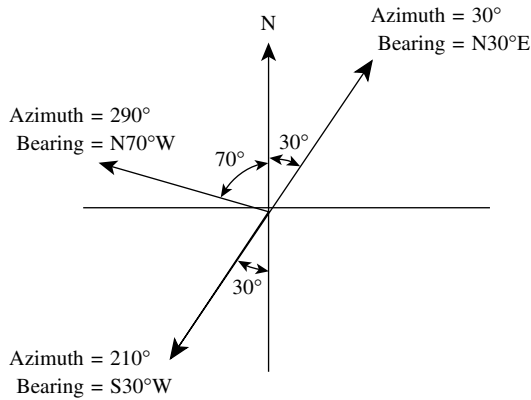


FIGURE 4.12
Azimuths and bearings.

4.3.2 Circular Curves

Horizontal curves are normally circular. Figure 4.13 illustrates several of their important features. Horizontal curves are described by *radius* (R), *central angle* (Δ) (which is equal to the deflection angle between the tangents), *length* (L), *semitangent distance* (T),

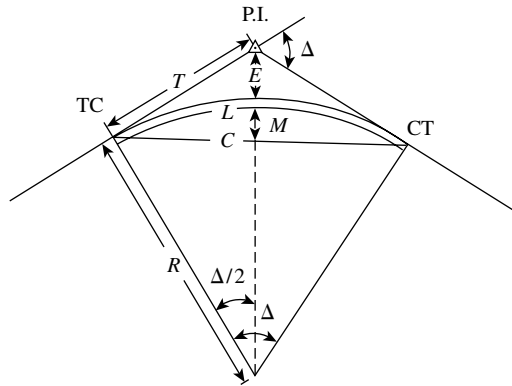


FIGURE 4.13
Horizontal curve features.

middle ordinate (M), *external distance (E)*, and *chord (C)*. The curve begins at the tangent-to-curve point (TC) and ends at the curve-to-tangent point (CT).

In the past, severity of curvature was sometimes expressed in degree of curvature. Although obsolete in the metric system, degree of curvature may still be encountered in some situations. Degree of curvature may be defined in two ways. The *arc definition* is the angle subtended by a 100 ft arc. The *chord definition* is the angle subtended by a 100 ft chord. The relationship between radius (in feet) and degree of curvature (arc definition) is

$$D = \frac{36,000}{2\pi R} = \frac{5729.58}{R} \quad (4.16)$$

where D = degree of curvature and R = radius of curvature, in feet. The length of a circular curve is given by

$$L = \frac{2\pi R \Delta}{360^\circ} = R \Delta_{\text{rad}} \quad (4.17)$$

where Δ is the central angle of the curve; Δ_{rad} refers to Δ measured in radians. The semitangent T of a circular curve is given by

$$T = R \tan \frac{\Delta}{2} \quad (4.18)$$

The middle ordinate M is given by

$$M = R - R \cos\left(\frac{\Delta}{2}\right) \quad (4.19)$$

The external distance E is given by

$$E = \frac{R}{\cos(\Delta/2)} - R \quad (4.20)$$

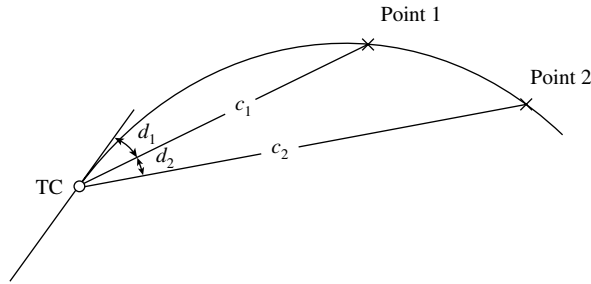


FIGURE 4.14
Layout of circular curve.

The chord C is given by

$$C = 2R \sin\left(\frac{\Delta}{2}\right) \quad (4.21)$$

Circular curves are usually laid out in the field by occupying the tangent-to-curve point TC with a transit and then establishing successive points by turning deflection angles and measuring chords, as shown in Figure 4.14.⁵ The deflection angle in radians d_x to a point on the curve at a distance x from the TC is given by

$$d_x = \left(\frac{x}{2R}\right)_{\text{rad}} \quad (4.22)$$

The chord c_x to this point is given by

$$c_x = 2R \sin d_x \quad (4.23)$$

Table 4.3 gives deflection angles and chords at 20 m intervals for a 500 m radius curve with a deflection angle of 15° and a TC at station 17 + 25.

Design standards for horizontal curves establish their minimum radii and, in some cases, their minimum lengths. Minimum radius of horizontal curve is most commonly

TABLE 4.3
Circular curve layout table
Radius = 500 m

	Station	x , m	Deflection angle		Chord, m
			Radians	Degrees	
TC	17 + 25	0.0	0.0000	$0^\circ 00' 00''$	0.000
	17 + 40	15.0	0.0150	$0^\circ 51' 34''$	14.999
	17 + 60	35.0	0.0350	$2^\circ 00' 19''$	34.993
	17 + 80	55.0	0.0550	$3^\circ 09' 05''$	54.972
	18 + 00	75.0	0.0750	$4^\circ 17' 50''$	74.930
	18 + 20	95.0	0.0950	$5^\circ 26' 35''$	94.857
	18 + 40	115.0	0.1150	$7^\circ 09' 43''$	114.474
CT	18 + 55.9	130.9	0.1309	$7^\circ 30' 00''$	130.526

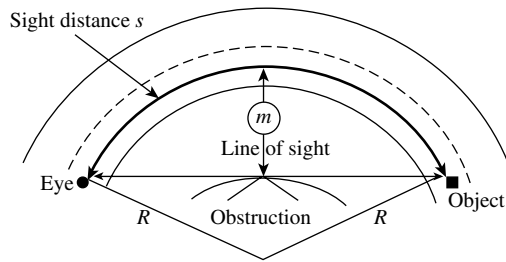


FIGURE 4.15
Stopping sight distance on horizontal curves.

established by the relationship between design speed, maximum rate of superelevation, and curve radius, which is discussed in Section 4.4. In other cases, minimum radii or curve lengths for highways may be established by the need to provide stopping sight distance or by appearance standards.

Figure 4.15 illustrates the relationship between curve radius, stopping sight distance, and the setback distance to obstructions to vision. The relationship between the radius of curvature R , the setback distance m , and the sight distance s is given by

$$m = R \left[1 - \cos \left(\frac{28.65s}{R} \right) \right] \quad (4.24)$$

and

$$s = \frac{R}{28.65} \left[\cos^{-1} \left(\frac{R - m}{R} \right) \right] \quad (4.25)$$

where the angles in the formulas are measured in degrees. Since these formulas are hard to solve for R , design charts or tables are normally used to find the minimum radius of curvature that will provide stopping sight distance.

Minimum lengths or radii of horizontal curves may also be based on appearance criteria. Where deflection angles are small, a short horizontal curve may give the appearance of a kink. To prevent this, minimum horizontal curve lengths may be prescribed for curves with small deflection angles.

4.3.3 Transition Curves

Transition curves are used to connect tangents to circular curves. Several forms of curve have been used for this purpose. The most logical choice from a theoretical standpoint, and the only one discussed here, is the clothoid spiral, for which the radius of curvature varies as the inverse of the distance along the curve from its beginning.

Spirals are used both for esthetic reasons and because they provide a “rational” superelevation transition. In the case of railways, such rational superelevation transitions are virtually necessary for reasons of vehicle dynamics. The use of spirals in conjunction with superelevation transitions is discussed in Section 4.4.

In the case of highways, spirals are used primarily for esthetic purposes. They are most appropriate for roadways with relatively high design standards, where large-radius curves are used. Under these circumstances, drivers can often see a considerable way ahead on the roadway, and can detect the difference between the more smooth, flowing lines provided by the transition curves and the more abrupt alignment that results in their absence. For roadways with lower design standards, some recent research reports indicate that use of spirals may increase accident rates. It is surmised that the reason is that drivers have a harder time judging the severity of curves where spirals are used.⁶

Figure 4.16 shows how spirals connect circular curves to tangents, and illustrates the nomenclature of spirals. Critical points in moving through the curve are the tangent to spiral point (TS), the spiral to curve point (SC), the curve to spiral point (CS), and the spiral to tangent point (ST). The effect of using the spirals is to shift the circular portion of the curve inwards, so that it no longer fits to the original tangents. It now fits to what are called *offset tangents*, which are shifted in by a distance p , measured perpendicular from the original tangents. The distance from the TS to any point on the curve is L ; if L is measured for the entire distance from the TS to the SC, it is referred to as L_s . Likewise, the angle between the tangent and a line tangent to the spiral at any distance L is referred to as the spiral angle θ ; if the spiral angle is measured at the SC, it is referred to as θ_s . The distance along the extended tangent from the TS to a point opposite that at which the circular curve is tangent to the offset tangents is referred to as k , and the distance from this point to the P.I. as T' . The length of the circular portion of the curve is L_c , and the coordinates of any point on the spiral, measured relative to the tangent, are X and Y .

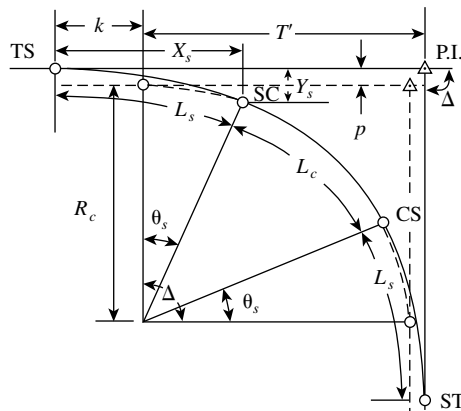


FIGURE 4.16

Spiral transition curve nomenclature.

Since the spiral is defined as the curve such that the reciprocal of the radius varies linearly from zero at the TS to $1/R_c$ at the SC,

$$\frac{1}{R} = \left(\frac{1}{R_c}\right)\left(\frac{L}{L_s}\right) \quad (4.26)$$

or

$$RL = R_c L_s = A^2 \quad (4.27)$$

The spiral angle θ is given by

$$\theta = \frac{L}{2R} \quad (4.28)$$

In particular,

$$\theta_s = \frac{L_s}{2R_c} \quad (4.29)$$

The coordinates X and Y of any point on the spiral may be expressed as functions of L :

$$X = L - \frac{L^5}{40A^4} + \frac{L^9}{3,456A^8} + \dots \quad (4.30)$$

and

$$Y = \frac{L^3}{6A^2} - \frac{L^7}{336A^6} + \frac{L^{11}}{42,240A^{10}} + \dots \quad (4.31)$$

Other measurements of interest are

$$p = Y_s - R_c (1 - \cos \theta_s) \quad (4.32)$$

$$k = X_s - R_c \sin \theta_s \quad (4.33)$$

$$T' = (R_c + p) \tan\left(\frac{\Delta}{2}\right) \quad (4.34)$$

and

$$L_c = R_c (\Delta_{\text{rad}} - 2\theta_s) = R_c \Delta_{\text{rad}} - L_s \quad (4.35)$$

Tables exist which allow X and Y to be calculated without directly evaluating Equations (4.28) and (4.29); however, since these equations are readily evaluated by computers or programmable calculators, such tables are of less importance than formerly, and will not be discussed further here.

Spirals are laid out in the field in a manner similar to that for circular curves. In this case the TS is occupied by the transit, and successive points along the spiral are established by turning deflection angles and measuring chords. For a point on a spiral

TABLE 4.4
Spiral curve layout table

	Station	L	X	Y	Spiral angle θ		Deflection angle d		c
					Radians	Degrees	Radians	Degrees	
TS	8 + 05	0	0.000	0.000	0.0000	0°00'00"	0.0000	0°00'00"	0.000
	8 + 20	15	15.000	0.014	0.0003	0°01'02"	0.0009	0°03'13"	15.000
	8 + 40	35	34.999	0.179	0.0153	0°52'36"	0.0051	0°17'35"	35.000
	8 + 60	55	54.992	0.693	0.0378	2°09'57"	0.0126	0°43'19"	54.997
	8 + 80	75	74.963	1.757	0.0703	4°01'40"	0.0234	1°20'34"	74.984
SC	8 + 85	80	79.949	2.132	0.0800	4°35'01"	0.0267	1°31'39"	79.977

whose coordinates have previously been calculated, the deflection angle d is

$$d = \tan^{-1}\left(\frac{Y}{X}\right) \quad (4.36)$$

and the chord c is

$$c = \sqrt{X^2 + Y^2} \quad (4.37)$$

Table 4.4 gives X , Y , θ , d , and c for 20 m intervals along an 80 m spiral that connects a tangent with a 500 m radius circular curve, with the TS at station 8 + 05.

In order to determine the stations of the critical points on the curves, it is necessary to remember that stationing runs along the curves and not the tangents. The station of the TS may be calculated measuring from the ST point of the last curve to the P.I. (which is actually not on the centerline of the facility), establishing a temporary station at the P.I., and subtracting $T' + k$ from this temporary station. The station of the SC may be found by adding the length of the spiral to the station of the TS; that of the CS is found by adding the length of the circular portion of the curve, as given by Equation (4.35), to that of the SC; and the station of the ST is found by adding the length of the spiral to that of the CS.

4.4 SUPERELEVATION

The purpose of superelevation or banking of curves is to counteract the centripetal acceleration produced as a vehicle rounds a curve. The term itself comes from railroad practice, where the top of the rail is the profile grade. In curves, the profile grade line follows the lower rail, and the upper rail is said to be "superelevated." Since most railways are built to a standard gage, the superelevations are given as the difference in elevation between the upper and lower rail. In the case of highways, somewhat more complicated modifications of the cross section are required, and, because widths vary, superelevation is expressed as a slope.

Consider the force diagram in Figure 4.17. If the vehicle is traveling around a curve with a radius R at a constant speed v , there will be a radial acceleration toward

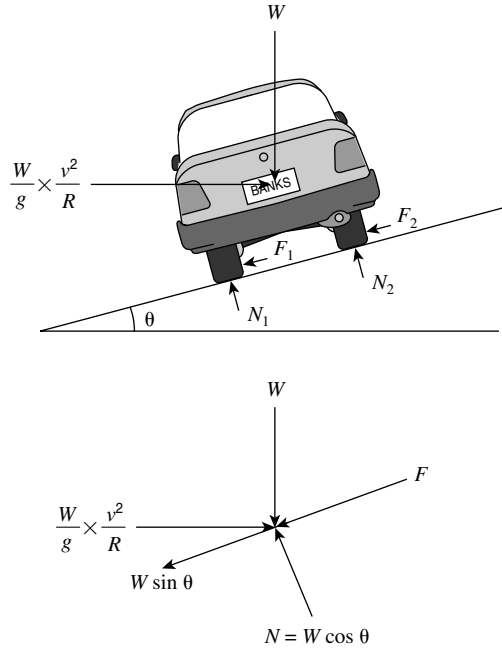


FIGURE 4.17
Force diagram for superelevation.

the center of the curve (toward the left in the diagram) of v^2/R , which will be opposed by a force of $(W/g)(v^2/R)$. Other forces acting on the vehicle are its weight W and the forces exerted against the wheels by the roadway surface. These forces are represented by two components: the normal forces N_1 and N_2 and the lateral forces F_1 and F_2 . For highway vehicles F_1 and F_2 are friction forces, so

$$F_1 \leq \mu N_1 \quad \text{and} \quad F_2 \leq \mu N_2 \quad (4.38)$$

where μ is the coefficient of friction between the tires and the roadway.

Summing forces parallel to the roadway gives

$$\cos \theta \left(\frac{W}{g} \right) \left(\frac{v^2}{R} \right) = F + W \sin \theta \quad (4.39)$$

Defining a factor f so that

$$f \equiv \frac{F}{N} \quad (4.40)$$

and noting that

$$N = W \cos \theta + \sin \theta \left(\frac{w}{g} \right) \left(\frac{v^2}{R} \right) \quad (4.41)$$

Equation (4.37) may be rewritten as

$$\cos \theta \left(\frac{W}{g} \right) \left(\frac{v^2}{R} \right) = W \sin \theta + fW \cos \theta + f \sin \theta \left(\frac{w}{g} \right) \left(\frac{v^2}{R} \right) \quad (4.42)$$

Dividing by $W \cos \theta$ leads to

$$\frac{v^2}{gR} = \tan \theta + f + f \tan \theta \left(\frac{v^2}{gR} \right) \quad (4.43)$$

But $\tan \theta$ is the cross-slope of the roadway, which is the same as the superelevation rate e . Consequently,

$$\frac{v^2}{gR} = e + f + ef \left(\frac{v^2}{gR} \right) \quad (4.44)$$

or

$$\frac{v^2}{gR} (1 + ef) = e + f \quad (4.45)$$

The term ef is small compared to one, and may be omitted, so the relationship can be simplified to

$$\frac{v^2}{gR} = f + e \quad (4.46)$$

Also,

$$R = \frac{v^2}{g(f + e)} \quad (4.47)$$

A commonly used mixed-unit version of Equation (4.45) is

$$R = \frac{V^2}{127(f + e)} \quad (4.47a)$$

where V is in km/h and R is in meters. Alternatively,

$$e = \frac{v^2}{gR} - f \quad \text{or} \quad e = \frac{V^2}{127R} - f \quad (4.48)$$

Values of f recommended by AASHTO are conservative relative to the actual friction factor between the tires and the roadway under most conditions, and vary with design speed. These are given in Table 4.5.

Maximum rates of superelevation are limited by the need to prevent slow-moving vehicles from sliding to the inside of the curve and, in urban areas, by the need to keep parking lanes relatively level and to keep the difference in slope between the roadway and any streets or driveways that intersect it within reasonable bounds. AASHTO recommends that maximum superelevation rates be limited to 12 percent for rural

TABLE 4.5
Values of side friction recommended by AASHTO

Design speed, km/h	Maximum side friction factor
30	0.17
40	0.17
50	0.16
60	0.15
70	0.14
80	0.14
90	0.13
100	0.12
110	0.11
120	0.09

Source: From *A Policy on Geometric Design of Highways and Streets*.
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 Transportation Officials, Washington, DC. Used by permission.

roadways; 8 percent for rural roadways for which snow or ice are likely to be present; and 6 percent or 4 percent for urban streets. In addition, there is a tradeoff between the maximum rate of superelevation and the minimum curve radius permitted at any design speed. AASHTO recommends the minimum curve radii shown in Table 4.6. For the higher design speeds, the superelevation rates for these minimum curve radii, as calculated by Equation (4.48), are less than the maximum superelevation rates given above. Consequently, the maximum superelevation rates really apply only to fairly low design speeds.

EXAMPLE PROBLEM 4.5 What is the minimum radius of curvature allowable for a roadway with a 100 km/h design speed, assuming that the maximum allowable superelevation rate is 0.12? Compare this with the minimum curve radius recommended by AASHTO. What is the actual maximum superelevation rate allowable under AASHTO recommended standards for a 100 km/h design speed, if the value of f is the maximum allowed by AASHTO for this speed? Round the answer down to the nearest whole percent.

Minimum radius of curvature for 100 km/h design speed:

$$R = \frac{V^2}{127(f + e)} = \frac{100^2}{127(0.12 + 0.12)} = 328 \text{ m}$$

Minimum radius recommended by AASHTO is 490 m. Actual maximum superelevation rate for AASHTO recommended standards for 100 km/h is

$$e = \frac{V^2}{127R} - f = \frac{100^2}{127(490)} - 0.12 = 0.041$$

Rounding,

$$e_{\max} = 0.04 = 4\%$$

TABLE 4.6
Recommended minimum radius of curvature

Design speed, km/h	Manimum curve raidus, m
30	35
40	60
50	100
60	150
70	215
80	280
90	375
100	490
110	635
120	870

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Superelevation transitions involve modification of the roadway cross section from normal crown to full superelevation, at which point the entire roadway width has a cross-slope of e . The manner in which this transition is accomplished is expressed by a superelevation diagram, which is a graph of superelevation (cross-slope) versus distance measured in stations. As an alternative, the diagram may show the difference in elevation between the profile grade and the edge versus distance.

Figure 4.18 is an example superelevation diagram, showing the transition from normal crown with 2 percent cross-slopes to 6 percent superelevation for a roadway with a spiral transition curve. Figure 4.19 is the alternative form of the diagram, assuming a two-lane highway with 3.6 m lanes. Figure 4.20 presents an interpretation of the superelevation diagram, showing the appearance of the cross section at intervals

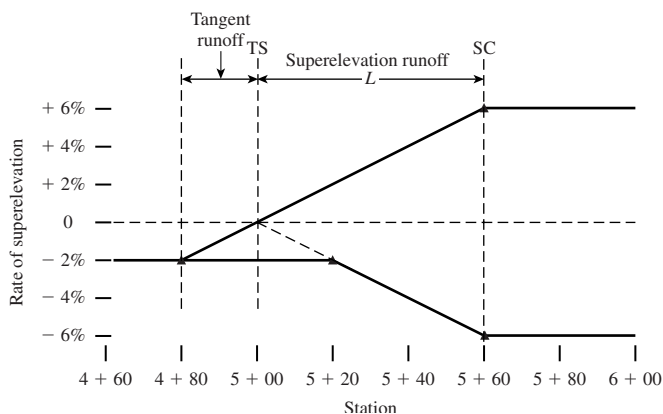


FIGURE 4.18

Superelevation diagram, showing rate of superelevation.

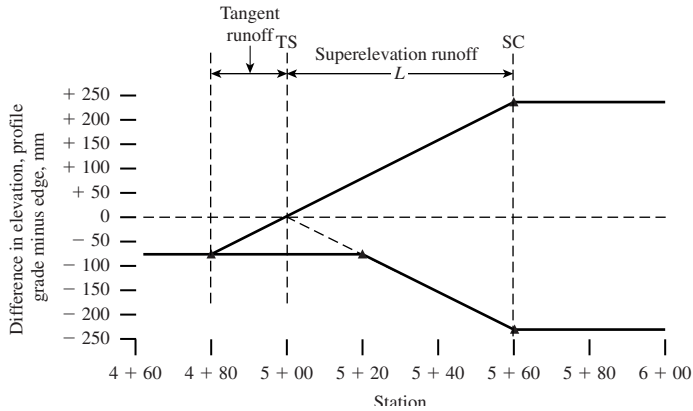


FIGURE 4.19
Superelevation diagram, showing difference in elevation.

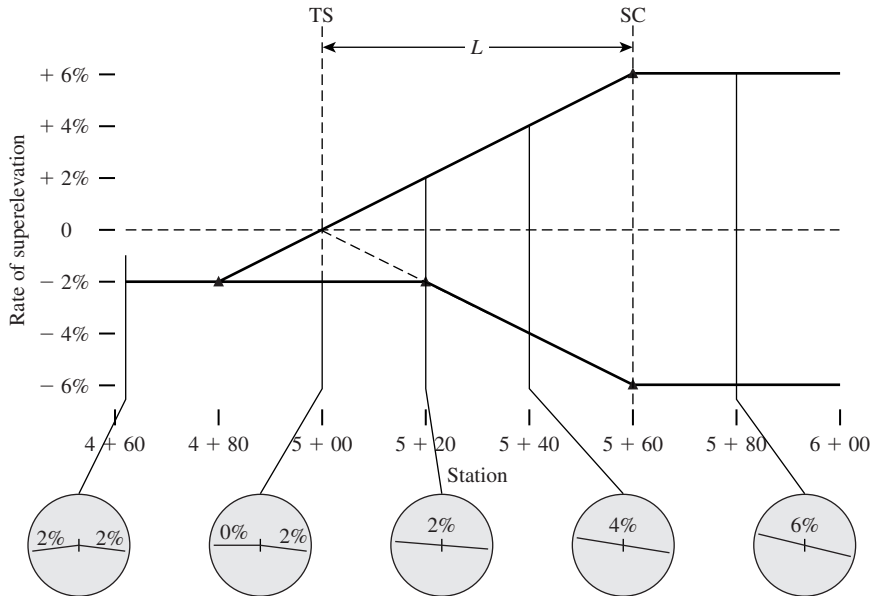


FIGURE 4.20
Superelevation diagram, showing roadway cross sections.

through the transition. As shown in Figure 4.18, the superelevation transition is normally linear; that is, the rate of rotation of the cross section is constant with respect to distance through the transition. The distance marked L , which runs from the point at which the outside half of the roadway (that is, the half on the outside of the curve) is at zero cross-slope to the P.I. at full superelevation (or from the tangent-to-spiral point TS

to the spiral-to-curve point SC), is called *superelevation runoff*. The distance from the point at which the outside half of the roadway first begins to rotate to the TS is referred to as *tangent runoff*.

As previously mentioned, spiral transition curves are often used in conjunction with superelevation transitions, and normally coincide with the superelevation runoff. The clothoid spiral provides for a rational superelevation transition because $1/R$ varies linearly with L . From Equation (4.44),

$$f + e \propto \frac{V^2}{R} \quad (4.49)$$

Consequently, for constant V , $f + e$ is proportional to $1/R$. If a linear superelevation transition is used, e is a linear function of L , where L is the distance from the beginning of the transition to any point. Meanwhile, $1/R$ is also proportional to L ; therefore, f must also be a linear function of L . Consequently, side friction increases linearly through a clothoid spiral superelevation transition if it is traversed at constant speed.

In the case of railways, use of transition curves in conjunction with superelevation transitions is virtually necessary. Railway superelevation is normally designed so that $f = 0$ if the curve is traversed at design speed. The reason for this is that nonzero side forces may lead to binding of the flanges of the wheels against the rails. More importantly, lateral forces tend over time to force the track out of its proper alignment and gage. Consequently, the usual practice is to design for zero side friction. Note that the clothoid spiral is the only superelevation transition curve that can meet this criterion fully. If f is set to zero, e is proportional to $1/R$; since $1/R$ varies linearly with L , so can e .

For highways, superelevation is usually applied by rotating the cross section about the profile grade line. This will be the centerline in the case of two-lane highways, undivided multilane highways, and multilane divided highways with paved medians. For multilane divided highways with wide medians, the inside edge of traveled way is often used as the axis of rotation. For freeway ramps, the axis of rotation will normally be the edge of traveled way closer to the freeway; however, it may be shifted to avoid sags in the edge grade of the ramp. For railways, the axis of rotation is the top of the lower rail.

The length of the superelevation runoff L is determined by either vehicle dynamics or appearance criteria. Where spiral transition curves are used, the superelevation transition will normally coincide with the spiral, as shown in Figure 4.18. In the case of railways, the minimum spiral length may be established by the need to limit the rate of increase of centripetal acceleration in the spiral, which leads to the following formula:

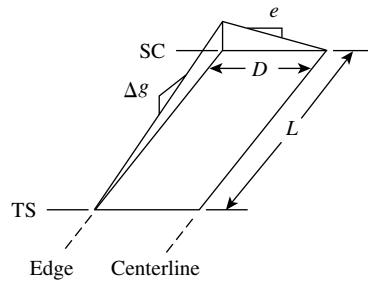
$$L = \frac{0.0702V^3}{RC} \quad (4.50)$$

where L = minimum length of spiral, m

V = speed, km/h

R = curve radius, m

C = rate of increase of centripetal acceleration, m/s^3

**FIGURE 4.21**

Difference in grade resulting from superelevation transition.

A value of C of 1 has usually been used in railroad practice. The formula has also sometimes been used for highways, with values of C ranging from 1 to 3.

More commonly, superelevation transition lengths for highways are based on appearance or comfort criteria. One such criterion is a rule that the difference in longitudinal slope (grade) between the centerline and edge of traveled way of a two-lane highway should not exceed $1/200$.

Figure 4.21 illustrates the application of this rule. L is measured from the TS to the SC, as in the superelevation diagram. At the TS the difference in elevation between the centerline and edge is zero. At the SC it is the superelevation rate e times the distance D from the centerline to the edge. Thus the difference in grade between the centerline and the edge is

$$\Delta g = \frac{De}{L} \quad (4.51)$$

Since the criterion that the difference in grade not exceed $1/200$ implies that

$$\frac{De}{L} \leq \frac{1}{200} \quad (4.52)$$

L is given by

$$L \geq 200De \quad (4.53)$$

L is normally rounded up to some convenient length, such as a multiple of 20 m.

EXAMPLE PROBLEM 4.6 A two-lane highway (3.6 m lanes) with a design speed of 100 km/h has a 400 m radius horizontal curve connecting tangents with bearings of $N75E^\circ$ and $S78E^\circ$. Determine the superelevation rate, the length of spiral if the difference in grade between the centerline and edge of traveled way is limited to $1/200$, and the stations of the TS, SC, CS, and ST, given that the temporary station of the P.I. is $150 + 00$. The length of the spiral should be rounded up to the next highest 20 m interval.

Determine superelevation rate:

$$e = \frac{V^2}{127R} - f = \frac{100^2}{127(400)} - 0.12 = 0.08$$

Determine length of superelevation transition and spiral:

$$L_s = 200De = 200(3.6)(0.08) = 57.6 \text{ m} \quad \text{Round to 60 m}$$

Determine spiral angle and coordinates of SC point:

$$\theta_s = \frac{L_s}{2R_c} = \frac{60}{2(400)} = (0.075) \text{ rad}$$

$$A = \sqrt{L_s R_c} = \sqrt{(60)(400)} = 154.9$$

$$\begin{aligned} X_s &= L_s - \frac{L_s^5}{40A^4} + \frac{L_s^9}{3,456A^8} = 60 - \frac{60^5}{40(154.9)^4} + \frac{60^9}{3,456(154.9)^8} \\ &= 60 - 0.034 + 0.000 = 59.966 \text{ m} \end{aligned}$$

$$\begin{aligned} Y_s &= \frac{L_s^3}{6A^2} - \frac{L_s^7}{336A^6} + \frac{L_s^{11}}{42,240A^{10}} \\ &= \frac{60^3}{6(154.9)^2} - \frac{60^7}{336(154.9)^6} + \frac{60^{11}}{42,240(154.9)^{10}} \\ &= 1.500 - 0.001 + 0.000 = 1.499 \text{ m} \end{aligned}$$

Determine p , k , Δ , T' , and L_c :

$$p = Y_s - R_c (1 - \cos \theta_s) = 1.499 - 400[1 - \cos(0.075)] = 0.375 \text{ m}$$

$$k = X_s - R_c \sin \theta_s = 59.996 - 400 \sin(0.075) = 30.024 \text{ m}$$

$$\Delta = (90^\circ - 75^\circ) + (90^\circ - 78^\circ) = 27^\circ = 0.471 \text{ rad}$$

$$T' = (R_c + p) \tan\left(\frac{\Delta}{2}\right) = (400 + 0.375) \tan\left(\frac{27^\circ}{2}\right) = 96.122 \text{ m}$$

$$L_c = R_c \Delta_{\text{rad}} - L_s = 400(0.471) - 60 = 128.4 \text{ m}$$

Determine stations of critical points:

$$\begin{aligned} \text{TS station} &= \text{P.I. station} - (T' + k) \\ &= (150 + 00) - [(0 + 96.1) + (0 + 30.0)] \\ &= 148 + 73.9 \end{aligned}$$

$$\begin{aligned} \text{SC station} &= \text{TS station} + L_s \\ &= (148 + 73.9) + (0 + 60) \\ &= 149 + 33.9 \end{aligned}$$

$$\begin{aligned}
 \text{CS station} &= \text{SC station} + L_c \\
 &= (149 + 33.9) + (1 + 28.4) \\
 &= 150 + 62.3 \\
 \text{ST station} &= \text{CS Station} + L_s \\
 &= (150 + 62.3) + (0 + 60) \\
 &= 151 + 22.3
 \end{aligned}$$

4.5 COORDINATION OF HORIZONTAL AND VERTICAL ALIGNMENT

Transportation facilities such as highways and railways are three-dimensional objects. Although many aspects of their design can be determined by considering horizontal and vertical alignment separately from one another, it is important to understand the relationship between them. Proper coordination of horizontal and vertical alignment is important for reasons related to the esthetics, economics, and safety of the facility.

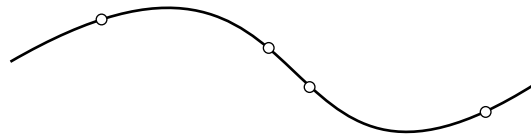
As a general rule, horizontal curvature and grades should be kept in balance. That is, the designer should avoid both the provision of minimal curvature at the expense of long, steep grades and the provision of level vertical alignment at the expense of excessive horizontal curvature.

Where there is both horizontal and vertical curvature, it is normally best from an esthetic standpoint to provide the impression of a single three-dimensional curve in both the horizontal and vertical planes. This means that horizontal and vertical curves should normally coincide. In some cases, however, safety considerations may suggest that horizontal curves be extended beyond vertical curves in order to avoid hiding the beginning of the horizontal curve from drivers approaching it. This is especially important where horizontal curves coincide with rather sharp crest vertical curves.

In addition, use of long, relatively gentle curves and short tangents will normally produce a more flowing line than will use of long tangents and short, sharp curves. This is especially true when the deflection angle of horizontal tangents or the difference between grades is small. This effect is illustrated by Figure 4.22.

At the same time, too much curvature may also pose problems. On two-lane highways, for instance, it is also important to provide an adequate number of sections with passing sight distance, and these sections need to be of adequate length to prevent drivers from becoming impatient when following slow-moving vehicles. Finally, on intersection approaches, highway alignments should provide adequate sight distance and should be as flat and straight as possible.

In addition to these rules, there are a number of guidelines related to the positioning of curves (either horizontal or vertical) relative to one another. These are based on esthetics and in the case of horizontal curves, on the necessity of providing adequate superelevation transitions. Figure 4.23 illustrates several special curve combinations.



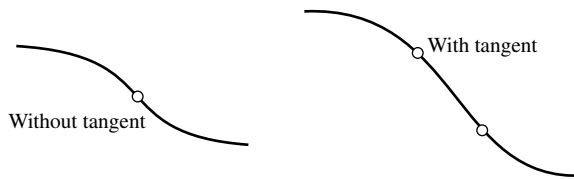
a. Long curves, short tangents



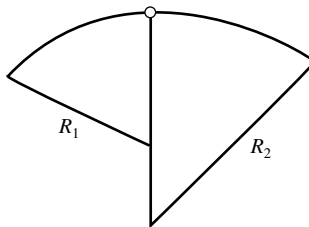
b. Short curves, long tangents

FIGURE 4.22

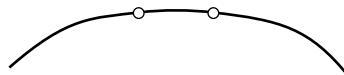
Long curves and short tangents provide a more fluid alignment.



a. Reversing curves



b. Compound curve



c. Broken-back curve

FIGURE 4.23

Special curves.

Reversing horizontal curves must be separated by a tangent or by transition curves to allow for the development of superelevation. The superelevated cross-slopes of the roadway will be in opposite directions in the two curves, so that some distance must be provided for rotation of the cross section. Where transition curves are used, however,

the ST of the first curve may coincide with the TS of the second. *Reversing vertical curves* pose no problem. *Compound curves* result when two curves of differing radius (or for vertical curves, different rates of change of grade) join one another. Such curves are normally avoided for centerline alignment, although use of three-centered compound curves for pavement edges in intersections is common. Problems with compound curves include esthetics and, in the case of horizontal curves, difficulty in developing the necessary superelevation transition, as well as possible deception of the driver as to the severity of the curve. Compound curves may be acceptable if the difference in radius is small or if they occur on a one-way roadway and the radius of curvature increases in the direction of travel. *Broken-back curves* consist of two curves in the same direction separated by a short tangent. Such curves (whether horizontal or vertical) are objectionable on esthetic grounds and should be replaced by a single, larger-radius curve.

4.6 INTERSECTIONS AND INTERCHANGES

Geometric design of transportation facilities must provide for the resolution of traffic conflicts. In general, these may be classified as *merging*, *diverging*, *weaving*, and *crossing* conflicts, illustrated in Figure 4.24. *Merging conflicts* occur when vehicles enter a traffic stream; *diverging conflicts* occur when vehicles leave the traffic stream; *weaving conflicts* occur when vehicles cross paths by first merging and then diverging; and *crossing conflicts* occur when they cross paths directly.

There are three basic ways of resolving crossing conflicts. *Time-sharing* solutions involve assignment of the right-of-way to particular movements for particular times. An example of this type of solution is the signalized intersection. *Space-sharing* solutions convert crossing conflicts into weaving conflicts. An example of this is the traffic circle or rotary. *Grade separation* solutions eliminate the crossing conflict by placing the conflicting traffic streams at different elevations at their point of intersection. Examples of this solution are freeway interchanges and highway–railway grade separations.

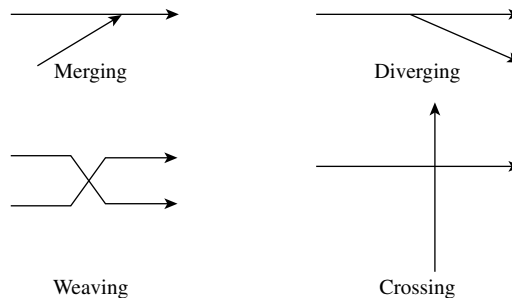


FIGURE 4.24
Types of traffic conflicts.

Freeway interchanges are classified primarily according to the way in which they handle left-turning traffic. *Diamond interchanges* employ *diamond ramps*, which connect to the cross road by means of an at-grade intersection. Left turns are accomplished by having vehicles turn left across traffic on the cross road. *Cloverleaf intersections* employ *loop ramps*, in which vehicles turn left by turning 270° to the right. *Partial cloverleaf* (or *parclo*) *interchanges* involve various combinations of diamond and loop ramps. *Direct interchanges* employ direct ramps, in which vehicles turn left by means of a left-turning ramp that is grade-separated as it crosses both traffic streams. *Frontage roads* are used to provide access to adjacent property, and *collector–distributor roads* are used to intercept traffic from local streets that do not cross the freeway. Figure 4.25 illustrates a variety of interchange configurations.

Interchange configurations are selected on the basis of structural costs, right-of-way costs, and ability to serve traffic. In general, diamond ramps have the lowest structural and land use costs, but also have the lowest capacities. Loop ramps provide higher capacities at moderate structural costs, but have high right-of-way costs because they take up more space. Direct ramps have the highest capacities, and often have moderate right-of-way costs, but often have very high structural costs due to the extensive structures required. Direct ramps are often used in freeway-to-freeway interchanges, where they are referred to as *freeway-to-freeway connectors* (or *branch connectors*) rather than ramps.

Freeway ramp junctions are normally treated as standard details, but may differ from agency to agency. In general, there are two types: tapered and parallel. Figures 4.26 and 4.27 illustrate these types of ramp junctions.

Highway intersections at grade also pose a number of special geometric problems. Pavement edges in intersections must be rounded to accommodate the wheel tracking paths of large vehicles. This is normally done by use of templates for particular design vehicles, such as that reproduced from the *California Highway Design Manual* as Figure 4.28. In addition, care must be taken to ensure that building setbacks and landscaping provide adequate sight distance for vehicles approaching the intersection, and pavement crowns must be flattened and warped so as to allow for drainage while at the same time providing as smooth a crossing as possible. There are no set rules for design of vertical alignment and cross section through intersections; rather each case must be analyzed individually in order to provide the best possible combination of smooth ride and drainage within whatever constraints may be present. As an example of the layout of a highway intersection, Figure 4.29 shows details for intersections between two-lane highways and minor roads, as given by the *California Highway Design Manual*.

4.7 DESIGN SOFTWARE

Most public agencies and consulting firms engaged in a large volume of design work now employ computer software to assist in the design of transportation facilities. Such design packages are used in conjunction with computer-aided drawing (CAD) software to produce facility plans. The most common arrangement is for the design and

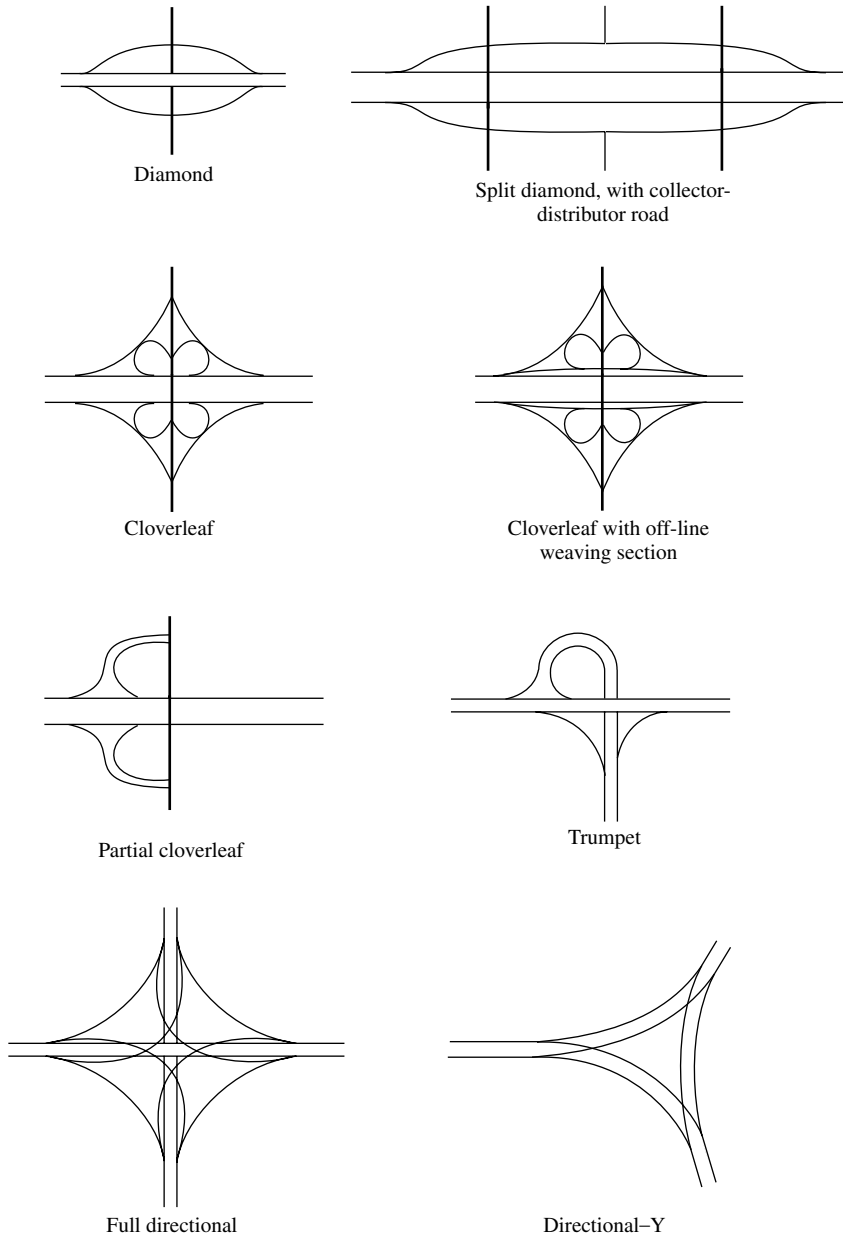
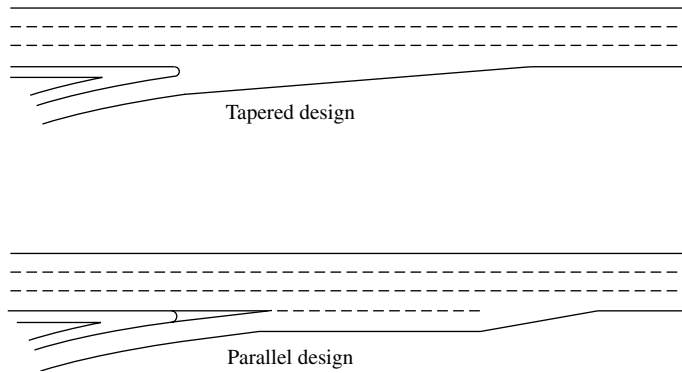
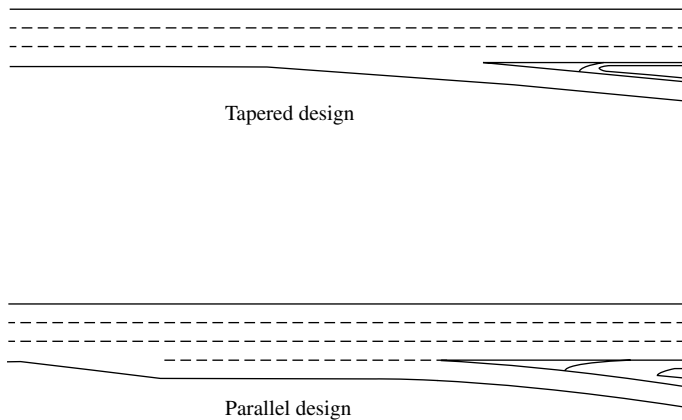


FIGURE 4.25
Interchange configurations.

**FIGURE 4.26**

Types of on-ramp junctions. (Source: Redrawn from *A Policy on Geometric Design of Highways and Streets*, Copyright 1994 by the American Association of State Highway and Transportation Officials, Washington, DC. Used by permission.)

**FIGURE 4.27**

Types of off-ramp junctions. (Source: Redrawn from *A Policy on Geometric Design of Highways and Streets*, Copyright 1994 by the American Association of State Highway and Transportation Officials, Washington, DC. Used by permission.)

drawing programs to run simultaneously, with the design program calling routines from the drawing program as needed to construct the drawings. It is also possible to have the design and drawing programs communicate by exchanging graphics files.

The process of designing a transportation facility with a design software package usually begins with a terrain file. If a digital terrain model is available, this may be used to construct a contour map to be used as a base map. Other features, such as existing structures, may be added to the base map by means of the drawing program.

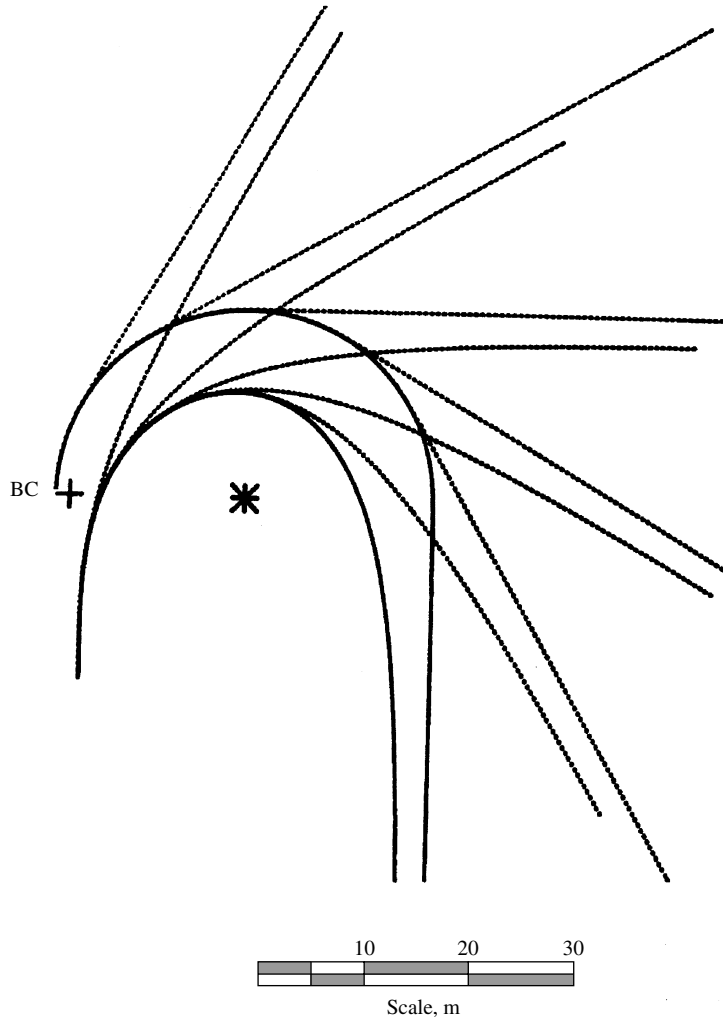
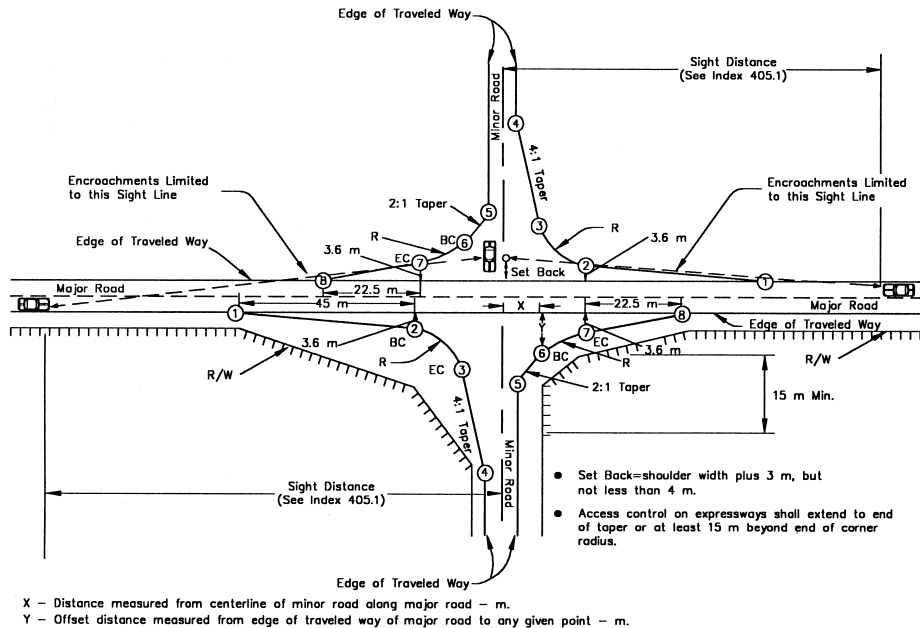


FIGURE 4.28

Turning radius template. California semitrailer wheel tracks, 18 m radius.
 (*California Highway Design Manual, 5th ed., 1995.*)

Once the base map is complete, the user begins to design the facility by defining horizontal and vertical *alignments*. As discussed in the introduction to this chapter, the horizontal alignment will be defined first; based on this, the design program will display existing elevations along the centerline or other primary reference line. A single project may have multiple alignments; for instance, divided roadways with wide medians can be designed with separate horizontal and vertical alignments, and separate ditch profiles may be specified. In constructing alignments, critical points (such as P.I.s) may be established either by specifying coordinates or (in the case of vertical alignment) stations and elevations, by projecting known directions or grades and distances from existing points, or visually, by dragging the point with a mouse or other



Radius of Curve	Design Vehicle	Pt ①		Pt ②		Pt ③		Pt ④		Pt ⑤		Pt ⑥		Pt ⑦		Pt ⑧	
		X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y	X	Y
9 m	Bus	61.26	0.0	16.26	3.6	8.25	10.39	3.6	28.97	3.6	12.20	5.47	8.46	12.10	3.6	34.60	0.0
12 m	California	64.52	0.0	19.52	3.6	8.84	12.65	3.6	33.61	3.6	16.01	6.56	10.08	15.40	3.6	37.90	0.0
15 m	STAA	67.85	0.0	22.83	3.6	9.47	14.91	3.6	38.39	3.6	22.69	9.09	11.70	20.14	3.6	42.64	0.0

FIGURE 4.29

California standard intersection, minor road with state highway. (*California Highway Design Manual, 5th ed., 1995.*)

digitizing device. The design software also will usually provide for automatic stationing of alignments and fitting of standard features such as circular horizontal curves, spirals, and parabolic vertical curves.

The user also defines *templates*, describing the facility cross section. These define the geometric cross section, ditch shapes, and earthwork cross-slopes (see Chapter 5). Multiple templates may be specified. If so, the design program has the capability of calculating transitions between successive templates. Where there are superelevated curves, the user enters the location and other details of superelevation transitions, and the program modifies the cross-section templates to account for the superelevation.

Once alignments and templates are defined, the program constructs a single three-dimensional mathematical representation of the facility from them. This can be used to produce three-dimensional views. These may be rotated and zoomed to produce three-dimensional drawings of the facility from any point of view.

Also, once alignments and templates are established, the program calculates earthwork cross sections by means of a digital terrain model. Where digital terrain models are not available, existing elevations for cross sections may be read from files. Once earthwork cross sections are established, the design program can calculate earthwork volumes (see Chapter 5).

Final versions of plans may be prepared by using the drawing program to add any features or notes not supplied by the design program.

Advantages of using software packages to design transportation facilities include:

1. Design software provides for rapid redesign of facilities; in particular, earthwork quantities for new alignments may be calculated very rapidly. This is a major advantage, since calculation of earthwork quantities by hand is a very laborious process (see Chapter 5).
2. Design software provides the ability to easily visualize facilities in three dimensions. Because three-dimensional representations may be constructed from any viewpoint and on any orientation, it is easy to evaluate the appearance of the facility.
3. With the design and drawing software, drawings may be rescaled rapidly, and multiple versions of drawings emphasizing different features may be readily produced. Drawing programs allow graphical objects to be grouped in different *layers*. These layers are usually represented on the screen by different colors. They may be faded or eliminated altogether (either on the screen or in prints or plots) to emphasize particular features.

4.8 SUMMARY

Geometric design of transportation facilities includes specification of cross sections, vertical alignments, horizontal alignments, and various design details. Standard cross sections for tangent horizontal alignment are specified by most design organizations. Vertical alignment consists of vertical tangents and parabolic vertical curves. Maximum grades for vertical tangents are determined by the effects of vehicle power/weight ratios on speeds on various grades. Both the length and steepness of the grade are important. Minimum grades for highways are determined by drainage requirements. Minimum lengths of vertical curves are determined by sight distance, comfort, or appearance criteria. Horizontal alignment consists of horizontal tangents; circular horizontal curves; and, in some cases, spiral transition curves. Superelevation or banking of curves is used to counteract the centrifugal forces developed by vehicles going around curves. Superelevation rates are limited by the need to prevent slow-moving vehicles from sliding to the inside of the curve or by roadside culture. Minimum horizontal curve radii are limited by maximum superelevation rates and by the relationship between design speed, superelevation rate, and curve radius; by sight distance; or by appearance criteria. Spiral transition curves, when used, coincide with the superelevation transition. In the case of railways they are required for reasons of vehicle dynamics. In the case of highways, they are used mainly for aesthetic reasons. Coordination of horizontal and vertical alignment is important for reasons of esthetics, economics, and safety. Several rules of thumb for the coordination of horizontal and vertical alignment have been provided. Intersections and interchanges provide for the resolution of traffic conflicts. Various interchange configurations have been presented,

along with standard details of ramp junctions and intersection layouts. Computer-aided design and drawing software packages are commonly used for the design of transportation facilities. These provide for rapid redesign, three-dimensional visualization of the facility, and flexible production of drawings representing various design features.

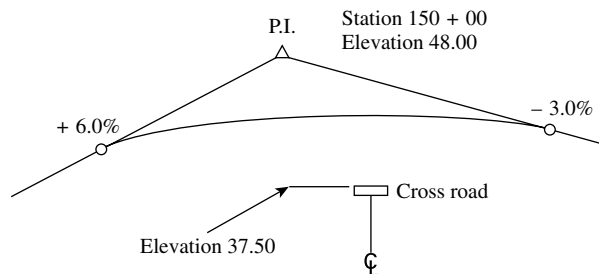
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6. Tom, G. K. J. *Accidents on Spiral Transition Curves in California*. Sacramento, CA: California Department of Transportation, Division of Traffic Operations, 1992.

PROBLEMS

- 4.1 Compute the minimum length of vertical curve to provide passing sight distance for a design speed of 100 km/h at the intersection of a +1.40% grade with a -0.60% grade.
- 4.2 Compute the minimum length of vertical curve that will provide 190 m stopping sight distance for a design speed of 100 km/h at the intersection of a +2.60% grade and a -2.40% grade.
- 4.3 Compute the minimum length of vertical curve that will provide 220 m stopping sight distance for a design speed of 110 km/h at the intersection of a +3.50% grade and a -2.70% grade.
- 4.4 Compute the minimum length of vertical curve that will provide 130 m stopping sight distance for a design speed of 80 km/h at the intersection of a +2.30% grade and a -4.80% grade.
- 4.5 Compute the minimum length of vertical curve that will provide 190 m stopping sight distance for a design speed of 100 km/h at the intersection of a -2.60% grade and a +2.40% grade.
- 4.6 Compute the minimum length of vertical curve that will provide 220 m stopping sight distance for a design speed of 110 km/h at the intersection of a -3.50% grade and a +2.70% grade.

- 4.7** Compute the minimum length of vertical curve that will provide 130 m stopping sight distance for a design speed of 80 km/h at the intersection of a -2.30% grade and a $+4.80\%$ grade.
- 4.8** (a) Compute curve elevations and offsets from tangents at 25 m intervals, including full stations, for a 350 m vertical curve joining a $+2.70\%$ grade with a -1.50% grade. Assume the P.I. is at station 150 + 00 and elevation 25.00 m. Results should be in tabular form, with columns for stations, tangent elevations, offsets, and curve elevations starting at the BVC and ending at the EVC of the curve.
(b) Plot the profile for the curve data in part a.
- 4.9** (a) Compute curve elevations and offsets from tangents at 25 m intervals, including full stations, for a 250 m vertical curve joining a $+2.60\%$ grade with a -2.40% grade. Assume the P.I. is at station 200 + 00 and elevation 30.00 m. Results should be in tabular form, with columns for stations, tangent elevations, offsets, and curve elevations starting at the BVC and ending at the EVC of the curve.
(b) Plot the profile for the curve data in part a.
- 4.10** (a) Compute curve elevations and offsets from tangents at 25 m intervals, including full stations, for a 300 m vertical curve joining a $+1.50\%$ grade with a -3.30% grade. Assume the P.I. is at station 100 + 00 and elevation 60.00 m. Results should be in tabular form, with columns for stations, tangent elevations, offsets, and curve elevations starting at the BVC and ending at the EVC of the curve.
(b) Plot the profile for the curve data in part a.
- 4.11** A 350 m vertical curve connects a $+3.00\%$ grade with a -2.00% grade. If the station of the BVC is 150 + 00, what is the station of the highest point on the curve?
- 4.12** A 400 m vertical curve connects a -2.00% grade to a $+4.00\%$ grade. The P.I. is located at station 150 + 00 and elevation 60.00 m above sea level. A pipe is to be located at the low point on the vertical curve. The roadway at this point consists of two 3.6 m lanes with a normal crown slope of 2%. If the lowest point on the surface of the roadway must clear the pipe by 0.75 m, what is the station and maximum elevation of the pipe?
- 4.13** Given the profile below, determine:
(a) The length of vertical curve needed to make the highest point on the vertical curve come out exactly over the centerline of the cross road at station 150 + 70.
(b) The vertical clearance between the profile grade on the vertical curve and the centerline of the cross road.



- 4.14** A vertical curve joins a -0.5% grade to a $+1.0\%$ grade. The P.I. of the vertical curve is at station $200 + 00$ and elevation 150.00 m above sea level. The centerline of the roadway must clear a pipe located at station $200 + 70$ by 0.75 m. The elevation of the top of the pipe is 150.40 m above sea level. What is the minimum length of vertical curve that can be used?
- 4.15** A vertical curve joins a -2.0% grade to a $+0.5\%$ grade. The P.I. of the vertical curve is at station $100 + 00$ and elevation 69.50 m above sea level. The centerline of the roadway must clear an overhead structure located at station $99 + 20$ by 5.67 m. The elevation of the bottom of the structure is 77.45 m above sea level. What is the maximum length of vertical curve that can be used?
- 4.16** Compute the minimum radius of a circular curve for a highway designed for 110 km/h. The maximum superelevation rate is 12% .
- 4.17** Compute the minimum radius of a circular curve for a highway designed for 80 km/h. Because snow and ice are present, the maximum superelevation rate is 8% .
- 4.18** Compute the minimum radius of a circular curve for a highway designed for 100 km/h. The maximum superelevation rate is 12% .
- 4.19** (a) A two-lane highway (one 3.6 m lane in each direction) goes from normal crown with 2% cross-slopes to 10% superelevation by means of a spiral transition curve. Determine the minimum length of the transition if the difference in grade between the centerline and edge of traveled way is limited to $1/200$. Round up to the next largest 20 m interval.
(b) Draw the superelevation diagram for the transition described in part *a*. The station of the TS is $160 + 00$.
- 4.20** (a) A two-lane highway (one 3.6 m lane in each direction) goes from normal crown with 2% cross-slopes to 8% superelevation by means of a spiral transition curve. Determine the minimum length of the transition if the difference in grade between the centerline and edge of traveled way is limited to $1/200$. Round up to the next largest 20 m interval.
(b) Draw the superelevation diagram for the transition described in part *a*. The station of the TS is $120 + 00$.
- 4.21** (a) A two-lane highway (one 3.6 m lane in each direction) goes from normal crown with 2% cross-slopes to 6% superelevation by means of a spiral transition curve. Determine the minimum length of the transition if the difference in grade between the centerline and edge of traveled way is limited to $1/200$. Round up to the next largest 20 m interval.
(b) Draw the superelevation diagram for the transition described in part *a*. The station of the TS is $75 + 00$.
- 4.22** Prepare a table giving chords and deflection angles for staking out a 450 m radius circular curve with a total deflection angle of 17° . The TC point is at station $22 + 40$. Give deflection angles and chords at 20 m intervals, including full stations.
- 4.23** Prepare a table giving chords and deflection angles for staking out a 650 m radius circular curve with a total deflection angle of 13° . The TC point is at station $13 + 25$. Give deflection angles and chords at 20 m intervals, including full stations.
- 4.24** Prepare a table giving chords and deflection angles for staking out a 480 m radius circular curve with a total deflection angle of 22° . The TC point is at station $10 + 32$. Give deflection angles and chords at 20 m intervals, including full stations.

- 4.25** (a) A roadway goes from tangent alignment to a 250 m circular curve by means of an 80 m long spiral transition curve. The deflection angle between the tangents is 45° . Use formulas to compute X_s , Y_s , p , and k . Assume that the station of the P.I., measured along the back tangent, is $250 + 00$, and compute the stations of the TS, SC, CS, and ST.
- (b) Prepare a table giving coordinates, spiral angles, deflection angles and chords (from the TS) at 20 m intervals, including full stations.
- 4.26** (a) A roadway goes from tangent alignment to a 275 m circular curve by means of a 100 m long spiral transition curve. The deflection angle between the tangents is 60° . Use formulas to compute X_s , Y_s , p , and k . Assume that the station of the P.I., measured along the back tangent, is $200 + 00$, and compute the stations of the TS, SC, CS, and ST.
- (b) Prepare a table giving coordinates, spiral angles, deflection angles and chords (from the TS) at 20 m intervals, including full stations.
- 4.27** (a) A roadway goes from tangent alignment to a 380 m circular curve by means of a 60 m long spiral transition curve. The deflection angle between the tangents is 40° . Use formulas to compute X_s , Y_s , p , and k . Assume that the station of the P.I., measured along the back tangent, is $100 + 00$, and compute the stations of the TS, SC, CS, and ST.
- (b) Prepare a table giving coordinates, spiral angles, deflection angles and chords (from the TS) at 20 m intervals, including full stations.
- 4.28** A circular curve with a radius of 350 m is connected by 60 m spiral transition curves to tangents with a deflection angle of 0.349 rad. If the station of the TS is $105 + 40$, determine the station of the ST.
- 4.29** A horizontal curve is connected by two spiral transition curves to tangents with a deflection angle of 0.26 rad. Stations of critical points are as follows: TS, $105 + 00$; SC, $105 + 80$; CS, $107 + 50$; ST, $108 + 30$. The roadway is a two-lane highway with one 3.6 m lane in each direction. If the difference in grade between the centerline and edge of traveled way in the superelevation transition is exactly $1/200$, at what speed can the curve be taken with no side friction?
- 4.30** A horizontal curve is connected by two spiral transition curves to tangents with a deflection angle of 0.30 rad. Stations of critical points are as follows: TS, $308 + 00$; SC, $308 + 40$; CS, $310 + 40$; ST, $310 + 80$. The roadway is a two-lane highway with one 3.6 m lane in each direction. If the difference in grade between the centerline and edge of traveled way in the superelevation transition is exactly $1/200$, what is the maximum speed that can be maintained on the curve if the side friction is limited to 0.10?
- 4.31** The allowable side friction factor for horizontal curves with a design speed of 100 km/h is 0.12.
- (a) What superelevation rate would you use for a curve with a design speed of 100 km/h and a radius of 420 m? Round to the nearest whole percent.
- (b) A spiral transition curve is used to go from a normal crown slope with 2% cross-slopes to full superelevation for the curve described above. If the maximum difference in grade between the centerline and the edge is $1/200$ and the roadway consists of two 3.6 m lanes, what is the minimum length of spiral? Round up to the next integral multiple of 20 m.
- (c) The station of the TS is $501 + 00$. Draw the superelevation diagram.

COMPUTER EXERCISES

- 4.1. Programming.** Write a computer program to compute the characteristics of vertical curves. The program should be interactive. The user is to enter the grades and the design speed. From this the program calculates the minimum length of vertical curve according to AASHTO standards for stopping sight distance and the California appearance criteria for minimum length of vertical curve. Use stopping sight distance values halfway between the extremes given in Table 3.3. The user then enters the station and elevation of the P.I. of the vertical curve and a length that must be at least as great as that calculated by the program. The program then calculates a table giving the station and elevation of points on the vertical curve at 20 m intervals. Output should also document the design speed; sight distance; minimum vertical curve length; grades; station and elevation of the P.I., BVC, and EVC; and the actual length of the vertical curve.
- 4.2. Programming.** Write a computer program to design superelevation transitions and spiral transition curves. The program should be interactive. The user is to enter the design speed, the radius of curvature for the horizontal curve, the deflection angle for the horizontal tangents, the distance from the axis of rotation to the edge of traveled way, and the imaginary station of the P.I. From this, the program calculates the superelevation rate by means of the formula

$$R = \frac{V^2}{127(f + e)}$$

Superelevation rates may range from 0.02 to 0.10 and should be rounded up to the next highest multiple of 0.01. If the calculated superelevation rate exceeds 0.10, the program should display a message indicating that the radius of curvature is too small for the design speed and prompt entry of a new curve radius. Once a suitable superelevation rate is determined, the program should calculate the length of the superelevation transitions (and the spirals) based on the criterion that the difference in grade between the axis of rotation and the edge of traveled way should be no greater than 0.005. Transition lengths should be rounded up to the next 20 m interval, with a minimum transition length of 40 m. The program should then calculate the coordinates, spiral angles, deflection angles, and chords at 20 m intervals along the spirals and the stations of the TS, SC, CS, and ST. Output should document in suitable form the design speed, radius, deflection angle, distance from axis of rotation to edge of traveled way, superelevation rate and length of the circular portion of the curve; the length of the spirals as well as the values of p and k for the spirals; the spiral coordinates, spiral angles, deflection angles, and chords for 20 m intervals; and the stations of the TS, SC, CS, and ST.

- 4.3. Programming.** Write a computer program to calculate the minimum length of a vertical curve connecting two known grades g_1 and g_2 , which must clear either an existing elevated structure (in the case of a crest curve) or an underground utility (in case of a sag curve) by a known distance C . The user should supply the program with stations and elevations of the P.I. and the critical point, the two grades, and the clearance C . Test your program using data from Problem 4.14.
- 4.4. Spread sheet.** Use a spread sheet to calculate and report the stations and elevations of the P.I., BVC, and EVC of one of the curves in Problems 4.8 to 4.10 and to construct a table showing stations and roadway elevations at 25 m intervals. Also use the spread sheet to plot the vertical curve.

- 4.5. *Spread sheet.* Use a spread sheet to determine deflection angles and chords from the TC point at 20 m intervals, including full stations, for a 400 m radius circular curve with a total deflection angle of 20° . The station of the TC is $32 + 65$.
- 4.6. *Spread sheet.* Use a spread sheet to calculate X_s , Y_s , p , and k for one of the spiral transition curves in Problems 4.25 to 4.27. Also use the spread sheet to construct a table showing the X and Y coordinates, the spiral angle, the deflection angle, and the chord at 10 m intervals.

DESIGN EXERCISES

- 4.1. A profile of existing ground is reproduced on page 107. Design a profile for a two-lane conventional state highway to connect with previously-designed segments at points A and B . You may select any design standards you believe appropriate for this type of roadway. The section you are designing must match the grade and elevation of previously designed segments at points A and B . Point A is at station $1 + 00$ and elevation 218.000 m; the roadway approaching this point is on a -1.00% grade. Point B is at station $34 + 00$ and elevation 170.500 m; the grade beyond this point is -1.50% . At station $19 + 00$, the road crosses a stream that requires a 1 m diameter reinforced pipe culvert. At station $11 + 50$, it intersects an existing road. Document your design by means of a profile showing grades, P.I. stations and elevations, BVCs, EVCs, lengths of vertical curves, and notes of any other control points involved in the design. Defend your design decisions in a brief written report discussing design standards, design objectives and constraints, major design decisions, and your rationale for your decisions.
- 4.2. Design a horizontal alignment for the centerline of a conventional state highway, using the topo map reproduced on page 108. The roadway should connect point A on the map to point B , and the alignment approaching A and B should match that of the tangents shown to the left of A and the right of B . The bearing of the tangent left of A is $N66^\circ21'E$ and that of the tangent to the right of B is $N74^\circ15'E$. You may select any design standards you believe to be appropriate for this type of roadway. Document your design by means of a drawing showing stationing, bearings of tangents, deflection angles and radii of horizontal curves, lengths of spiral transition curves, etc. Defend your design decisions in a short report discussing design standards, design objectives, major design decisions, and your rationale for your decisions.
- 4.3. Design a horizontal alignment for a freight railway to connect points A and B on the topo map reproduced on page 109. To reduce operating costs, it is desired to keep the absolute value of the grade at all points on the line to 0.50 percent or less. The alignment of the railway approaching A and B should match that of the existing tangents shown to the left of A and the right of B . The bearing of each of these tangents is $N45^\circ00'E$. You may select any design standards you believe to be appropriate for this type of facility. Document your design by means of a drawing showing stationing, bearings of tangents, deflection angles and radii of horizontal curves, lengths of spiral transition curves, etc. Defend your design decisions in a short report discussing design standards, design objectives, major design decisions, and your rationale for your decisions.

