Superelevation

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Curve Resistance

- When a vehicle takes a curve, external forces act on the front wheels of the vehicle. These forces have components that retard the forward motion of the vehicle. This resistance depends on the radius of curvature and the speed of the vehicle. This curve resistance can be given as:

$$R_c = 0.5 * \frac{\left(\frac{1000}{3600} v\right)^2 m}{R}$$

- where
  - R_c = Curve Resistance (N)
  - v = vehicle speed (km/hr)
  - m = gross vehicle mass (kg)
  - g = acceleration due to gravity (9.8 m/sec^2)
  - R = Radius of curvature (m)

Example: Curve Resistance

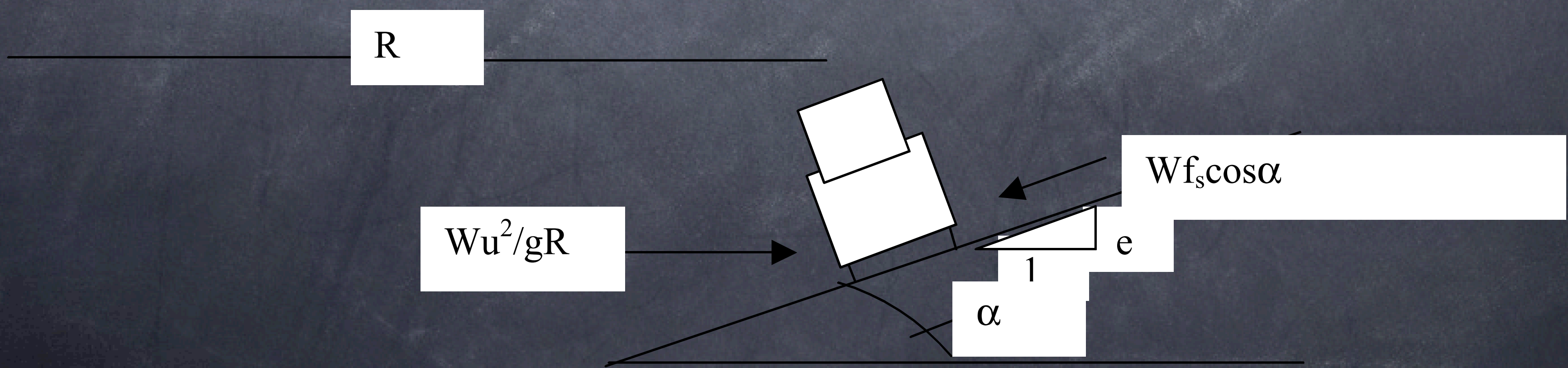
- A 1000 kg vehicle is traveling at 100 km/hr around a curve with a radius of 250 m. What is the curve resistance?

$$R_c = 0.5 * \frac{\left(\frac{1000}{3600} v\right)^2 m}{R}$$

$$= 0.5 * \frac{\left(\frac{1000}{3600} 100\right)^2 1000}{250} = 1543N$$

Radius of Curvature

- Vertical Curves on roads are parabolic,
- Horizontal Curves are based on circles.
- When a vehicles moves around a horizontal curve, it is subject to the outward radial force (centrifugal force) and the inward radial force. The inward force is not due to gravity, but rather because of the friction between tires and the roadway. At high speeds, the inward force is inadequate to balance the outward force without some help.
- That help arises from banking the road, what transportation engineers call superelevation (e). This banking, an inclination into the center of the circle, keeps vehicles on the road at high speed.



"Centrifugal Force"

- The minimum radius of circular curve (R) for a vehicle traveling at u kph can be found by considering the equilibrium of a vehicle with respect to moving up or down the incline. Let alpha (α) be the angle of incline, the component of weight down the incline is $W*\sin(\alpha)$, the frictional force acting down the incline is $W*f*\cos(\alpha)$. The "centrifugal" force F_c is

$$F_c = \frac{Wa_c}{g}$$

- where
  - a_c = acceleration for curvilinear motion = v^2/R
  - W = weight of the vehicle
  - g = acceleration due to gravity

Equilibrium of Forces

- When the vehicle is in equilibrium with respect to the incline (the vehicle moves forward along the road, but neither up nor down the incline), the forces may be equated as follows:

$$\frac{mv^2}{R} = \frac{Wv^2}{gR}$$

$$= W \sin\alpha + Wf_s \cos\alpha$$

- where
  - f_s = coefficient of side friction and
  - $v^2/g = R\ (\tan(\alpha) + f_s)$

Computing Radius of Curvature

- Let tan(α)=e, g=9.8 m/sec^2, u is in km/hr (and we need R in meters)

$$R = \frac{(v)^2}{g(e+f_s)} = \frac{\left(v\frac{1000}{3600}\right)^2}{9.81(e+f_s)}$$

$$= \frac{(v)^2}{127(e+f_s)}$$

- So to reduce R for a given speed, you must increase e or f_s.

Standards

- There are maximum values for e and f_a, which depend on the location of the highway (whether it is urban or rural), weather (dry or wet on a regular basis, snow), and distribution of slow vehicles.
  - In rural areas with no snow or ice, a maximum superelevation (e) of 0.10 is used.
  - In urban areas, a maximum of 0.08 is used.
  - Less is used in places like Minnesota, where it is 0.06 (see MN Design Guidelines). Values for f_s vary with design speed.

Side-Friction (Mn)

Design Speed (km/hr)	Coefficient of Side Friction (f_s) Urban	Coefficient of Side Friction (f_s) Rural	Minimum Radius (m) Urban	Minimum Radius (m) Rural
30	0.312	0.17	20	30
40	0.252	0.17	40	55
50	0.214	0.16	70	90
60	0.186	0.15	115	135
70	0.162	0.14	175	195

Design Speed (km/hr)	Coefficient of Side Friction (f_s) All High Speed	Minimum Radius (m) All High Speed
80	0.147	250
90	0.14	340
100	0.128	450
110	0.115	590
120	0.102	775

Example

- An existing horizontal curve has a radius of 85 meters, which restricts the maximum speed on this section of road to only 60% of the design speed of the highway. Highway officials want to improve the road to eliminate this bottleneck. Assume coefficient of side friction is 0.15 and rate of superelevation is 0.08. Compute the existing speed, design speed, and find the new radius of curvature.

Solution

- Existing Speed

$$R = \frac{\left(v\frac{1000}{3600}\right)^2}{9.81\left(e+f_s\right)}$$

$$85 = \frac{\left(v\frac{1000}{3600}\right)^2}{9.81\left(0.08+0.15\right)} = \frac{v^2 * 0.077}{2.254}$$

$$v = 50km/hr$$

- Design Speed

50/.6=83.33km/hr

Find the radius of the new curve, using the value of f_s for 83.33 kph (f_s=0.14)

$$R = \frac{\left(83.33\frac{1000}{3600}\right)^2}{9.81(0.08+0.14)} = 248m$$

Problem

- An existing horizontal curve has a radius of 105 meters, which restricts the maximum speed on this section of road to only 75% of the design speed of this rural highway. Highway officials want to improve the road to eliminate this bottleneck. What does the new radius need to be? Assume superelevation is a maximum of 6%.

Questions

- Questions?

Key Terms

- Curvature
- Superelevation
- Radius of curvature
- Curve Resistance

Variables

- R_c = Curve Resistance (N)
- v = vehicle speed (km/hr)
- m = gross vehicle mass (kg)
- g = acceleration due to gravity (9.8 m/sec^2)
- R = Radius of curvature (m)
- a_c = acceleration for curvilinear motion = v2/R
- W = weight of the vehicle
- e = superelevation
- f_s = coefficient of side friction